Effective theory approaches to the trans-Planckian problem

presented by

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Inflation was developed to overcome some of
the problems of the old “big bang” model …

—

… and though it succeeded in doing so,
inflation is not without its own problems.

—

We shall discuss one of these problems today,
describing how it might provide the chance to look
at how nature behaves at extremely short distances

Overview:

- ☐ the horizon problem
- ☐ inflation
- ☐ the trans-Planckian problem
- ☐ effective-theory approaches

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The standard cosmological picture

(old version)

The development of general relativity together with the observation that the universe is expanding led to a standard cosmology

a few preliminaries

space-time metric: $ds^2 = dt^2 - a^2(t)\, d\mathrm{x} \cdot dx$

$a(t)$ is the *scale factor*

$H = \dot{a}/a$ is the *Hubble scale*

Einstein's Equation (space-time dynamics)

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G\, T_{\mu\nu}$$

The standard cosmological picture

(old version)

Initially, the only known ingredients were matter and radiation

both have a retarding effect on the expansion

density (ρ) and pressure (p)

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p)$$

matter:	$p \approx 0$	$a(t) \propto t^{2/3}$
radiation:	$p = \rho/3$	$a(t) \propto t^{1/2}$

Some consequences:

the universe grew out of a much hotter and denser state

it had a "beginning" ($a = 0$)

A successful prediction: the CMB

A little reasoning:

earlier, the universe was hotter and denser

gas ignites when heated and compressed

far away = long ago

we should be able to see the glow of this early plasma phase

Success!   cosmic microwave background radiation

it is *extremely* close to a perfect black body

The fact that it is so uniform everywhere presents a problem for our theory

The problem with having a beginning

A conundrum

How big is a causally connected patch
when the CMB formed?

the particle horizon
(or how far a signal can travel)

null signal: $ds^2 = 0 = dt^2 - a^2(t)\, dx^2$

$$x_{\mathrm{part}}(t) = \int_{t_0}^{t} \frac{dt'}{a(t')} = \eta$$

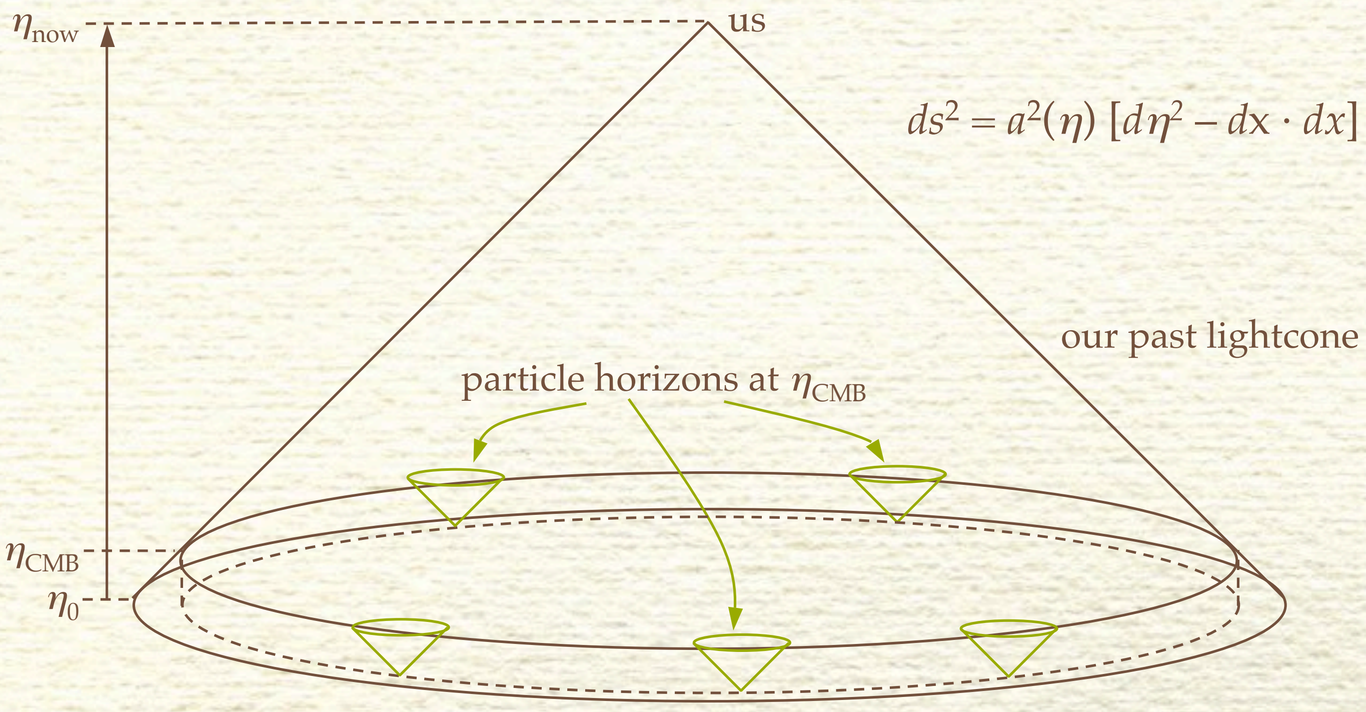
particle horizon = elapsed conformal time

$$dt = a\, d\eta \quad \Rightarrow \quad ds^2 = a^2(\eta)\, [d\eta^2 - d\mathrm{x} \cdot d\mathrm{x}]$$

For the old picture, the casually connected regions
at the time of the CMB are about 1° of the sky(BBN)

The problem with having a beginning

Or, as a picture (in conformal time)



We need $\eta_{now} - \eta_{CMB} < \eta_{CMB} - \eta_0$

How is this possible?

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Evading the horizon problem

(or how to move back η_0)

Different horizons

particle horizon	η	casually disconnected
Hubble horizon	$1/H$	only *currently* hidden

η can only increase, but $1/H$ can increase or decrease

what if there were an early phase when the comoving Hubble horizon was decreasing?

the particle horizon (again)

$$\eta = \int_{t_0}^{t} \frac{dt'}{a(t')} = \int_0^a \frac{d\tilde{a}}{\tilde{a}} \frac{1}{\tilde{a}\, H(\tilde{a})}$$

the comoving Hubble scale should be increasing

$$\frac{d}{dt}[\, a\, H\,] = \frac{d}{dt}\left[a\,\frac{\dot{a}}{a}\right] = \ddot{a} > 0$$

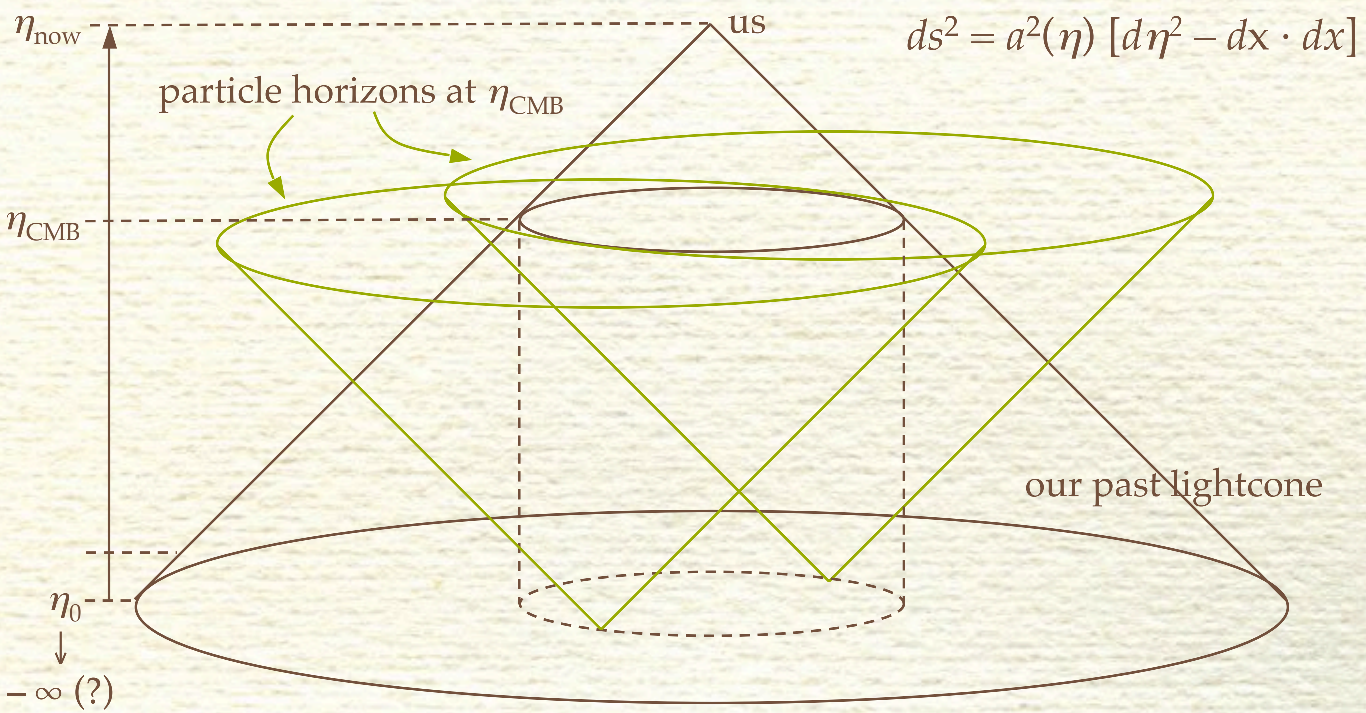
An accelerating (inflating) universe

Inflation:

most of η occurred very early in the universe

or

more of the universe was once in causal contact than we can currently "see"



A vacuum-energy phase

How do we get the universe to inflate ($\ddot{a} > 0$)?

Recall that (using normal time, t)

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p)$$

Try a scalar field: $\phi_0(t)$

$$\rho = \frac{1}{2}\dot{\phi}_0^2 + V(\phi_0) \qquad p = \frac{1}{2}\dot{\phi}_0^2 - V(\phi_0)$$

$$\frac{\ddot{a}}{a} = \frac{8\pi G}{3}(V - \dot{\phi}_0^2)$$

The potential energy must be greater than the kinetic

A quantum field during inflation

(or, How to make structure)

Inflation allows us to evade the horizon problem

but it does something far more important too

a quantum scalar field

$$\Phi(\eta,\mathrm{x}) = \phi_0(\eta) + \varphi(\eta,\mathrm{x})$$

classical zero mode

quantum fluctuation

Expand the quantum part in eigenmodes:

$$\varphi(\eta,\mathrm{x}) = \int \frac{d^3\mathrm{k}}{(2\pi)^3} \left[\varphi_\mathrm{k}(\eta)\, e^{i\mathrm{k}\cdot\mathrm{x}}\, a_\mathrm{k} + \varphi_\mathrm{k}^*(\eta)\, e^{-i\mathrm{k}\cdot\mathrm{x}}\, a_\mathrm{k}^\dagger \right]$$

A quantum field is fluctuating all the time,

$$\langle 0 | \varphi(\eta,\mathrm{x}) \varphi(\eta,\mathrm{y}) | 0 \rangle \neq 0,$$

but what happens when it is in an inflating space-time?

The power spectrum

(of the primordial perturbations)

As the universe inflates, a random pattern of fluctuations fills the universe

How do we characterize this pattern?

Correlation functions: how fluctuations are correlated at different places

n-point function

$$\langle 0|\varphi(\eta,\mathrm{x}_1)\varphi(\eta,\mathrm{x}_2)\ \dots\ \varphi(\eta,\mathrm{x}_n)|0\rangle$$

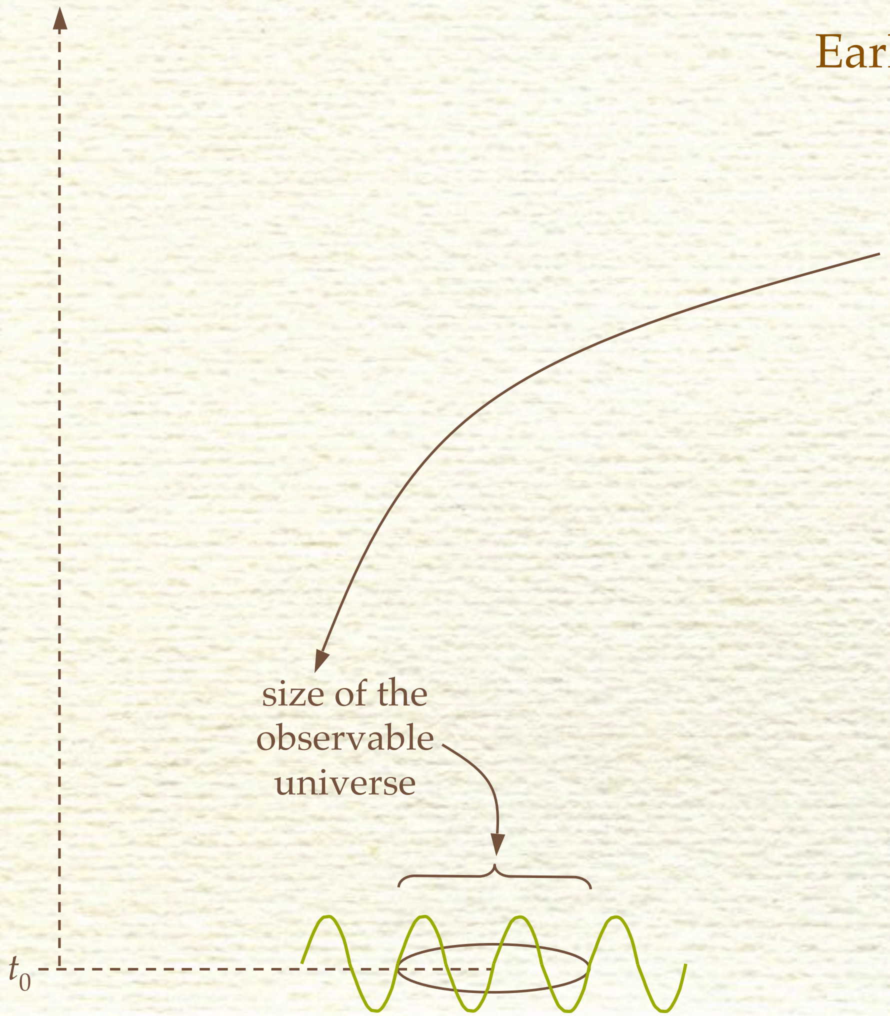
The simplest is the two-point function (power spectrum)

$$\langle 0|\varphi(\eta,\mathrm{x})\varphi(\eta,\mathrm{y})|0\rangle = \int \frac{d^3\mathrm{k}}{(2\pi)^3} e^{i\mathrm{k}\cdot(\mathrm{x}-\mathrm{y})} \quad \frac{2\pi^2}{k^3} P_{\mathrm{k}}(\eta)$$

Let us follow the evolution of a single mode, $\varphi_k(t)$

Following a mode through time

(in physical coordinates)



Early during inflation

$k_{\text{phys}} > H$

Hubble horizon:

$1/H \propto$ constant

Particle horizon:

$$\eta \propto e^{H(t_{\text{end}} - t_0)} - e^{H(t_{\text{end}} - t)}$$

mode: $\varphi_k(t)$

$$\lambda_{\text{phys}}(t) = 2\pi\, a(t)/k \propto e^{Ht}/k$$

Following a mode through time

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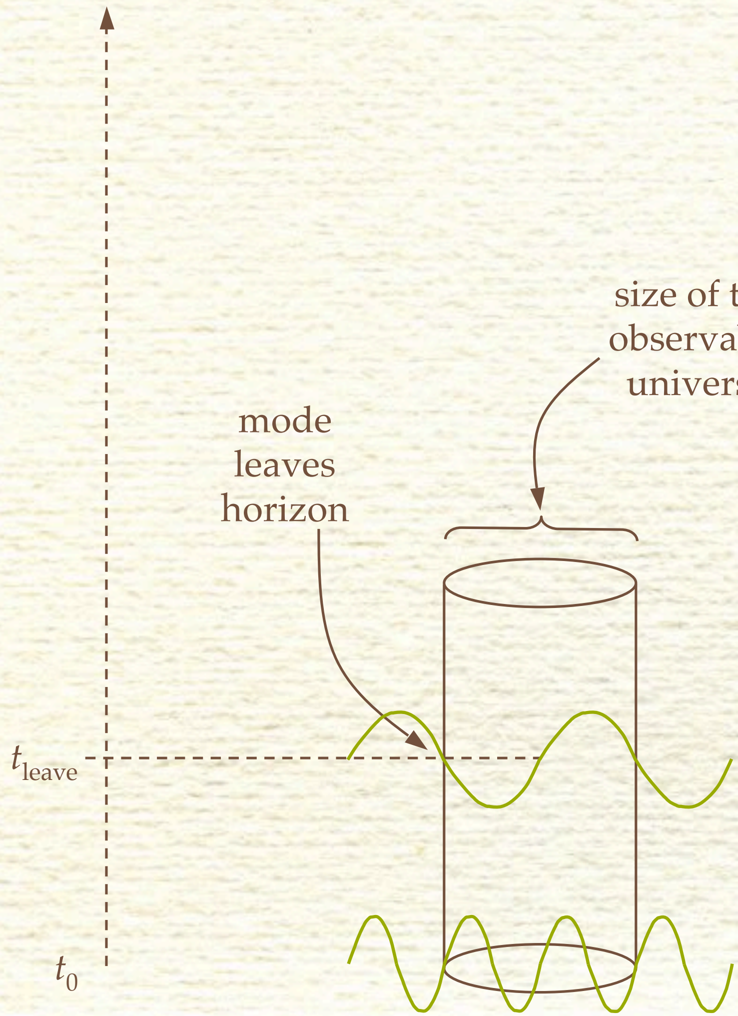

Later during inflation

$$k_{\text{phys}} = H$$

Hubble horizon:

$$1/H \propto \text{constant}$$

Particle horizon:

$$\eta \propto e^{H(t_{\text{end}} - t_0)} - e^{H(t_{\text{end}} - t)}$$

mode: $\varphi_k(t)$

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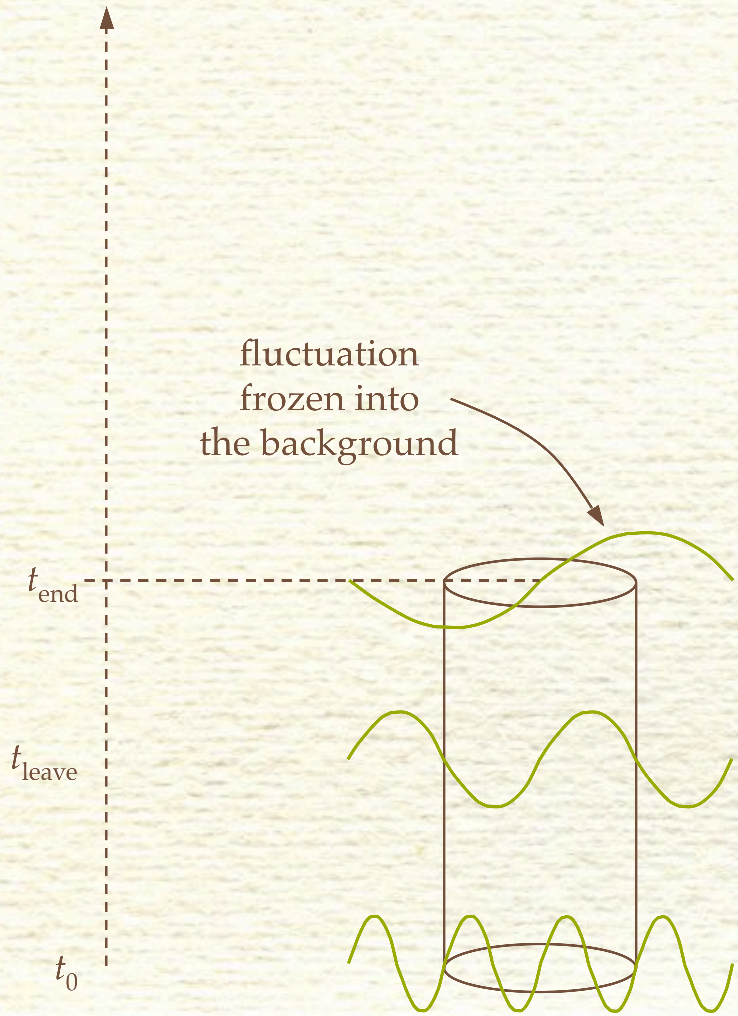

Inflation ends

$k_{\text{phys}} < H$

Hubble horizon:

$1/H \propto$ constant

Particle horizon:

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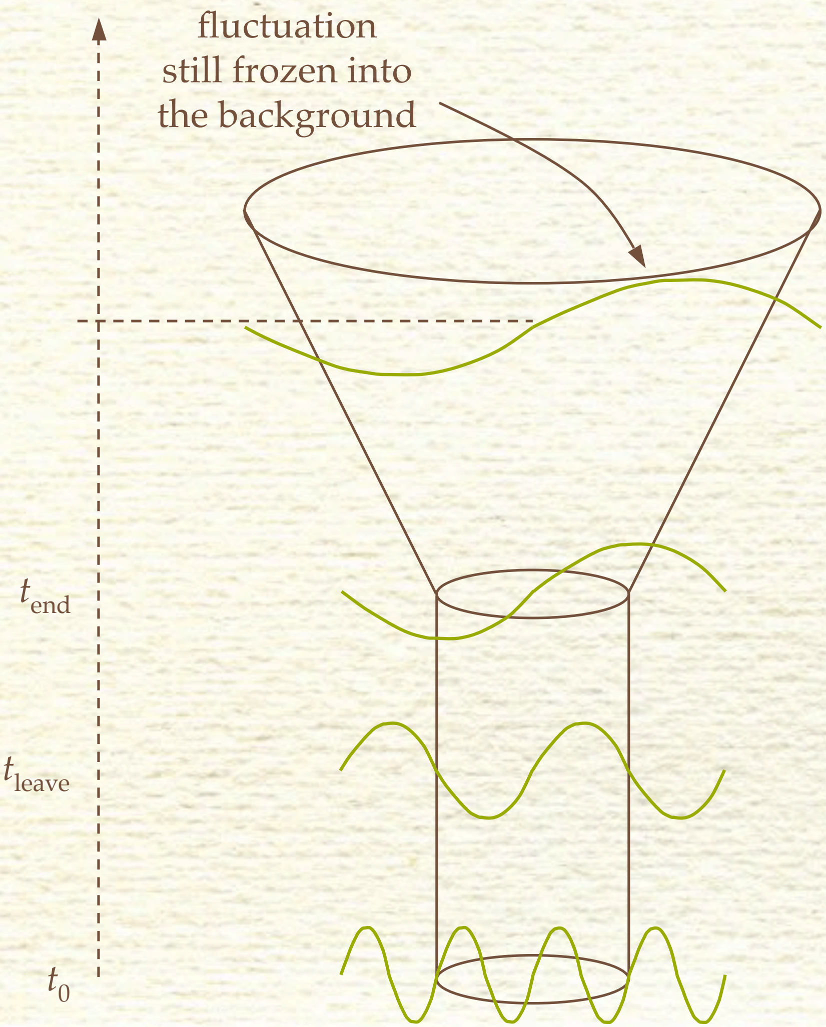


Radiation domination

$k_{phys} < H$

Hubble horizon:

$$1/H \propto 2t$$

Particle horizon:

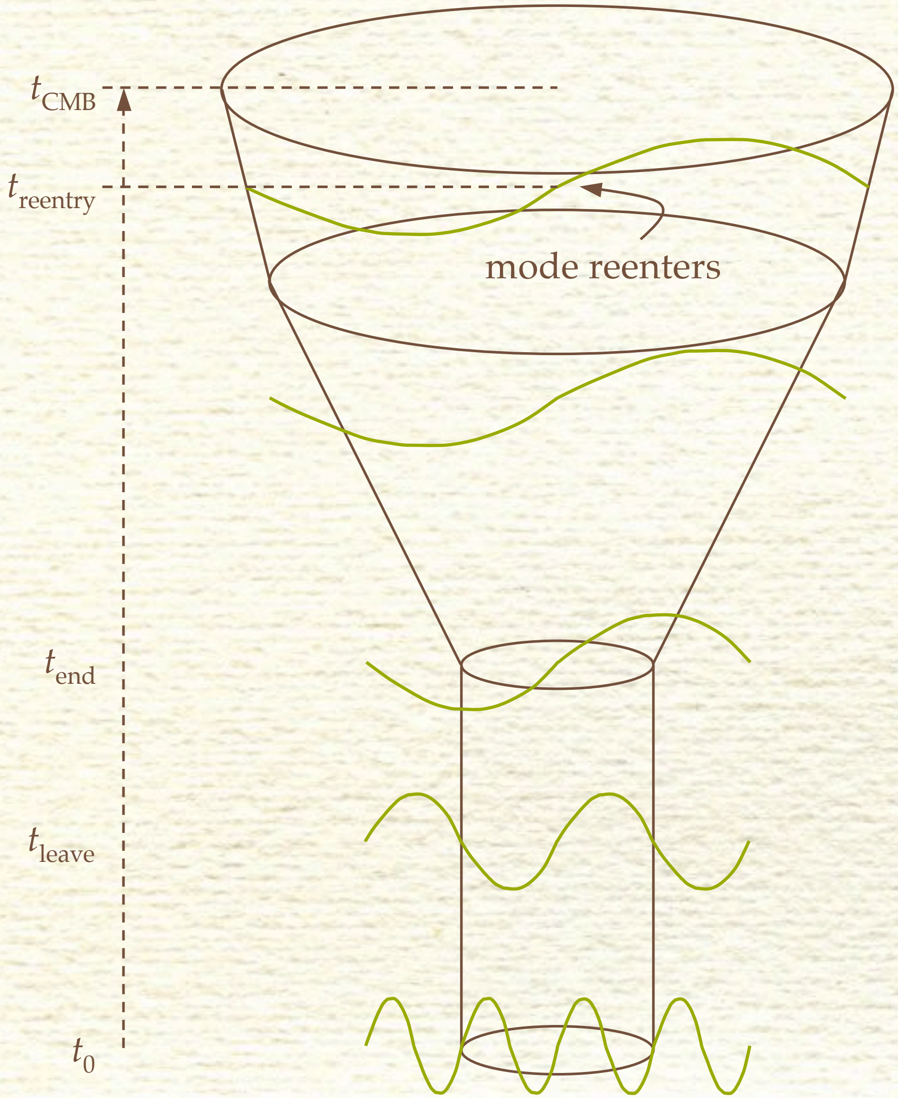
$$\eta \propto t^{1/2}$$

mode: $\varphi_k(t)$

$$\lambda_{phys}(t) = 2\pi\, a(t)/k$$
$$\propto t^{1/2}/k$$

Following a mode through time

(in physical coordinates)



Matter domination

$k_{\text{phys}} > H$ (again)

Hubble horizon:

$$1/H \propto \frac{3}{2}\, t$$

Particle horizon:

$$\eta \propto t^{1/3}$$

mode: $\varphi_k(t)$

$$\lambda_{\text{phys}}(t) = 2\pi\, a(t)/k$$
$$\propto t^{2/3}/k$$

Following a mode through time

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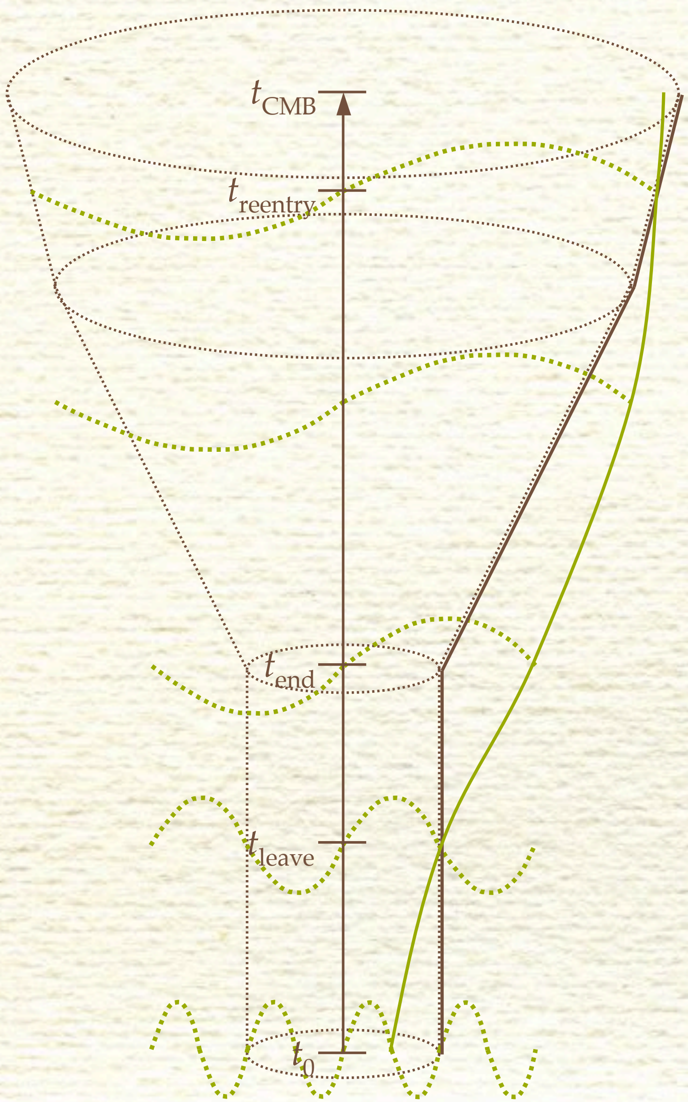

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$k_{\mathrm{phys}} > H$ (again)

Hubble horizon:

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mode: $\varphi_k(t)$

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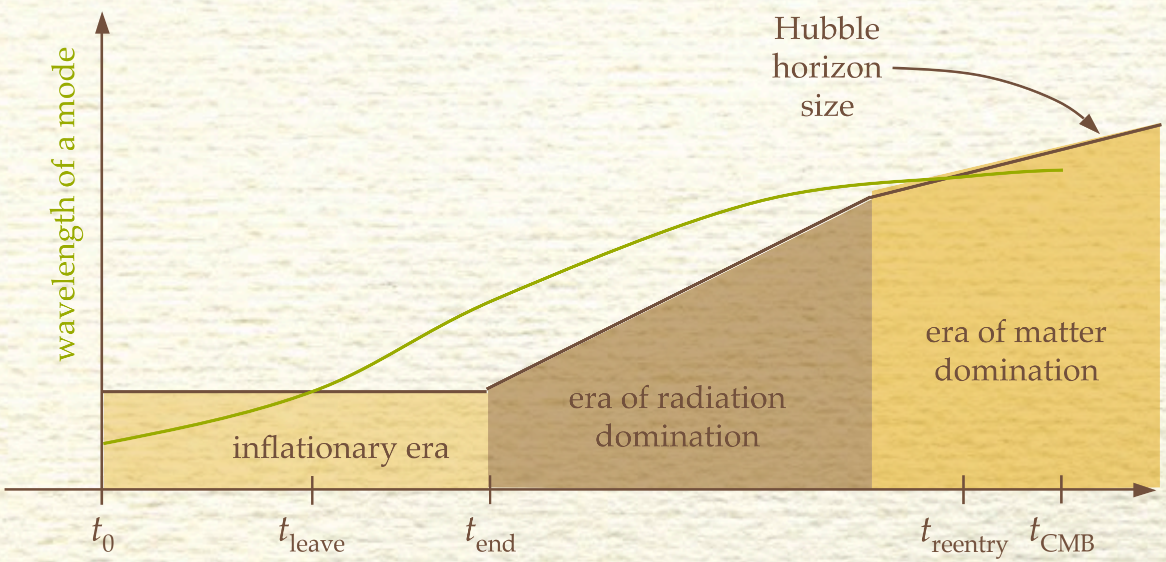
Primordial Perturbations

Not only does inflation solve the horizon problem,

it also fills the universe with
small fluctuations in the space-time background



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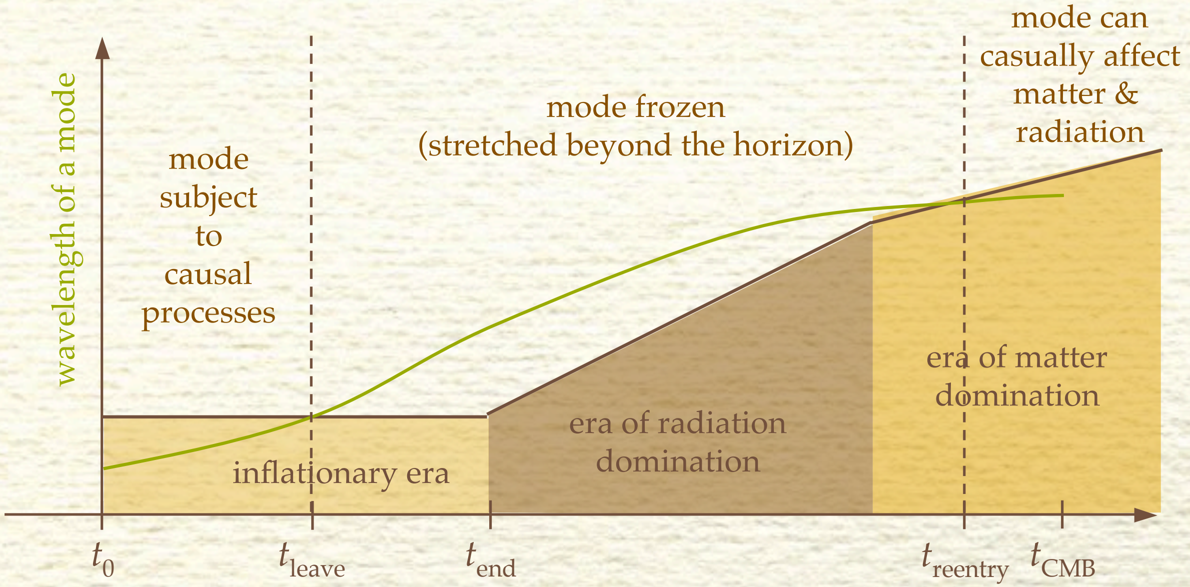

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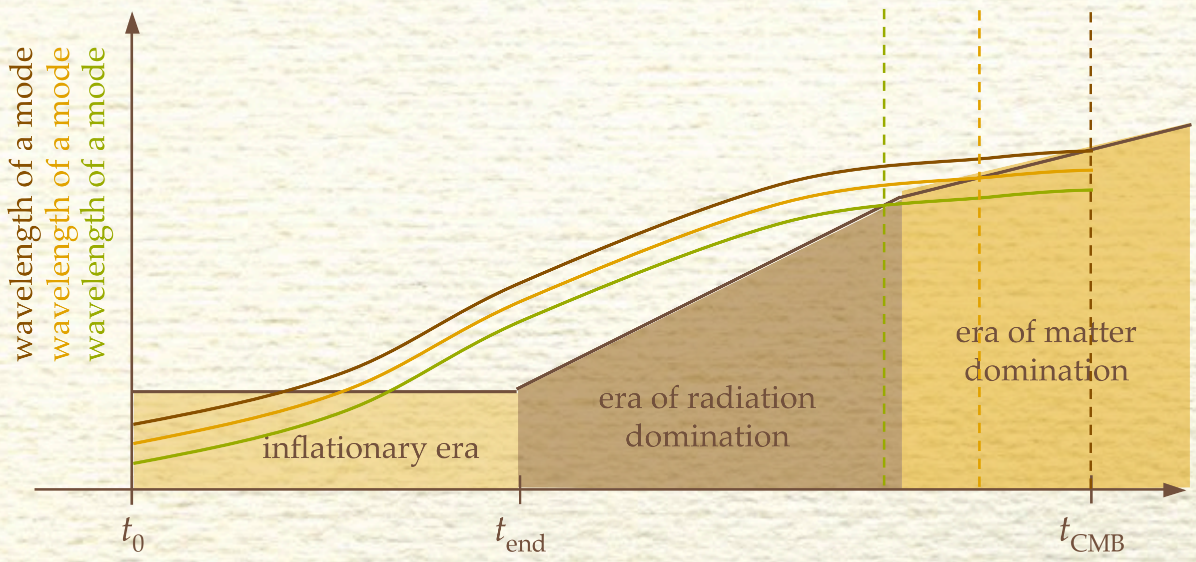
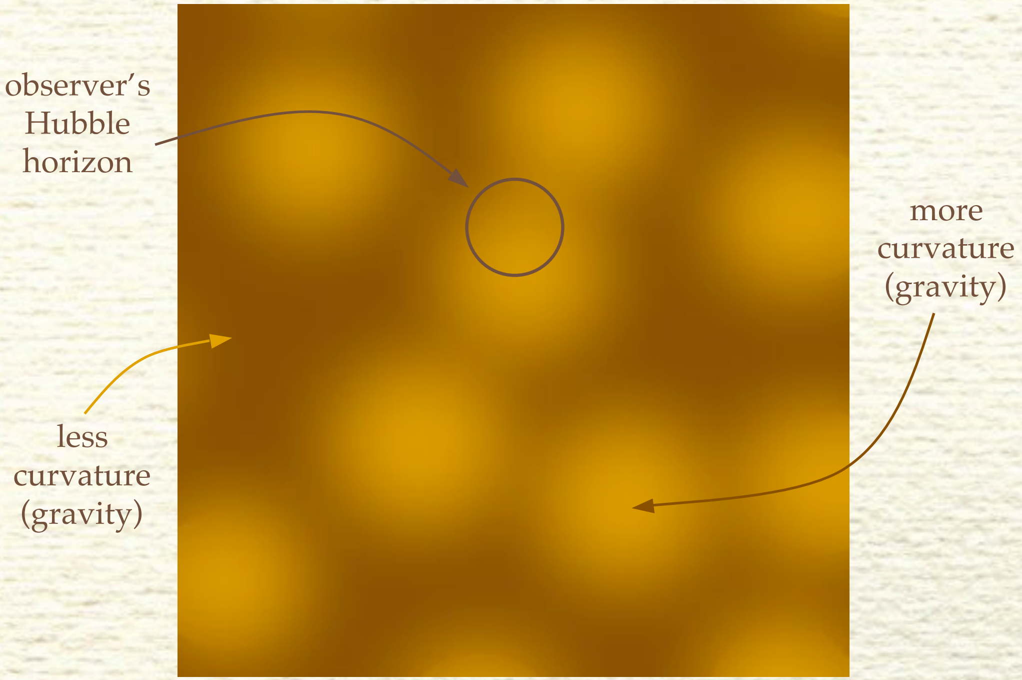


Different modes leave and reenter at different times

reenter earlier ⇒ more time to influence matter

Primordial Perturbations & the CMB

Let us look at how a particular mode influences the distribution (and temperature) of matter



Start with $k(t) < H(t)$

matter is not able to sense the fluctuations

Primordial Perturbations & the CMB

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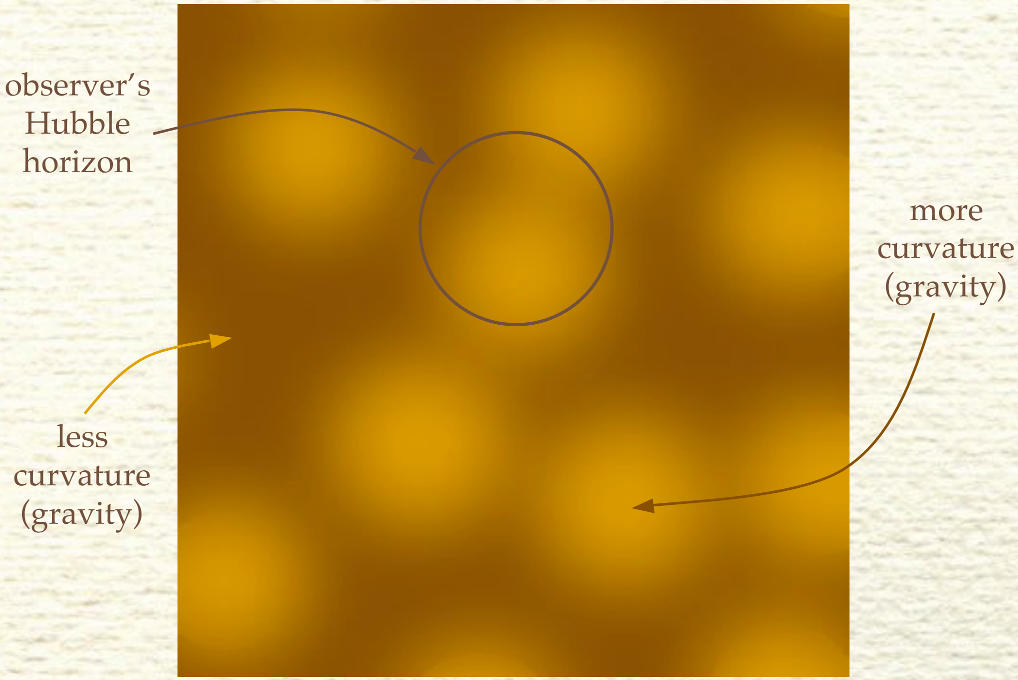
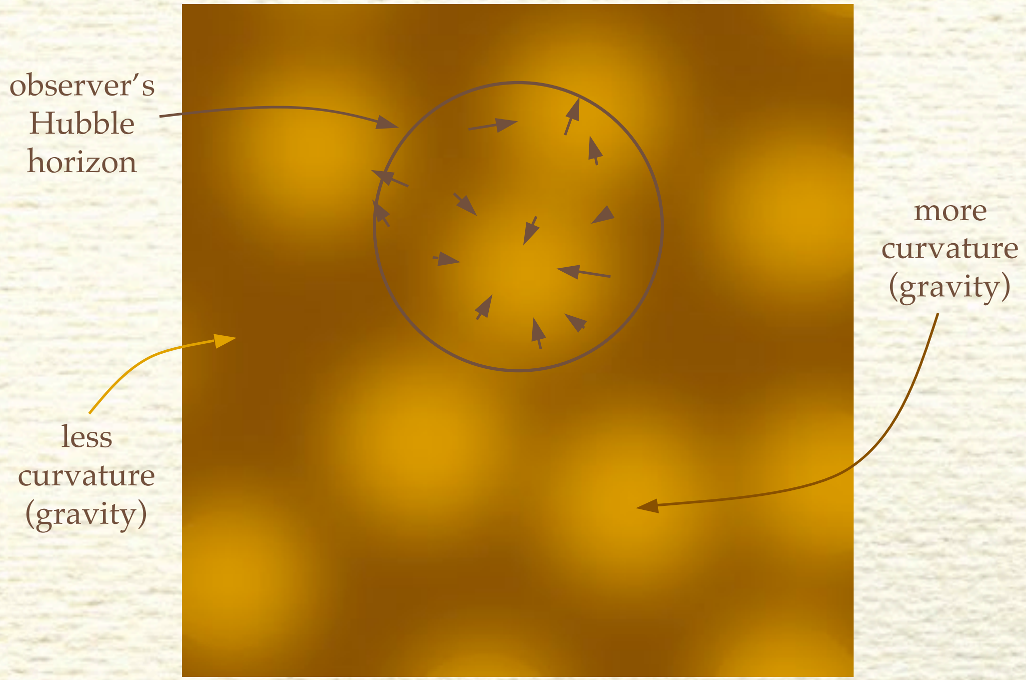


$$k(t) = H(t)$$

matter begins to sense the fluctuations

Primordial Perturbations & the CMB

Let us look at how a particular mode influences the distribution (and temperature) of matter

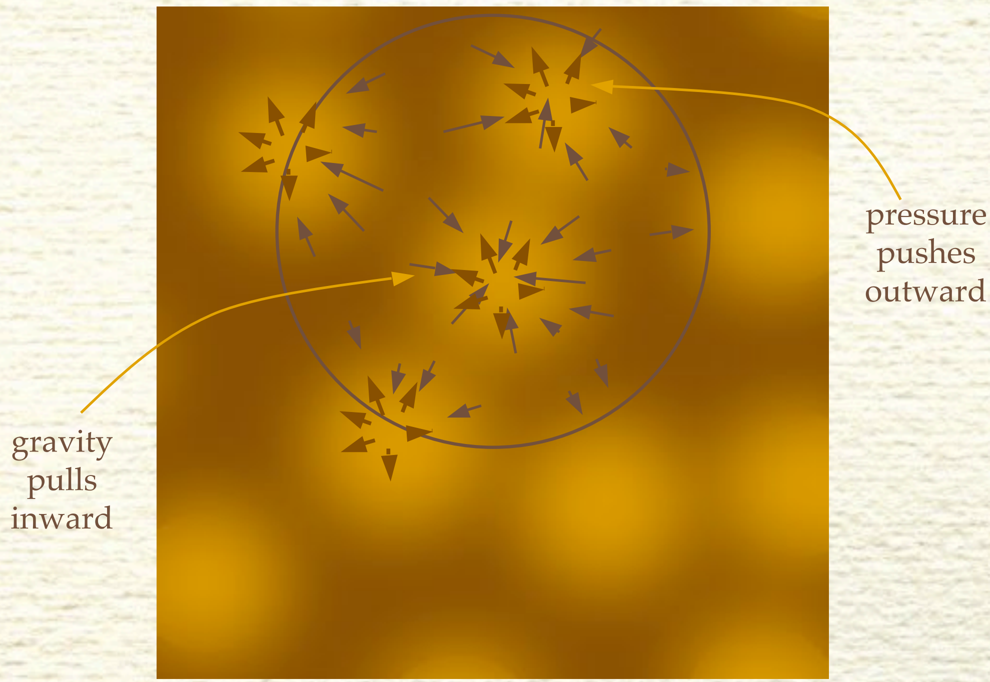


$$k(t) > H(t)$$

matter collapses into areas with more curvature

Primordial Perturbations & the CMB

At some point the condensing matter heats and bounces back



$k(t) > H(t)$

acoustic oscillations

Primordial Perturbations

So inflation provides the "initial" input …

… that produces the pattern …

… in the cosmic microwave background (CMB) or the large-scale structure (LSS)

Perform an angular "Fourier transform"

$$c_l = \int_0^\infty dk\, k^2\, P_k\, T_l(k)$$

"initial input"
primordial perturbations

"transfer function"
Einstein & Boltzmann equations

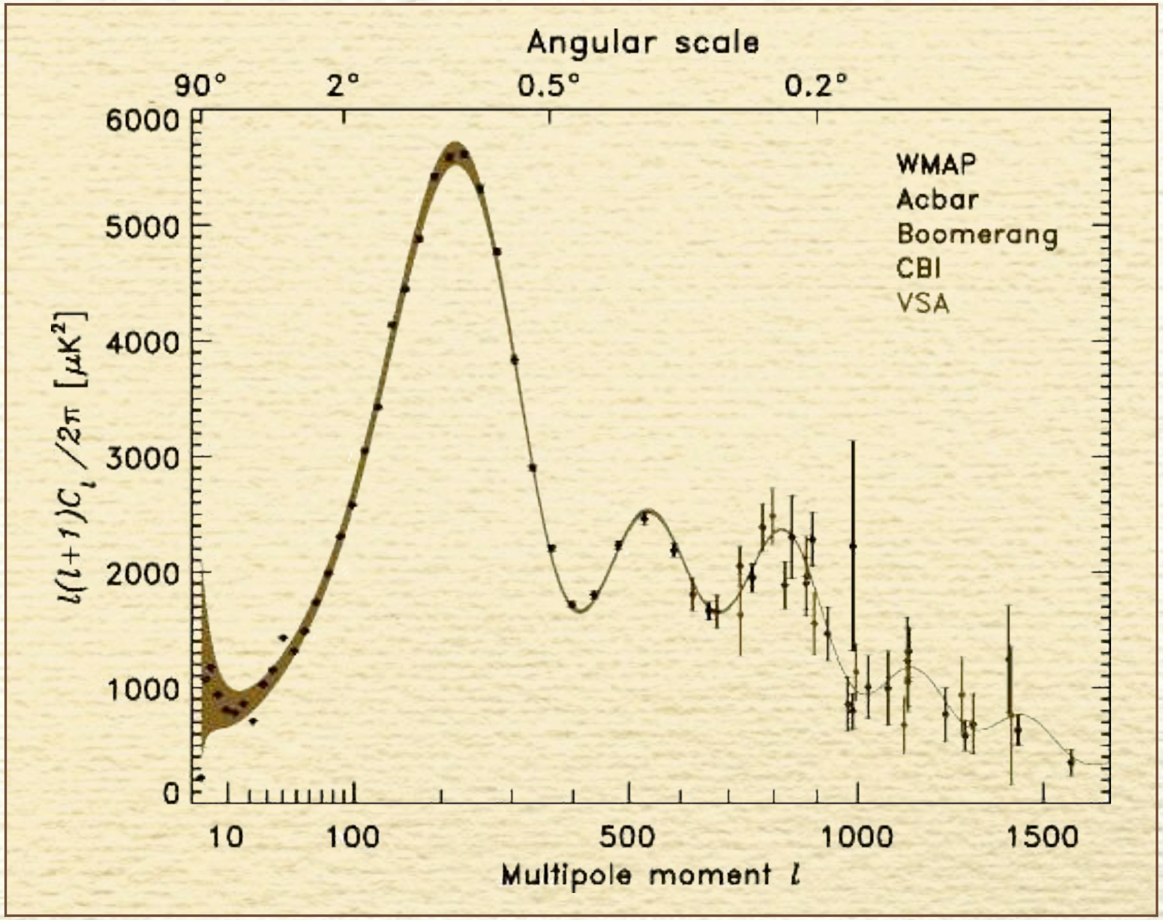
Modes that entered earlier underwent more acoustic oscillations

Predictions from inflation

This basic picture produces several general expectations:

1. structures at all scales (even the largest)
2. acoustic oscillations in CMB (and LSS)
3. synchronized oscillations (phase)
4. nearly Gaussian initial noise
5. primordial gravity waves

Observations of the CMB



from the WMAP Science team

1. structures at all scales
2. acoustic oscillations in CMB
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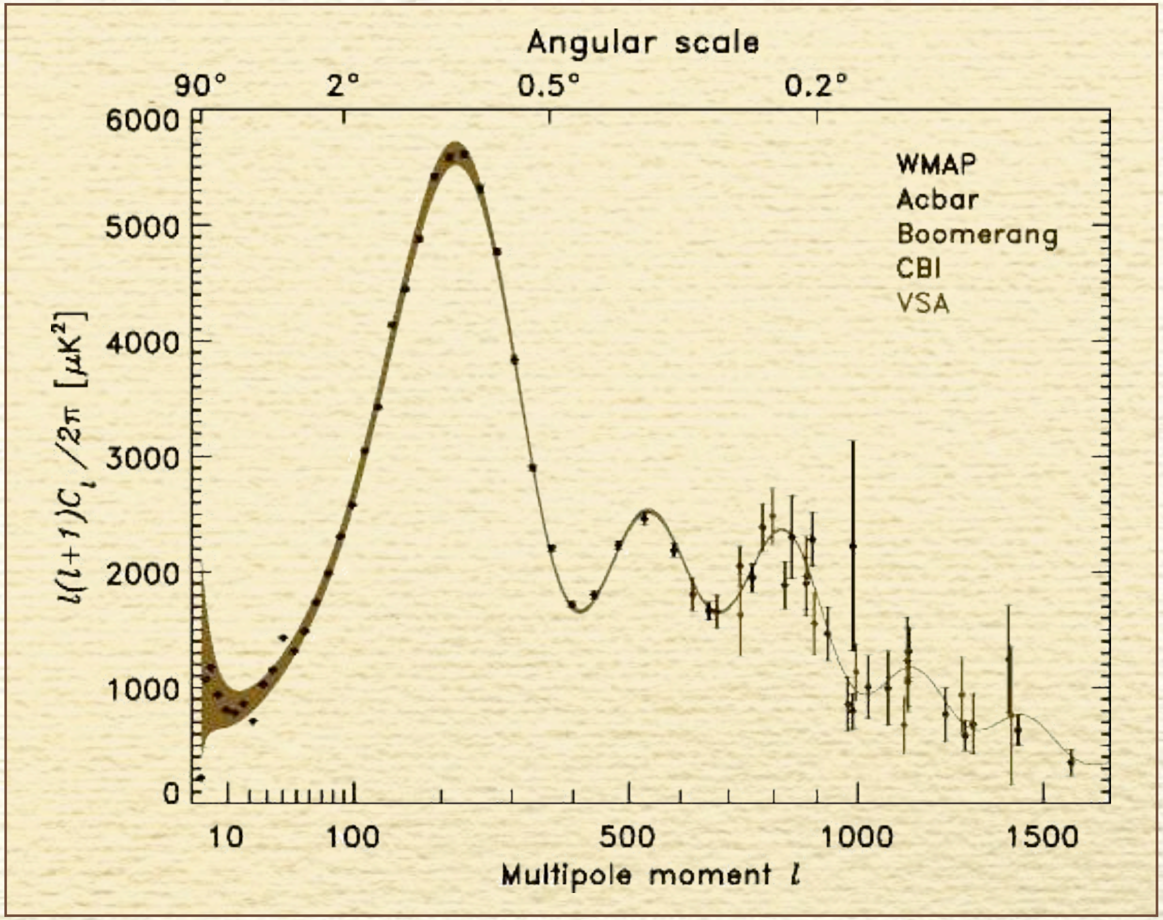

from the WMAP Science team

- ✓ structures at all scales
- ✓ acoustic oscillations in CMB
- ✓ synchronized oscillations
- ✓ nearly Gaussian initial noise
- ? primordial gravity waves

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Inflation (in a little more detail)

So far we have been explaining how inflation works
without mentioning its more unsettling properties

Let us sketch a simple inflationary model

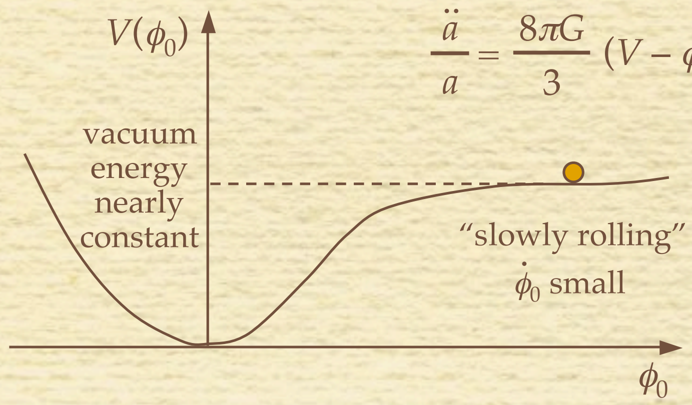
begin with a quantum scalar field

$$\Phi(\eta, x) = \phi_0(\eta) + \varphi(\eta, x)$$

classical zero mode

quantum fluctuation

$$\frac{\ddot{a}}{a} = \frac{8\pi G}{3}\left(V - \dot{\phi}_0^{\,2}\right)$$



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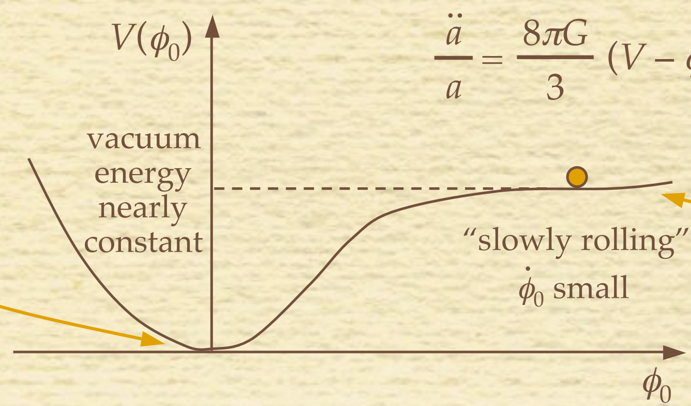
Let us sketch a simple inflationary model

begin with a quantum scalar field

$$\Phi(\eta,x) = \underbrace{\phi_0(\eta)}_{\text{classical zero mode}} + \underbrace{\varphi(\eta,x)}_{\text{quantum fluctuation}}$$

What is this scalar field?

$$\frac{\ddot{a}}{a} = \frac{8\pi G}{3}\left(V - \dot{\phi}_0^2\right)$$



Why is this potential so flat?

cosmological constant problem

The eigenmodes

The quantum side is important for producing the primordial perturbations

expand the operator in eigenmodes

$$\varphi(\eta,\mathrm{x}) = \int \frac{d\mathrm{k}}{(2\pi)^3}\left[\varphi_{\mathrm{k}}(\eta)\, e^{i\mathrm{k}\cdot\mathrm{x}}\, a_{\mathrm{k}} + \varphi_{\mathrm{k}}^{*}(\eta)\, e^{-i\mathrm{k}\cdot\mathrm{x}}\, a_{\mathrm{k}}^{\dagger}\right]$$

In a conformally flat metric: $ds^2 = a^2(\eta)\,[d\eta^2 - d\mathrm{x}\cdot d\mathrm{x}]$

what are the eigenmodes?

in de Sitter space: $a(\eta) = -1/H\eta$

$$\frac{d^2\varphi_k}{d\eta^2} - \frac{2}{\eta}\frac{d\varphi_k}{d\eta} + \left(k^2 + \frac{1}{\eta^2}\frac{m^2}{H^2}\right)\varphi_k = 0$$

solution ($\nu^2 = {}^9\!/_4 - {}^{m^2}\!/_{H^2}$):

$$\varphi_k(\eta) = A_k\, \eta^{3/2}\, H_\nu^{(2)}(k\eta) + B_k\, \eta^{3/2}\, H_\nu^{(1)}(k\eta)$$

Choosing the initial conditions

How do we choose the initial state?

$$[\ \varphi(\eta,\mathrm{x}),\ \pi(\eta,\mathrm{y})\ ] = i\ \delta^3(\mathrm{x}-\mathrm{y})$$

fixes the normalization

$$\varphi_k(\eta) = N_k\ [\ \frac{\sqrt{\pi}}{2} H\eta^{3/2}\, H_\nu^{(2)}(k\eta) + f_k\, \frac{\sqrt{\pi}}{2} H\eta^{3/2}\, H_\nu^{(1)}(k\eta)\ ]$$

But what determines f_k?

Possibilities:

1. match flat modes at $k >> H$
2. fix modes at $\eta_0 \to -\infty$
3. impose infinitesimal Lorentz symmetry

Choosing the initial conditions

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But what determines f_k?

Possibilities:

but is $k \approx M_{\mathrm{pl}} >> H$?

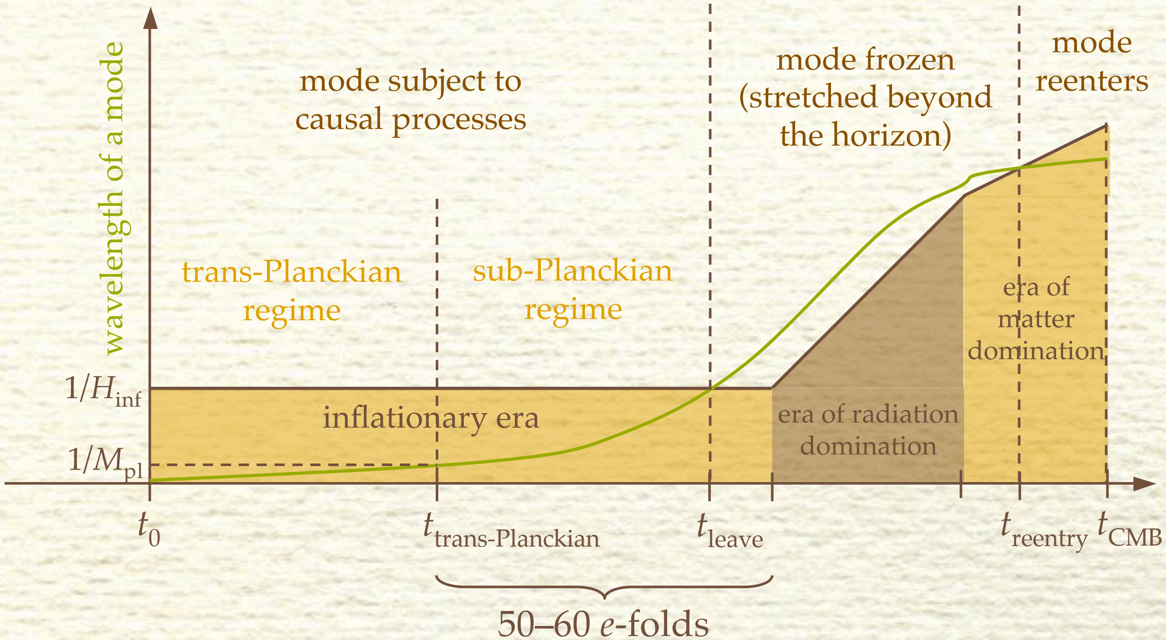
1. match flat modes at $k >> H$
2. fix modes at $\eta_0 \to -\infty$ not asymptotically free?
3. impose infinitesimal Lorentz symmetry

assumes "particle" symmetries for $k >> M_{\mathrm{pl}}$

The trans-Planckian problem

Unless just enough—and no more—inflation occurred, we must fix the observable modes for

$$k >> M_{\mathrm{pl}}$$



Differing philosophies

Different theories lead to different expectations,

1. minimal: point-like beyond Planck-scale
2. finite resolution begins near Planck-scale
   – modified uncertainty relation
   – space-time non-commutativity
   – exited states too

We choose *different initial states* (f_k) for each case

$$\varphi_k(\eta) = N_k \Big[\frac{\sqrt{\pi}}{2} H\eta^{3/2} H_\nu^{(2)}(k\eta) + f_k \frac{\sqrt{\pi}}{2} H\eta^{3/2} H_\nu^{(1)}(k\eta) \Big]$$

1. minimal case: $f_k \rightarrow 0$ (faster than $1/k^4$)
2. finite resolution: $f_k \neq 0$ as $k \rightarrow \infty$

The standard picture

If we follow the usual reasoning, we would choose

$$f_k = 0$$

which actually agrees well with experiment

Recall that the power spectrum is

$$\langle 0 | \varphi(\eta, \mathrm{x}) \varphi(\eta, \mathrm{y}) | 0 \rangle = \int \frac{d^3\mathrm{k}}{(2\pi)^3} \mathrm{e}^{i\mathrm{k}\cdot(\mathrm{x}-\mathrm{y})} \; \frac{2\pi^2}{k^3} P_{\mathrm{k}}(\eta)$$

in the massless limit: $\nu \to {}^3/_2$

$$P_k(\eta) = \frac{H^2}{4\pi^2} [\, 1 + (k\eta)^2 \,]$$

when stretched outside the horizon: $k\eta \to 0$

So whatever we do,
this should be the 'tree-level' result
($f_k \to 0$ as $k \to 0$)

Observability

The most important question is
whether such effects can be observed

Inflation seems to break *decoupling* rather strongly

So look for these effects in the relics left from inflation:
the primordial perturbations

$$P_k(\eta) = \frac{H^2}{4\pi^2}\left(1 + \text{slow-roll} + \mathrm{O}\left(\frac{H^n}{M^n}\right)\right)$$

1. minimal case: $n = 2$
2. finite resolution: $n = 1$

Note that M might not be $M_{\rm pl}$

Different models

1. The minimal picture
   - Lorentz-invariant, point-like, "vacuum state"

     Kaloper, Kleban, Lawrence, Shenker, & Susskind, 2001–2003
   - Adiabatic vacua

     Anderson, Molina-Paris, Mottola, 2005
2. Something new
   - modified dispersion relation

     Brandenberger & Martin, 2001–2003
   - a stringy uncertainty relation

     Easther, Greene, Kinney, & Shiu, 2001–2004
   - cut-off states

     Niemeyer & Kempf, 2001–2006; Collins & M. Martin, 2004
   - minimal length scale

     Danielsson, 2002–2006
   - coupling to an excited state

     Burgess, Cline, Lemieux, & Holman, 2003

Is there a more general approach that simultaneously incorporates all of these possibilities?

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Effective theory formulations

Let us be a little less ambitious,
but in the process say something much more general

Do not attempt to explain nature at all scales (times)
but only up to some initial time, η_0

1. The effective initial state
   Collins & Holman, 2005–
2. Boundary operator formulation
   Greene, Schalm, Shiu, & van der Schaar, 2004–2005
3. Lorentz-breaking boundary operators
   Collins & Holman, 2007–

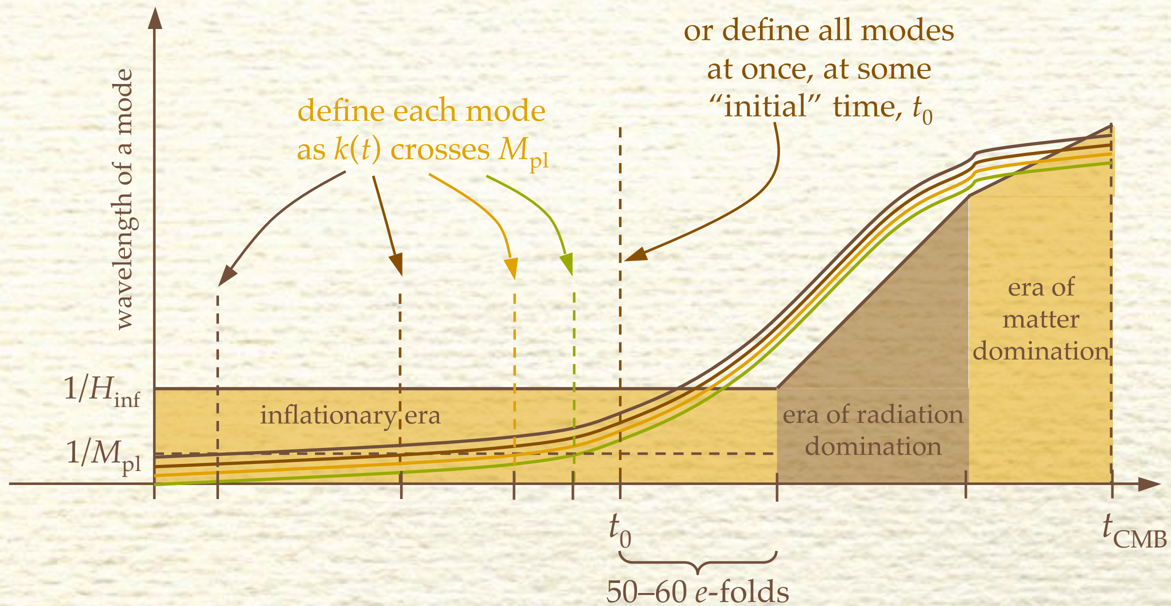
Questions:

How large is the 'trans-Planckian' signal?
What is its *typical* shape?

A subtle point

There is an important difference in how we set up the modes in either case

which leads to a difference in the predictions



mode-by mode: defined on a time-like surface (?)

initial state: defined on a space-like surface

A partially addressed challenge: loops

1. The minimal picture
   - no new ingredients
   - Perturbative stability requires $f_k < k^{-4}$

2. Something new
   - very few approaches have considered loops

     Greene, Schalm, Shiu, & van der Schaar, 2004–2005
     Collins & Holman, 2005–
   - inconsistencies between two-point function and the propagator?
   - cut-offs make loops finite, but may give the wrong power counting

     Porrati, 2004

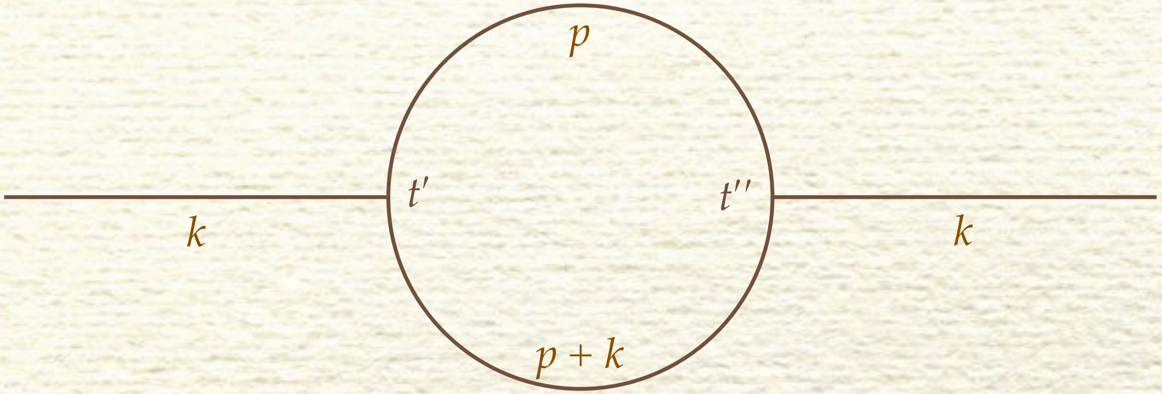
To establish the consistency,
we must show renormalizability

Loops and perturbative stability

Write a Feynman propagator in its time-ordered form,

$$\int \frac{dk_0}{2\pi} \frac{ie^{-ik_0(t-t')}}{k_0{}^2 - k^2 - m^2 + i\varepsilon} = \Theta(t-t') \frac{e^{-i\omega_k(t-t')}}{2\omega_k} + \Theta(t'-t) \frac{e^{i\omega_k(t-t')}}{2\omega_k}$$

Θ-functions keep the phases from canceling in the UV



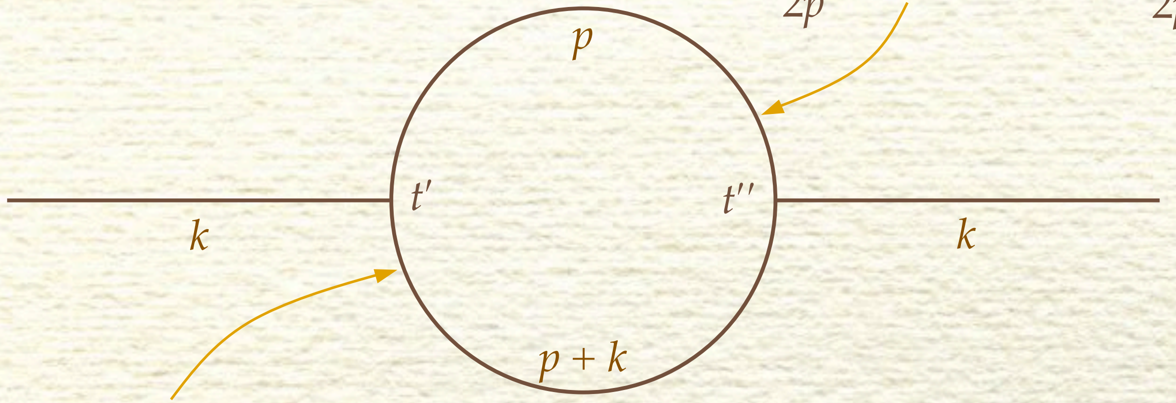
Loops and perturbative stability

Write a Feynman propagator in its time-ordered form,

$$\int \frac{dk_0}{2\pi} \frac{i e^{-ik_0(t-t')}}{k_0{}^2 - k^2 - m^2 + i\varepsilon} = \Theta(t-t') \frac{e^{-i\omega_k(t-t')}}{2\omega_k} + \Theta(t'-t) \frac{e^{i\omega_k(t-t')}}{2\omega_k}$$

Θ-functions keep the phases from canceling in the UV

$$\Theta(t'-t'') \frac{e^{-ip(t'-t'')}}{2p} + \Theta(t''-t') \frac{e^{ip(t'-t'')}}{2p}$$



$$\Theta(t'-t'') \frac{e^{-i(p+k)(t'-t'')}}{2(p+k)} + \Theta(t''-t') \frac{e^{i(p+k)(t'-t'')}}{2(p+k)}$$

The f_k terms have the opposite phase —

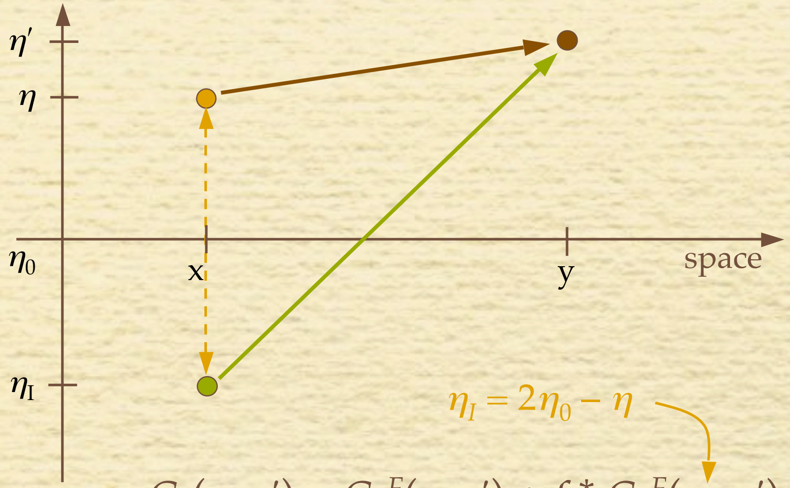
modify the time-ordering to preserve the phase separation

Interference in the propagator

The propagator for the effective theory has two terms,

point source + boundary influence

In flat space, the second piece can be written as the effect of a fictitious 'image' source



$$\eta_I = 2\eta_0 - \eta$$

$$G_k(\eta,\eta') = G_k^F(\eta,\eta') + f_k^* \, G_k^F(\eta_I,\eta')$$

Boundary renormalization

Consider a simple effective description of
the initial structure in the state (f_k)

Collins & Holman, 2005–2006

$$f_k = \sum_{n=1}^{\infty} d_n \frac{k^n}{(a(\eta_0)M)^n}$$

Loops create new divergences when
we sum over the UV behavior of the state

These occur *only* where the state is defined (η_0)

We find a beautiful correspondence between

UV structures in the initial state $\longleftrightarrow$ irrelevant boundary counterterms φ^4, etc.

Boundary renormalization

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Collins & Holman, 2005–2006

$$f_k = \sum_{n=1}^{\infty} d_n \frac{k^n}{(a(\eta_0)M)^n} + \sum_{n=0}^{\infty} c_n \frac{(aH(\eta_0))^n}{\omega_k{}^n}$$

Loops create new divergences when we sum over the UV behavior of the state

These occur *only* where the state is defined (η_0)

We find a beautiful correspondence between

UV structures in the initial state	⟷	irrelevant boundary counterterms	φ^4, etc.
IR structures in the initial state	⟷	relevant / marginal boundary counterterms	φ^2, $H\varphi^2$, $\varphi\varphi'$, etc.

Primordial perturbations—corrections

As an example, examine the leading correction

$$f_k = d_1 \, \frac{k}{a(\eta_0)M} + \cdots$$

This produces a correction to the power spectrum

$$\boxed{P_k(\eta) = \frac{H^2}{4\pi^2}\left\{1 + d_1 \frac{k}{k_*} \sin\left(2\,\frac{k}{k_*}\,\frac{M}{H}\right)\right\}}$$

where $k_*/a(\eta_0) = M$

Because we use a space-like surface to define the state, k/k_* terms arise naturally (counterterms $\Rightarrow H/M$ too)

States defined mode-by-mode (time-like surface) more typically give H/M

$$\boxed{P_k(\eta) = \frac{H^2}{4\pi^2}\left\{1 + \mathrm{O}(1)\,\frac{H}{M} \sin\left(2\,\frac{M}{H} + \phi\right)\right\}}$$

Aside: symmetry-breaking operators

Models with local Lorentz violation—and the usual vacuum state—can produce the same signal

$$L_{\mathrm{LV}} = \frac{d_1}{aM}\varphi\,(-\nabla\cdot\nabla)^{3/2}\varphi + \frac{d_2}{(aM)^2}\nabla\varphi\cdot\nabla\varphi$$

This Lagrangian yields

Collins & Holman, 2007

$$P_k(\eta) = \frac{H^2}{4\pi^2}\left\{1 - d_1\frac{k}{k_*}\cos\left[2\,\frac{k}{k_*}\,\frac{M}{H}\right]\right\}$$

where $k_*/a(\eta_0) = M$

Note that k_* here is really defined at the beginning of inflation (*unlike* the effective state)

Conclusions & Open Questions

Inflation provides a successful explanation for the source of the primordial perturbations in the universe

but the picture still is very incomplete

e.g., the trans-Planckian problem

—

is there a way to circumvent this apparent violation of decoupling?

—

observable signals (H/M or k/k_*)

CMB precisions: 0.1% LSS precisions: 0.001%

—

Much more to investigate:
initial state matrix elements,
non-Gaussianites,
detailed experimental fits, . . .

the end