A few unanswered questions about the inflationary picture

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What is the purpose of inflation?

<u>I</u>

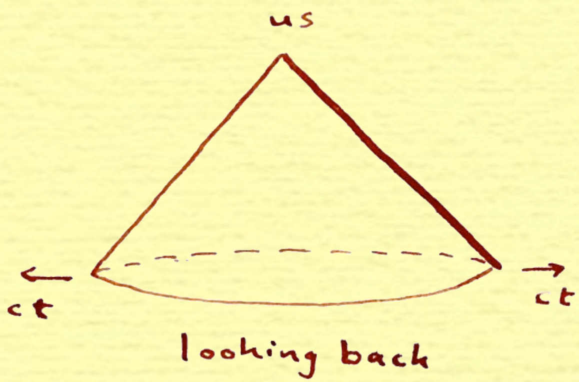
It extends the causal range of
primordial processes

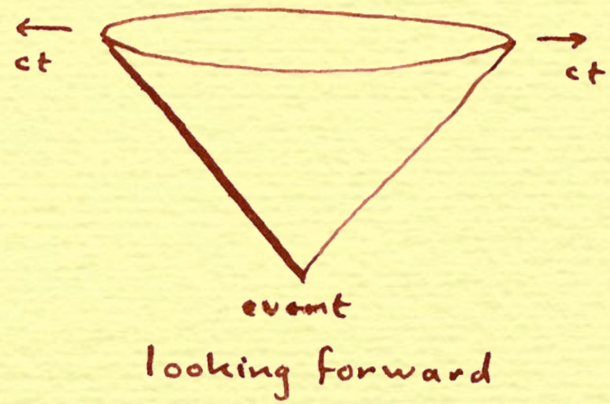
<u>II</u>

It explains why we see something
(structures) rather than nothing
(perfect uniformity)

Horizons

There are two sorts of horizons in relativity:





Hubble horizon

the farthest distance
that we can "see"

Causal horizon

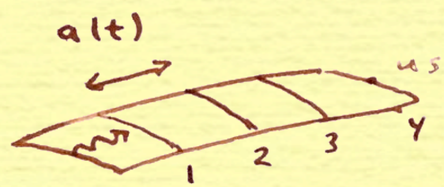
the farthest subsequent
reach of an event

In special relativity they grow at equal rates;

in general relativity their growth can differ

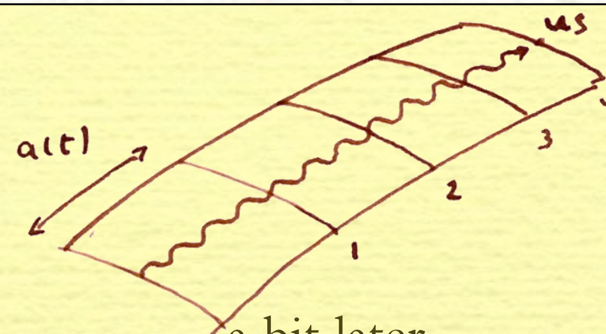
Horizons

Causal horizon: because the space-time is itself expanding, a signal can travel farther than $c\Delta t$ *globally*



start

the local speed is always c



a bit later

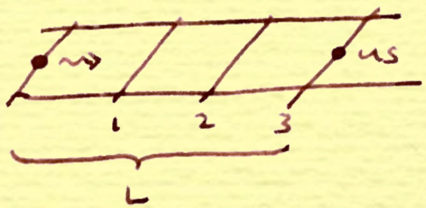
but globally it rides along with the expansion (redshift)

$$ds^2 = dt^2 - a^2(t)\,[dx^2 + dy^2 + dz^2]$$

$$\text{causal horizon} = \int \frac{dt'}{a(t')}$$

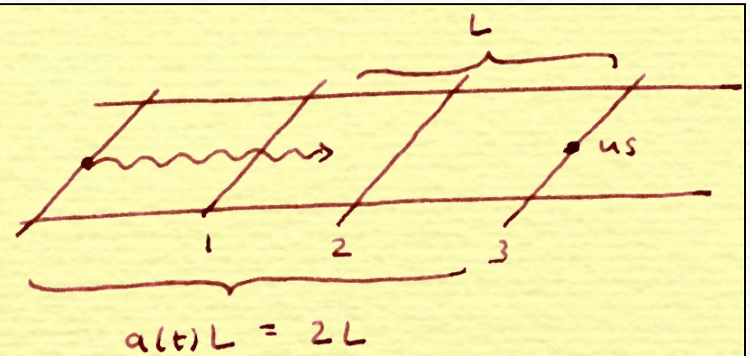
Horizons

Hubble horizon: in addition to a finite age for the universe, how far we can "see" is limited in very rapid expansions



start

it must travel a physical distance L



a bit later

the expansion is such that it still has just as far to go

$$\text{Hubble horizon} = \frac{a}{\dot{a}}$$

What happens in a matter universe

In a universe composed of matter/radiation, it is the Hubble horizon that grows faster

metric: $ds^2 = dt^2 - a^2(t)\,[dx^2 + dy^2 + dz^2]$

matter: $a(t) \propto t^{2/3}$ radiation: $a(t) \propto t^{1/2}$

universe	scale factor	causal horizon	Hubble horizon
matter	$t^{2/3}$	$t^{1/3}$	$\frac{3}{2}t$
radiation	$t^{1/2}$	$t^{1/2}$	$2t$

So the distance *that we can see* <u>grows faster</u> than the range *that could have influenced us*

How inflation solves this problem

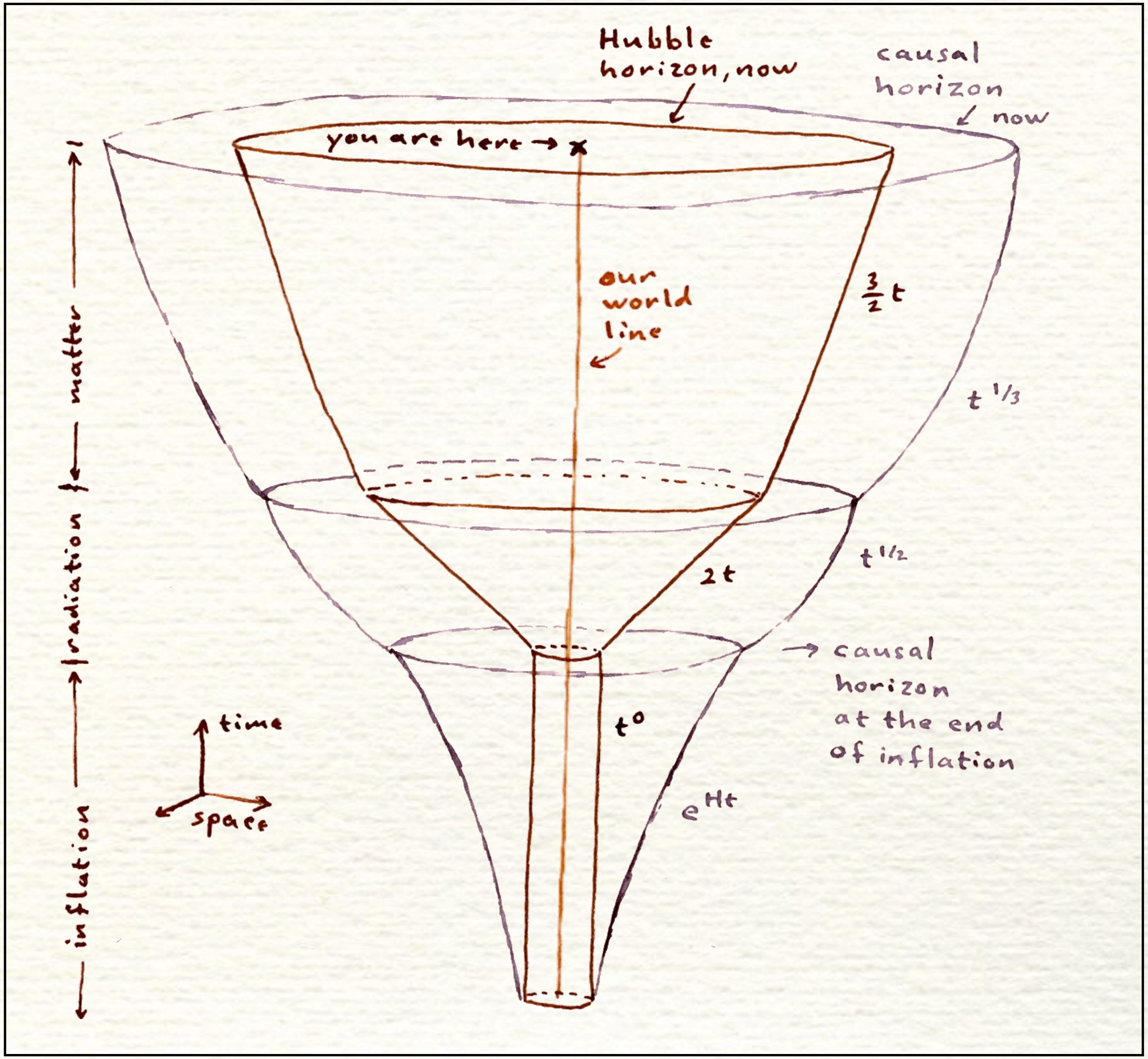
When the expansion accelerates (*inflation*), it is the causal horizon that grows faster

For example, an expansion very close to a de Sitter expansion (pure cosmological constant) vacuum energy: $a(t) \propto e^{Ht}$ $[\ddot{a}/a = H^2 > 0]$

universe	scale factor	causal horizon	Hubble horizon
matter	$t^{2/3}$	$t^{1/3}$	$\frac{3}{2}t$
radiation	$t^{1/2}$	$t^{1/2}$	$2t$
inflation	e^{Ht}	$e^{H\Delta t}$	$\frac{1}{H}$

$N = H\Delta t$ is the number of *e-folds* of inflation

How the horizons evolve



Horizons

An unsettling question:

A finite amount of inflation only puts off the horizon problem for a time (though it is a very long one)

Eventually we shall encounter the very same problem in the future

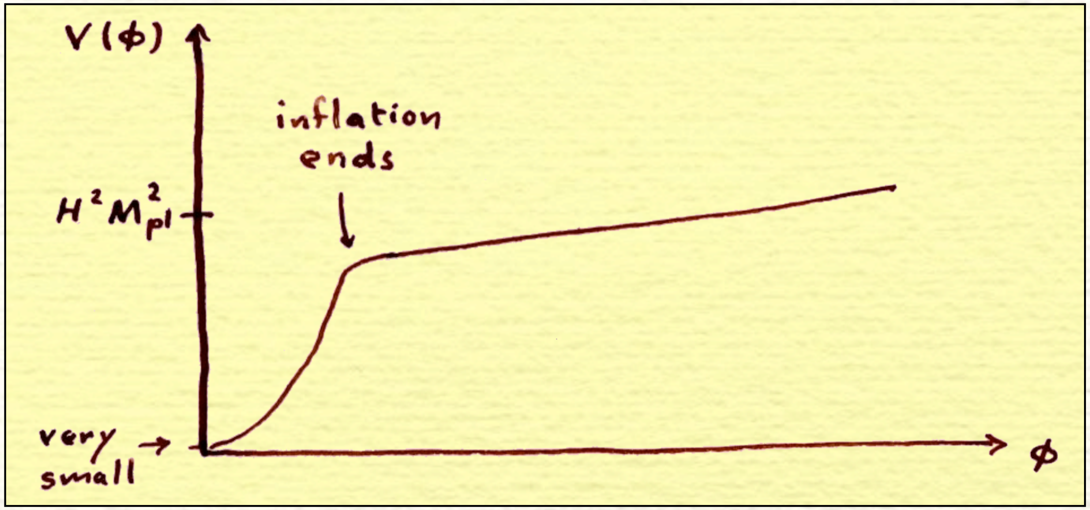
Does the universe experience periodic spurts of accelerated growth?

Why? What causes them and when?

A typical inflationary model

Introduce a scalar field called the inflaton, ϕ

$$S = \int d^4x \; |g|^{1/2} \left[\frac{1}{2} M_{\rm pl}{}^2 R + \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) \right]$$



This model raises a few more questions:

Why is the potential so flat?

Why is V at the end of inflation so small?

What *is* the inflaton?

Making inhomogeneities

An important accomplishment:

The main feat of inflation is that it can explain the "initial conditions" of the conventional (matter/radiation) expansion

We need a tiny amount of *inhomogeneity*:

$$\phi = \phi_0(\eta) + \delta\phi(\eta,\mathrm{x})$$

classical fields quantum fields

$$g_{\mu\nu} = g^{(0)}_{\mu\nu}(\eta) + \delta g_{\mu\nu}(\eta,\mathrm{x})$$

We have switched to conformal time, $\eta = \int \frac{dt}{a(t)}$

Inflation makes inhomogeneities through two ingredients,

– the accelerated expansion
– quantum fluctuations of a field

Quantum fluctuations

Note, there is only one *physical* scalar mode,

$$\varphi = \begin{pmatrix} \text{inflaton} \\ \text{fluctuation} \end{pmatrix} + \frac{\phi_0'}{aH} \begin{pmatrix} \text{spatial metric} \\ \text{fluctuation} \end{pmatrix}$$

Expanding the field in its 'Fourier modes,'

$$\varphi = \int \frac{d^3\mathrm{k}}{(2\pi)^3} \{ \varphi_k(\eta) e^{i\mathrm{k}\cdot\mathrm{x}}\, a_{\mathrm{k}} + \varphi_k{}^*(\eta) e^{-i\mathrm{k}\cdot\mathrm{x}}\, a_{\mathrm{k}}{}^\dagger \}$$

The equation of motion for φ_k is

$$\varphi_k'' + 2aH\varphi_k' + (k^2 + a^2m^2)\varphi_k = 0$$

This is a 2nd order equation, so we need two 'boundary' conditions

Quantum fluctuations

Note, there is only one *physical* scalar mode,

$$\boxed{\varphi = \delta\phi + \frac{\phi_0'}{aH}\Psi \quad \text{or} \quad \zeta = \frac{aH}{\phi_0'}\varphi}$$

(for the knowledgeable)

Expanding the field in its 'Fourier modes,'

$$\boxed{\varphi = \int \frac{d^3\mathrm{k}}{(2\pi)^3}\{\varphi_k(\eta)e^{i\mathrm{k}\cdot\mathrm{x}}\, a_{\mathrm{k}} + \varphi_k{}^*(\eta)e^{-i\mathrm{k}\cdot\mathrm{x}}\, a_{\mathrm{k}}{}^\dagger\}}$$

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Boundary conditions

#1 $\Rightarrow$ <u>equal time commutator</u>

$$[\varphi(\eta,\mathrm{x}),\ \pi(\eta,\mathrm{y})] = i\,\delta^3(\mathrm{x}-\mathrm{y})$$

This fixes the normalization for $\varphi_k(\eta)$

#2 $\Rightarrow$ <u>Minkowski-space vacuum modes</u>

Typically, the second condition is to choose the modes to match the vacuum modes for Minkowski space at short distances,

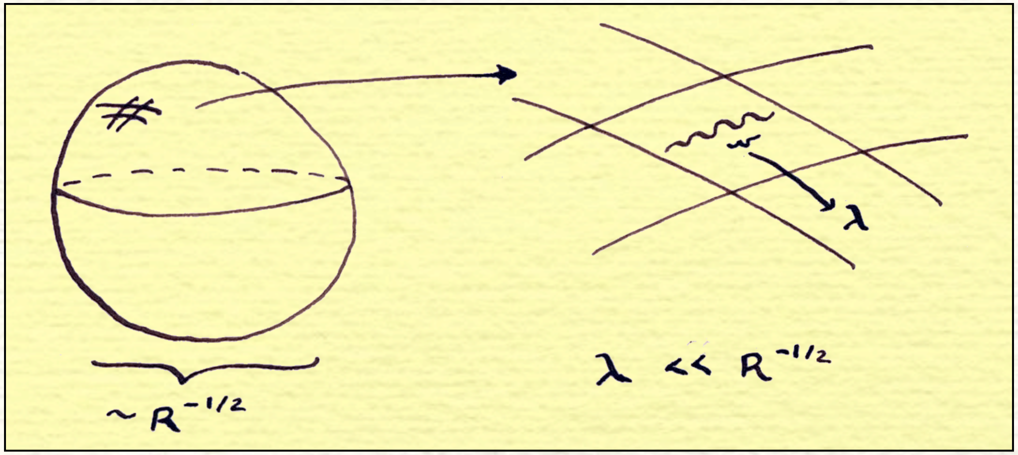
which raises several more questions:

What is a short distance?
When is a distance short?
Is space-time flat on small enough scales?

Quantum fluctuations

What is a short distance?

In *classical* general relativity, space-time looks flat when the curvature is small



$$\text{wavelength} << (\text{curvature})^{-1/2}$$

$$\lambda_{\text{phys}} \propto 1/k_{\text{phys}} \qquad R \propto H^2$$

A short-distance is one for which, $k_{\text{phys}} >> H$

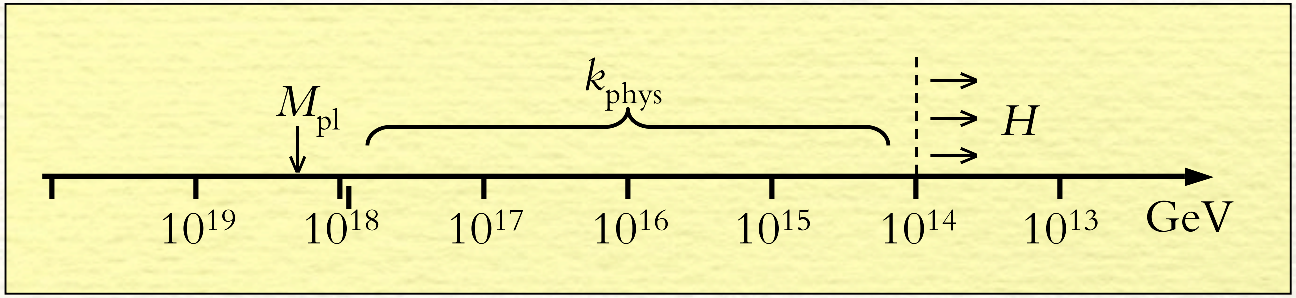
Quantum fluctuations

What range of k's are needed?

The microwave background covers about 3 orders of magnitude; the large scale structure covers even more

The inflationary energy scale is H

$$10^3 \text{ GeV} < H < 10^{14} \text{ GeV}$$

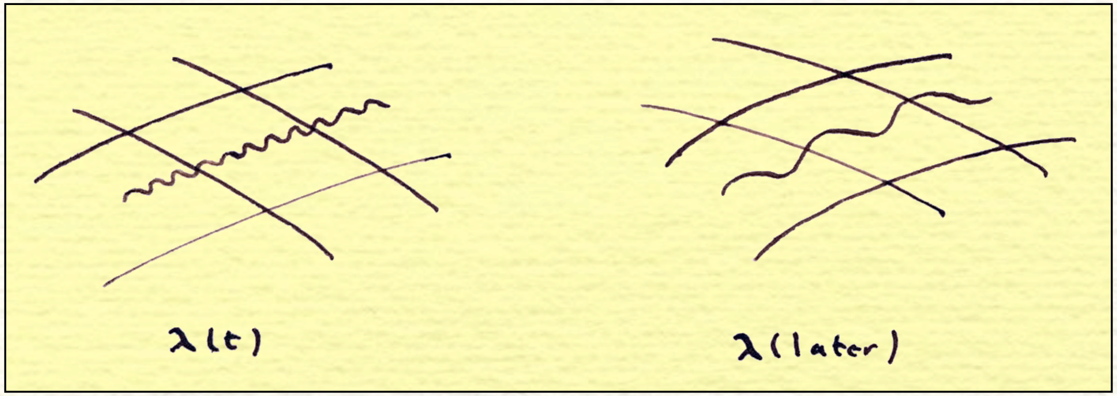


There is not much space before $k_{\text{phys}} > M_{\text{pl}}$

Quantum fluctuations

When is a distance short?

A physical wavelength, $\lambda_{\rm phys}(\eta)$, is not fixed, but is stretched by the expansion



$$\lambda_{\rm phys} = a(\eta)\lambda \quad \Rightarrow \quad k_{\rm phys} = k/a(\eta)$$

We must choose a sufficiently early time, such that

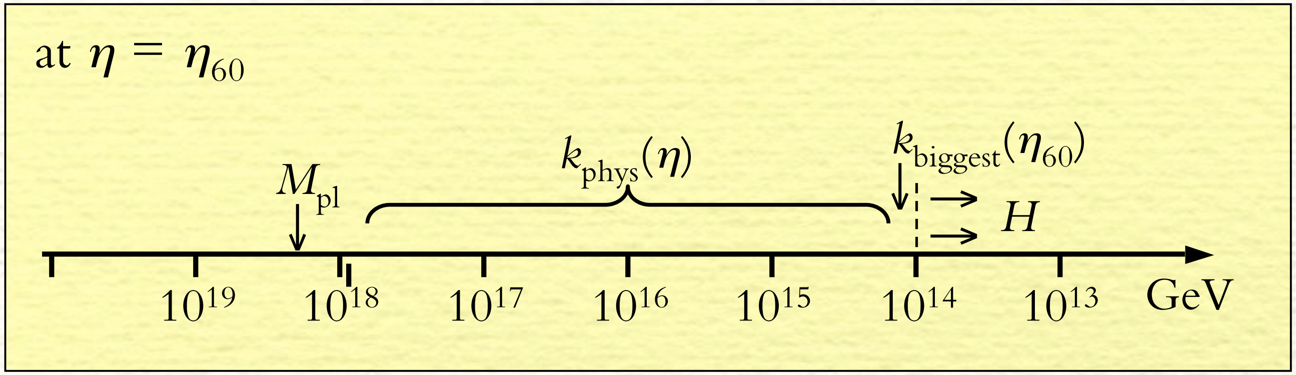
$$\boxed{k >> a(\eta)H(\eta)}$$

Quantum fluctuations

Can we choose an initial time too early?

(more than the 'just enough' of inflation)

let η_{60} be the time when a wavelength the size of the universe today (k_{biggest}) was just leaving the horizon during inflation



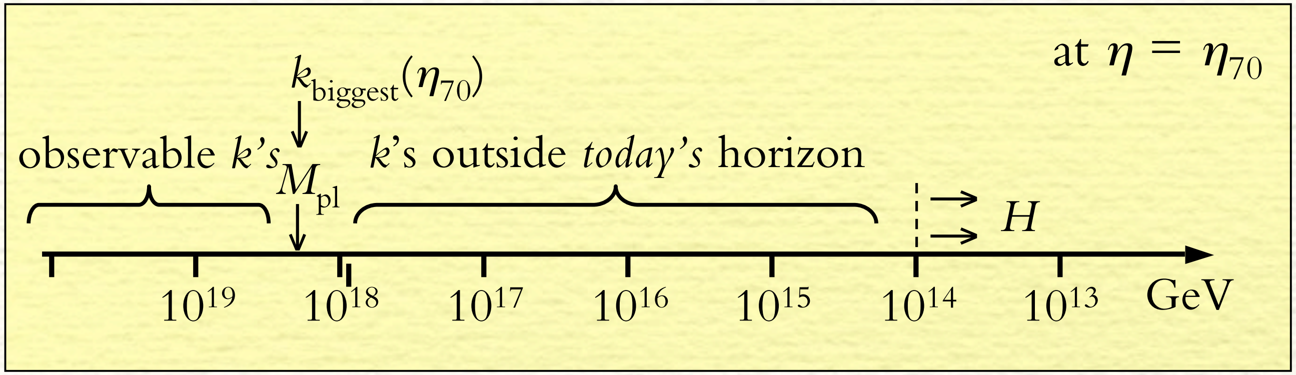
Now suppose that inflation lasted just 10 e-folds longer than the minimal amount needed

Quantum fluctuations

Can we choose an initial time too early?

(more than the 'just enough' of inflation)

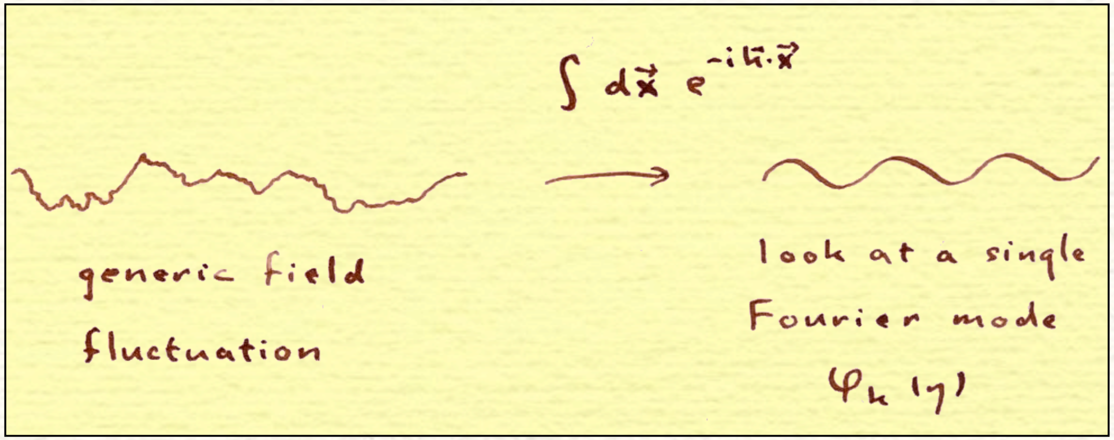
let η_{70} be 10 *e-folds* earlier than η_{60} (when a wavelength the size of the universe today was just leaving the horizon during inflation)



If we start earlier, more of the *observationally relevant* wavelengths were smaller than M_{pl}

How fluctuations evolve

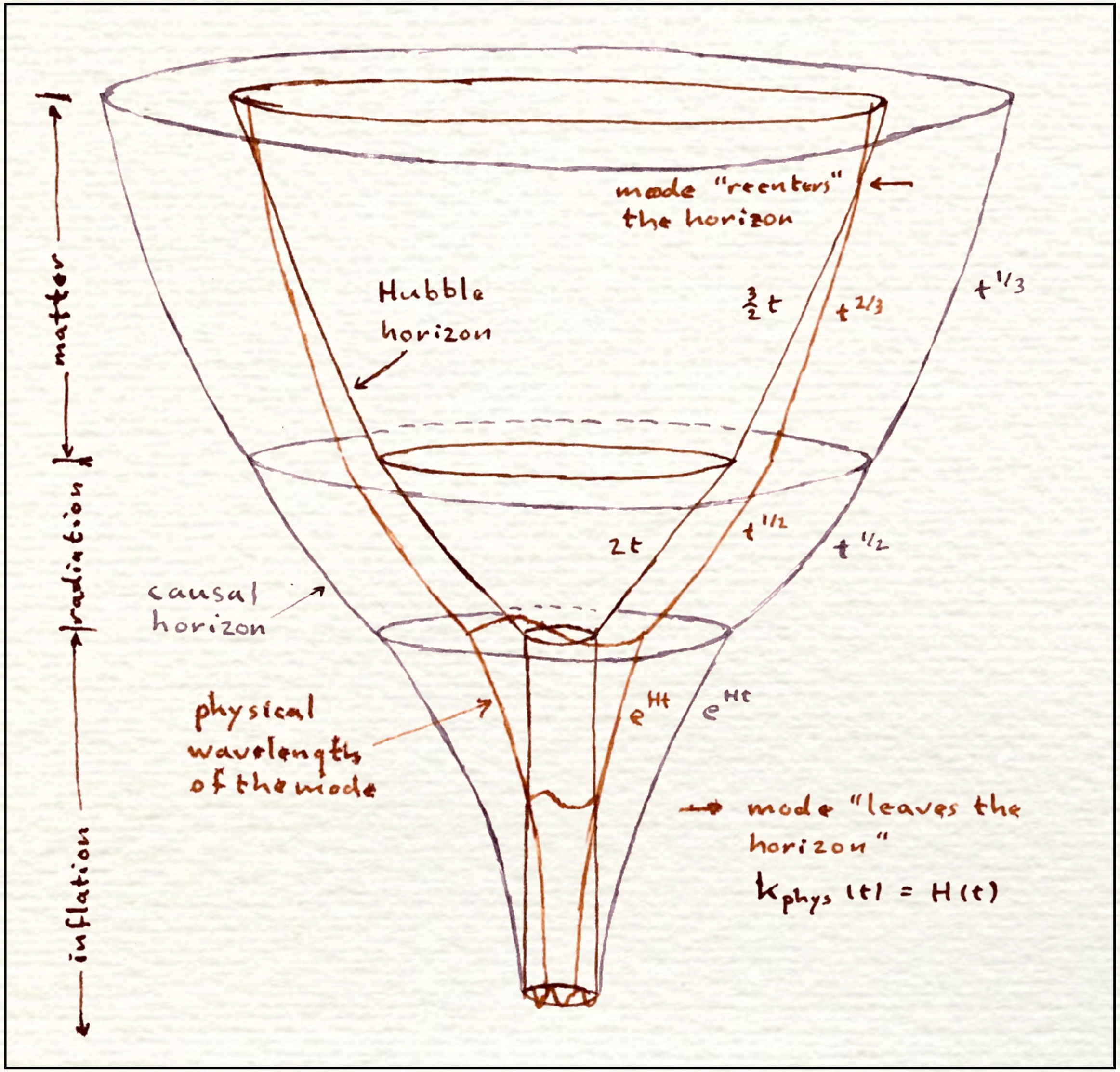
Let us follow a particular Fourier mode from the “beginning” until today



The physical wavelength is proportional to the scale factor, $\lambda_{\text{phys}}(t) = a(t)\lambda$

A space-time fluctuation can only influence material when it is <u>*inside*</u> the Hubble horizon

How fluctuations evolve



A ‘trans-Planckian’ problem

an even more unsettling question:

If we would like to cover many scales for structure formation and if there was a bit more than the minimal amount of inflation,

Then *some* of the fluctuations responsible for observations today had a wavelength *smaller than* $1/M_{\rm pl}$ during inflation

Do we need to understand quantum gravity before we can make sense of inflation?

or is it an opportunity?

Can quantum gravity influence inflationary predictions?

stringy uncertainty:	Easther, Greene, Kinney & Shiu, PRD **64**, 103502 (2001); PRD **67**, 063508 (2003)
modified kinetic:	G. Shiu & Wasserman, Phys. Lett. B **536**, 1 (2002)
UV cut-off:	Greene, Kinney & G. Shiu, PRD **66**, 023518 (2002); Niemeyer, PRD **63**, 123502 (2001); Kempf, PRD **63**, 083514 (2001); PRD **63**, 083514 (2001); Kempf & Niemeyer, PRD **64**, 103501 (2001); Kempf & Lorenz, PRD **74**, 103517 (2006)
minimum length:	Danielsson, PRD **66**, 023511 (2002); JHEP **0207**, 040 (2002)
modified dispersion:	Niemeyer & Parentani, PRD **64**, 101301 (2001); Brandenberger & Martin, Int. J. Mod. Phys. A **17**, 3663 (2002)

What can we say without a particular model?

"effective state" idea:	H. Collins & R. Holman, PRD **71**, 085009 (2005); hep-th/0507081; PRD **74**, 045009 (2006); hep-th/0609002
effective operators:	Schalm, Shiu & van der Schaar, JHEP **0404**, 076 (2004); AIP Conf. Proc. **743**, 362 (2005); Greene, Schalm, Shiu, & van der Schaar, JCAP **0502**, 001 (2005)
	H. Collins & R. Holman, PRD **77**, 105016 (2008); PRD **80**, 043524 (2009)

Or are we missing some important principle?

– is the quantum gravitational information forgotten or "washed out"?
– does some decoupling occur (appropriate for an *expanding* background)?

Questions rather than Conclusions

Has the trans-Planckian problem been resolved?

Certainly, we can make some assumptions about nature at scales shorter than a Planck length (even consistent ones), but must we do so and why should we trust them?

Most work has been to assess the size of observable effects for a set of trans-Planckian assumptions

However, this is not truly a solution to the problem, unless we can show that these are the correct assumptions for our universe (*a consistent theory is not necessarily the right one*)

Inflation only puts off the horizon problem

Is this satisfactory? Can we do better?

One last aside

Thinking about short-distance problems has almost always been very fruitful

A historical example: quantum field theory

Quantum field theory seemed to provide a sensible perturbative framework, except that its loop corrections were infinite

Resolving its problems led to
- the running of coupling "constants"
- the renormalization group
- effective field theories

the space-time expansion adds a new complication to this picture *the end*