The influence of inhomogeneities on the large-scale expansion of the universe

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Three observations

I

the appearance of the universe today

II

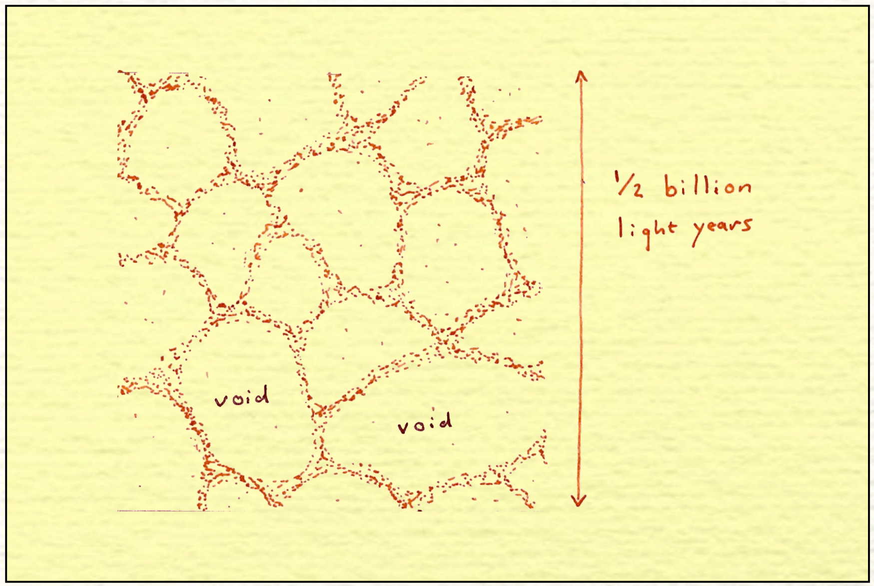
the universe long ago

III

something odd happened about
four billion years ago

First Observation: the universe today

The universe today is very inhomogeneous, even over quite large distances



At the scale of a few hundred million light-years, the universe is an intricate web of filaments, walls and voids

First Observation: the universe today

The voids are still small compared with with the size of the observable universe

cosmological distances

horizon size	47 billion light years
void width	200 million light years
distance to M31	2.5 million light years
size of the galaxy	100,000 light years

$$\frac{\text{horizon}}{\text{void}} \approx 200$$

It is thought they should have a negligible effect on the large-scale expansion, but is this a good assumption?

First Observation: the universe today

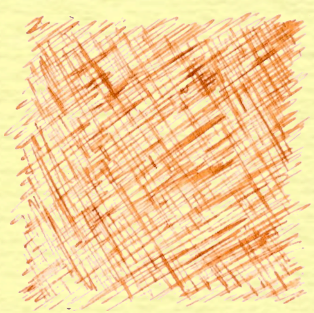
One reason to question this assumption is because Einstein's equations are highly non-linear,

$$R_{\mu\nu} - \frac{1}{2}\, g_{\mu\nu} R = 8\pi G\, T_{\mu\nu}$$

$$R = g^{\mu\nu} g^{\lambda\sigma} \left[\partial_\mu \partial_\nu g_{\lambda\sigma} - \partial_\mu \partial_\sigma g_{\lambda\nu}\right] + \cdots$$

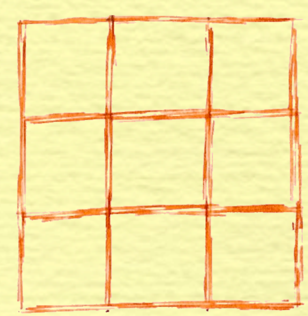
The uniform and non-uniform parts grow at very different rates

So why should



evolve at the same rate as

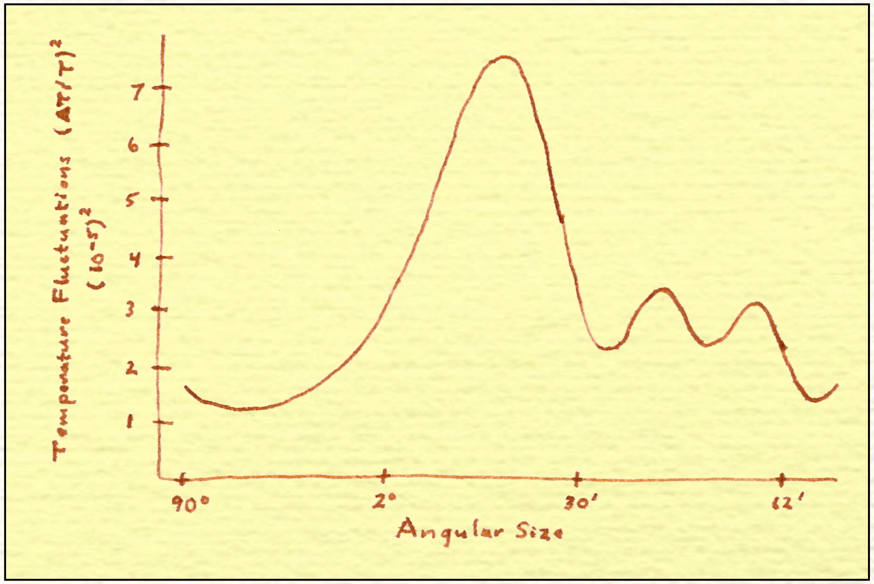
$\langle\rho\rangle = \langle\rho\rangle$



?

Second Observation: the universe long ago

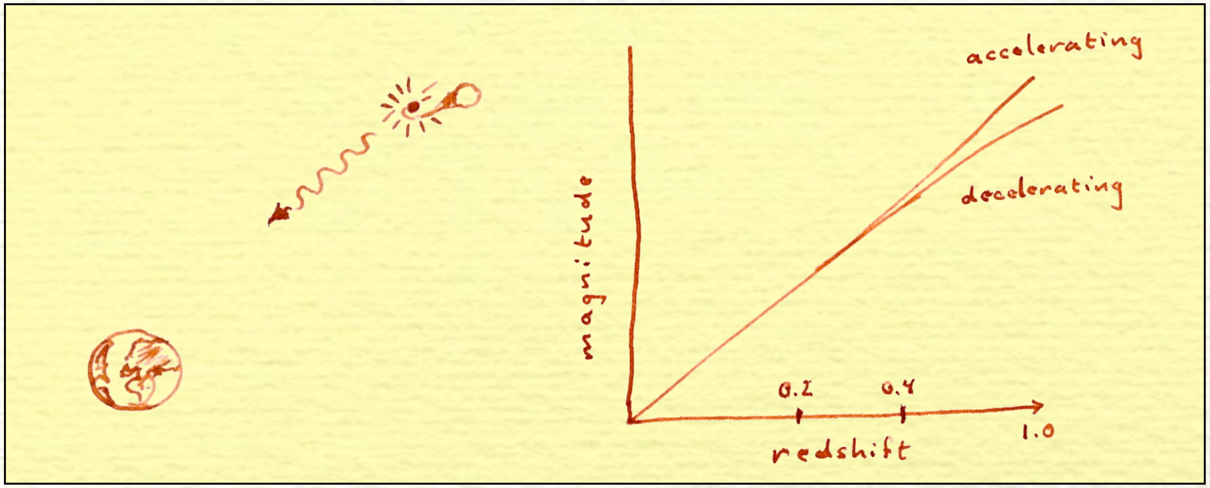
At much earlier times, the universe appears to have been nearly uniform on all scales



Observations suggest that early fluctuations in the density, relative to the average, were about $\delta\rho/\langle\rho\rangle = 10^{-5}$

Third Observation: four billion years ago

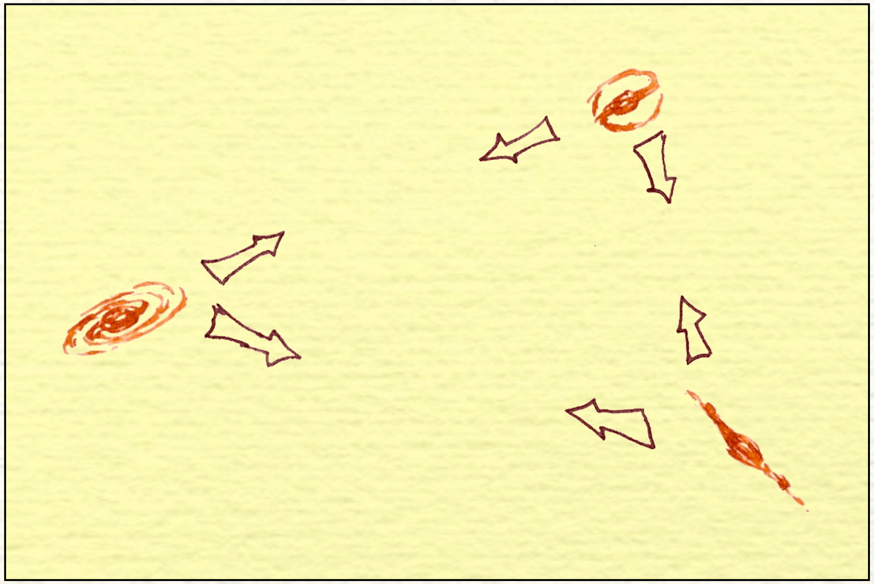
A bit more than a decade ago, measurements of supernovae implied that the expansion rate started to accelerate about 4 billion years ago



What is actually seen is that the supernovae were dimmer than would be expected for a decelerating universe

Third Observation: four billion years ago

This is difficult to understand since the attraction of matter should lead to a slowing down of the expansion, which never changes



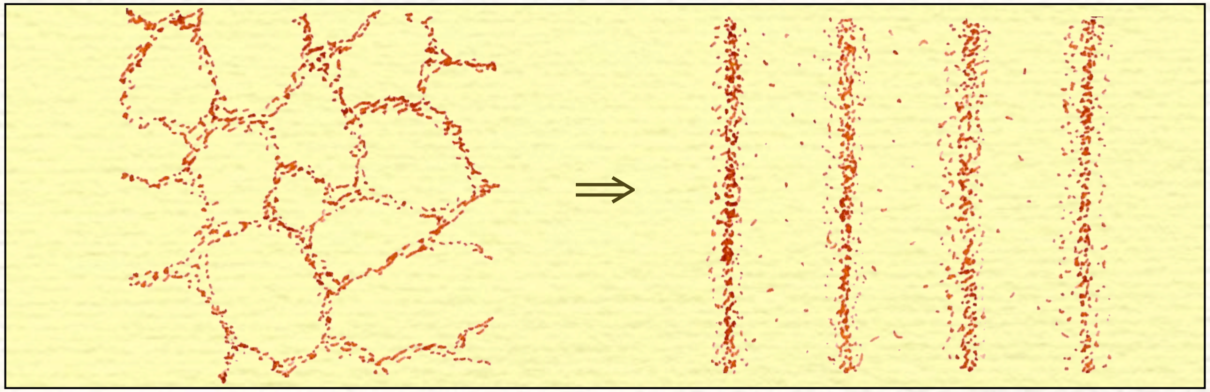
Why did this acceleration happen?
Why did it happen when it did?

Perhaps these three observations are related.

Does the growth of inhomogeneities influence the expansion on larger scales?

A simple, though not too simple, example

Consider a simple example to test whether and when the growth of inhomogeneities can influence the large-scale expansion



Properties kept – small initial amplitude
– fluctuations in one dimension
– single wavelength, $\lambda = k^{-1}$
– forms a network of walls and voids

A simple, though not too simple, example

The metric and physical density consistent with these requirements are

$$ds^2 = dt^2 - b^2(t,x)\,dx^2 - a^2(t,x)\,[dy^2 + dz^2]$$

$$\rho(t,x) = \frac{\rho_0(x)}{a^2\ b}$$

Choose an periodic comoving density,

$$\rho_0(x) = \bar{\rho_0}\ [1 + \varepsilon \cos(2\pi kx)]$$

Initially, everything is nearly homogeneous,

$$a(t,x) = \bar{a}_0\, t^{2/3} + O(\varepsilon), \quad b(t,x) = \bar{a}_0\, t^{2/3} + O(\varepsilon)$$

Over time, this evolves into a periodic network of walls and "voids"

A simple, though not too simple, example

The smallness of the amplitude provides a good parameter for describing the analytic solution, at least until the inhomogeneous era

Solving Einstein’s equation to fourth order,

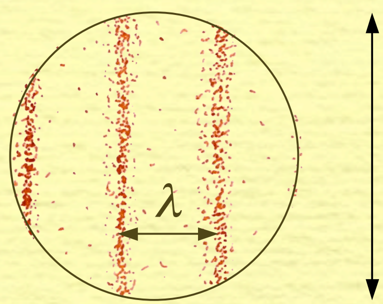
$$
\begin{aligned}
b(t,x) = a_0 t^{2/3} \Big[1 &+ \tfrac{1}{3}\varepsilon\cos(2\pi k x) - \tfrac{1}{3}\varepsilon^2\cos^2(2\pi k x) + \tfrac{5}{27}\varepsilon^3\cos^3(2\pi k x) - \tfrac{7}{81}\varepsilon^4\cos^4(2\pi k x)\Big] \\
&- \frac{1}{20}\frac{(2\pi k)^2}{\bar{a}_0} t^{4/3} \big[6\varepsilon\cos(2\pi k x) + \varepsilon^2[3\sin^2(2\pi k x) - 4\cos^2(2\pi k x)] \\
&\qquad + \varepsilon^3[7\sin^2(2\pi k x) - 4\cos^2(2\pi k x)]\cos(2\pi k x) \\
&\qquad + \varepsilon^4[11\sin^2(2\pi k x) - 4\cos^2(2\pi k x)]\cos^2(2\pi k x)\big] \\
&- \frac{9}{700}\frac{(2\pi k)^4}{\bar{a}_0^{\,3}} t^2\, \varepsilon^3 \big[\cos(2\pi k x) + \tfrac{1}{12}\varepsilon[7\sin^2(2\pi k x) - 24\cos^2(2\pi k x)]\sin^2(2\pi k x)\big] + O(\varepsilon^5)
\end{aligned}
$$

The inhomogeneous terms grow at a faster rate ($t^{4/3}$,t^2) than the homogeneous one ($t^{2/3}$)

Yardsticks

Measure time in a more intuitive unit by counting the number of wavelengths per (homogeneous Hubble) horizon,

$$n(t) = \frac{\text{Hubble horizon}}{\text{wavelength}} = \frac{d_H/a|_{\varepsilon=0}}{\lambda} = \frac{3}{2}\,\frac{k}{\bar{a}_0}\,t^{1/3}$$



$$d_H = \frac{a}{\dot{a}}$$

Define the physical amplitude, δ, to be that when the wavelength equals the horizon,

$$\delta = \frac{8\pi^2}{15}\,\varepsilon$$

<u>Question</u> –

What mistake is made by assuming that the inhomogeneities do not matter for the average large-scale evolution?

Misattribution

It is a simple matter to define averages

$$\bar{f}(t) = \langle f(t,x) \rangle = k \int_0^{1/k} dx\, f(t,x)$$

The scaling factor in the x-direction, on average, evolves as

$$\bar{b}(t) = \bar{a}_0\, t^{2/3} \left[1 + \frac{1}{4} n^4 \delta^2 - \frac{1}{288} n^8 \delta^4 + \cdots \right]$$

When $n^2\delta \approx \frac{2}{3}$, the expansion begins to accelerate as it passes from

$$t^{2/3} \quad \rightarrow \quad t^{2/3} n^4 = t^2$$

Of course the perturbative solution is a poor approximation once $n^2\delta \approx 1$

Misattribution

From a slightly different perspective, look at the averaged Friedmann's equation,

$$\bar{G}_{00}(t) = \frac{4}{3}\frac{1}{t^2}\left[\, 1 + \frac{1}{2}\, n^4\delta^2 + \frac{3}{8}\, n^8\delta^4 + \cdots \,\right]$$

If one insists that this is produced by uniformly distributed materials, matter alone would appear not to be sufficient

$$G_{00}(t) = 8\pi G\, \rho(t)$$

$$\rho_{\mathrm{mat}}^{\mathrm{uniform}}(t) \propto t^{-2}$$

The terms produced by the inhomogeneities dilute more slowly

Physical Scales

When do the inhomogeneities become important? When does $n^2\delta \approx 1$?

$$\text{for } \delta \approx 5 \times 10^{-5},\ n \approx 140$$

If this occurred about 4 billion years ago, then very roughly $n \approx 160$ today

This is the number of times the wavelength fits within the Hubble horizon today,

$$\frac{c/H_0}{160} = \frac{13.5 \text{ billion light years}}{160}$$

$$\approx 84 \text{ million light years}$$

which is very similar to the typical sizes of the voids, 150-300 million light years.

Next Steps

Calculational side –

- add more details: variations in all dimenions, randomly distributed voids, full spectrum of fluctuations
- calculate (numerically) the detailed expansion history during the inhomogeneous era

Observational side –

- several future proposals will very precisely measure the recent expansion history
- experiments: EUCLID, WFIRST (JDEM)…

the end