

An effective theory of initial conditions in inflation

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flat: [hep-th/0501158](#)

expanding: [hep-th/0507081](#)

back-reaction: [hep-th/0605107](#)

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We begin with a simple question

Why are we able to explain what happens at long distances without knowing what happens at short distances?

In quantum field theory we have an answer: the details at short distances do not matter . . . at least not much!

⇒ decoupling & effective field theory

Overview

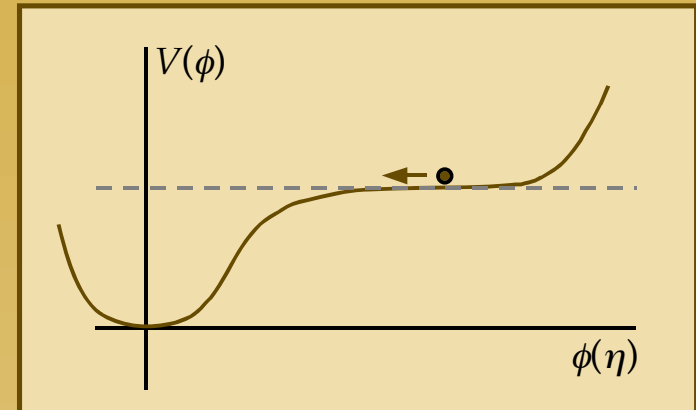
- Primordial perturbations in inflation
- The trans-Planckian problem of inflation
- An effective theory of initial conditions
- Boundary renormalization
- Observational outlook and conclusions

Primordial perturbations from inflation

- Let us briefly review the origin of primordial perturbations in inflation
- In quantum field theory, there is always some inherent variation in a field, $\varphi(\eta, \mathbf{x})$
 - The pattern of fluctuations is then characterized by the variance of φ
- To calculate the variance, expand the field in its operator eigenmodes

$$\varphi = \int \frac{d^3 \mathbf{k}}{(2\pi)^3} \left[\varphi_k e^{i\mathbf{k} \cdot \mathbf{x}} a_{\mathbf{k}} + \varphi_k^* e^{-i\mathbf{k} \cdot \mathbf{x}} a_{\mathbf{k}}^\dagger \right]$$

- The Fourier transform of the variance is the power spectrum
- The time-dependent eigenmode $\varphi_k(\eta)$ satisfies the Klein-Gordon equation
 - one constant of integration is fixed by equal-time commutation relation
 - but how do we choose the other, f_k ?



$$\langle 0 | \varphi(\eta, \mathbf{x}) \varphi(\eta, \mathbf{y}) | 0 \rangle \neq 0$$

$$\begin{aligned} & \langle 0 | \varphi(\eta, \mathbf{x}) \varphi(\eta, \mathbf{y}) | 0 \rangle \\ &= \int \frac{d^3 \mathbf{k}}{(2\pi)^3} e^{i\mathbf{k} \cdot (\mathbf{x} - \mathbf{y})} \frac{2\pi^2}{k^3} P_k^\varphi(\eta) \end{aligned}$$

$$\varphi_k = \frac{\varphi_k^0 + f_k \varphi_k^{0*}}{\sqrt{1 - f_k f_k^*}}$$

Choosing the vacuum state

- At very short distances, $\ll 1/H$, the background curvature is not very apparent and space-time looks flat
- Therefore a natural choice is the state that matches with the flat space vacuum as $k \rightarrow \infty$ with η fixed; this choice fixes $f_k = 0$
- At some stage we might worry about some of our underlying assumptions
 - $H \ll k \ll M_{\text{pl}}$
 - sometimes η is taken to ∞
 - complicated dynamics/other fields
- We have encountered the question posed at the very beginning:
 - how do we know what happens at very short length scales (or any scale $< 1/M_{\text{pl}}$)?
- If we assume that—to some degree—these details decouple, the leading result should be that given by this “vacuum”

de Sitter example:

$$\varphi_k = \varphi_k^0 = \frac{iH}{k\sqrt{2k}} (1 + ik\eta)e^{-ik\eta}$$

$$P_k^\varphi(\eta) = \frac{k^3}{2\pi^2} |\varphi_k^0|^2$$

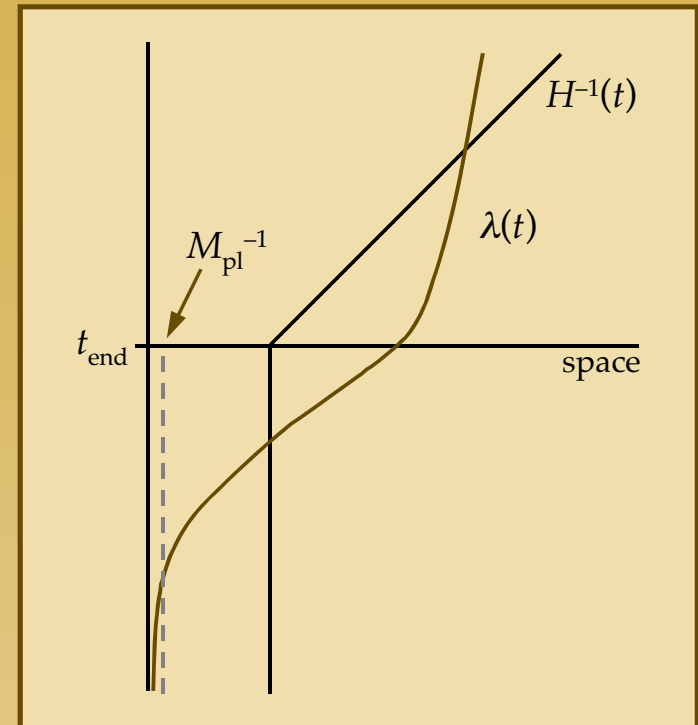
$$P_k^\varphi(k\eta \rightarrow 0) = \frac{H^2}{4\pi^2}$$

a flat primordial power spectrum

This behavior is more or less observed in the CMB; so to *leading order*, choosing of the standard vacuum seems to have been justified

The trans-Planckian problem

- We would like to be able to calculate the corrections to this leading result, but there is a subtlety to decoupling during inflation
- The expansion of the background means that what may be a large scale in the primordial background was smaller and smaller the earlier we follow it back during inflation
- So some perturbation that produces, for example, a feature in the CMB was much smaller when it arose during inflation
 - 60–70 e -folds to solve the horizon problem
 - a bit more and the wavelength of that mode would have been smaller than the Planck length at some time
- What we need is an effective theory description of the possible differences between our assumed vacuum state and the true vacuum
 - Collins & Holman, 2005
 - Greene, Schalm, Shiu & van der Schaar, 2004–2005



- Brandenberger & J. Martin, 2001–2003
- Easter, Greene, Kinney & Shiu, 2001–2002
- Niemeyer & Kempf, 2001
- Danielsson, 2002
- Goldstein & Lowe, 2003
- Collins & M. Martin, 2004
- Kaloper, Kleban, Lawrence, Shenker & Susskind, 2002
- Burgess, Cline, Lemieux & Holman, 2003

An effective initial state—boundary conditions

- Let us return to the point where we chose a particular initial state
- We shall examine the case of flat space
 - the regime in which the new effects will appear should be at much shorter lengths than the Hubble horizon
 - FRW case is in hep-th/0507081
- Earlier we mentioned that a state is defined up to one k -dependent constant of integration
- Let us define our state by imposing an initial condition at $t = t_0$ and evolve forward
 - Notice that this initial condition includes the standard vacuum state, $f_k = 0$
- In an effective theory, there is always an inherent error between predictions based on our theory and those of a better description of nature
 - e.g. Feynman–Gell-Mann ($V - A$) theory compared with electroweak theory

$$S = \int d^4x \left[\frac{1}{2} \partial_\mu \varphi \partial_\nu \varphi - \frac{1}{2} m^2 \varphi^2 \right]$$

$$\varphi = \int \frac{d^3\mathbf{k}}{(2\pi)^3} \left[\varphi_k e^{i\mathbf{k}\cdot\mathbf{x}} a_{\mathbf{k}} + \varphi_k^* e^{-i\mathbf{k}\cdot\mathbf{x}} a_{\mathbf{k}}^\dagger \right]$$

$$\frac{\partial^2 \varphi_k}{\partial t^2} = -\omega_k^2 \varphi_k, \quad \omega_k = \sqrt{k^2 + m^2}$$

$$\frac{\partial \varphi_k}{\partial t}(t_0) = -i\omega_k \frac{1 - f_k}{1 + f_k} \varphi_k(t_0)$$

$$\varphi_k = \frac{\varphi_k^0 + f_k \varphi_k^{0*}}{\sqrt{1 - f_k f_k^*}}$$

$$\varphi_k^0(t) = \frac{e^{-ik(t-t_0)}}{\sqrt{2\omega_k}}$$

An effective initial state—short-distance structure

- If we could solve for the “true vacuum” it might not be the same as our low energy idea of the vacuum; an effective state parameterizes this difference
 - non-localities? non-commutative space-time? strongly interacting gravity?
- To our “vacuum” state this difference appears as new short-distance structure
- The propagator should also be consistent with our initial condition
 - this condition results in an extra term in the propagator associated with the structure of the initial state
- For an general initial state, a loop will also introduce sums of over the short-distance structure of the state
 - new divergences require *boundary counterterms*

$$f_k = \text{“IR important”} + \text{“UV important”}$$

for vacuum, $\rightarrow 0$

possible for $k > M$ \leftarrow

$$f_k = \sum_{n=1}^{\infty} c_n \frac{k^n}{M^n}$$

UV important features
of the state



irrelevant counterterms
on the initial boundary

A brief overview of the initial state renormalization

- What emerges is an effective theory with many familiar properties
 - the long distance features are fixed empirically and any divergences are cancelled by *relevant* or *marginal* counterterms with respect to the boundary action
 - we include a general set of short distance features consistent with the symmetries of the state; their divergences also require *irrelevant* boundary counterterms hep-th/0501158, hep-th/0507081
- Note that when regulating the theory, there is a single cutoff so both “bulk” and “boundary” counterterms depend on a single renormalization scale μ
 - Callan-Symanzik equation
- An effective theory of a state provides a model-independent description of the trans-Planckian effects
 - typical effect scales as H/M
- But can such effects be seen? Spergel (ISCAP, 5/2005)
 - CMB precision measurements (WMAP/Planck): 10^{-3}
 - LSS/galaxy surveys (Square kilometre array, ...): 10^{-5}
 - note that we should include other subleading effects too, so it is important to determine both the *amplitude* (H/M) and the *shape* of the effective initial state signal

Renormalization of a state and its evolution

- Let us summarize what we have found, both for a flat and a completely general Robertson-Walker background,

	IR/long distance			UV/short distance		
	structure	renormalization		structure	renormalization	
		operators	examples		operators	examples
bulk (evolution)	observed long distance degrees of freedom	relevant, marginal (dim ≤ 4)	$\nabla_\mu \varphi \nabla^\mu \varphi,$ $\varphi^2, \varphi^4,$ $R\varphi^2$	completely free, up to assumed symmetries of background	irrelevant (dim > 4)	$(\nabla_\mu \varphi \nabla^\mu \varphi)^p,$ $\varphi^6, \varphi^8,$ $R^2\varphi^2, \dots$
boundary (state)	appropriate state of long distance effective free theory	relevant, marginal (dim ≤ 3)	$\varphi^2,$ $\varphi \nabla_n \varphi, K\varphi^2$	completely free, up to assumed symmetries of state	irrelevant (dim > 3)	$\varphi^4, (\nabla_n \varphi)^2,$ $\nabla_i \varphi \nabla^i \varphi,$ $K^2\varphi^2, \dots$

- Here, $\nabla_n = n^\mu \nabla_\mu$ is a derivative normal to the initial surface and $K_{\mu\nu} = h_\mu^\lambda \nabla_\lambda n_\nu$ is the extrinsic curvature along the surface

Further work

- So we find an elegant correspondence between the long and short distance features of the initial state and the sorts of operators that appear in their renormalization

- This is still rather a young subject so there are many aspects which should be studied further
 - back-reaction (size of effect, types of operators that appear)
 - RG flow (de Sitter space?)
 - decoherence of quantum effects
 - generating effective states by integrating out excited heavy fields
 - calculation of the amplitude and the generic shape of the trans-Planckian correction to the power spectrum
 - ...

Back-reaction and naturalness:

- Porrati, 2004–2005
- Greene, Schalm, Shiu, & van der Schaar, 2004–2005

Somewhat related work on RG flows in de Sitter space:

- Larsen & McNees, 2003–2004

Fits to the CMB data:

- Easter, Kinney & Peiris, 2004–2005