# Invisible axion in a Randall-Sundrum universe

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We study the problem of integrating an invisible axion into the Randall-Sundrum scenario as an example of how to generate new energy scales between the extremes of the Planck mass and the electroweak scale. In this scenario, the axion corresponds to the phase of a complex bulk scalar field. We show how to generate an intermediate energy scale by including a third brane in the scenario. We discuss the stabilization of this brane in detail to demonstrate that no additional fine tunings arise.

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## I. INTRODUCTION

One of the features which the Randall-Sundrum scenario [1] shares with other solutions to the hierarchy problem is that it assumes a desert between the scale of electroweak physics and the scale of gravity. Although new phenomena—strong gravity and bulk Kaluza Klein modes—appear above the electroweak scale, all the physics in this scenario can be expressed in terms of these two scales. Such a picture is usually adequate since we have no direct evidence of phenomena between these energies. Yet in some cases we may wish to introduce some new physics whose dynamics occurs at an intermediate scale. The difficulty in the Randall-Sundrum brane world is to understand how a scale can naturally arise, surviving in the low energy theory, that is not either of these two natural extremes.

A specific instance of where such an intermediate scale is needed occurs in the invisible axion solution to the strong *CP* problem [2]. The vacuum structure of QCD combined with the *CP* violation in the weak interactions permits an interaction of the form

$$\bar{\theta}\frac{\alpha_s}{8\pi}\mathrm{Tr}[\epsilon_{\mu\nu\lambda\rho}F^{\mu\nu}F^{\lambda\rho}] \tag{1.1}$$

where $F^{\mu\nu}$ is the QCD field strength. This interaction violates *P* and *CP* and is highly constrained by measurements of the neutron dipole moment which require $\bar{\theta}\lesssim 10^{-9}$. As a free parameter, $\bar{\theta}$ must thus be finely tuned for an acceptable theory. Peccei and Quinn [3] showed that if $\bar{\theta}$ is promoted to a dynamical field $a(x^\mu)$, which is the Goldstone boson associated with a spontaneously broken global $U(1)_{\mathrm{PQ}}$ symmetry, then $\bar{\theta}$ is dynamically driven to zero. Although this field, the axion, is a Goldstone mode, it does acquire a mass of the order $\Lambda_{\mathrm{QCD}}^2/f$, where $f$ is scale at which the $U(1)_{\mathrm{PQ}}$ breaking occurs. An $f$ of the order of the electroweak scale produces too massive an axion for experimental constraints.

An acceptable axion mass does occur when the $U(1)_{\mathrm{PQ}}$ breaks at some high scale $f\gg$TeV. From astrophysical observations, this scale should lie within the interval [4,5]

$$10^{10}\ \mathrm{GeV}\lesssim f\lesssim 10^{13}\ \mathrm{GeV}. \tag{1.2}$$

Since the invisible axion models do not address the hierarchy problem, they do not attempt to explain whether such a scale can arise naturally.

In the Randall-Sundrum scenario, the only natural scales are the bulk Planck mass, $M_5$, and the AdS curvature, $k$. Other, exponentially smaller scales do arise when the physics responsible for them is confined to a region at some distance from the UV brane. Although the mass scales for the fields confined to the IR brane are also of the order $M_5$, when the fields there are rescaled to remove redshift factors introduced by the induced metric on the brane, the apparent mass scales on the IR brane can be naturally of the order of the electroweak scale. Goldberger and Wise [6] showed that the position of the IR brane relative to the UV brane can be stabilized—and thus the electroweak-gravity hierarchy—without finely tuning the parameters of the stabilization mechanism. The observed Planck mass in low energy, four dimensional effective theory, determined by $M_4^2\approx M_5^3/k$, remains large.

In this article we shall use the invisible axion as a case study of how to introduce new intermediate scales into the Randall-Sundrum scenario. For this purpose it provides an ideal subject—the scale $f$ is experimentally constrained to not be that associated with either of the branes. These constraints arise, moreover, from low energy physics with respect to the electroweak scale so that bulk effects do not allow us to modify these bounds as in scenarios with large extra dimensions [7]. We shall see that adding a further brane in the bulk produces an experimentally reasonable value for $f$ without any new excessive fine tunings.

Section II discusses the origin of the scale $f$ associated with the breaking of $U(1)_{\mathrm{PQ}}$ when the axion is the phase of a complex bulk scalar field. Section III discusses how the scale $f$ arises when a third brane is added to the bulk and analyzes the problem of stabilizing all the branes with a single real scalar field. In Sec. IV we show that it is more difficult to generate an acceptable value for $f$ using a bulk potential for the complex field containing the axion. Section V concludes.

## II. THE INVISIBLE AXION AS A BULK FIELD

The action for the original Randall-Sundrum model contains an Einstein-Hilbert term and cosmological constant for

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the bulk as well as tension terms for the branes

$$S_{\mathrm{RS}}=M_5^3\int d^4x dy\sqrt{-g}[-2\Lambda+R]$$
$$+M_5^3\int_{\mathrm{UV}}d^4x\sqrt{-h_0}[-2\sigma_0+4K_0]$$
$$+M_5^3\int_{\mathrm{IR}}d^4x\sqrt{-h_1}[-2\sigma_1+4K_1+M_5^{-3}\mathcal{L}_{\mathrm{sm}}]. \tag{2.1}$$

Here $h_{0,1}$ and $K_{0,1}$ are the determinant of the induced metric and the trace of the extrinsic curvature on the UV and IR branes. In terms of the AdS curvature $k$, the cosmological constant is $\Lambda=-6k^2$ and the brane tensions should be $\sigma_0=-\sigma_1=6k$. $\mathcal{L}_{\mathrm{sm}}$ represents the standard model Lagrangian. The UV and IR branes are located at $y=0$ and $y=\Delta y$, respectively.

To solve the strong *CP* problem in the low energy theory, we introduce a global $U(1)_{\mathrm{PQ}}$ symmetry under which the brane quark and Higgs fields transform non-trivially [2]. Since the scale of Peccei-Quinn symmetry breaking does not lie near the scales associated with either brane, it is natural to attempt to break this symmetry through bulk dynamics. Thus, the axion will correspond to the phase of a bulk complex scalar field,

$$\sigma=\frac{\rho}{\sqrt{2}}e^{ia}. \tag{2.2}$$

The dynamics of this field will be determined by a $U(1)_{\mathrm{PQ}}$-symmetric potential:

$$S_\sigma=\int d^4x dy\sqrt{-g}[-\nabla_a\sigma^\dagger\nabla^a\sigma-V(\sigma^\dagger\sigma)]$$
$$+\int_{\mathrm{UV}}d^4x\sqrt{-h_0}\mathcal{V}_0(\sigma^\dagger\sigma)$$
$$+\int_{\mathrm{IR}}d^4x\sqrt{-h_1}\mathcal{V}_1(\sigma^\dagger\sigma). \tag{2.3}$$

Here, as in Goldberger and Wise [6], the potentials on the branes will be used to fix the value of the field $\rho$ on the UV and IR branes to be, respectively, $\rho_0 M_5^{3/2}$ and $\rho_1 M_5^{3/2}$.

The form of the $U(1)_{\mathrm{PQ}}$ symmetry breaking due to the bulk potential $V(\rho)$ is quite different from the invisible axion solution in $3+1$ dimensions. A vacuum solution in which $\rho=f$ for the bulk theory would necessarily require some fine tuning of the bulk potential to obtain a realistic $f$. Instead, $\rho$ can have some non-trivial dependence on the extra dimension which also breaks the $U(1)_{\mathrm{PQ}}$. In going to the low energy effective theory, integrating out the bulk field $\rho$ will induce a scale $f$ for the axion which plays the same role as the symmetry breaking scale in the 4d invisible axion models.

The field equations for the components of $\sigma$ are

$$\nabla^2 a+\frac{2}{\rho}\nabla_a\rho\nabla^a a=0$$
$$\nabla^2\rho-\frac{\delta V}{\delta\rho}-\rho\nabla_a a\nabla^a a=0. \tag{2.4}$$

The important dynamics of the axion occurs at energies well below the TeV scale, beyond which bulk effects become important. In this low energy regime, we shall neglect the higher-order Kaluza-Klein modes of the axion which, since it is a Goldstone mode, will have a massless mode which remains in the effective theory. Thus we shall consider only the lowest mode in the Kaluza-Klein tower, $a\to a(x^\mu)$. This situation differs greatly from a bulk axion in models with large, flat extra dimensions where the Kaluza-Klein modes of the axion are of the order of the inverse compactification radius and are important in the low energy ($\ll$TeV) theory [7]. The field $\rho$ is not protected by any symmetry and its vacuum state is determined by the bulk potential $V(\rho)$ so we shall neglect any $x^\mu$-dependent fluctuations about the vacuum configuration, $\rho=\rho(y)$, as small in the effective theory,

$$\rho''-4k\rho'\approx\frac{\delta V}{\delta\rho},\quad \partial_\mu\partial^\mu a\approx 0. \tag{2.5}$$

The axion from this perspective becomes a massless field while the scalar field $\rho$ has its dynamics set by the scale of the bulk physics. At energies below a TeV, we can integrate out $\rho(y)$ to obtain an effective description of the axion dynamics,

$$S_\sigma=\int d^4x\left[-\frac{1}{2}\left(\int_0^{\Delta y}dy\,2e^{-2ky}\rho^2(y)\right)\partial_\mu a\partial^\mu a\right]$$
$$=\int d^4x\left[-\frac{1}{2}f^2\partial_\mu a\partial^\mu a+\cdots\right], \tag{2.6}$$

where we have defined

$$f^2\equiv\int_0^{\Delta y}dy\,2e^{-2ky}\rho^2(y) \tag{2.7}$$

which sets the scale associated with the axion by rescaling

$$a(x^\mu)\to\frac{a(x^\mu)}{f}. \tag{2.8}$$

After this rescaling, the axion has the proper dimensions for a scalar field in the 4d effective theory.

The remaining components needed to implement a solution to the strong *CP* problem closely resemble those found in standard invisible axion models. Typically such models introduce heavy quarks which carry $U(1)_{\mathrm{PQ}}$ charge and couple to $\sigma$—Kim-Shifman-Vainshtein-Zakharov (KSVZ) axions [8]—or an extra Higgs doublet is added which couples to $\sigma$—Dine-Fischler-Srednicki-Zhitnitsky (DFSZ) axions [9]. To obtain the latter model within a Randall-

Sundrum scenario, we add an interaction between a pair of brane Higgs doublets $\Phi_1$ and $\Phi_2$ and the bulk complex field $\sigma$:

$$S_{\rm int}=\int_{\rm IR} d^4x\sqrt{-h_1}\,[\kappa M_5^{-1}\epsilon_{ij}\Phi_1^i\Phi_2^j(\sigma^\dagger(\Delta y))^2+\text{H.c.}]. \tag{2.9}$$

Here we have extracted a factor of the Planck mass so that $\kappa$ is a dimensionless coupling. Using that on the IR brane, $\sigma(\Delta y)=(1/\sqrt{2})\rho_1 M_5^{3/2}e^{ia}$, and rescaling the axion using Eq. (2.8) and the Higgs fields by $\Phi_{1,2}\to e^{k\Delta y}\Phi_{1,2}$ so that they have canonically normalized kinetic terms, the leading behavior from Eq. (2.9) in the low energy limit is

$$S_{\rm int}=\int d^4x[\kappa_{\rm eff}\epsilon_{ij}\Phi_1^i\Phi_2^j e^{-2ia/f}+\text{H.c.}], \tag{2.10}$$

where

$$\kappa_{\rm eff}\equiv\frac{1}{2}\kappa\rho_1^2(e^{-k\Delta y}M_5)^2\sim\mathcal{O}(\text{TeV}^2). \tag{2.11}$$

The standard model fields confined to the IR brane also have $U(1)_{\rm PQ}$ charges which we shall choose to be $+\frac{1}{2}$ for the right-handed fermions and $-\frac{1}{2}$ for the left-handed $SU(2)$ doublet fermions. With these assignments, the Higgs fields have $U(1)_{\rm PQ}$ charges $+1$ so that Eq. (2.10) is an invariant interaction. The fact that the Higgs fields transform nontrivially under $U(1)_{\rm PQ}$ allows some of their degrees of freedom to mix with the massless mode in the effective theory that arises when we integrate out the extra dimension (2.6). At this point the theory is essentially indistinguishable from the 4d invisible axion model.

## III. AN INTERMEDIATE BRANE

The energy scale associated with the standard model fields remains naturally light since they are confined to the IR brane at which the redshift suppresses the strength gravity by an exponential factor. Similarly, the introduction of another brane, at some intermediate distance in the bulk, $0<y_a<\Delta y$, will produce a new energy scale $e^{-ky_a}M_4$. A simple mechanism for achieving a reasonable value for the axion scale occurs when the bulk complex scalar field is free with a mass of $m_\rho$. If the brane potentials mainly act to force $\rho$ to assume natural values on the intermediate and IR branes, $\rho(y_a)=\rho_a M_5^{3/2}$ and $\rho(\Delta y)=\rho_1 M_5^{3/2}$, respectively, with $\rho_a,\rho_1\sim\mathcal{O}(1)$, and to vanish on the UV brane, $\rho(0)=0$, then integrating over the bulk yields a scale

$$\frac{f}{M_4}\approx\frac{\sqrt{2}(2+m_\rho^2k^{-2})^{1/4}}{(3+m_\rho^2k^{-2})^{1/2}}\rho_a e^{-ky_a}. \tag{3.1}$$

Since we have assumed that the brane potentials in Eq. (2.3) are $U(1)_{\rm PQ}$ symmetric, as long as they are analytic functions of $\rho$, $\rho=0$ will be an extremum on the branes so we do not need to fine tune $\rho(0)=0$ to be a minimum.

To generate the scale for the axion, the relative positions of all three branes must be stabilized. In this section we shall show how the introduction of a *single* real Goldberger-Wise [6] field stabilizes *both* radion degrees of freedom that correspond to the two independent distances between pairs of branes. For simplicity we shall neglect the effect of the bulk complex scalar $\rho$ when analyzing the brane stabilization.

Consider a bulk space-time in which the UV and IR branes reside as usual at the fixed points of the orbifold, $y=0$ and $y=\Delta y$, respectively, while an intermediate brane partitions the bulk into two regions with cosmological constants $\Lambda_0=-6k_0^2$ $(0\leqslant y\leqslant y_a)$ and $\Lambda_1=-6k_1^2$ $(y_a\leqslant y\leqslant\Delta y)$. Matching the induced metric on both sides of the axion brane, the bulk metric can be written in the form

$$ds^2=e^{-2k_0y}\eta_{\mu\nu}dx^\mu dx^\nu+dy^2 \tag{3.2}$$

for $0\leqslant y\leqslant y_a$ and

$$ds^2=e^{-2k_1y}e^{-2(k_0-k_1)y_a}\eta_{\mu\nu}dx^\mu dx^\nu+dy^2 \tag{3.3}$$

for $y_a\leqslant y\leqslant\Delta y$. The Israel jump conditions across the branes require the UV, IR and axion branes to have tensions, respectively, of

$$\sigma_0=6k_0,\quad\sigma_1=-6k_1,\quad\sigma_a=3(k_0-k_1). \tag{3.4}$$

Note that when the cosmological constants are equal, $k_0=k_1=k$, the axion brane becomes a tensionless "probe" brane. For simplicity, we shall consider this case in the following analysis.

Equation (3.4) summarizes the three fine tunings necessary for this model. One of these fine tunings is equivalent to the vanishing of the cosmological constant in the low energy effective theory. As in the Randall-Sundrum scenario, we shall not attempt to resolve this fine tuning. In a scenario with a further extra dimension, this vanishing can be reduced to the tuning of the initial conditions rather than a tuning of the parameters in the gravitational action [10].

The remaining two fine tunings in Eq. (3.4) correspond to tuning the two potentials for the positions of the IR and axion brane, relative to the UV brane, to be flat. The introduction of a bulk scalar produces an effective potential, $V_{\rm eff}(y_a,\Delta y)$, which breaks both of the symmetries associated with arbitrarily changing $y_a$ and $\Delta y$.

Let us examine a single massive bulk scalar field,

$$S_\phi=M_5^3\int d^5x\,\sqrt{-g}\left[-\frac{1}{2}\nabla_a\phi\nabla^a\phi-\frac{1}{2}m^2\phi^2\right] \tag{3.5}$$

with a mass $m=k\sqrt{\nu^2-4}$; we also define $\nu=2+\epsilon$. Note that we have extracted a factor of $M_5^3$ so that $\phi(y)$ is dimensionless. As in the standard Goldberger-Wise mechanism, we assume that the actions on the three branes,

$$S_\phi^{\rm brane}=M_5^3\int_{y=0} d^4x\sqrt{-h}[-\lambda_0(\phi^2-v_0^2)^2]$$
$$+M_5^3\int_{y=y_a} d^4x\sqrt{-h}[-\lambda_a(\phi^2-v_a^2)^2]$$
$$+M_5^3\int_{y=\Delta y} d^4x\sqrt{-h}[-\lambda_1(\phi^2-v_1^2)^2] \quad (3.6)$$

essentially act to fix the value of the scalar field to be $v_0$, $v_a$ and $v_1$ on the UV, axion and IR branes, respectively. $h$ represents the determinant of the induced metric on the appropriate brane.

The scalar field satisfies a Klein-Gordon equation in the bulk and its solution in each of the two bulk regions is

$$\phi(y)=\begin{cases}\phi_0(y) & \text{for } 0\leqslant y\leqslant y_a,\\ \phi_1(y) & \text{for } y_a\leqslant y\leqslant \Delta y,\end{cases} \quad (3.7)$$

where

$$\phi_0(y)=-\frac{v_0e^{-(\nu-2)ky_a}-v_a}{1-e^{-2\nu ky_a}}e^{(\nu+2)k(y-y_a)}$$
$$+\frac{v_0-v_ae^{-(\nu+2)ky_a}}{1-e^{-2\nu ky_a}}e^{-(\nu-2)ky}$$
$$\phi_1(y)=-\frac{v_ae^{-(\nu-2)k(\Delta y-y_a)}-v_1}{1-e^{-2\nu k(\Delta y-y_a)}}e^{(\nu+2)k(y-\Delta y)}$$
$$+\frac{v_a-v_1e^{-(\nu+2)k(\Delta y-y_a)}}{1-e^{-2\nu k(\Delta y-y_a)}}e^{-(\nu-2)k(y-y_a)}. \quad (3.8)$$

Integrating the scalar field action over the extra dimension produces an effective potential for $y_a$ and $\Delta y$,

$$V_{\rm eff}(y_a,\Delta y)=M_5^3\int dy e^{-4ky}\,[\nabla_a\phi\nabla^a\phi+m^2\phi^2], \quad (3.9)$$

which becomes, in terms of $z_a\equiv e^{-ky_a}$ and $z_1\equiv e^{-k\Delta y}$,

$$\frac{V_{\rm eff}(z_a,z_1)}{kM_5^3}=\frac{(v_0z_a^{\nu-2}-v_a)^2}{1-z_a^{2\nu}}z_a^4(\nu+2)$$
$$+\frac{(v_0-v_az_a^{\nu+2})^2}{1-z_a^{2\nu}}(\nu-2)$$
$$+\frac{[v_a(z_1/z_a)^{\nu-2}-v_1]^2}{1-(z_1/z_a)^{2\nu}}z_1^4(\nu+2)$$
$$+\frac{[v_a-v_1(z_1/z_a)^{\nu+2}]^2}{1-(z_1/z_a)^{2\nu}}z_a^4(\nu-2). \quad (3.10)$$

If this potential is to stabilize both of the radion parameters, then $z_a$ and $z_a$ are set by

$$\frac{\partial V_{\rm eff}}{\partial\Delta y}=-kz_1\frac{\partial V_{\rm eff}}{\partial z_1}=0$$
$$\frac{\partial V_{\rm eff}}{\partial y_a}=-kz_a\frac{\partial V_{\rm eff}}{\partial z_a}=0. \quad (3.11)$$

These first partial derivatives are

$$\frac{z_a}{kM_5^3}\frac{\partial V_{\rm eff}}{\partial z_a}=-\frac{z_1}{kM_5^3}\frac{\partial V_{\rm eff}}{\partial z_1}+\frac{2z_a^4}{1-z_a^{2\nu}}$$
$$\times\Bigg\{(\nu^2-4)z_a^{\nu-2}[v_0^2z_a^{\nu-2}-2v_0v_a+v_a^2z_a^{\nu+2}]$$
$$+2(\nu+2)[v_0z_a^{\nu-2}-v_a]^2$$
$$+\nu(\nu+2)z_a^{2\nu}\frac{[v_0z_a^{\nu-2}-v_a]^2}{1-z_a^{2\nu}}$$
$$+\nu(\nu-2)z_a^{2(\nu-2)}\frac{[v_0-v_az_a^{\nu+2}]^2}{1-z_a^{2\nu}}\Bigg\}$$
$$+\frac{4z_a^4}{1-(z_1/z_a)^{2\nu}}$$
$$\times\Bigg\{(\nu+2)\left(\frac{z_1}{z_a}\right)^4\left(v_a\left[\frac{z_1}{z_a}\right]^{\nu-2}-v_1\right)^2$$
$$+(\nu-2)\left(v_a-v_1\left[\frac{z_1}{z_a}\right]^{\nu+2}\right)^2\Bigg\} \quad (3.12)$$

and

$$z_1\frac{\partial V_{\rm eff}}{\partial z_1}=\frac{2z_1^4}{1-(z_1/z_a)^{2\nu}}$$
$$\times\Bigg\{(\nu^2-4)\left[\frac{z_1}{z_a}\right]^{\nu-2}\left(v_a^2\left[\frac{z_1}{z_a}\right]^{\nu-2}-2v_av_1\right)$$
$$+(\nu^2-4)v_1^2\left[\frac{z_1}{z_a}\right]^{2\nu}$$
$$+2(\nu+2)\left(v_a\left[\frac{z_1}{z_a}\right]^{\nu-2}-v_1\right)^2$$
$$+\nu(\nu-2)\left[\frac{z_1}{z_a}\right]^{2(\nu-2)}\frac{[v_a-v_1(z_1/z_a)^{\nu+2}]^2}{1-(z_1/z_a)^{2\nu}}$$
$$+\nu(\nu+2)\left[\frac{z_1}{z_a}\right]^{2\nu}\frac{[v_a(z_1/z_a)^{\nu-2}-v_1]^2}{1-(z_1/z_a)^{2\nu}}\Bigg\}. \quad (3.13)$$

Both $z_a$ and $z_1$ are exponentially small so higher powers of these factors in Eqs. (3.12) and (3.13) contribute negligibly. The structure of the matrix of second derivatives is such that as long as the intermediate brane is not too close to the UV brane, one eigenvalue will be of the order $z_a^4$ while the other will be of the order $z_1^4$. This structure breaks down when $z_a^3 \sim z_1$ in which case the smaller eigenvalue receives corrections of the order $z_a^{12} \sim z_1^4$. For the scale required for the axion, the intermediate brane will be sufficiently far from the UV brane that we are well within the $z_1 \gg z_a^3$ regime. Retaining terms up to order $z_1^4$, we have

$$\frac{z_a}{kM_5^3}\frac{\partial V_{\text{eff}}}{\partial z_a} = 4(2+\epsilon)z_a^4[(2+\epsilon)v_0^2 z_a^{2\epsilon} - (4+\epsilon)v_0 v_a z_a^{\epsilon} + 2v_a^2] - 4\epsilon(2+\epsilon)z_1^4 v_a\left[\frac{z_1}{z_a}\right]^{\epsilon}\left(v_a\left[\frac{z_1}{z_a}\right]^{\epsilon} - v_1\right) + \mathcal{O}(z_a^8, z_1^8),$$

$$\frac{z_1}{kM_5^3}\frac{\partial V_{\text{eff}}}{\partial z_1} = 4z_1^4\left((2+\epsilon)^2 v_a^2\left[\frac{z_1}{z_a}\right]^{2\epsilon} - (2+\epsilon)(4+\epsilon)v_a v_1\left[\frac{z_1}{z_a}\right]^{\epsilon} + (4+\epsilon)v_1^2\right) + \mathcal{O}(z_1^8). \tag{3.14}$$

The second equation determines the equilibrium distance between the intermediate and IR branes,

$$\left[\frac{z_1}{z_a}\right]^{\epsilon} = \frac{1}{2(2+\epsilon)}\frac{v_1}{v_a}[(4+\epsilon) + \sqrt{\epsilon(4+\epsilon)}], \tag{3.15}$$

while the first equation, after using Eq. (3.15), determines the distance between the intermediate and UV branes,

$$z_a^{\epsilon} = \frac{v_a}{v_0}\left(1 + \frac{\sqrt{\epsilon(4+\epsilon)}}{(2+\epsilon)^2}\left[\frac{z_1}{z_a}\right]^4 \frac{v_1^2}{v_a^2}\right). \tag{3.16}$$

In Eqs. (3.15) and (3.16) we have implicitly chosen the root which corresponds to a minimum in both the $z_a$ and $z_1$ directions. At these extrema, the second partial derivatives are, to leading order in $\epsilon$ and in the small exponentials,

$$\frac{\partial^2 V_{\text{eff}}}{\partial y_a^2} = 8\epsilon^2 v_a^2 M_4^2 e^{-4ky_a}$$
$$\frac{\partial^2 V_{\text{eff}}}{\partial \Delta y^2} = 16\epsilon^{3/2} v_1^2 M_4^2 e^{-4k\Delta y} \tag{3.17}$$
$$\frac{\partial^2 V_{\text{eff}}}{\partial y_a \partial \Delta y} = -16\epsilon^{3/2} v_1^2 M_4^2 e^{-4k\Delta y}.$$

The eigenvalues of this matrix of second derivatives are both positive,

$$8\epsilon^2 v_a^2 M_4^2 e^{-4ky_a} \quad \text{and} \quad 16\epsilon^{3/2} v_1^2 M_4^2 e^{-4k\Delta y}, \tag{3.18}$$

so the relative positions of the branes are stable.

The bulk complex scalar which contains the axion might disrupt the stabilization of the branes if it produces too large a contribution to the total effective potential. However, the contribution from the complex field will be negligible compared to that arising from integrating out the field $\phi$ as long as $v_0, v_a, v_1 \gg \rho_a, \rho_1$.

### Multiple branes

In the preceding example with a single intermediate brane, we observed that for each brane we have one fine tuning. Aside from the one fine tuning for the effective cosmological constant at low energies, these fine tunings can be interpreted as requiring the potentials for the radions—for the relative separations among the branes—to vanish. A single bulk scalar field was sufficient to break the flatness with respect to both independent interbrane distances. This technique can be extended to scenarios with multiple intermediate branes which could then have several large mass scales appearing in the low energy theory.

As an example, consider a theory with two intermediate branes at positions $y = y_a$ and $y = y_b$. Including a free massive scalar field in the bulk with potentials on these branes such that $\phi(y_a) = v_a$ and $\phi(y_b) = v_b$ but otherwise identical to the case above, we find that the branes are stabilized at

$$ky_a = \frac{1}{\epsilon}\ln\left[\frac{v_0}{v_a}\right], \quad ky_b = \frac{1}{\epsilon}\ln\left[\frac{v_0}{v_b}\right] \tag{3.19}$$

and

$$k\Delta y = -\frac{1}{\epsilon}\ln\left(\frac{1}{2}\frac{1}{2+\epsilon}\frac{v_1}{v_0}[4+\epsilon+\sqrt{\epsilon(4+\epsilon)}]\right) \tag{3.20}$$

to leading order in powers of the small exponentials. The matrix of second derivatives for $y_a$, $y_b$ and $\Delta y$, given by Eqs. (3.19) and (3.20), has eigenvalues which are all positive,

$$\lambda_a \approx 8\epsilon^2 v_a^2 M_4^2 e^{-4ky_a}$$
$$\lambda_b \approx 8\epsilon^2 v_b^2 M_4^2 e^{-4ky_b} \tag{3.21}$$
$$\lambda_1 \approx 16\epsilon^{3/2} v_1^2 M_4^2 e^{-4k\Delta y},$$

to leading order in $\epsilon$. In deriving Eq. (3.21) we have assumed that the branes are sufficiently separated, $e^{-ky_b} \gg e^{-3ky_a}$ and $e^{-k\Delta y} \gg e^{-3ky_b}$, for the eigenvalues to assume this form.

## IV. BULK POTENTIALS

While we can arrange for an invisible axion with the correct symmetry breaking scale $f$ without any unnatural constraints on the theory, it becomes more difficult to avoid some fine tuning when we attempt to use a bulk potential to produce this behavior. In this section we shall determine how carefully we need to tune the form of simple potentials for the bulk complex field to produce a reasonable value for this scale.

As we saw for the inclusion of an intermediate brane, if $\rho \ll 1$ until some intermediate position $y_a$, then the warping factor from the bulk metric yields $f \sim e^{-ky_a} M_4$. We can obtain some intuition as to the necessary form for the bulk potential by noting that the field equation for $\rho$,

$$\rho'' - 4k\rho' = \frac{\delta V}{\delta \rho}, \tag{4.1}$$

is that of a particle rolling in the inverted potential, $-V(\rho)$, under the influence of a *negative* friction term. If the particle starts at $\rho=0$ with a small initial velocity, it tends to accelerate. Thus, in regions where the potential is approximately constant, its value will be exponentially larger after a finite interval or, conversely, throughout most of the interval $\rho(y)$ will be exponentially small. This evolution should not occur throughout the entire bulk since then the integral (2.7) would then only produce an $f \sim e^{-k\Delta y} M_4 \sim \mathrm{TeV}$. If $V(\rho)$ decreases substantially after $\rho$ has become sufficiently large, this change will act to dissipate the "kinetic energy" produced by the friction term and $\rho$ will grow more slowly so that $\rho$ is not exponentially weighted toward that latter end of this stage of its evolution in the bulk. After this dissipative stage, we could follow it with another region in which $V(\rho)$ is approximately flat—as long as $\rho$ does not grow exponentially larger than its values during the prior stage before it reaches the IR brane, the exponential factor in Eq. (2.7) ensures that the integral will be dominated by intermediate values of $y$.

### A. A free massive bulk field

The simplest potential is a mass term for the bulk scalar. From the preceding arguments, a positive mass squared term will have the effect of accelerating the growth of the field already produced by the friction term as we move from the UV to the IR brane. Although it might seem that a negative mass term could slow the effects of the friction term, we shall see that this case also does not produce an acceptable value for $f$ without some fine tuning. Since the field equations for this potential can be solved exactly, we present both cases.

Consider a generalized mass term, to allow for either a stable or unstable extremum at $\rho=0$,

$$V(\rho) = \frac{1}{2}(\mu-4)k^2\rho^2, \tag{4.2}$$

where $\mu$ is a dimensionless parameter. When $\mu>0$, the solution is

$$\rho(y) = \rho_1 M_5^{3/2} \frac{e^{2ky}}{e^{2k\Delta y}} \frac{\sinh(\sqrt{\mu}ky)}{\sinh(\sqrt{\mu}k\Delta y)} \tag{4.3}$$

while for $\mu<0$,

$$\rho(y) = \rho_1 M_5^{3/2} \frac{e^{2ky}}{e^{2k\Delta y}} \frac{\sin(\sqrt{-\mu}ky)}{\sin(\sqrt{-\mu}k\Delta y)}. \tag{4.4}$$

Here we have imposed the boundary conditions $\rho(0)=0$ and $\rho(\Delta y)=\rho_1$. For $\mu>0$, the scale $f$ is always of the order

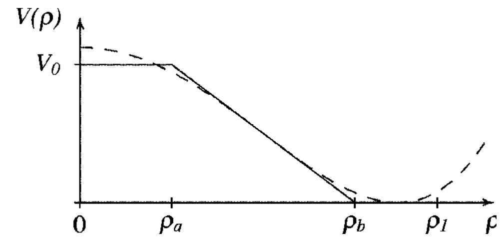


FIG. 1. We shall study the profile of a bulk field $\rho$ whose potential is given by the toy model shown by the solid line. It can be regarded as a crude model for $0 \leq \rho \leq \rho_1$ of a quartic, double-well potential shown by the dashed line.

$$f/M_4 \sim \rho_1 e^{-k\Delta y} \tag{4.5}$$

which is too small for $\rho_1$ of a natural size, $\mathcal{O}(1)$. In fact $\rho_1$ should be somewhat small if the presence of the scalar field is not to distort the background $\mathrm{AdS}_5$ geometry. For $\mu<0$ the zeros of the denominator can generate much larger scales than (4.5); when $\sqrt{-\mu}k\Delta y = n\pi - \delta$ ($n=1,2,3,\ldots$),

$$f/M_4 \sim \rho_1 \frac{\sqrt{-\mu}}{\sqrt{1-\mu}} \frac{e^{-k\Delta y}}{\delta}, \tag{4.6}$$

but only if we finely tune $\delta \lesssim 10^{-6}$.

Note that in either case, if we relax the requirement $\rho(0)=\rho_0=0$ then $\rho_0$ must itself be tuned to be of the order $\leq 10^{-6}$. In this case $\mu$ should also satisfy $\mu>1$ since otherwise the large negative mass squared favors an exponentially large value of $\rho(y)$ within the bulk so that we would find $f \gg M_4$.

### B. A potential well

The reason that a mass term alone does not succeed is that the potential contains no feature which might allow a brief growth of $\rho(y)$ which appears in the vicinity of $0<y_a<\Delta y$ but which is damped soon after so that the field assumes an $\mathcal{O}(1)$ value at the IR brane.

To model this behavior with a potential which we can solve exactly, we shall study the following toy potential:

$$V(\rho(y)) = \begin{cases} V_0 & \text{for } \rho \leq \rho_a \\ V_0\left[1 - \dfrac{\rho(y)-\rho(y_a)}{\Delta\rho}\right] & \text{for } \rho_a \leq \rho \leq \rho_b \\ 0 & \text{for } \rho \geq \rho_b \end{cases} \tag{4.7}$$

where

$$\Delta\rho = \rho_b - \rho_a. \tag{4.8}$$

The parameters specifying this potential are $\rho_a$, $\rho_b$ and $V_0$. We next shall estimate how carefully the form of the potential must be tuned to achieve an acceptable value for $f$.

Naively, Eq. (4.7) resembles a portion of a double well potential seen in the vicinity of the origin, as shown in Fig. 1, and we assume that $V(-\rho)=V(\rho)$. Note that $V(\rho)$ could grow again for larger values of $\rho$, but as long as this growth occurs for values $\rho(y) > \rho_1$, it will not affect our derivation. Note also that we are implicitly assuming that $\rho(y)$ is a monotonically increasing function of $y$, which occurs provided the well is not so deep that all the "kinetic energy" is dissipated and the particle rolls back toward $\rho=0$. It is also important that $\rho(0)=0$ at the UV brane, which as in the intermediate brane case can be arranged without any additional fine tuning, since it is difficult for any natural potential to suppress it quickly enough to prevent the small $y$ region from dominating Eq. (2.7). We shall assume hereafter that $\rho_0=0$ and $\rho_1 \approx \mathcal{O}(1)$.

Let us define positions $y_a < y_b$ such that $\rho(y_a)=\rho_a$ and $\rho(y_b)=\rho_b$. The solution to Eq. (4.1) for this toy potential with the boundary conditions $\rho(0)=0$ and $\rho(\Delta y)=\rho_1$ is then

$$\rho(y)=c(e^{4ky}-1) \tag{4.9}$$

for $0<y<y_a$,

$$\rho(y)=c(e^{4ky}-1)-\frac{1}{16k^2}\frac{V_0}{\Delta\rho}(e^{4k(y-y_a)}-1)+\frac{1}{4k}\frac{V_0}{\Delta\rho}(y-y_a) \tag{4.10}$$

for $y_a<y<y_b$ and

$$\rho(y)=c(e^{4ky}-1)-\frac{1}{16k^2}\frac{V_0}{\Delta\rho}(e^{-4ky_a}-e^{-4ky_b})e^{4ky}+\frac{1}{4k}\frac{V_0}{\Delta\rho}(y_b-y_a) \tag{4.11}$$

for $y_b<y$. For convenience we have defined the constant $c$ to be

$$c \equiv \frac{\rho_1 M_5^{3/2}}{e^{4k\Delta y}-1}+\frac{1}{16k^2}\frac{V_0}{\Delta\rho}(e^{-4ky_a}-e^{-4ky_b})\frac{e^{4k\Delta y}}{e^{4k\Delta y}-1}-\frac{1}{4k}\frac{V_0}{\Delta\rho}\frac{y_b-y_a}{e^{4k\Delta y}-1}. \tag{4.12}$$

To learn whether the potential requires any fine tunings, we can reparametrize the slope of the potential in terms of the natural scales available, $k$, $M_5$,

$$\frac{V_0}{\Delta\rho} \equiv 16k^2 M_5^{3/2}\alpha \tag{4.13}$$

where $\alpha$ should be some constant of order one. To leading order in powers of the exponential factors, the integral (2.7) is then

$$\begin{aligned} f^2/M_4^2 = {} & \frac{40}{3}\alpha^2 e^{-2ky_a}-\frac{32}{3}\alpha^2[1+3k(y_b-y_a)]e^{-2ky_b} \\ & -\frac{8}{3}\alpha^2 e^{-2k(y_b-y_a)}e^{-2ky_b}+\frac{1}{3}[\rho_1^2+16\rho_1 k \\ & \times(y_b-y_a)\alpha-128k^2(y_b-y_a)^2\alpha^2]e^{-2k\Delta y} \\ & -\frac{8}{3}\alpha[\rho_1-4k(y_b-y_a)\alpha] \\ & \times(1-e^{-2k(y_b-y_a)})e^{-2k(\Delta y-y_b)}e^{-2k\Delta y}+\cdots. \end{aligned} \tag{4.14}$$

For $e^{-ky_a} \gg e^{-ky_b} \gg e^{-k\Delta y}$ we have

$$\frac{f}{M_4} \approx 2\sqrt{\frac{10}{3}}\alpha e^{-ky_a}. \tag{4.15}$$

The chief contribution to Eq. (2.7) comes from the region $y \sim y_a$.

We can now show that to achieve a realistic value for $y_a$ requires finely tuning the potential. In terms of the parameters $y_a, y_b$ of the solution we have

$$\begin{aligned} \rho_a M_5^{-3/2} &= \alpha[1-e^{-4k(y_b-y_a)}]+\cdots \\ \rho_b M_5^{-3/2} &= 4k\alpha(y_b-y_a)+e^{-4k(\Delta y-y_b)} \\ & \quad \times[\rho_1-4k\alpha(y_b-y_a)]+\cdots. \end{aligned} \tag{4.16}$$

Note that $\rho_a$, $\rho_b$ and $\alpha$ are the parameters specifying the shape of the potential. The first line in Eq. (4.16) indicates that we must tune $\alpha^{-1}\rho_a M_5^{-3/2} \approx 1$ to within a fractional correction of the order $e^{-4k(y_b-y_a)}$. We can evade this fine tuning if $y_b \sim y_a$; however, then we must finely tune the value of $\rho_b$ to within an order $e^{-4k(\Delta y-y_b)}$ correction. From Eq. (4.15) and Eq. (1.2), $16 \lesssim ky_a \lesssim 23$ and an electroweak-Planck hierarchy of $10^{-16}$ requires $k\Delta y \sim 37$. Assuming $y_b \sim y_a$ yields then an exponentially small correction.

## V. CONCLUSIONS

The original Randall-Sundrum scenario contains only two, widely separated, energy scales associated, respectively, with the bulk and the IR brane physics. Since these scales could naturally be exponentially different, this model provides an attractive alternative explanation of the hierarchy between the gravitational and electroweak physics. In this article we have shown that other, intermediate scales can be incorporated into the Randall-Sundrum scenario, such as are needed for the invisible axion solution to the strong *CP* problem.

The basic requirement for generating a scale much lower than the Planck mass from a bulk field is to find a mechanism which excludes this field from the region near the UV brane. The introduction of an intermediate brane produces this behavior. When the potential on the UV brane causes the field to vanish there, most of the contribution to the effective symmetry breaking scale comes from the region between the intermediate and the IR brane. The position of the intermediate brane then determines the scale needed by the invisible axion. In this scenario, the $U(1)_{\mathrm{PQ}}$ symmetry breaking proceeds differently than in standard $3+1$ picture since it results from the non-trivial profile of the bulk complex scalar field whose phase is the axion.

A single additional scalar field is needed to stabilize the two independent distances between the pairs of branes. From a low energy perspective, this field produces an effective potential which breaks the necessity to tune the potentials for the two radion parameters to be flat. This stabilization only requires a very mild tuning to ensure that the complex scalar does not disrupt the mechanism. The resulting radions have masses of the order of the $U(1)_{\mathrm{PQ}}$ breaking scale and near the electroweak scale, respectively.

While the invisible axion is not the only solution to the strong *CP* problem in extra dimensional scenarios—for example, the QCD gauge fields could be promoted to bulk fields [12,13]—it provides an intriguing example of a warped geometry with multiple scales exponentially below the Planck mass. More generally, the existence of intermediate scales allows the possibility of a hidden sector which is naturally suppressed by a large, but not-Planckian, mass. It would also be interesting to understand the origin of multiple mass scales from the perspective of the AdS/CFT correspondence [11].

## ACKNOWLEDGMENT

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