Cosmic signals from the breaking of local Lorentz invariance

presented by

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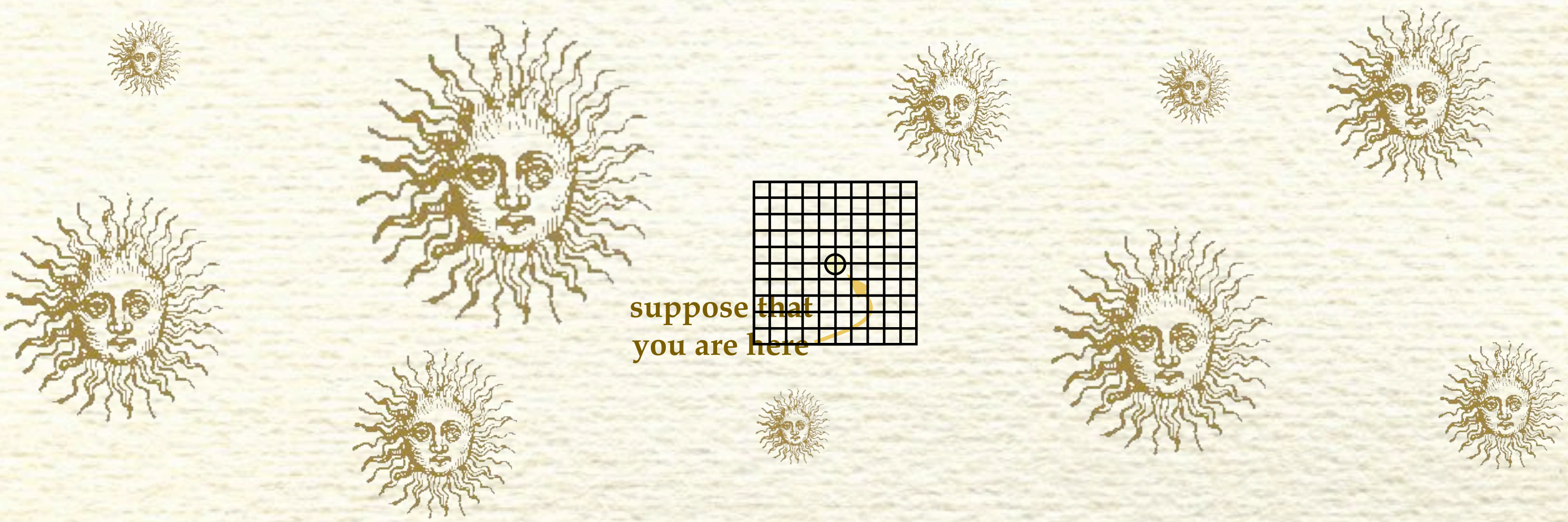
Local Lorentz invariance

We take for granted that it is always possible to treat the vicinity of a place and time as though it were free of the influence of gravity

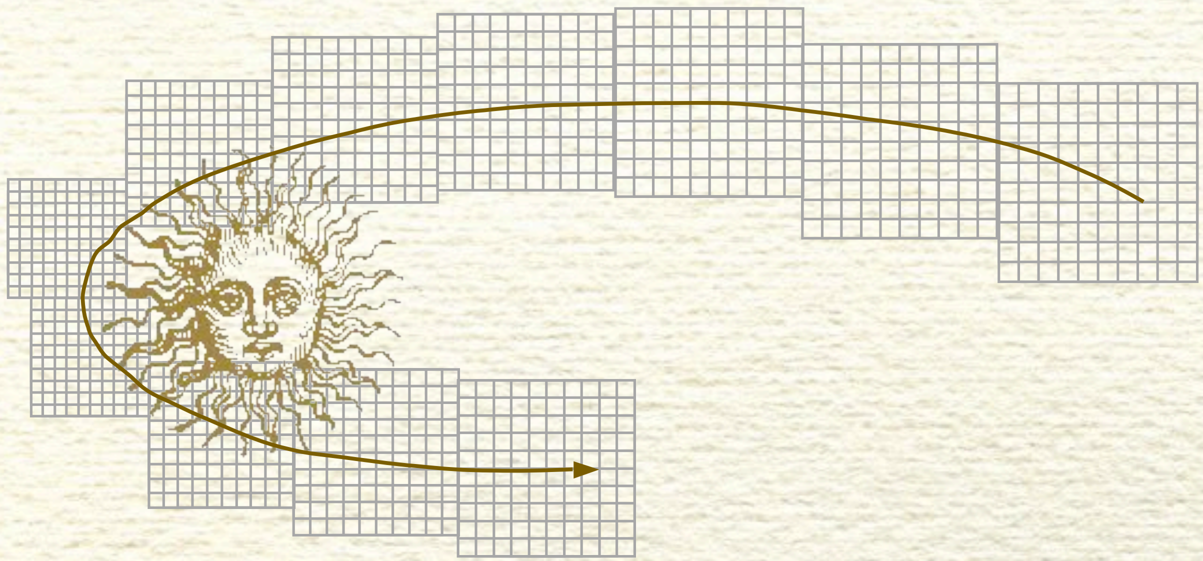


Physically: we can choose a locally Lorentzian frame

Mathematically: space-time has a manifold structure

Local Lorentz invariance

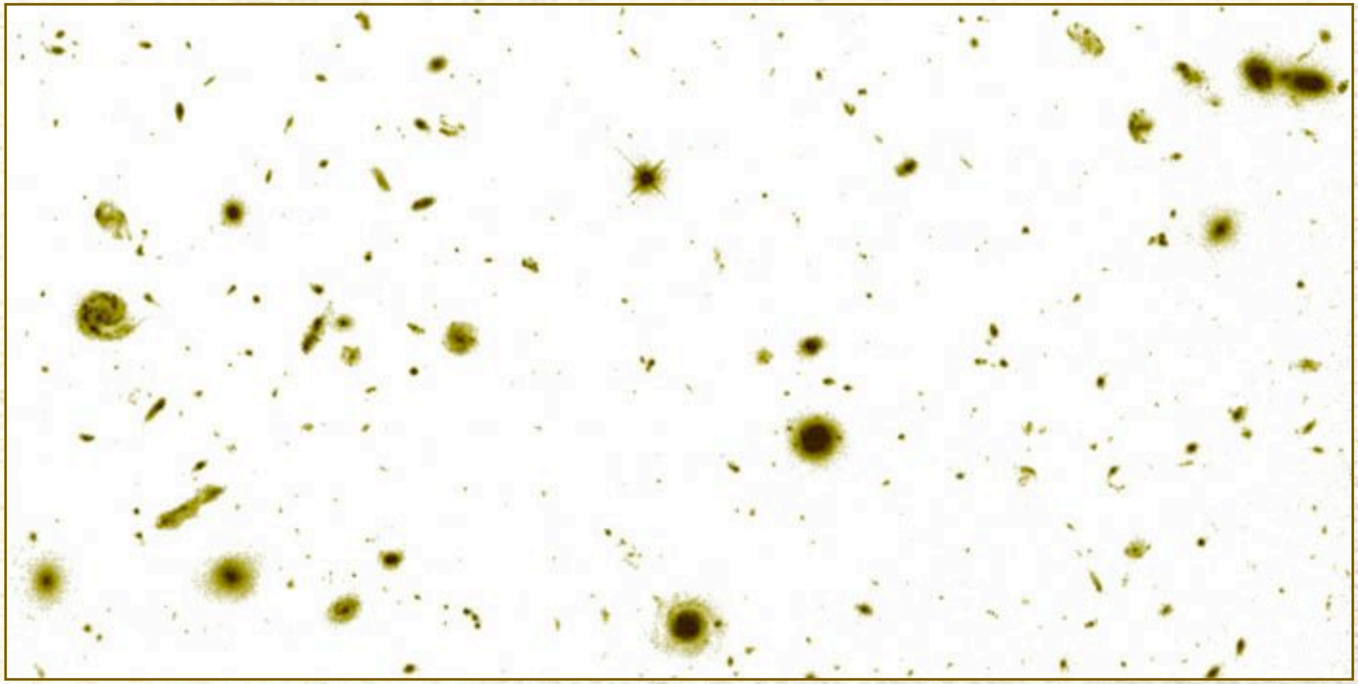
Looking from one place to another,
general relativity how
these flat frames all fit together



The theory works well from terrestrial distances
up to the size of the observable universe*

Troubles at large scales (?)

For the past five or so billion years the universe has been expanding at an *accelerating* rate

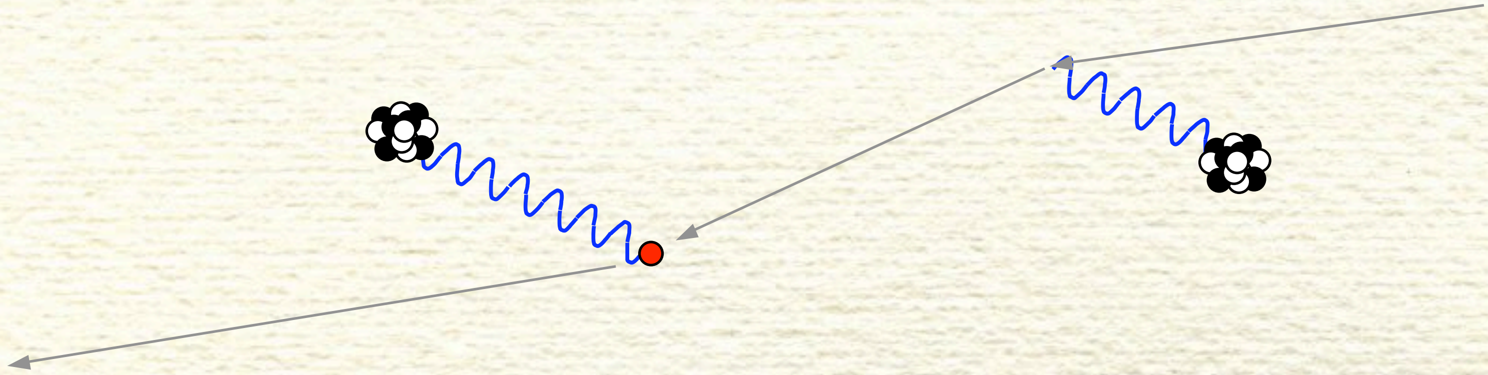


Is this the failure of our theoretical ideas or our understanding of the ingredients of the universe?

The opposite extreme

Quantum field theory is usually formulated in flat space

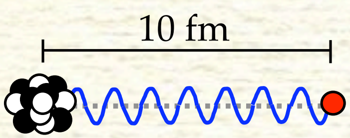
which is a good approximation for
most experimental settings



But what happens if we proceed to ever smaller distances?

The Planck threshold

At typical experimental scales, gravity is completely negligible compared to other interactions



$$V_{\mathrm{EM}} = 144\ \mathrm{keV}$$

$$V_{\mathrm{gravity}} = 76 \times 10^{-37}\ \mathrm{keV}$$

More importantly, gravitational *self-interactions* are small

But something interesting happens at distances smaller than a Planck length

$$\boxed{L_{\mathrm{pl}} = \sqrt{\frac{\hbar G_N}{c^3}} \approx 1.6 \times 10^{-33}\ \mathrm{cm}}$$

The Planck threshold

If we treat gravity as an effective quantum theory

$$S_G = \int d^4x \sqrt{-g}\left[2M_{\rm pl}^2\Lambda + M_{\rm pl}^2 R + \alpha R^2 + \beta R_{\mu\nu}R^{\mu\nu} + \cdots\right]$$

$$M_{\rm pl} = \sqrt{\frac{\hbar c}{G_N}} = \frac{1}{L_{\rm pl}}\frac{\hbar}{c} \approx 1.22\times 10^{19}\ \mathrm{GeV}c^{-2}$$

At Planck scales ($\Delta x \approx L_{\rm pl}$) it would be strongly interacting

$$R \approx \left(\frac{d}{dx}\right)^2 \quad \Rightarrow \quad R \approx \frac{1}{L_{\rm pl}^2} \approx M_{\rm pl}^2$$

that is, all the terms in the action are equally important

The Planck threshold

But gravity = the dynamics of space-time itself

So beyond the Planck scale, is it sensible to treat space-time as locally flat?

Two approaches:

I. Look for symmetry breaking effects at long distances

"High energy tests of Lorentz invariance" – Coleman & Glashow

add relevant local symmetry-breaking operators

II. Look for symmetry breaking effects at short distances

if decoupling holds, how can we possibly see such things?

inflation and the early universe

Overview:

§ Choosing a preferred frame

§ Inflation (and de Sitter space)

§ Corrections to the power spectrum

§ Comments and comparisons

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A preferred reference frame

At large scales and at early times, the universe appears highly isotropic and homogeneous

The metric for such a space-time can be written in a standard 'Robertson-Walker' form

$$ds^2 = g_{\mu\nu}\, dx^\mu dx^\nu = a^2(\eta)[d\eta^2 - d\mathbf{x}\cdot d\mathbf{x}]$$
$$= dt^2 - a^2(t)\, d\mathbf{x}\cdot d\mathbf{x}$$

$a(\eta)$ is the scale factor; the rate at which it changes defines

$$\text{Hubble scale} = H(\eta) = \frac{a'}{a^2} = \frac{1}{a^2}\frac{da}{d\eta} \quad \left(= \frac{1}{a}\frac{da}{dt}\right)$$

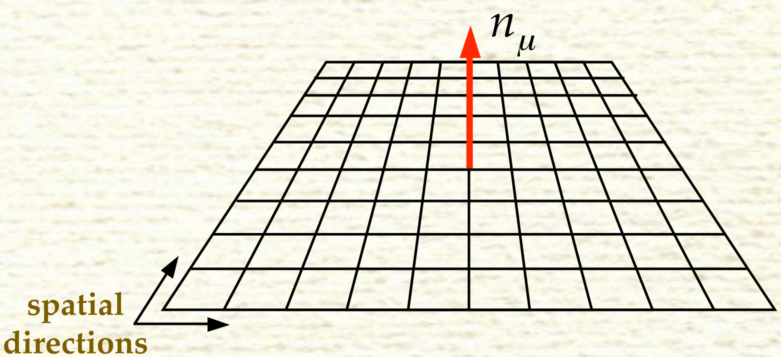
Let us use this metric to define a 'preferred frame'

Building symmetry-breaking operators

Our preferred frame as fewer symmetries (only six)

we have translations & rotations in every surface orthogonal to

$$n_\mu = \big(a(\eta),0,0,0\big)$$



the *induced metric* along these surfaces is

$$h_{\mu\nu} = g_{\mu\nu} - n_\mu n_\nu \quad \Rightarrow \quad h_{\mu\nu}\, dx^\mu dx^\nu = -a^2(\eta)\, d\mathbf{x}\cdot d\mathbf{x}$$

how these surfaces are embedded in the full space-time is encoded in the *extrinsic curvature*

$$K_{\mu\nu} = h_\mu^\lambda \nabla_\lambda n_\nu \quad \Rightarrow \quad K_{\mu\nu}\, dx^\mu dx^\nu = -a^2 H\, d\mathbf{x}\cdot d\mathbf{x}$$

Building symmetry-breaking operators

So, in addition to the usual tensors

$$g_{\mu\nu}, \nabla_\mu, R_{\lambda\mu\nu\sigma}, \ldots$$

we shall also use

$$n_\mu, h_{\mu\nu}, K_{\mu\nu}, \ldots$$

to construct the operators for our theory

thus, for example, we can build
dimension three operators

$$K\varphi^2$$

φ is a scalar field, e.g. the inflaton

A peculiar derivative operator

To model some signals from effective states,
we include one further ingredient

a peculiar one-derivative operator

$$D = \left(h^{\mu\nu}\nabla_\mu\nabla_\nu - Kn^\mu\nabla_\mu\right)^{1/2} \Rightarrow \frac{1}{a}\left(-\nabla\cdot\nabla\right)^{1/2}$$

In a momentum representation,
it just extracts a power of the momentum

$$D\varphi = \int \frac{d^3\mathbf{k}}{(2\pi)^3}\frac{k}{a}\left[U_k(\eta)e^{i\mathbf{k}\cdot\mathbf{x}}a_{\mathbf{k}} + U_k^*(\eta)e^{-i\mathbf{k}\cdot\mathbf{x}}a_{\mathbf{k}}^\dagger\right]$$

The leading irrelevant operators

For our (invariant) free theory, we take

$$L_{\mathrm{C}} = \tfrac{1}{2} g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi - \tfrac{1}{2} m^2 \varphi^2$$

consider a fairly general set of dimension five operators that are quadratic in φ (and assuming $H' << H^2$)

$$L_{\mathrm{NR}} = \frac{d_1}{27}\frac{1}{M} K^3 \varphi^2 + \frac{d_2}{9}\frac{1}{M} K^2 \varphi D \varphi - \frac{d_3}{3}\frac{1}{M} K h^{\mu\nu} \nabla_\mu \varphi \nabla_\nu \varphi + \frac{d_4}{M} \varphi D^3 \varphi$$

M is a scale where the symmetry breaking occurs

The leading irrelevant operators

The free theory

$$L_{\mathrm{C}} = \tfrac{1}{2} g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi - \tfrac{1}{2} m^2 \varphi^2$$

The interacting part

$$L_{\mathrm{NR}} = \frac{d_1}{M} H^3 \varphi^2 + \frac{d_2}{aM} H^2 \varphi (-\nabla \cdot \nabla)^{1/2} \varphi + \frac{d_3}{a^2 M} H \nabla \varphi \cdot \nabla \varphi + \frac{d_4}{a^3 M} \varphi (-\nabla \cdot \nabla)^{3/2} \varphi$$

Goals:

I. test the limits of preferred frame effects during inflation

II. mimic signals from effective vacuum states

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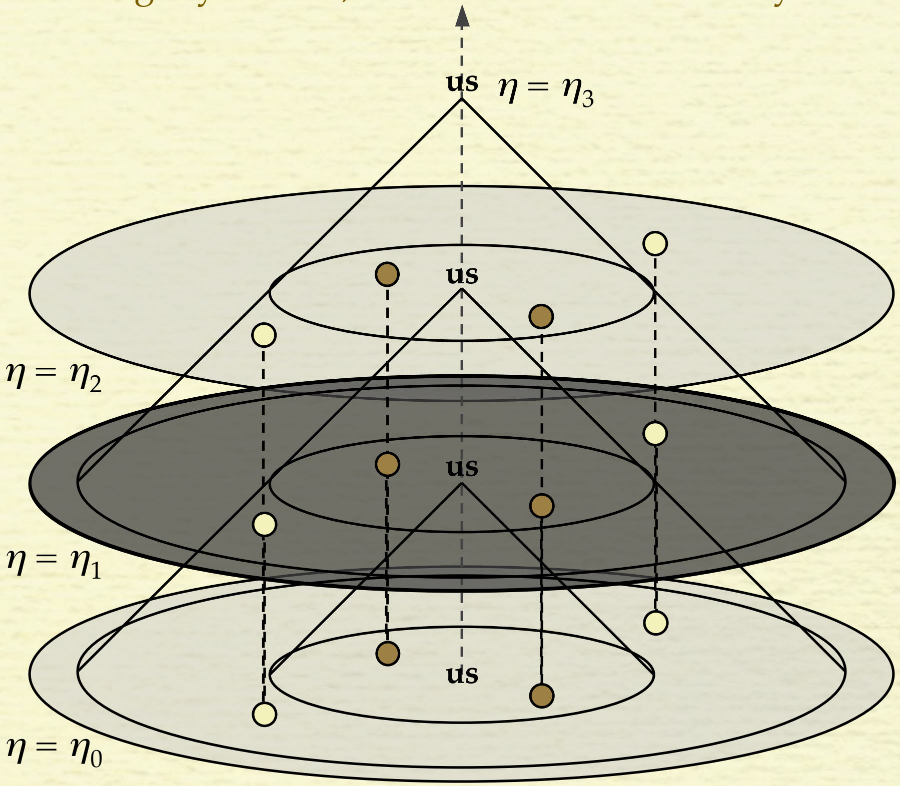
What inflation accomplishes

Two types of universes

In flat space, $a(\eta) \to 1$ $\quad ds^2 = d\eta^2 - d\mathbf{x} \cdot d\mathbf{x}$

the longer you wait, the more of the universe you see



In a matter or radiation-dominated universe, over time, we see farther and farther

how far we see defines a 'horizon'

things only can enter our horizon

so why does the early universe look so uniform?

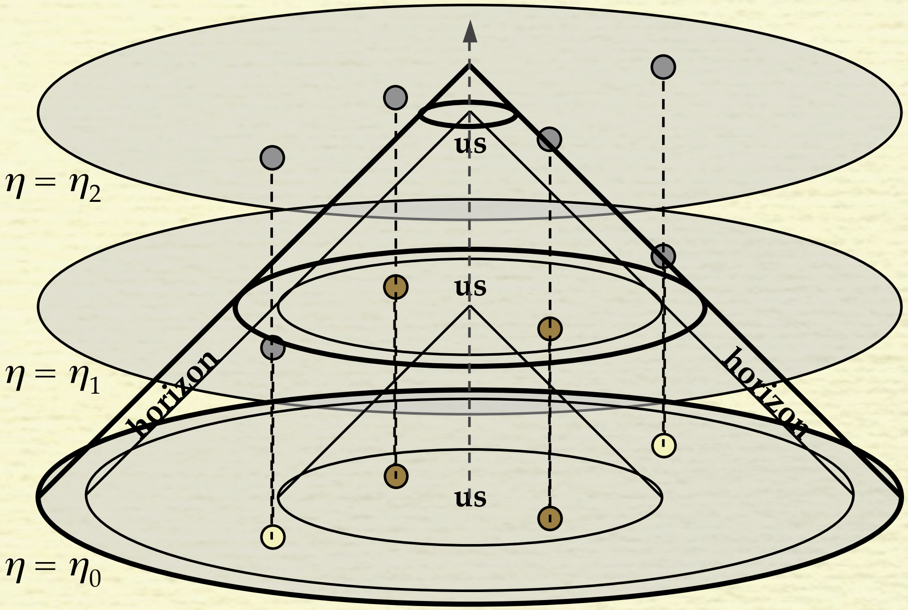
What inflation accomplishes

Two types of universes

In an inflating space, $ds^2 = a^2(\eta)[d\eta^2 - d\mathbf{x}\cdot d\mathbf{x}]$

the longer you wait, the *less* universe you see

In co-moving coordinates [note $\eta = -\infty, \ldots, 0$]



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Inflation is a mechanism for hiding stuff behind the horizon

- requires a stage of accelerating expansion
- things can now leave the horizon
- once something has left, we shall never see it again as long as the inflation lasts

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Two types of universes

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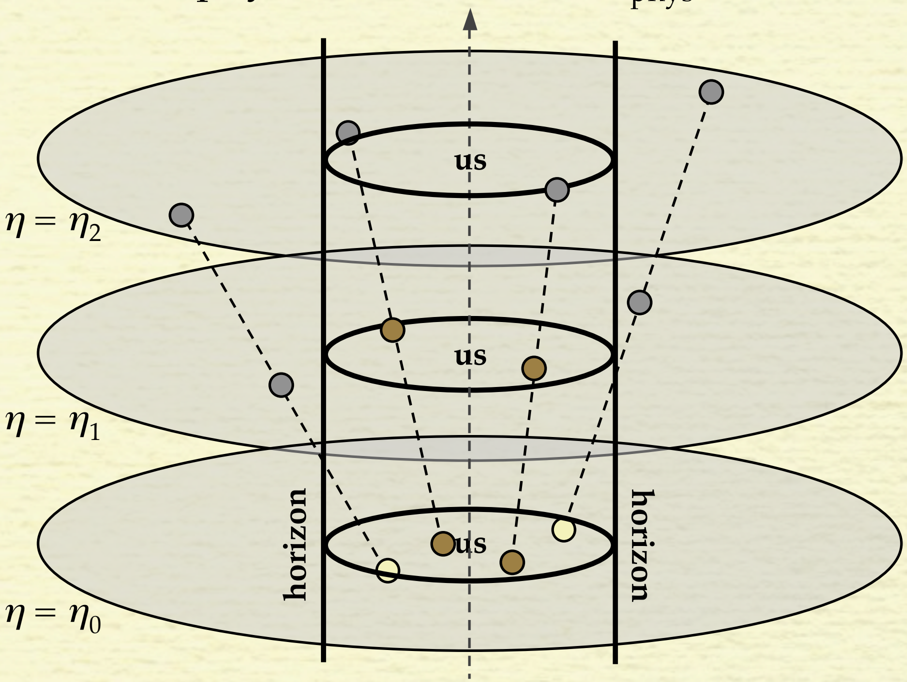
the longer you wait, the less universe you see

In physical coordinates $[\mathbf{x}_{\text{phys}} = a\mathbf{x}]$



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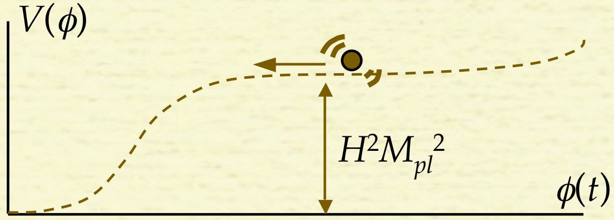
Ingredients for inflation

Implementing inflation



divide the field into classical and quantum parts

$$\Phi(\eta,\mathbf{x}) = \phi(\eta) + \varphi(\eta,\mathbf{x})$$

$\phi(\eta) =$ classical zero mode
$\varphi(\eta,x) =$ quantum fluctuation

the Klein-Gordon equation

$$[\nabla^2 + m^2]\varphi = 0$$

An enormous variety of models with a few common elements

In a typical inflationary model

One (or more) scalar field—the inflaton

Slowly rolling down a nearly flat potential

Nearly constant vacuum energy $\quad \dfrac{d^2a}{dt^2} > 0$

At the end of inflation (reheating)

Vacuum energy $\to$ 0 (almost)

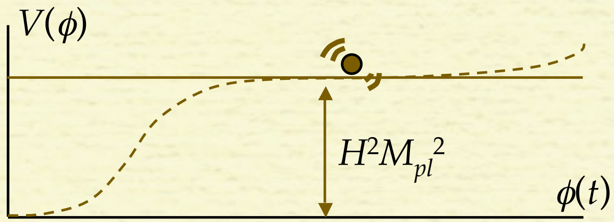
Inflaton decays to other fields

For simplicity, we shall consider the de Sitter limit

Constant vacuum energy density $[H(\eta) \to H]$

de Sitter space

the de Sitter space limit



$$\varphi(\eta, x) = \int \frac{d^3\mathbf{k}}{(2\pi)^3}\Big[U_k(\eta)e^{i\mathbf{k}\cdot\mathbf{x}}a_{\mathbf{k}} + U_k^*(\eta)e^{-i\mathbf{k}\cdot\mathbf{x}}a_{\mathbf{k}}^\dagger\Big]$$

the de Sitter scale factor is

$$a(\eta) = -\frac{1}{H\eta}$$

and the metric becomes

$$ds^2 = a^2(\eta)[d\eta^2 - d\mathbf{x}\cdot d\mathbf{x}] = \frac{d\eta^2 - d\mathbf{x}\cdot d\mathbf{x}}{H^2\eta^2}$$

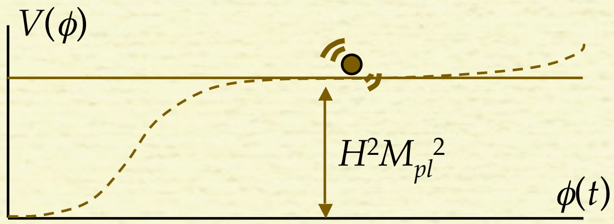
Choose the “no-roll” limit—a space-time with constant vacuum energy (de Sitter space)

The Klein-Gordon equation becomes

$$U_k'' - \frac{2}{\eta}U_k' + \left(k^2 + \frac{1}{\eta^2}\frac{m^2}{H^2}\right)U_k = 0$$

de Sitter space

the de Sitter space limit



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Change variables ($z = k\eta$) and rescale ($U_k = \eta^{3/2} Z_\nu$)

$$\frac{d^2 Z_\nu}{dz^2} + \frac{1}{z}\frac{dZ_\nu}{dz} + \left(1 - \frac{\nu^2}{z^2}\right)Z_\nu = 0$$

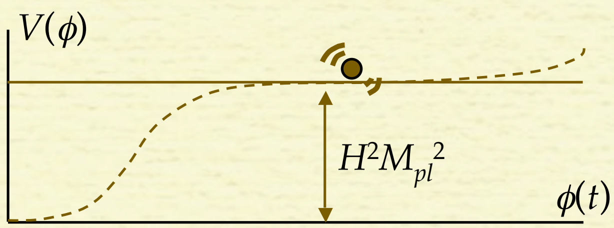
$$\nu^2 = \frac{9}{4} - \frac{m^2}{H^2}$$

which is Bessel's equation

$$U_k(\eta) = \eta^{3/2}\left[\alpha H_\nu^{(2)}(k\eta) + \beta H_\nu^{(1)}(k\eta)\right]$$

de Sitter space

the de Sitter space limit



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The Klein-Gordon equation becomes

$$U_k'' - \frac{2}{\eta}U_k' + \left(k^2 + \frac{1}{\eta^2}\frac{m^2}{H^2}\right)U_k = 0$$

Choose the standard "vacuum" state solution

$$U_k(\eta) = \frac{\sqrt{\pi}}{2}H\eta^{3/2}H_\nu^{(2)}(k\eta)$$

$$\nu^2 = \frac{9}{4} - \frac{m^2}{H^2}$$

In the massless limit ($\nu = 3/2$)

$$U_k(\eta) = \frac{H}{k\sqrt{2k}}(i - k\eta)e^{-ik\eta}$$

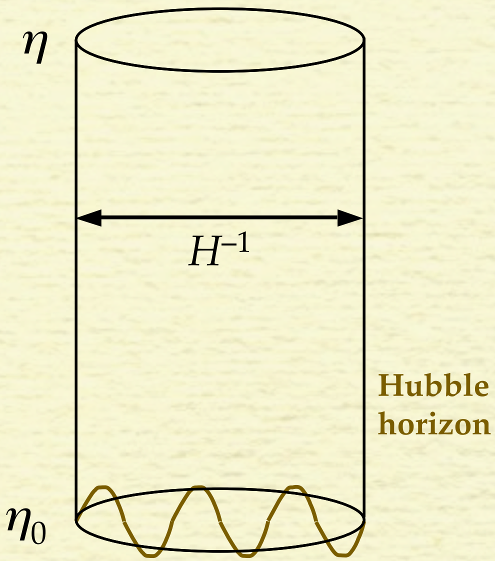
de Sitter space—limits

Inflationary redshifting

scale factor (de Sitter space)

$$a(\eta) = -\frac{1}{H\eta}$$

which are the interesting modes?



At the beginning of inflation, the interesting modes should be smaller than the horizon,

$$\frac{k}{|a(\eta_0)|} > H$$

$$\frac{k}{|a(\eta_0)|} = k\cancel{H}|\eta_0| > \cancel{H} \quad \Rightarrow \quad |k\eta_0| > 1$$

$$k\eta_0 \to -\infty$$

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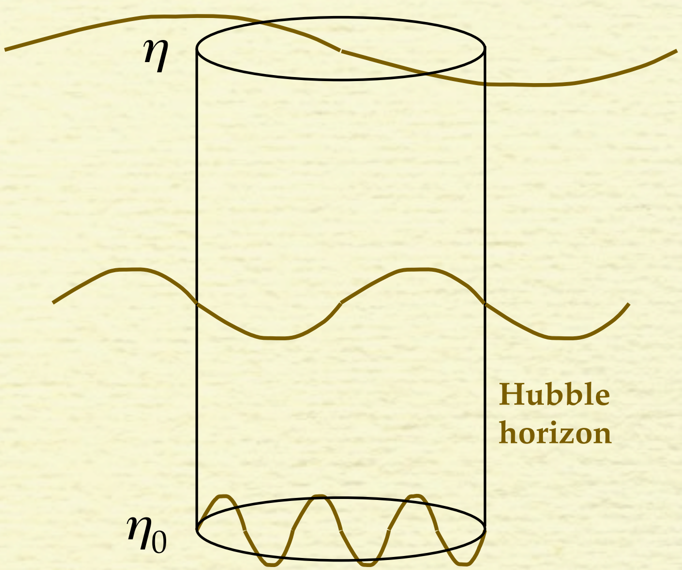

At the beginning of inflation, the interesting modes should be smaller than the horizon,

$$k\eta_0 \to -\infty$$

But by the end, they should be much larger than the horizon,

$$\frac{k}{|a(\eta)|} << H$$

$$\frac{k}{|a(\eta)|} = k\cancel{H}|\eta| << \cancel{H} \quad \Rightarrow \quad |k\eta| << 1$$

$$k\eta \to 0$$

de Sitter space—limits

Inflationary redshifting

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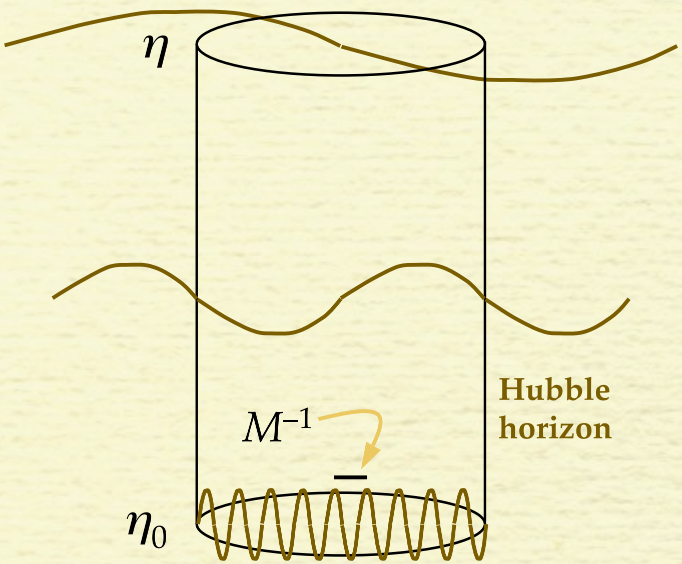

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$$k\eta_0 \to -\infty$$

But by the end, they should be much larger than the horizon,

$$k\eta \to 0$$

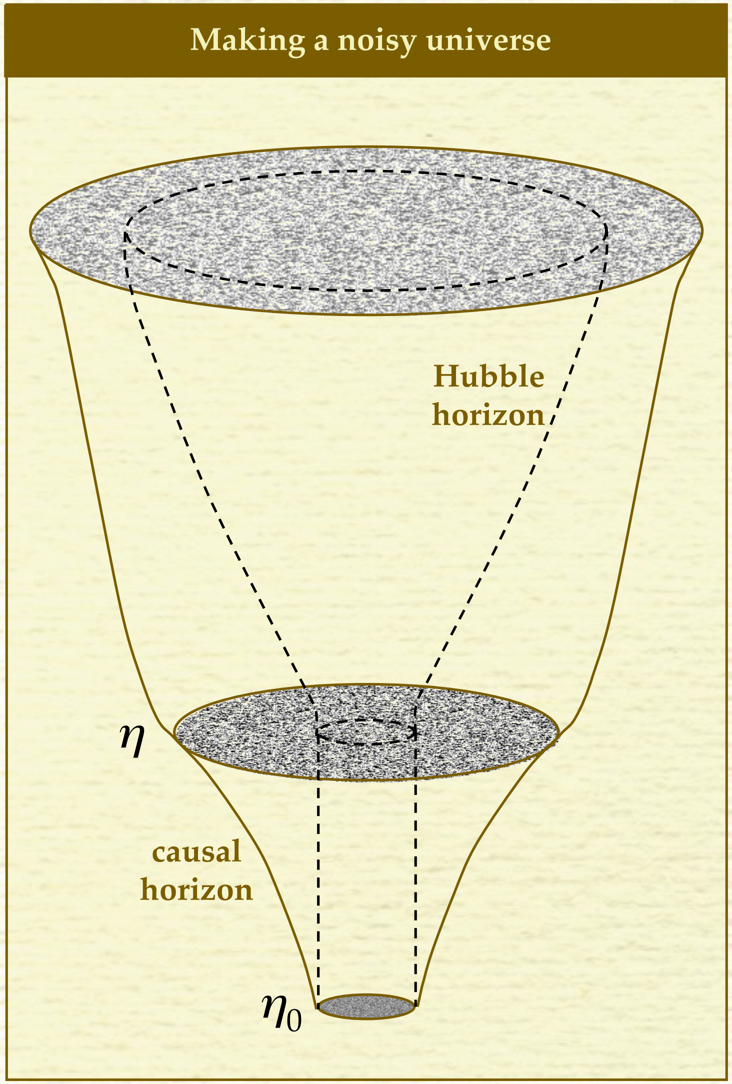
Define a threshold wave number, k_*,

$$\frac{k_*}{|a(\eta_0)|} = M$$

$$\eta_0 = -\frac{1}{k_*}\frac{M}{H}$$

"Trans-Planckian" modes: $k > k_*$

Making a lot of noise



Inflation converts

quantum noise inside a causal patch

into

classical noise spread throughout space-time

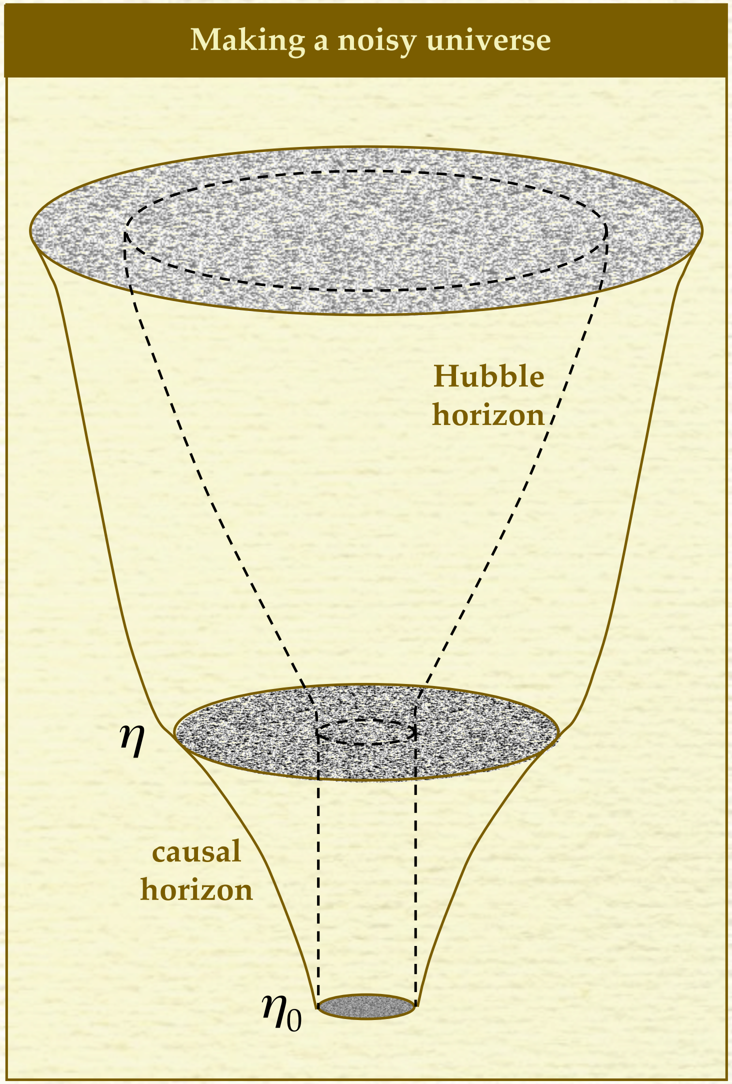
The simplest measure of this noise is its two-point function

$$\left\langle 0(\eta)\middle|\varphi(\eta,\mathbf{x})\varphi(\eta,\mathbf{y})\middle|0(\eta)\right\rangle = \int \frac{d^3\mathbf{k}}{(2\pi)^3} e^{i\mathbf{k}\cdot(\mathbf{x}-\mathbf{y})} \frac{2\pi^2}{k^3} P_k(\eta)$$

or its "power spectrum" $P_k(\eta)$

(of course, there are higher moments or n-point functions too)

Making a lot of noise



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or its "power spectrum" $P_k(\eta)$

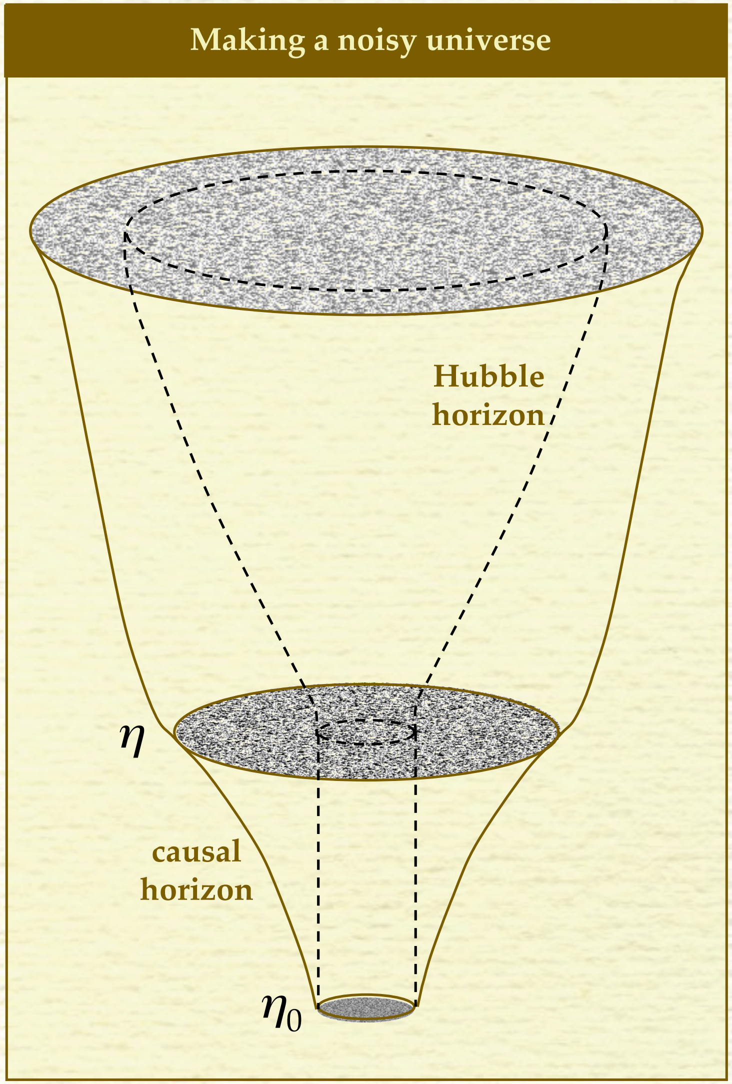
To see this noise we fold it into something we can measure, *e.g.* the microwave background

$$c_l = \int \frac{d^3\mathbf{k}}{(2\pi)^3} P_k(\eta) \times [T_l^{\text{CMB}}(k)]^2$$

a "transfer function"

or the distributions of galaxies, etc.

Making a lot of noise



The simplest measure of this noise is its two-point function

$$\langle 0(\eta) | \varphi(\eta,\mathbf{x})\varphi(\eta,\mathbf{y}) | 0(\eta) \rangle$$
$$= \int \frac{d^3\mathbf{k}}{(2\pi)^3} e^{i\mathbf{k}\cdot(\mathbf{x}-\mathbf{y})} \frac{2\pi^2}{k^3} P_k(\eta)$$

or its "power spectrum" $P_k(\eta)$

As a 0th order result (for later comparisons)

$$P_k(\eta) = \frac{k^3}{2\pi^2} U_k(\eta) U_k^*(\eta)$$

In de Sitter space (and the $k\eta \to 0$ limit)

$$P_k(\eta) = \frac{k^3}{2\pi^2} \left| \frac{H}{k\sqrt{2k}} (i - k\eta) e^{-ik\eta} \right|^2$$
$$= \frac{H^2}{4\pi^2}(1 + k^2\eta^2) \to \frac{H^2}{4\pi^2}$$

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The effects of non-invariant operators

Now let us calculate the corrections from the irrelevant operators

$$L_{\mathrm{NR}} = \frac{d_1}{M} H^3 \varphi^2 + \frac{d_2}{aM} H^2 \varphi (-\nabla\cdot\nabla)^{1/2} \varphi + \frac{d_3}{a^2 M} H \nabla\varphi\cdot\nabla\varphi + \frac{d_4}{a^3 M} \varphi(-\nabla\cdot\nabla)^{3/2}\varphi$$

to the two-point function, or power spectrum

$$\langle 0(\eta) | \varphi(\eta,\mathbf{x})\varphi(\eta,\mathbf{y}) | 0(\eta)\rangle = \int \frac{d^3\mathbf{k}}{(2\pi)^3} e^{i\mathbf{k}\cdot(\mathbf{x}-\mathbf{y})} \frac{2\pi^2}{k^3} P_k(\eta)$$

The effects of non-invariant operators

Time-evolve the state from the initial time

$$|0(\eta)\rangle = T e^{-i\int_{\eta_0}^{\eta} d\eta'\, H_I(\eta')}|0\rangle \qquad \left(|0\rangle = |0(\eta_0)\rangle\right)$$

where the interaction Hamiltonian, H_I,

$$H_I(\eta) = -\int d^3\mathbf{x}\sqrt{-g}\,L_{\mathrm{NR}}$$

is given by the new non-renormalizable operators

$$L_{\mathrm{NR}} = \frac{d_1}{M}H^3\varphi^2 + \frac{d_2}{aM}H^2\varphi(-\nabla\cdot\nabla)^{1/2}\varphi + \frac{d_3}{a^2M}H\nabla\varphi\cdot\nabla\varphi + \frac{d_4}{a^3M}\varphi(-\nabla\cdot\nabla)^{3/2}\varphi$$

The effects of non-invariant operators

The result —

$$P_k(\eta) = \frac{H^2}{4\pi^2}[1+k^2\eta^2] + \frac{H^2}{4\pi^2}\frac{H}{M}[d_1 I_4(k\eta,k\eta_0) - d_2 I_3(k\eta,k\eta_0) - d_3 I_2(k\eta,k\eta_0) - d_4 I_1(k\eta,k\eta_0)]$$

where $I_n(z,z_0)$ denotes a family of dimensionless integrals

$$I_n(z,z_0) = \int_{z_0}^{z}\frac{dz'}{z'^n}\Big[[1-z^2+4zz'-z'^2+z^2z'^2]\sin[2(z-z')] - 2(z-z')[1+zz']\cos[2(z-z')]\Big]$$

Remember that the interesting limit is where

$$k\eta \to 0 \text{ and } k\eta_0 \to -\infty$$

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Modifying the vacuum state

Minkowski vacuum modes

the flat space massless Klein-Gordon equation

$$\left(\frac{d^2}{dt^2}+k^2\right)U_k^{\text{flat}}(t)=0$$

flat space vacuum modes

$$\boxed{U_k^{\text{flat}}(t)=\frac{e^{-ikt}}{\sqrt{2k}}}$$

Switch to other de Sitter coordinates

$$-H\eta=e^{-Ht}$$

$$ds^2=dt^2-e^{2Ht}\,d\mathbf{x}\cdot d\mathbf{x}$$

Even in a curved space, assuming that space-time looks flat locally strongly constrains the state

Look again at a *general solution* to the massless Klein-Gordon equation in de Sitter space

$$\varphi_k(\eta)=\frac{1}{\sqrt{1-f_kf_k^*}}\frac{H}{k\sqrt{2k}}\times\left[(i-k\eta)e^{-ik\eta}-f_k(i+k\eta)e^{ik\eta}\right]$$

At very short distances, $k/H\to\infty$,

$$\varphi_k(t)\approx\frac{1}{\sqrt{1-f_kf_k^*}}\frac{e^{-Ht}}{\sqrt{2k}}\left[e^{i\frac{k}{H}\exp(-Ht)}+f_ke^{-i\frac{k}{H}\exp(-Ht)}\right]$$

and very short intervals, $Ht\to 0$,

$$\varphi_k(t)\approx\frac{1}{\sqrt{1-f_kf_k^*}}\left[e^{ik/H}\boxed{\frac{e^{-ikt}}{\sqrt{2k}}}+f_ke^{-ik/H}\frac{e^{ikt}}{\sqrt{2k}}\right]$$

A somewhat disturbing logic

We define the vacuum state by appealing to
its behavior at short distances

that is, short compared to the curvature scale, H,

$H \quad << \quad k \qquad M_{\mathrm{pl}}$

$< 10^{14}$ GeV $\qquad 10^{19}$ GeV

So, we are defining the (quantum) vacuum using
a locally flat limit near distances where the idea of
a classical geometry may even not make sense

Moreover, the background constantly and
dramatically red-shifts scales

An effective vacuum state

So, perhaps we should allow more general states, once $k > M$ ($= M_{\rm pl}$?)

$$\varphi_k(\eta) \approx \frac{1}{\sqrt{1 - f_k f_k^*}} \left[U_k(\eta) + f_k U_k^*(\eta) \right]$$

the standard "vacuum" modes

A few requirements

1. $f_k \to 0$ as $k \to 0$

2. the new state should be renormalizable

3. theory is perturbative at large scales

This is the idea behind an effective vacuum state

Two approaches (and their predictions)

Effective state signal

$$P_k(\eta) = \frac{H^2}{4\pi^2}\left\{1 + O(1)\frac{k}{k_*}\sin\left(2\frac{k}{k_*}\frac{M}{H}\right)\right\}$$

is the power spectrum from

$$f_k \approx O(1) \times \frac{k}{M}$$

(any mode $k > k_*$ is "trans-Planckian")

A 'minimal length' signal

$$P_k(\eta) = \frac{H^2}{4\pi^2}\left\{1 - O(1)\frac{H}{M}\cos\left(2\frac{M}{H} + \theta\right)\right\}$$

a phase

when the slow-roll parameters are set to zero (de Sitter); more realistically,

$$H \to H(k)$$

Effective vacuum states (just described)

- new structure in the modes
- boundary operators
- boundary renormalization

Requires initial data in the propagator (?)

Particular models (a top-down approach)

- some defined mode-by-mode
- some defined on a time-like surface

Are these approaches renormalizable?

Can symmetry-breaking operators mimic either of these signals?

Typical 'modified vacuum' signals

Representative trans-Planckian signals
for a modified vacuum states

Both signals have some 'ringing'

1. a growing, amplitude and frequency
(eventually becomes non-perturbative)

$$P_k(\eta) = \frac{H^2}{4\pi^2}\left\{1 + O(1)\frac{k}{k_*}\sin\left(2\frac{k}{k_*}\frac{M}{H}\right)\right\}$$

2. an inversely correlated amplitude and frequency

$$P_k(\eta) = \frac{H^2}{4\pi^2}\left\{1 - O(1)\frac{H}{M}\cos\left(2\frac{M}{H} + \phi\right)\right\}$$

Overview:

- § Choosing a preferred frame
- § Inflation (and de Sitter space)
- § Corrections to the power spectrum
- § Comments and comparisons

1. The correction from $K^3\varphi^2 \to H^3\varphi^2$

What is the effect on the power spectrum from the operator

$$L_{NR} = \frac{d_1}{27}\frac{1}{M}K^3\varphi^2 + \cdots$$

for the physically interesting modes ($k\eta \to 0$ and $k\eta_0 \to -\infty$)

The corrected power spectrum

$$P_k(\eta) = \frac{H^2}{4\pi^2}\left\{1 + \frac{4}{3}d_1\frac{H}{M}\left[\ln(2k\eta) - 2 + \gamma\right] + \cdots\right\}$$

<u>Comments</u>:

The logarithm shows some long-distance sensitivity

The divergence should be tamed as we let $m \neq 0$

This signal is distinct from altered vacuum states

2. The correction from $K^2\varphi D\varphi \to H^2\varphi(-\nabla\cdot\nabla)^{1/2}\varphi$

What is the effect on the power spectrum from the operator

$$L_{NR} = \frac{d_2}{9}\frac{1}{M}K^2\varphi D\varphi + \cdots$$

for the physically interesting modes ($k\eta \to 0$ and $k\eta_0 \to -\infty$)

The corrected power spectrum

$$P_k(\eta) = \frac{H^2}{4\pi^2}\left\{1 + \pi d_2 \frac{H}{M} + \cdots\right\}$$

Comments:

The H/M correction is *not* accompanied by any ringing

Including a spatial derivative does not automatically produce $k \to \infty$ divergences

3. The correction from $K\, h^{\mu\nu}\, \nabla_\mu\varphi\, \nabla_\nu\varphi \to H\, \nabla\varphi \cdot \nabla\varphi$

What is the effect on the power spectrum from the operator

$$L_{NR} = \frac{d_3}{3}\frac{1}{M} K h^{\mu\nu} \nabla_\mu \varphi \nabla_\nu \varphi + \cdots$$

for the physically interesting modes ($k\eta \to 0$ and $k\eta_0 \to -\infty$)

The corrected power spectrum

$$P_k(\eta) = \frac{H^2}{4\pi^2}\left\{1 + d_3 \frac{H}{M}\left[3 + \cos\left(2\frac{k}{k_*}\frac{M}{H}\right)\right] + \cdots\right\}$$

Comments:

Again, the usual H/M suppression

However, the ringing is *not* inversely correlated
with the amplitude

4. The correction from $\varphi D^3\varphi \to \varphi(-\nabla\cdot\nabla)^{3/2}\varphi$

What is the effect on the power spectrum from the operator

$$L_{NR} = d_4 \frac{1}{M}\varphi D^3\varphi + \cdots$$

for the physically interesting modes ($k\eta \to 0$ and $k\eta_0 \to -\infty$)

The corrected power spectrum

$$P_k(\eta) = \frac{H^2}{4\pi^2}\left\{1 - d_4\frac{k}{k_*}\cos\left(2\frac{k}{k_*}\frac{M}{H}\right) + \cdots\right\}$$

<u>Comments</u>:

This is essentially the same sort of signal as arose from an effective vacuum state

Note that k_* here is really associated with the beginning of inflation (unlike the effective state)

Overview:

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Higher order operators

There is nothing special about the derivative operator D

For example the dimension six operator

$$\frac{d_5}{M^2}\frac{(\nabla\varphi\cdot\nabla\varphi)^2}{a^4}$$

also produces terms that become non-perturbative
for trans-Planckian modes

$$\boxed{P_k(\eta)=\frac{H^2}{4\pi^2}\left\{1-d_5\frac{k^2}{k_*^2}\cos\left(2\frac{k}{k_*}\frac{M}{H}\right)+\cdots\right\}}$$

A summary of what we have found

A fairly general set of dimension five operators that violate local Lorentz invariance produce a variety of signatures

no ringing ringing

$$P_k(\eta) = \frac{H^2}{4\pi^2}\left\{1 + \frac{H}{M}\left[\frac{4}{3}d_1\left[\ln(2k\eta) - 2 + \gamma\right] + \pi d_2 + d_3\left[3 + \cos\left(2\frac{k}{k_*}\frac{M}{H}\right)\right]\right] - d_4\frac{k}{k_*}\cos\left(2\frac{k}{k_*}\frac{M}{H}\right)\right\}$$

H/*M* suppression trans-Planckian divergence

Some of these signals look very much like those of an effective state (to a degree, a state does break local Lorentz invariance)

$$P_k(\eta) = \frac{H^2}{4\pi^2}\left\{1 + O(1)\frac{k}{k_*}\sin\left(2\frac{k}{k_*}\frac{M}{H}\right)\right\}$$

Others are not quite reproduced

$$P_k(\eta) = \frac{H^2}{4\pi^2}\left\{1 - O(1)\frac{H}{M}\cos\left(2\frac{M}{H} + \phi\right)\right\}$$

A few concluding questions and comments

We used the standard Bunch-Davies vacuum throughout
(& the standard propagator)

Can we reproduce all possible ‘trans-Planckian’ signals
in a conventional effective theory?

What must we sacrifice (symmetries, etc.) to do so?

Provides another constraint on (local) Lorentz violation
(in addition to high energy tests)

We already cannot allow simultaneously
certain operators and too much inflation

What can we learn about nature at the very shortest scales?

How does this knowledge constrain our ideas for a
quantum theory of gravity?

the end