An effective theory of initial conditions in inflation

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references

scalar field: hep-th/0501158, hep-th/0507081
gravity/backreaction: hep-th/0605107, hep-th/0609002
power spectrum: *in progress*

collaborator: Rich Holman (*Carnegie Mellon University*)

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Inflation and structure formation

- Inflation provides a very elegant and economical explanation for the observed structure, relying upon two essential ingredients:
  - quantum fluctuations
  - an accelerating expansion

- A general set of predictions shared across many particular models
  - nearly flat primordial power spectrum
  - small non-Gaussianities
  - correlations of features (TT-TE) on super-(Hubble)-horizon scales
  - synchronized acoustic oscillations

- This picture is not, however, without its problems (Brandenberger)

<u>The ingredients of inflation</u>:

a quantum field, $\Phi(t,\mathrm{x}) = \phi(t) + \varphi(t,\mathrm{x})$

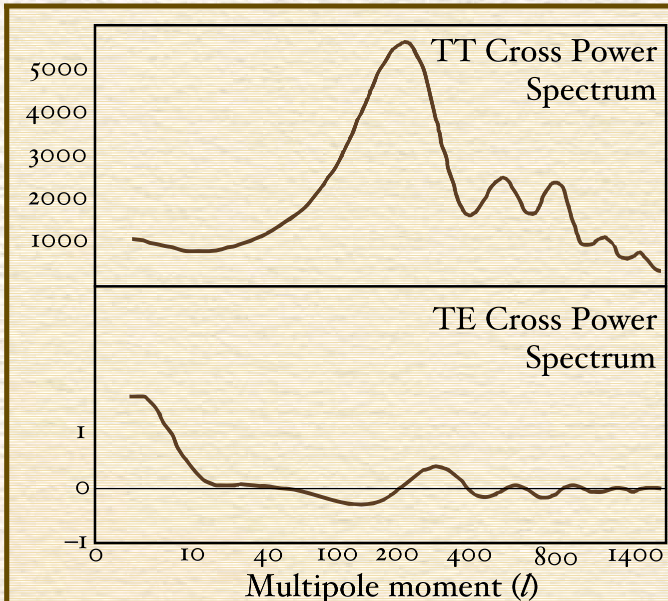
$$\langle 0|\varphi(x)\varphi(y)|0\rangle \neq 0$$

and an expanding background,

$$ds^2 = dt^2 - a^2(t)\, d\mathrm{x}\cdot d\mathrm{x}$$

$$= a^2(\eta)\left[d\eta^2 - d\mathrm{x}\cdot d\mathrm{x}\right]$$

$$\frac{1}{a}\frac{d^2 a}{dt^2} = \left(1/6M_{\mathrm{pl}}^2\right)\left[V(\phi) - \left[\frac{d\phi}{dt}\right]^2\right]$$



adapted from the WMAP Science Team

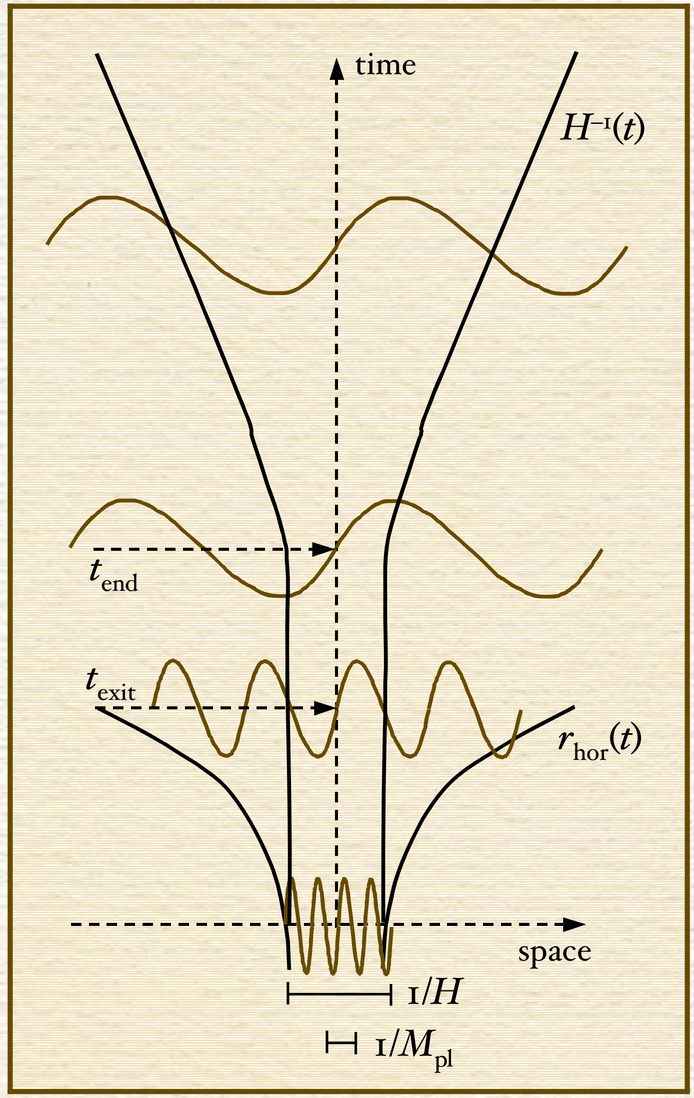
The trans-Planckian problem of inflation

- One problem in particular arises directly from the ingredients which make inflation so appealing: its quantum fluctuations and its expansion
  - 60–70 *e*-folds of expansion solves the horizon problem
  - but most models have much more

- A little thought-experiment:

  consider some feature in the primordial power spectrum at the end of inflation that later will cause a feature in the microwave background radiation
  - starts outside horizon (t_{end})
  - going backwards: left the horizon (t_{exit})
  - still earlier: it was smaller than $1/M_{pl}$

- A breakdown of the principle of decoupling



An effective description of the initial state

- How do we address this problem?
  - One method is to assume something about the behavior above M_{pl}
    - stringy uncertainty relation, shortest distance, noncommuting space-time, etc.
  - Another approach is more in keeping with the effective theory principle
    - is the nominal vacuum is the true vacuum at short distances?

- Divide the features of the initial state according to whether they vary much on a length scale $1/M$
  - here M is the scale of new physics
  - fit the long distances to observations
  - leave short distances completely general
  - we define the state at an "initial time" η_0
  - is this picture perturbatively consistent?

Martin & Brandenberger — Easther, Greene, Kinney & Shiu — Kaloper, Kleban, Lawrence, Shenker & Susskind — Burgess, Cline, Lemieux & Holman — Niemeyer — Kempf — Starobinsky — Goldstein & Lowe — Danielsson — among many others

$$\varphi = \int \frac{d^3\mathbf{k}}{(2\pi)^3}\left[\varphi_k(\eta)e^{i\mathbf{k}\cdot\mathbf{x}}a_{\mathbf{k}} + \text{h.c.}\right]$$

$$\varphi_k = \frac{1}{\sqrt{1-f_k f_k^*}}\left[U_k + f_k U_k^*\right]$$

Bunch-Davies modes

Greene, Schalm, van der Schaar & Shiu — Collins & Holman — Easther, Kinney & Peiris — Anderson, Molina-Paris & Mottola

$$f_k = \sum_{n=1}^{\infty} d_n \frac{k^n}{(aM)^n}$$

Boundary divergences & renormalization

- The new short-distance structure describes how the true state departs from the standard vacuum,

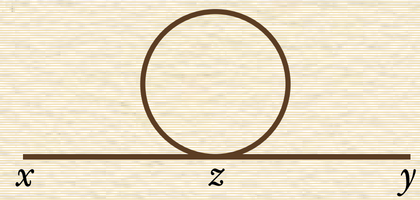
$$\varphi_k(\eta_0) - \varphi_k^{BD}(\eta_0) \to \sum d_n \frac{k^n}{M^n}$$

- Because these effects grow at short distances, they produce new divergences in processes summing over all scales
  - loop processes in interacting theories
  - the energy-momentum tensor

- This second example relates to a further requirement: the *backreaction* on the energy-momentum should not be larger than the original vacuum energy

Greene, Schalm, van der Schaar & Shiu
Porrati, Nitti & Rombouts
Collins & Holman: hep-th/0605107

Internal vertices, such as $z = (\eta', \mathbf{z})$, range over the entire space-time,



including the initial boundary, $\eta' = \eta_0$

Backreaction from leading term:

$$\rho_{\mathrm{tPl}} \approx \frac{1}{8\pi^2}\frac{H}{M}H^4$$

Compare this to the vacuum energy sustaining the inflationary expansion:

$$\rho_{\mathrm{vac}} \approx M_{\mathrm{pl}}^2 H^2$$

So the backreaction is small.

Loop corrections in an interacting theory

hep-th/0501158
hep-th/0507081

- We shall use a formalism that time-evolves the entire matrix element forward from a specified initial state
  - Schwinger-Keldysh and not the *S*-matrix
- A loop correction sums over all points, including over all the trans-Planckian structure of the state
  - This produces divergences confined to the initial boundary ($\eta = \eta_0$)
  - They are cancelled by adding a 3d boundary counterterm action
- Upon renormalization,
  - trans-Planckian corrections are small and finite
  - obeys an extended Callan-Symanzik Equation

In the Schwinger-Keldysh approach, we evolve both the state and its dual,

$$\langle 0_{\rm eff}(\eta)|\varphi(\eta,{\rm x})|0_{\rm eff}(\eta)\rangle$$

becomes

$$\langle 0_{\rm eff}|T\left(\varphi\, e^{-i\int_{\eta_0}^{\eta} d\eta'\left[H_I^+ - H_I^-\right]}\right)|0_{\rm eff}\rangle$$

$H_I^\pm$ is the interaction Hamiltonian

Long or short distance structures in the state accordingly require relevant or irrelevant counterterm on the boundary

$$f_k = d_1 \frac{k}{aM}$$

e.g., requires dimension four operators,

$$S_\delta \propto \frac{1}{M}\int d^3x\sqrt{-g_3}\left\{\tfrac{1}{8}\varphi^4 + \tfrac{1}{2}\nabla_n^2\varphi^2 + \cdots\right\}$$

The energy-momentum tensor

- The approach, in brief: the classical equations of motion arise as a tadpole condition on the quantum theory
  - small quantum metric fluctuations
  - $g_{\mu\nu} = a^2(\eta)\,[\eta_{\mu\nu} + h_{\mu\nu}]$
- The expectation value of the energy-momentum tensor sums over all scales
  - two types of divergences: "bulk" and "boundary" (bulk = standard divergences)
- The extra $d\eta'$ integral is useful for treating the boundary divergences
  - isolate the potentially divergent terms
  - integrate by parts with respect to η'
  - add appropriate boundary counterterms
  - renders the theory finite
  - the backreaction is small: $\frac{\rho_{\rm tPl}}{\rho_{\rm vac}} \approx \frac{1}{8\pi^2}\frac{H^2}{M_{\rm pl}^2}\frac{H}{M}$

A renormalization condition:

$$\langle 0_{\rm eff}(\eta)|h_{\mu\nu}(\eta,\mathbf{x})|0_{\rm eff}(\eta)\rangle = 0$$

To leading order in a small fluctuation:

$$0 = \frac{1}{2}\int d\eta'\,\mathrm{leg}(\eta,\eta')\Big\{2M_{\rm pl}^2\big[G_{\mu\nu} - g_{\mu\nu}\Lambda\big] - T_{\mu\nu} + \cdots\Big\}$$

$T_{\mu\nu}$ is the expectation value of the energy-momentum in a scalar field φ

$$T_{\mu\nu} = \langle 0_{\rm eff}|T_{\mu\nu}[\varphi]|0_{\rm eff}\rangle$$

Expand the counterterm to first order in the fluctuation too:

$$-\int d^3x\Big\{a^3 h^{\mu\nu}\,\delta T_{\mu\nu}\,\delta(\eta' - \eta_0)\Big\}$$

Observability

- We shall conclude by considering the observability of a trans-Planckian signal
- The natural place to look for a trans-Planckian signal is in the CMB
  - adds a modulation about the standard, flat primordial power spectrum
  - typical size of the signal scales as H/M
  - depends on an amplitude and phase, but the frequency is not independent (unlike a signal from the inflaton potential)
  - a source for non-Gaussianities (?)
- A better place to look is in the LSS
  - CMB accuracy: about 0.1%
  - long-term LSS accuracy: about 0.001%

(from D. Spergel's ISCAP talk)

CMB experiments:

– the 3yr WMAP data set
– 6yr final WMAP data set
– Planck satellite

The data will be measured in any event, so we should understand the role of a trans-Planckian signal

The same acoustic peaks have been seen in the SDSS data

Large scale structure (LSS) experiments:

– Square kilometre array (10 x SDSS)
– 21 cm high redshift gas
– Cosmic inflation probe

More & more of the matter is still in a linear regime the farther we look back