Energy-momentum divergences and the renormalization of gravity

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see also chapter 6 of Birrell & Davies

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We begin with a simple question

How is the *classical* gravitational action affected by the presence of a *quantum* field?

The various divergences in field—since it is a source for gravity—lead to a need to renormalize the parameters of the gravitational theory

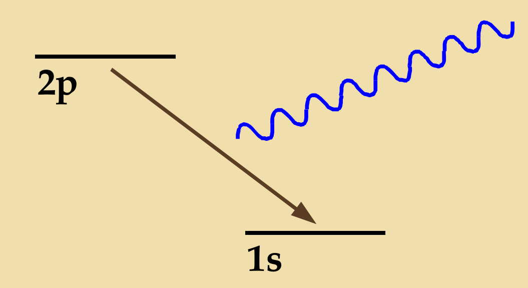
(as long as the space-time symmetries are unbroken)

Overview

- Flat space—the cosmological constant
- An adiabatic description of a field
- Renormalization of classical gravity
- The energy-momentum tensor in an expanding background
- A few words on boundary renormalization

Flat space & the cosmological constant, Λ

- We begin with a simple and familiar example, the renormalization of the vacuum energy, or cosmological constant
- In ordinary quantum field theories there is not usually an idea of absolute scales
- what we typically measure are transitions from one state to another
  - e.g. a decay, a scattering from one 2-particle state to a different 2-particle state, etc.
- However, gravity—or *classical* gravity, at any rate—is acutely sensitive to absolute scales
  - different values of the vacuum energy produce dramatically different cosmologies



both the sign

$$\begin{aligned} \Lambda > 0, &\quad \text{de Sitter} \\ \Lambda = 0, &\quad \text{Minkowski} \\ \Lambda < 0, &\quad \text{anti-de Sitter} \end{aligned}$$

and the magnitude

$\Lambda <$ (few meV)^4
for a reasonable
cosmology

A free scalar field in flat space

- This talk will examine how we can still treat a quantum field theory consistently even in a curved, classical background
  - divergences in the quantum theory lead to a renormalization of the parameters of the gravitational theory
  - ultimate goal: to understand the analogous renormalization of the boundary divergences of a particular initial state
- Consider the simplest possible example, a free scalar field in Minkowski space
- The source for gravity is provided by the distribution of matter and energy, which in Einstein's equations corresponds to the energy-momentum tensor, $T_{\mu\nu}$
- In particular, the energy density contained in the field corresponds to $T_0{}^0$

action

$$S_\varphi = \int d^4x \left[\tfrac{1}{2}\partial_\mu\varphi\partial^\mu\varphi - \tfrac{1}{2}m^2\varphi^2 \right]$$

energy-momentum tensor

$$T_{\mu\nu} = \partial_\mu\varphi\partial_\nu\varphi - \tfrac{1}{2}\eta_{\mu\nu}\left(\partial_\lambda\varphi\partial^\lambda\varphi - m^2\varphi^2\right)$$

the energy density

$$T_0^0 = \frac{1}{2}\left(\frac{d\varphi}{dt}\right)^2 + \frac{1}{2}\nabla\varphi\cdot\nabla\varphi + \frac{1}{2}m^2\varphi^2$$

The energy density

- At this point, to relate the presence of the field to its effect on the classical gravitational background, we must reduce the *operator* $T_0^{\ 0}$ to a *classical* quantity
  – if the field is in a particular state, for example the standard vacuum, we can evaluate the expectation value of $T_0^{\ 0}$

- If we expand the field as a sum of creation and annihilation operators,

$$\varphi = \int \frac{d^3\mathbf{k}}{(2\pi)^3}\left[u_k(t)e^{i\mathbf{k}\cdot\mathbf{x}}a_{\mathbf{k}} + u_k^*(t)e^{-i\mathbf{k}\cdot\mathbf{x}}a_{\mathbf{k}}^{\leq}\right]$$

the standard vacuum eigenmodes are

$$u_k(t) = \frac{e^{-i\omega_k t}}{\sqrt{2\omega_k}}, \qquad \omega_k = \sqrt{k^2 + m^2}$$

energy density

$$\rho = \langle 0|T_0^{\ 0}|0\rangle$$

three assumptions fix the form of the vacuum modes:

1. they obey the Klein-Gordon equation

$$\frac{d^2 u_k}{dt^2} = -\omega_k^2 u_k$$

2. the field satisfies the equal time commutation relation

$$\left[\dot\varphi(t,\mathbf{x}),\varphi(t,\mathbf{y})\right] = -i\delta^3(\mathbf{x}-\mathbf{y})$$

3. positive energy eigenstates

A divergent vacuum energy

- Evaluating the energy density contained within the scalar field, we find an infinite result

- What does this mean with respect to gravity?

- The result becomes more transparent if we also evaluate the pressure, p

- In this form, the pressure and density are not particularly suggestive; but if we dimensionally regularize the momentum integrals we discover that

$$\rho = -p = -\frac{m^4}{64\pi^2}\frac{1}{\varepsilon} + \cdots$$

energy density

$$\rho = \langle 0 | T_0^0 | 0 \rangle$$
$$= \int \frac{d^3\mathbf{k}}{(2\pi)^3}\left\{\frac{1}{2}\left|\frac{du_k}{dt}\right|^2 + \frac{1}{2}\omega_k^2 |u_k|^2\right\}$$
$$= \int \frac{d^3\mathbf{k}}{(2\pi)^3}\left\{\frac{1}{2}\omega_k\right\}$$

pressure

$$p\delta_i^j = -\langle 0 | T_i^j | 0 \rangle$$
$$p = \frac{1}{6}\int \frac{d^3\mathbf{k}}{(2\pi)^3}\left\{\omega_k - \frac{m^2}{\omega_k}\right\}$$

Renormalization of the cosmological constant

- The divergent part of the pressure is exactly the same as the density in magnitude but with the opposite sign

$$\rho = -p = -\frac{m^4}{64\pi^2}\frac{1}{\varepsilon} + \cdots$$

- Both are divergent, but both are also time-independent so they have precisely the form of a cosmological constant

- We can restore our flat background by assuming that there is actually a "bare" cosmological constant that precisely cancels this vacuum energy density from the scalar field

gravity (Λ, G, …)

$+$

scalar field: $\langle 0 | \mathrm{T}_{\mu\nu} | 0 \rangle$

$\Downarrow$

renormalized
gravity (Λ_{R}, G, …)

- Thus the presence of a quantum field φ and the divergences associated with its energy-momentum (ρ, p) has led to a need to renormalize the gravitational part of the theory

$$2\Lambda + \frac{m^4}{64\pi^2}\frac{1}{\varepsilon} = 2\Lambda_R = 0$$

An isotropically expanding universe

- Let us next generalize to an isotropically expanding background, described by a conformally flat metric

- When a quantum scalar field is placed in this background, the spatial flatness still allows an expansion in plane waves

$$\varphi(\eta,\mathbf{x}) = \int \frac{d^3\mathbf{k}}{(2\pi)^3}\Big[\varphi_k(\eta)e^{i\mathbf{k}\cdot\mathbf{x}}a_{\mathbf{k}} + \varphi_k^*(\eta)e^{-i\mathbf{k}\cdot\mathbf{x}}a_{\mathbf{k}}^{\leq}\Big]$$

where the modes satisfy

$$\varphi_k'' + 2aH\varphi_k' + (k^2 + a^2m^2)\varphi_k = 0$$

- We again can ask how this field affects the gravitational background, particularly through its short-distance divergences

metric

$$ds^2 = a^2(\eta)\big[d\eta^2 - d\mathbf{x}\cdot d\mathbf{x}\big]$$

$$S_\varphi = \int d^4x\sqrt{-g}\Big[\tfrac{1}{2}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - \tfrac{1}{2}m^2\varphi^2\Big]$$

Hubble scale

$$H = \frac{a'}{a^2} = \frac{1}{a^2}\frac{\partial a}{\partial\eta}$$

The standard vacuum modes

- For now we shall choose the "vacuum" modes; yet we seem to have reached an impasse, since we do not have an explicit solutions for even these modes in general
- However, since the divergences occur only at short distances, it is sufficient to solve only for the $k \to \infty$ behavior of the modes
- First, we write the modes in a form reminiscent of the flat space vacuum modes
- $\Omega_k(\eta)$ generalizes the flat space frequency and satisfies its own Klein-Gordon eqn

$$\Omega_k^2 = k^2 + a^2 m^2 - \frac{a''}{a} - \frac{1}{2}\frac{\Omega_k''}{\Omega_k} + \frac{3}{4}\frac{\Omega_k'^2}{\Omega_k^2}$$

vacuum modes

$$\varphi_k(\eta) \to U_k(\eta)$$

$$U_k(\eta) = \frac{e^{-i\int_{\eta_0}^{\eta} d\eta' \, \Omega_k(\eta')}}{a(\eta)\sqrt{2\Omega_k(\eta)}}$$

define a 0th order frequency

$$\omega_k^2 = k^2 + a^2 m^2$$

An adiabatic approximation

- To isolate the leading k dependence, we shall use an expansion based on assuming that time-derivatives are small—an adiabatic approximation

$$\Omega_k = \Omega_k^{(0)} + \Omega_k^{(1)} + \Omega_k^{(2)} + \cdots$$

- More precisely, the derivatives of the n^{th} term determine the form of the $(n+1)^{\text{st}}$ term
- For example, the 0th and 1st order terms in this approximation are shown to the right
- For the standard renormalization of gravity, the first term is sufficient
  - but more terms are needed when we extend to a general initial state

Klein-Gordon equation

$$\Omega_k^2 = k + a^2 m^2 - \frac{a''}{a} - \frac{1}{2}\frac{\Omega_k''}{\Omega_k} + \frac{3}{4}\frac{\Omega_k'^2}{\Omega_k^2}$$

zeroth order term

$$[\Omega_k^{(0)}]^2 = k^2 + a^2 m^2 - \frac{a''}{a} = \omega_k^2 - \frac{a''}{a}$$

first order term

$$\Omega_k^{(1)} = -\frac{1}{4}\frac{[\Omega_k^{(0)}]''}{[\Omega_k^{(0)}]^2} + \frac{3}{8}\frac{1}{\Omega_k^{(0)}}\frac{[\Omega_k^{(0)}]'^2}{[\Omega_k^{(0)}]^2}$$

The renormalization of gravity

- The renormalization of the cosmological constant is simply the first example of a general renormalization of the parameters of the gravitational action required in the presence of a quantum field
- As an effective theory, the gravitational action can be expressed as a series arranged by powers of derivatives
  - in four dimensions the Gauss-Bonnet term is purely topological
  - the Weyl tensor vanishes for a conformally flat metric

$$S_g = \int d^4x \sqrt{-g}\Big[2\Lambda + M_{\rm pl}^2 R + \alpha R^2 + \beta R_{\mu\nu}R^{\mu\nu} + \gamma R_{\lambda\mu\nu\sigma}R^{\lambda\mu\nu\sigma} + \cdots\Big]$$

- Thus, in practice we shall require only one of the the four-derivative terms

$$\Lambda,\ M_{\rm pl}^2 = \frac{1}{16\pi G}, \ldots$$

The Gauss-Bonnet term

$$R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\lambda\mu\nu\sigma}R^{\lambda\mu\nu\sigma}$$

Weyl^2 term

$$C_{\lambda\mu\nu\sigma}C^{\lambda\mu\nu\sigma} = \tfrac{1}{3}R^2 - 2R_{\mu\nu}R^{\mu\nu} + R_{\lambda\mu\nu\sigma}R^{\lambda\mu\nu\sigma}$$

$$\int d^4x \sqrt{-g}\left[2\Lambda + M_{\rm pl}^2 R + \alpha R^2\right]$$

Renormalization of gravity—equations of motion

$$\int d^4x \sqrt{-g}\left[2\Lambda + M_{\rm pl}^2 R + \alpha R^2\right]$$

- Varying this action with respect to $\delta g_{\mu\nu}$ yields the gravitational contribution to the equations of motion,

$$\frac{2}{\sqrt{-g}}\frac{\delta S_g}{\delta g^{\mu\nu}} = 2\Lambda g_{\mu\nu} - 2M_{\rm pl}^2\left[R_{\mu\nu} - \tfrac{1}{2} g_{\mu\nu} R\right] \\ -4\alpha\left[\nabla_\mu \nabla_\nu R - g_{\mu\nu}\nabla^2 R + RR_{\mu\nu} - \tfrac{1}{4} g_{\mu\nu} R^2\right]$$

- The other component comes from the scalar field. Since the energy-momentum tensor is an operator, we must take its expectation value to learn its effect on gravity,

energy-momentum tensor

$$T_{\mu\nu} = -\frac{2}{\sqrt{-g}}\frac{\delta S_\varphi}{\delta g^{\mu\nu}}$$

$$\langle 0 | T_{\mu\nu} | 0 \rangle = 2\Lambda g_{\mu\nu} - 2M_{\rm pl}^2\left[R_{\mu\nu} - \tfrac{1}{2} g_{\mu\nu} R\right] \\ -4\alpha\left[\nabla_\mu \nabla_\nu R - g_{\mu\nu}\nabla^2 R + RR_{\mu\nu} - \tfrac{1}{4} g_{\mu\nu} R^2\right]$$

Symmetries of the background

- Before evaluating the divergences in $\langle 0 | T_{\mu\nu} | 0 \rangle$, it is helpful to use the symmetries of the background to isolate the independent components of this tensor

$$\langle 0 | T_\mu^\nu | 0 \rangle = \mathrm{diag}\left[\rho(\eta), -p(\eta), -p(\eta), -p(\eta)\right]$$

- The spatial flatness implies that this tensor can be written—completely generally—as

- Although we shall evaluate both the density and the pressure, there is in fact some redundancy implied by $\nabla_\mu T^\mu{}_\lambda = 0$

conservation equation

$$\rho' = -3aH(\rho + p)$$

- Thus,

$$\rho = 2\Lambda + M_{\rm pl}^2 \frac{6}{a^2}\frac{a'^2}{a^2} + \frac{36\alpha}{a^4}\left[2\frac{a'''}{a}\frac{a'}{a} - \frac{a''^2}{a^2} - 4\frac{a''}{a}\frac{a'^2}{a^2}\right]$$

$$p = -2\Lambda + M_{\rm pl}^2 \frac{2}{a^2}\left[\frac{a'^2}{a^2} - 2\frac{a''}{a}\right] - \frac{24\alpha}{a^4}\left[\frac{a''''}{a} - 5\frac{a'''}{a}\frac{a'}{a} - \frac{5}{2}\frac{a''^2}{a^2} + 8\frac{a''}{a}\frac{a'^2}{a^2}\right]$$

A few assumptions

- Before proceeding, we should be a little more explicit about some of our assumptions for the field
- Both the action and the state chosen respect the same classical symmetries as the background
  - therefore the divergences should have the same structure as the previous gravitational terms
  - states that break the classical symmetries of the background can produce additional divergences that are not of the form of the terms we have included thus far (boundary terms)
- The standard approach, which we have been following, is to reduce the field contribution to a classical function
  - however, in a more general setting, this reduction may not itself be sufficient to understand the renormalization
  - we should instead treat the gravitational side quantum mechanically (at tree level) so that it can properly respond to the scalar field

$$\left[\nabla^2 + m^2\right]\varphi(\eta,\mathbf{x}) = 0$$

$$|0\rangle \rightarrow \begin{cases}\text{maximally}\\ \text{symmetric}\\ \text{state}\end{cases}$$

Divergences in the energy & momentum

- In a curved background, the energy density contained within the scalar field in is standard vacuum is

$$\rho = \tfrac{1}{2} a^{-2} \langle 0 | \left[\varphi'^2 + \nabla\varphi \cdot \nabla\varphi + a^2 m^2 \varphi^2 \right] | 0 \rangle$$

which becomes (point-splitting)

$$\rho = \frac{1}{2}\frac{1}{a^2}\int \frac{d^3\mathbf{k}}{(2\pi)^3}\left\{ |U'_k|^2 + (k^2 + a^2 m^2)|U_k|^2 \right\}$$

- To apply our adiabatic approximation, to isolate the leading short-distance behavior, we rewrite the integrand in terms of $\Omega_k(\eta)$,

$$\rho = \frac{1}{4}\frac{1}{a^4}\int \frac{d^3\mathbf{k}}{(2\pi)^3}\frac{1}{\Omega_k}\left\{ 2\Omega_k^2 + \frac{a''}{a} + \frac{a'^2}{a^2} + \frac{a'}{a}\frac{\Omega'_k}{\Omega_k} + \frac{1}{2}\frac{\Omega''_k}{\Omega_k} - \frac{1}{2}\frac{\Omega'^2_k}{\Omega_k^2} \right\}$$

vacuum modes

$$U_k(\eta) = \frac{e^{-i\int_{\eta_0}^{\eta} d\eta'\,\Omega_k(\eta')}}{a(\eta)\sqrt{2\Omega_k(\eta)}}$$

Adiabatic approximation

$$\Omega_k = \omega_k - \frac{1}{2}\frac{1}{\omega_k}\frac{a''}{a} - \frac{1}{4}\frac{a^2 m^2}{\omega_k^3} + \frac{1}{8}\frac{1}{\omega_k^3}\left[\frac{a''''}{a} - 2\frac{a'''}{a}\frac{a'}{a} - 2\frac{a''^2}{a^2} + 2\frac{a''}{a}\frac{a'^2}{a^2} \right] + \cdots$$

$$\omega_k^2 = k^2 + a^2 m^2$$

The adiabatic approximation of the density

- The adiabatic approximation allows us to isolate the divergent behavior without solving for the exact form of the modes,

$$\rho = \frac{1}{2}\frac{1}{a^4}\int \frac{d^3\mathbf{k}}{(2\pi)^3}\omega_k + \frac{1}{4}\frac{1}{a^4}\frac{a'^2}{a^2}\int \frac{d^3\mathbf{k}}{(2\pi)^3}\left[\frac{1}{\omega_k} + \frac{a^2m^2}{\omega_k^3}\right]$$
$$-\frac{1}{16}\frac{1}{a^4}\left[2\frac{a'''}{a}\frac{a'}{a} - \frac{a''^2}{a^2} - 4\frac{a''}{a}\frac{a'^2}{a^2}\right]\int \frac{d^3\mathbf{k}}{(2\pi)^3}\frac{1}{\omega_k^3}$$
$$+ \text{ finite}$$

- Even before evaluating the integrals, we can see that the divergences have the same form as the gravitational terms,

$$\rho = 2\Lambda + M_{\text{pl}}^2\frac{6}{a^2}\frac{a'^2}{a^2}$$
$$+\frac{36\alpha}{a^4}\left[2\frac{a'''}{a}\frac{a'}{a} - \frac{a''^2}{a^2} - 4\frac{a''}{a}\frac{a'^2}{a^2}\right]$$

general form

$$\int \frac{d^3\mathbf{k}}{(2\pi)^3}\frac{1}{\omega_k^n}$$

$$\omega_k^2 = k^2 + a^2m^2$$

infrared regulator

A little aside on dimensional regularization

- The structure of the momentum integrals that diverge at short distances have a structure similar to the standard form encountered in an S-matrix calculation
  - they are already purely spatial, so no Wick rotation is necessary

$$I_n^0 = \int \frac{d^3\mathbf{k}}{(2\pi)^3} \frac{1}{\omega_k^n} = \int \frac{d^3\mathbf{k}}{(2\pi)^3} \frac{1}{(k^2 + a^2 m^2)^{n/2}}$$

- We regulate the integrals by extending the number of spatial dimensions to $3 - 2\varepsilon$
  - since k is actually the comoving momentum we include a factor of the scale a along with the renormalization scale μ
  - the logarithms thus do not depend on $a(\eta)$

$$I_n^\varepsilon = \int \frac{d^{3-2\varepsilon}\mathbf{k}}{(2\pi)^{3-2\varepsilon}} \frac{(a\mu)^{2\varepsilon}}{(k^2 + a^2 m^2)^{n/2}}$$

- We then extract the poles by taking $\varepsilon \to 0$

$$I_n^\varepsilon = \frac{\sqrt{\pi}}{8\pi^2} \frac{\Gamma(\varepsilon - \frac{3-n}{2})}{\Gamma(\frac{n}{2})} \left[\frac{4\pi\mu^2}{m^2}\right]^\varepsilon (am)^{3-n}$$

Renormalizing classical gravity

- Extracting just the pole terms we discover that the divergences in the energy density of the scalar field,

$$\rho = -\frac{m^4}{64\pi^2}\frac{1}{\varepsilon} + \frac{m^2}{192\pi^2}\frac{6}{a^2}\frac{a'^2}{a^2}\frac{1}{\varepsilon} - \frac{1}{2304\pi^2}\frac{36}{a^4}\left[2\frac{a'''}{a}\frac{a'}{a} - \frac{a''^2}{a^2} - 4\frac{a''}{a}\frac{a'^2}{a^2}\right]\frac{1}{\varepsilon} + \cdots$$

precisely match the scale-dependences the gravitational terms,

$$\rho = 2\Lambda + M_{\rm pl}^2\frac{6}{a^2}\frac{a'^2}{a^2} + \frac{36\alpha}{a^4}\left[2\frac{a'''}{a}\frac{a'}{a} - \frac{a''^2}{a^2} - 4\frac{a''}{a}\frac{a'^2}{a^2}\right]$$

e.g., in the MS scheme

cosmological constant

$$\Lambda_R = \Lambda + \frac{m^4}{128\pi^2}\frac{1}{\varepsilon}$$

Planck mass (Newton constant)

$$M_{{\rm pl},R}^2 = M_{\rm pl}^2 - \frac{m^2}{192\pi^2}\frac{1}{\varepsilon}$$

four-derivative terms

$$\alpha_R = \alpha + \frac{1}{2304\pi^2}\frac{1}{\varepsilon}$$

A few words on boundary renormalization

- Our ultimate interest is to derive the analogous renormalization of the background geometry that occurs when we chose a more general effective initial state for the scalar field
- Let us recall the philosophy behind an effective initial state
  - the usual vacuum is a derived based upon a particular choice for the low energy theory, which is extrapolated to arbitrary scales
- The true vacuum may differ from this state
  - new dynamics at short distances
  - broken or deformed symmetries
  - new physical principles
- The effective state then provides a general parameterizations of the possible differences between the standard and the true vacuum states

define the state at an initial time through

$$n^\mu \nabla_\mu \varphi_k \Big|_{\eta=\eta_0} = -i\varpi_k \varphi_k(\eta_0)$$

$$\varpi_k = \frac{1-f_k}{1+f_k}\frac{\Omega_k}{a} - iH - \frac{i}{2}\frac{\Omega_k'}{\Omega_k}$$

$$\varphi_k(\eta) = \frac{U_k(\eta) + f_k U_k^*(\eta)}{\sqrt{1-f_k f_k^*}}$$

e.g., for a "trans-Planckian" initial state,

$$f_k = \sum_{n=1}^{\infty} c_n \frac{\omega^n}{(aM)^n}$$

A few words on boundary renormalization

- A more general effective state produces new divergences in the energy-momentum tensor
  - because of the oscillatory factor, these divergences occur at the initial boundary
- These divergences then require modifying the gravitational side of the theory, by adding new boundary counterterms
  - For example, in Minkowski space there is only one possible surface counterterm—a surface tension—but that is enough
  - More generally we have terms built from the extrinsic and intrinsic curvatures
- The calculation is more complicated, but the approach is the same
  - expand to the appropriate adiabatic order
  - and counterterms with the same scale dependence (subtlety—dynamics)

including the new contribution

$$\rho \to \rho + \rho_{\text{surf}}$$

where

$$\rho_{\text{surf}} = \frac{1}{4}\frac{1}{a^4}\int \frac{d^3\mathbf{k}}{(2\pi)^3} f_k^* \frac{e^{-2i\int_{\eta_0}^{\eta} d\eta' \,\Omega_k(\eta')}}{\Omega_k} \times \left\{ 2i\frac{a'}{a}\Omega_k + \frac{a''}{a} + \frac{a'^2}{a^2} + i\Omega_k' + \frac{a'}{a}\frac{\Omega_k'}{\Omega_k} + \frac{1}{2}\frac{\Omega_k''}{\Omega_k} - \frac{1}{2}\frac{\Omega_k'^2}{\Omega_k^2} \right\}$$

for example, if $f_k \approx k/M$

$$\rho_{\text{surf}} \propto \int \frac{d^3\mathbf{k}}{(2\pi)^3}\frac{\omega_k}{M} \times \frac{1}{\omega_k^4} \times \text{four derivatives of } a$$

A controversy and its possible resolution

- Treating gravity completely classically, however, may not be sufficient to capture the effects we are seeking
  - the state explicitly breaks some of the background symmetries
  - allow the geometry to adjust to match that of the scalar field state

- However, by treating gravity on the same footing as the scalar field—and at tree level—it should be possible to resolve an important question about the naturalness of trans-Planckian initial conditions
  - in this framework, there would be no fine-tuning in the initial state as some groups have previously claimed
  - the subtle point concerns normal derivatives in the boundary counterterms

simplified back-reaction calculations:

- Porrati, 2004–2005
- Greene, Schalm, Shiu, & van der Shaar, 2004–2005

possible counterterm action

$$S_{\rm surf} = \int d^3x \sqrt{-h}\,\Big\{\sigma + g_1 K + g_2 K^2 + g_2'\, n^\mu \nabla_\mu K + \cdots\Big\}$$

extrinsic curvature

$$K_{\mu\nu} = h_\mu^\lambda \nabla_\lambda n_\nu$$
$$h_{\mu\nu} = g_{\mu\nu} - n_\mu n_\nu$$

Conclusions

- The purpose of this talk has been somewhat modest—to show how the presence of quantum fields in a classical gravitational background leads to a renormalization of the gravitational parameters

- as well as to provide a brief overview of the new elements for the energy-momentum associated with a general effective initial state

- Because the standard vacuum respects the symmetries at arbitrarily short-distances, the counterterms will be of exactly the same form as those already in the action
  - but for other states, the broken symmetries mean that we can include other counterterms consistent with the remaining unbroken symmetry
  - the field is, after all, the source for gravity

renormalizing "bulk" gravity

$$\Lambda_R = \Lambda + \frac{m^4}{128\pi^2}\frac{1}{\varepsilon}$$

$$M^2_{\mathrm{pl},R} = M^2_{\mathrm{pl}} - \frac{m^2}{192\pi^2}\frac{1}{\varepsilon}$$

$$\alpha_R = \alpha + \frac{1}{2304\pi^2}\frac{1}{\varepsilon}$$

the geometry must be allowed to adapt to the form of the state for a consistent treatment

$$S_{\mathrm{surf}} = \int d^3x\sqrt{-h}\,\Big\{\sigma + g_1 K + g_2 K^2 + g_2'\, n^\mu \nabla_\mu K + \cdots\Big\}$$