Radion-Induced Brane Preheating

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(Received 23 May 2002; published 10 June 2003)

When the interbrane separation in the compact Randall-Sundrum model is stabilized using the Goldberger-Wise mechanism, a potential is generated for the four-dimensional field, the radion, that encodes this separation. Coherent oscillations of the radion in the early universe will produce an exponential growth in the number of brane particles due to parametric amplification. We describe the conditions necessary for this process, which resembles the preheating phase in inflation, and show the exponential growth in the case of a scalar field confined to a brane.

DOI: 10.1103/PhysRevLett.90.231301 PACS numbers: 98.80.Cq, 04.50.+h, 11.10.Kk, 11.25.Mj

The compact Randall-Sundrum model [1] provides an extremely interesting relation between hierarchies in four-dimensional physics and the warping of a five-dimensional bulk geometry in which our universe could be embedded. The model consists of two 3-branes placed at the fixed points of an $S^1/\mathbb{Z}_2$ orbifold, which serve as the boundaries of a slice of a five-dimensional anti–de Sitter space-time (AdS_5). In order to balance the bulk cosmological constant, the branes must have equal and opposite tensions, which are in turn related to the value of the bulk cosmological constant.

The model contains a geometrical mode, the radion, that encodes the physical separation between the branes. We can write the bulk metric in a way that exhibits it explicitly [2]:

$$ds^2 = e^{-2k[y+f(x)e^{2ky}]}\hat{g}_{\mu\nu}(x)dx^\mu dx^\nu - [1 + 2kf(x)e^{2ky}]^2 dy^2. \tag{1}$$

The radion is given by the function $f(x)$ and the metric has been written to incorporate properly the symmetries of the geometry. In particular, this form guarantees that the radion will not mix with massless degrees of freedom such as the $3+1$-dimensional gravitons [2]. The bulk cosmological constant is equal to $-6k^2$.

The linearized Einstein equations,

$$\hat{R}_{\mu\nu} - \tfrac{1}{2}\hat{g}_{\mu\nu}\hat{R} = 0, \qquad \hat{\nabla}^2 f = 0, \tag{2}$$

where $\hat{R}_{\mu\nu}$ and $\hat{\nabla}_\mu$ are the Ricci tensor and covariant derivative associated with the metric $\hat{g}_{\mu\nu}(x)$, show that the radion behaves as a massless $3+1$-dimensional field. This behavior follows from the fact that the radion is the Goldstone boson associated with the invariance of the action under placing the second brane at an arbitrary distance from the first. Note that the radion has no associated Kaluza-Klein tower [2,3].

Since the radion is a Goldstone mode, the model does not by itself select a particular value for the interbrane separation. Physically, this separation determines the hierarchy between the scale of electroweak interactions and that of gravity, so that some means of fixing the expectation value of the radion must be added. Goldberger and Wise devised an elegant mechanism [4] for breaking the invariance of the theory under changes of the distance between the branes by introducing a bulk scalar field with a nontrivial profile in the extra dimension and with potentials on each of the branes. For simplicity, these potentials are chosen essentially to fix the value of the scalar field on each brane. Integrating the action of this scalar field over the bulk coordinate yields an effective potential for the radion, whose expectation value can then be fixed in terms of the parameters of the Goldberger-Wise action to take the correct value to enforce the required hierarchy.

The appearance of the radion in the metric implies that it naturally couples to any brane field whose action breaks Weyl invariance. At sufficiently early times, the radion will not yet have settled to the minimum of its effective potential and will oscillate. As with the inflaton at the end of the inflationary era [5], these oscillations can induce an explosive production of brane particles through parametric amplification, a process known as *preheating* [6].

In this Letter, we describe the mechanism of radion preheating by first deriving the effective action of the radion and its couplings to fields on the IR brane. In particular, we examine a scalar field on the brane and follow the growth of the number of scalar particles as a result of radion oscillations. We find not only an exponential growth of the scalar particle number but also that this growth occurs over a wide band of momenta resulting in an exponentially rapid production of brane particles.

The cosmology of a brane universe at temperatures of the order of a TeV, the scale associated with the IR brane, is not well understood since at such energies gravity becomes strongly interacting near the IR brane. One consequence is that, at such temperatures, the geometry is unstable to the formation of a horizon [7] in the region between the branes. To avoid this regime and to be able to use a simple four-dimensional effective action for the radion, we shall study the cosmology of a stabilized Randall-Sundrum model below this scale. Radion preheating is simplest to analyze in the limit where only the coherent, spatially independent, modes of the radion are important. A sufficient means to obtain this limit occurs if some inflationary era preceded the radion preheating. Since compelling reasons already exist for inflation, we shall assume in this Letter that the spatially dependent modes have been inflated away. We should note, however, that the question as to how to incorporate inflation into the Randall-Sundrum model remains unsolved; in particular, without any new ingredients, the radion cannot by itself serve as the inflaton [7].

Once the energy density of the universe is below the scale associated with the IR brane and the Kaluza-Klein masses, $e^{-k\Delta y}M_5$, the dynamics of the radion can be described by an effective action. The five-dimensional Planck mass is M_5, and Δy is the separation between the branes. The kinetic terms for the radion effective action arise from the five-dimensional Einstein action, which is supplemented by the Gibbons-Hawking terms on the boundary branes. The Goldberger-Wise action contains a free scalar field of mass m with potentials to fix the value of the field to be $v_0 M_5^{3/2}$, and $v_1 M_5^{3/2}$ on the UV and the IR branes, respectively. The calculation of the radion effective action obtained by integrating out the extra dimension results in an effective Lagrangian of the form

$$\begin{aligned}\mathcal{L}_{\text{radion}} &= \tfrac{1}{2}(1 - \Lambda_F^{-1}F + \tfrac{1}{2}\Lambda_F^{-2}F^2)\partial^\mu F \partial_\mu F - \tfrac{1}{2}m_F^2 F^2 \\ &\quad + \tfrac{4}{3}g_F \Lambda_F F^3 - g_F F^4 + \cdots \\ &= \tfrac{1}{2}\partial^\mu F \partial_\mu F - \tfrac{1}{2}m_F^2 F^2 + \cdots . \end{aligned} \tag{3}$$

We have defined the following quantities in this equation:

$$\begin{aligned} &F(x) = 2\sqrt{3}kM_4 e^{k\Delta y} f(x), \\ &\Lambda_F \equiv \sqrt{3}e^{-k\Delta y}M_4 \sim \mathcal{O}(\text{TeV}), \\ &m_F = \frac{v_1(1-\eta)^{1/4}}{3}\left(\frac{m}{M_5}\right)^{3/2}\Lambda_F, \qquad g_F \equiv \frac{1}{2}\frac{m_F^2}{\Lambda_F^2}. \end{aligned} \tag{4}$$

Note that Λ_F is the natural cutoff for the effective theory of the radion; at energies above Λ_F radion dynamics—as well as gravitation interactions on the IR brane—become strong. η (<1) parametrizes the change in the tension of the IR brane needed to counteract the effects of the Goldberger-Wise scalar field. At low energies, it is convenient to introduce the effective Planck mass M_4 seen by a low energy observer where $M_4^2 = (1 - e^{-2k\Delta y})M_5^3/k \approx M_5^3/k$. Reference [8] found a similar mass for the radion based on a calculation using an exact solution to a quartic potential for the Goldberger-Wise field, rather than the effective action approach for a free Goldberger-Wise field which yielded Eq. (3).

Only the leading terms of the effective potential are important when the amplitude of the radion is sufficiently smaller than the natural cutoff, Λ_F. To some extent, the Randall-Sundrum scenario already places the radion in this regime. A comparison of the free energies of a stabilized Randall-Sundrum scenario with two branes and a scenario with a single brane and a horizon, whose distance to the UV brane is smaller than the equilibrium position of the IR brane, suggests than the transition between these two phases occurs at a temperature [7]

$$T_c \approx \frac{1}{2^{3/4}\pi}\left(\frac{k}{M_4}\right)^{1/2}\left(\frac{m_F}{\Lambda_F}\right)^{1/2}\Lambda_F. \tag{5}$$

To be in the weak gravity regime in the bulk, we require $k \ll M_4$. We shall also generally have $m_F < \Lambda_F$ so that the critical temperature is typically about an order of magnitude below Λ_F. Since Λ_F is the natural scale that suppresses higher order operators in the effective Lagrangian in Eq. (3), once the two brane phase sets in, we can assume that the amplitude of the radion's fluctuations is small compared to Λ_F and that the self-interactions of Eq. (3) can be neglected.

To study the effect of excitations of the radion on brane fields, consider a scalar field $\Phi(x^\mu)$ confined to the IR brane. The induced metric on the IR brane couples this field with the radion. In the small amplitude regime for Φ, we can write the scalar action as

$$S_\Phi = \int_{y=\Delta y} d^4x\sqrt{-h}[\tfrac{1}{2}h^{ab}\partial_a\tilde{\Phi}\partial_b\tilde{\Phi} - \tfrac{1}{2}\tilde{m}_\Phi^2\tilde{\Phi}^2 + \cdots], \tag{6}$$

where h_{ab} is the induced metric on the IR brane. Using Eq. (1), we can rewrite this as

$$\begin{aligned} S_\Phi = \int d^4x\sqrt{-\hat{g}}[&\tfrac{1}{2}e^{-F/\Lambda_F}\hat{g}^{\mu\nu}\partial_\mu\Phi\partial_\nu\Phi \\ &- \tfrac{1}{2}e^{-2F/\Lambda_F}m_\Phi^2\Phi^2 + \cdots], \end{aligned} \tag{7}$$

where we have rescaled the fields $\Phi = e^{-k\Delta y}\tilde{\Phi}$ and $m_\Phi = e^{-k\Delta y}\tilde{m}_\Phi$ to obtain a canonically normalized kinetic term when the radion is at its equilibrium value $F = 0$. The couplings of the radion to standard model fields were also considered in [8] where precision electroweak observables were studied. In contrast, we are more concerned with dynamical aspects of these couplings of the radion to brane fields.

If we assume that the universe is homogenous and isotropic in the $3 + 1$ large dimensions, the excitations of the radion act as an oscillating background for the brane fields,

$$F(t) = F_0\Lambda_F\cos(m_F t), \tag{8}$$

where F_0 is dimensionless. Equation (5) indicates that F_0 will naturally be small and we can approximate the effective action in Eq. (3) by its quadratic terms. In this Letter, we shall neglect radion quantum fluctuations as well as the effects of the backreaction due to the produced particles or other fields. This assumption is reasonable during the initial stage of preheating since the backreaction is small. The radion's couplings to Φ imply that its energy is drained off as the radion produces Φ particles. In the absence of backreaction, this process apparently proceeds indefinitely, but more realistically the backreaction eventually damps the motion of the radion.

To compute the number of scalar field particles produced, we expand Φ in creation and annihilation operators, $a_{\vec{k}}^{\dagger}$ and $a_{\vec{k}}$, for a mode with momentum $\vec{k}$,

$$\Phi(t, \vec{x}) = \int \frac{d^3 k}{(2\pi)^{3/2}} [\Phi_k(t) e^{-i\vec{k}\cdot\vec{x}} a_{\vec{k}} + \Phi_k^*(t) e^{i\vec{k}\cdot\vec{x}} a_{\vec{k}}^{\dagger}]. \quad (9)$$

The mode functions are then determined by the equation of motion for the scalar field from Eq. (7),

$$\frac{d^2\Phi_{\vec{k}}}{dt^2} + F_0 m_F \sin(m_F t) \frac{d\Phi_{\vec{k}}}{dt} + \omega_{\vec{k}}^2(t)\Phi_{\vec{k}} = 0, \quad (10)$$

where

$$\omega_{\vec{k}}^2(t) \equiv [|\vec{k}|^2 + m_{\Phi}^2 e^{-F_0 \cos(m_F t)}]. \quad (11)$$

Although the universe is expanding, the effect of this expansion, set by the Hubble parameter $H < \mathrm{TeV}^2/M_4 \sim 10^{-16}$ TeV, is negligible during the relaxation of the radion oscillations, whose time scale is set by the radion mass, and thus the expansion has not been included in Eq. (10).

We shall follow the number $N_{\vec{k}}(t)$ of quanta of the initial state defined by

$$\dot{\Phi}_{\vec{k}}(0) = -i\omega_k(0)\Phi_{\vec{k}}(0). \quad (12)$$

This defines a vacuum state $|0\rangle$ that is annihilated by the operators $a_{\vec{k}}$.

The periodicity of the coefficient functions in Eq. (10) under $t \to t + 2\pi m_F^{-1}$ implies, by Floquet theory (see, for example, [9]), that there will be values of the dimensionless momentum $|\vec{k}|/m_{\Phi}$ for which $\Phi_{\vec{k}}(t)$ will undergo exponential growth—there are unstable bands. These instability bands are shown in Fig. 1.

The growth in the number of particles with momentum $\vec{k}$, $N_{\vec{k}}(t) = \langle 0|a_{\vec{k}}^{\dagger} a_{\vec{k}}|0\rangle$, is found by evolving the creation and annihilation operators with respect to the initial vacuum defined in Eq. (12). For momenta in the unstable bands, the particle number grows exponentially, as shown in Fig. 2, which plots the expression for $\ln N_{\vec{k}}(t)$:

$$N_{\vec{k}}(t) = \frac{e^{-F_0}}{2\omega_k(0)} [e^{2F_0[1-\cos(m_F t)]} |\dot{\Phi}_{\vec{k}}(t)|^2 + \omega_{\vec{k}}^2(0)|\Phi_{\vec{k}}(t)|^2] - \frac{ie^{-F_0 \cos(m_F t)}}{2} [\Phi_{\vec{k}}^*(t)\dot{\Phi}_{\vec{k}}(t) - \dot{\Phi}_{\vec{k}}^*(t)\Phi_{\vec{k}}(t)]. \quad (13)$$

This extremely rapid particle production will affect brane physics, producing a large nonthermal distribution of standard model particles starting when the brane temperature is $\mathcal{O}$ (100 GeV). Although by the time the universe cools down to nucleosynthesis temperatures the standard model fields should have thermalized, depending on their abundance as well as their equation of state they can modify the expansion law in the early universe. To follow this effect carefully would require computing the expectation value of the energy-momentum tensor of the produced particles and using it as a source in the relevant Robertson-Walker equations describing the expansion of the brane. We have estimated the density and pressure of the produced particles and plotted the behavior of the equation of state $w \equiv p/\rho$ as a function of time in Fig. 3.

While the initial radion amplitude determines the width of the instability band, we quite generally find that a rapid production of particles occurs when $m_F \gtrsim 2m_{\Phi}$. Thus, if Φ corresponds to a Higgs field with $m_{\Phi} \sim 150$ GeV, then the population of Higgs fields will receive a parametric amplification during the preheating phase as long as the mass of the radion is greater than about 300 GeV, although, depending on the actual Higgs boson mass and the amplitude of the radion oscillations, this limit can be substantially lower.

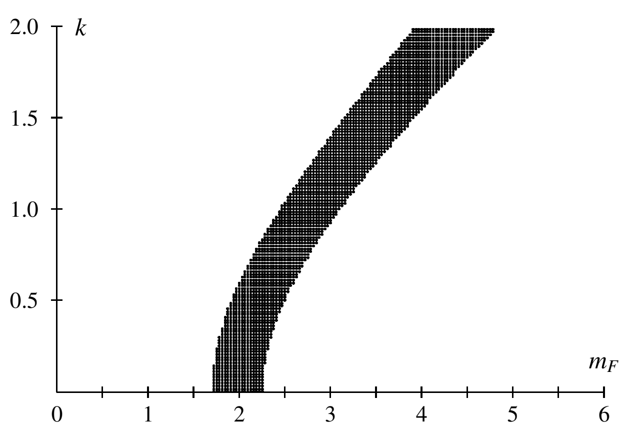

FIG. 1. The broad instability band for the differential equation, Eq. (10), when $F_0 = 0.2$. The dark region shows where the amplitude of $\Phi_{\vec{k}}(t)$ grows exponentially. Both m_F and $k = |\vec{k}|$ are expressed in units of m_{Φ}.

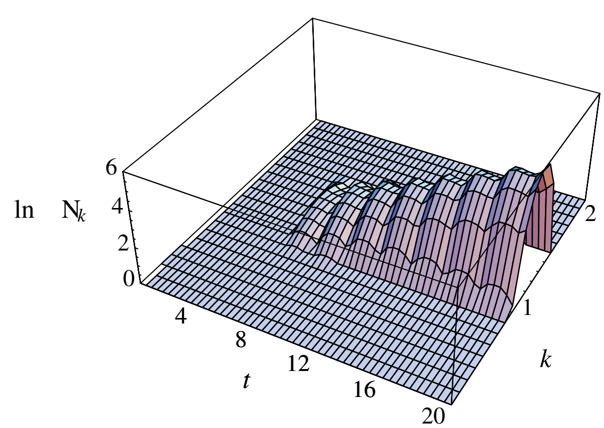

FIG. 2 (color online). The $\ln N_{\vec{k}}(t)$ for $m_F = 3m_{\Phi}$ and $F_0 = 0.2$. Note that, for momenta within the instability band of Fig. 1, the number of particles produced grows exponentially; t and k are expressed in units of m_{Φ}^{-1} and m_{Φ}, respectively.

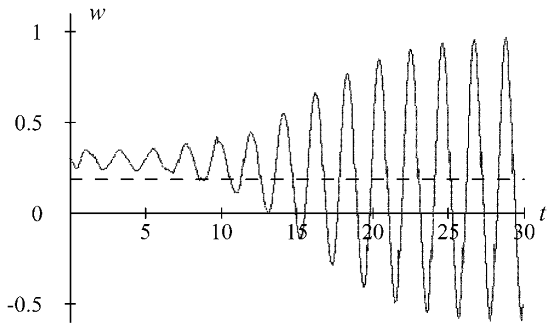

FIG. 3. The equation of state $w \equiv p/\rho$ as a function of time. The dotted line indicates an average value of w at later times. For this case, $m_F = 3m_\Phi$ and $F_0 = 0.2$, this average is approximately 0.189. t is given in units of m_Φ^{-1}.

The initial distribution of the standard model fields on the brane depends on the details of the mechanism that produces the IR brane. If the standard model fields are essentially in their vacuum state, then the subsequent evolution of the nonthermal particles could differ substantially from the standard radiation dominated universe when the temperature is near the electroweak scale. Even with an initial thermal population of fields, the parametric amplification due to the radion could still produce a significant fraction of the total energy density in a nonequilibrium distribution.

The production of a large component of the universe in a nonthermal distribution with an energy density very near or below the electroweak phase transition naturally provides some of the ingredients needed for electroweak baryogenesis.

For example, the explosive production of Higgs fields would generate winding configurations in a manner analogous to the production of topological defects. With the presence of CP violation, the decay of some windings produces baryons anomalously [10]. While past inflationary scenarios have attempted to produce the observed baryon asymmetry by not reheating above the electroweak scale [11], in radion preheating this scale emerges naturally. Moreover, unlike the couplings of the inflaton, the couplings of the radion to other fields is completely determined.

For the case of a light radion, a rough estimate using the sphaleron length scale and the Kibble mechanism gives enough Higgs windings to explain the observed baryon asymmetry with the addition of a modest amount of additional CP violation beyond the standard model [12].

A stage of radion-induced preheating represents a general feature of the Randall-Sundrum cosmology [13]. In this Letter, we have described its basic mechanism, showing how oscillations of the radion after the formation of the IR brane lead to the exponential growth of the fields on this brane. An appealing feature of this picture is that the full geometry completely determines the radion's couplings and fixes scales to be naturally near the electroweak scale so that definite predictions are possible. As an illustration, we mentioned how radion preheating could produce the observed baryon asymmetry. We shall address this baryogenesis and other consequences and study the damping of the radion during preheating in future work.

We thank Ira Rothstein for valuable comments. This work was supported in part by DOE Grant No. DE-FG03-91-ER40682.

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