THE RANDALL-SUNDRUM SCENARIO WITH AN EXTRA WARPED DIMENSION*

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We investigate a scenario with two four-branes embedded in six dimensions. When the gravitational action contains four derivative terms, the picture that emerges resembles the Randall-Sundrum model but with an extra warped dimension that allows solutions that are flat in the large $3+1$ dimensions *without* finely tuning the parameters in the underlying six dimensional theory.

Two of the most enigmatic features of the universe are the weakness of gravity compared to electroweak interactions—the hierarchy problem—and the small size of the cosmological constant. The recent Randall-Sundrum approach[1] to the hierarchy problem has observed that the presence of a small warped extra dimension could naturally produce an exponential hierarchy between the scales of gravitational and Standard Model interactions; however, this model still requires a fine-tuning to produce a low energy effective $3+1$ dimensional theory with no cosmological constant.

The presence of extra dimensions may also provide a mechanism for addressing the cosmological constant problem[2,3]. For example, in $4+1$ dimensions, the presence of a non-vanishing cosmological constant can distort, or warp, one of the dimensions while leaving the theory with a $3+1$ dimensional Poincaré symmetry. Moreover, since this warping is accomplished with a metric that is both smooth and periodic in the extra dimensions, there is no need to cut off the space or to encounter any singularities.

In this talk, I shall outline a scenario[4] with two parallel 4-branes in a six dimensional bulk that combines both of these ideas. The $6d$ space-time dynamics are determined by an effective action for gravity that includes terms with up to four derivatives of the metric. With some mild bounds on the parameters in the action, we find that the equations of motion allow the geometry of the resulting universe to contain a five-dimensional anti-de Sitter (AdS_5) subspace with a warped metric that is periodic in the sixth dimension.

The Randall-Sundrum scenario as an effective theory

Randall and Sundrum[1] proposed that if the universe were to consist of two 3-branes bounding a bulk region of five-dimensional anti-de Sitter space-time, then the redshift induced by the bulk metric at one of the branes could generate an exponential hierarchy between the Planck scale and the scale of electroweak symmetry breaking. Yet for the theory to be free of a cosmological constant in the low energy $3+1$ dimensional effective theory limit, the cosmological constant must be finely tuned to a value determined by the brane tensions. As the cosmological constant and the surface tension appear in the action, they represent fundamental parameters of the theory and we have no reason *a priori* that such a fine-tuning condition is satisfied. If instead these quantities arise from some more fundamental theory, then it becomes possible for a dynamical mechanism to exist that favors solutions in which the low energy, $3+1$ dimensional theory is nearly flat.

We can adapt the picture developed in [3] without branes to one which resembles the Randall-

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Sundrum construction but where the AdS_5 length is not uniquely determined by the higher-dimensional cosmological constant. The structure for such a model would include *two* extra dimensions—one small periodic dimension (y) to avoid fine-tuning the cosmological constant and a second (r) to generate the electroweak-Planck hierarchy.

We shall consider gravity as an effective theory, expanded in powers of derivatives, with a scalar field ϕ,

$$\begin{aligned} S_{\rm bulk} &= M_6^4 \int d^4x dr dy \sqrt{-g}\left(-2\Lambda + R - aR^2 - bR_{ab}R^{ab} - cR_{abcd}R^{abcd} + \cdots\right) \\ &+ M_6^4 \int d^4x dr dy \sqrt{-g}\left(-\tfrac{1}{2}\nabla_a\phi\nabla^a\phi + \Delta\mathcal{L}\right) \\ &+ M_6^4 \int_{r=0} d^4x dy \sqrt{-h}\left(-2\sigma^{(0)}\right) + M_6^4 \int_{r=r_c} d^4x dy \sqrt{-h}\left(-2\sigma^{(r_c)}\right). \end{aligned} \tag{1}$$

Here $a,b,\ldots = 0,1,2,3,r,y$ while Λ, M_6, $\sigma^{(0)}$, and $\sigma^{(r_c)}$ denote the total cosmological constant—including any quantum contributions—the $6d$ Planck mass and the tensions on the two 4-branes at $r = 0, r_c$ respectively. h denotes the determinant of the induced metric along the branes.

We focus on the simplest case in which $\Delta\mathcal{L}$ describes a Casimir effect, although the existence of metric solutions that are periodic in y appears to be a fairly generic feature of the action (1).[3] Since the finite y-direction explicitly breaks $5+1$ dimensional Poincaré symmetry, any quantum vacuum contribution can differ in the y and (x^λ, r) directions. We include this effect by adding the energy-momentum tensor, $T^{\rm vac}{}_a{}^b = {\rm diag}\,(C,C,C,C,C,-C)$, to the field equations. Satisfactory, periodic solutions exist as long as C is above a mild bound.

To produce a Randall-Sundrum scenario in five of the dimensions, we shall examine a metric of the form

$$ds^2 = g_{ab}(x^\lambda, r, y)\, dx^a dx^b = e^{A(y)}\hat{g}_{MN}(x^\lambda, r)\, dx^M dx^N + dy^2 \tag{2}$$

with an AdS_5 metric for the (x^λ, r)-subspace,

$$d\hat{s}^2 = \hat{g}_{MN}\, dx^M dx^N = e^{-2|r|/\ell}\eta_{\mu\nu}\, dx^\mu dx^\nu + dr^2. \tag{3}$$

When $A(y)$ is a periodic function of y, we can obtain a compact extra dimension with a very non-trivial y-dependence without any singularities. The shape of $A(y)$ determines the effective cosmological constant of the $\hat{g}_{MN}$ metric. Since this metric is conformally flat, the linear combinations of R^2 terms that represents the squared Weyl tensor will not contribute to the equations of motion. It is convenient to parameterize the two remaining linear combinations by

$$\tilde{\mu} \equiv 20a + 6b + 4c \qquad \tilde{\lambda} \equiv 15a + \tfrac{5}{2}b + c. \tag{4}$$

Integrating out the y-dimension in the warped background given by the metric (3), a Randall-Sundrum action emerges when gravity in the $5d$ effective theory is weak. The $5d$ effective bulk action due to the background given by (2) is

$$S^{\rm eff}_{\rm bulk} = M_5^3 \int d^4x dr \sqrt{-\hat{g}}\left(-2\Lambda_{\rm eff} + \hat{R} - a_{\rm eff}\hat{R}^2 - b_{\rm eff}\hat{R}_{MN}\hat{R}^{MN} - c_{\rm eff}\hat{R}_{MNPQ}\hat{R}^{MNPQ}\right). \tag{5}$$

This action should reproduce the leading behavior for small x^μ-dependent perturbations about AdS_5 (3), $\mathbf{R}^{4,1}$ or de Sitter space backgrounds. Here $\hat{R}$, $\hat{R}_{MN}$ and $\hat{R}_{MNPQ}$ correspond to the curvature tensors associated with the $5d$ metric $\hat{g}_{MN}$.

The new parameters that appear in this effective action depend partially upon the "fundamental" parameters of the original action but also upon the behavior of the warp function. Thus, in the low energy theory the $5d$ cosmological constant is

$$M_5^3\Lambda_{\rm eff} = M_6^4 \int_0^{y_c} dy\, e^{\frac{5}{2}A(y)} \left(\Lambda + \tfrac{1}{4}(\phi')^2 - \tfrac{5}{2}(A')^2 + \tfrac{5}{8}\tilde\mu(A'')^2 - \tfrac{5}{24}\tilde\lambda(A')^4\right) \tag{6}$$

while the $5d$ Planck mass is

$$M_5^3 = M_6^4 \int_0^{y_c} dy\, e^{\frac{3}{2}A(y)} \left(1 + \tfrac{1}{8}(3\tilde\mu - 4\tilde\lambda)(A')^2\right). \tag{7}$$

The coefficients of the $\hat R^2$ terms are

$$M_5^3 a_{\rm eff} = M_6^4 a \int_0^{y_c} dy\, e^{\frac{1}{2}A(y)}, \tag{8}$$

with analogous expressions for $b_{\rm eff}$ and $c_{\rm eff}$.

For the theory to resemble the standard Randall-Sundrum picture, the five dimensional theory of gravity should be weak, $M_5\ell \gg 1$. Since $\Lambda_{\rm eff} \sim \ell^{-2}$, we require the effective $5d$ cosmological constant to be small which can easily occur when the contribution from the bulk cosmological constant is partially cancelled by effects from the warp function in (6).

In the weak $5d$ gravity limit, $M_5\ell \gg 1$, the four-derivative terms become negligible and the leading behavior is governed by the Einstein-Hilbert terms in (5). Including the effective brane action, we recover the Randall-Sundrum action[1],

$$\begin{aligned} S^{\rm eff} &= M_5^3 \int d^4x dr \sqrt{-\hat g}\, \left(-2\Lambda_{\rm eff} + \hat R\right) \\ &\quad + M_5^3 \int_{r=0} d^4x \sqrt{-\hat h}\, \left(-2\sigma_{\rm eff}^{(0)}\right) + M_5^3 \int_{r=r_c} d^4x \sqrt{-\hat h}\, \left(-2\sigma_{\rm eff}^{(r_c)}\right) + \cdots. \end{aligned} \tag{9}$$

Here the effective tension on the $r=0$ brane, with an analogous expression at $r=r_c$, is

$$M_5^3\sigma_{\rm eff}^{(0)} = M_6^4 \sigma^{(0)} \int_0^{y_c} dy\, e^{2A(y)}. \tag{10}$$

$\hat h_{MN}$ represents the metric induced on the branes by $\hat g_{MN}$.

The required fine-tunings of the Randall-Sundrum scenario are

$$\sqrt{-6\Lambda_{\rm eff}} = \sigma_{\rm eff}^{(0)} = -\sigma_{\rm eff}^{(r_c)}. \tag{11}$$

Numerically, we find[4] solutions periodic in the y-direction provided that the effective cosmological constant is of the same order or smaller than the full cosmological constant, $|\Lambda_{\rm eff}| \lesssim O(\Lambda)$. Using the desired value of $\Lambda_{\rm eff}$ from (11) and applying (6), (7) and (10), we can find solutions that are periodic in y and satisfy (11) without finely tuning any of the parameters when

$$(\sigma^{(0)})^2 \lesssim O(\Lambda). \tag{12}$$

In making this transition from the effective parameters back to the $6d$ parameters, any exponential factors are either small or tend to cancel. In figure 1 we show a representative solution, generated numerically.

Although (11) actually contains two fine-tunings, Goldberger and Wise[5] showed that including a massive bulk scalar field with quartic couplings to the brane, thereby generating a non-trivial effective potential for r_c, eliminates one of these fine-tunings. Since we expect that some such mechanism can be adapted to our picture, we are left with one condition on $\Lambda_{\rm eff}$ in terms of $\sigma_{\rm eff}^{(0)}$.

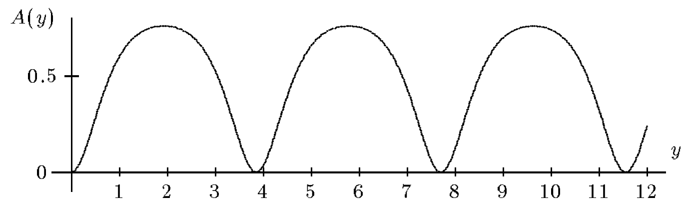

Figure 1. A periodic warp function $A(y)$ for $\Lambda = 1$, $\lambda = 2$ and $\mu = 0.9$. The initial condition is $A''(0) = 3.199870015$. The value of the AdS length is $\ell = 10$ which corresponds to $\sigma^{(0)} = 0.396$.

When (12) is satisfied, solutions of the equations of motion from (1) can satisfy this condition without any unnatural choices for the parameters in the action. The vanishing of the $4d$ effective cosmological constant in the Randall-Sundrum scenario in this picture thus reduces to a dynamical question as to why the flat solutions are favored.

Concluding remarks

An intriguing feature of this scenario is that for each choice of the parameters in the action that admits a periodic warp function, a family of solutions exists. Elements of this family are specified by the value of the $5d$ effective cosmological constant, $\Lambda_{\rm eff}$. While this scenario does not require any unnatural choices of the parameters in (1) to satisfy (11), this condition is not the unique solution to the equations of motion. Some further mechanism is still required that favors a bulk $\Lambda_{\rm eff}$ that obeys (11).

The picture that we have described allows solutions with a stable exponential electroweak-Planck hierarchy without an unnatural choice of the parameters in the action. Since this picture crucially relies on the presence of R^2 terms in the action, it might be worried whether it persists upon including $R^{n>2}$ terms. However, periodic solutions should exist generically for an action composed of general powers of the curvature tensors as was shown[3], in particular, for the small cosmological constant case, $M_6^2\Lambda^{-1} \gg 1$, with with $\Lambda_{\rm eff} = 0$. Including a small $5d$ $\Lambda_{\rm eff}$ does not greatly perturb this result.

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