Randall-Sundrum I cosmology as brane dynamics in an AdS-Schwarzschild bulk

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We explore various facets of the cosmology of the Randall-Sundrum scenario with two branes by considering the dynamics of the branes moving in a bulk AdS-Schwarzschild geometry. This approach allows us to understand both in more detail and from a different perspective the role of the stabilization of the hierarchy in the brane cosmology, as well as to extend to the situation where the metric contains a horizon. In particular, we explicitly determine how the Goldberger-Wise stabilization mechanism perturbs the background bulk geometry to produce a realistic cosmology.

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I. INTRODUCTION

The cosmology of Randall-Sundrum (RS) [1,2] models is of great interest for a variety of reasons, not the least of which is that these models allow us to understand some longstanding problems of particle physics, most notably the hierarchy problem of mass scales.

Kraus [3] (see in particular [4–7] as well as [8,9]) has made the interesting observation that, for the so-called RS2 models [2], the brane cosmology can be found by solving the Israel junction conditions [10] for a domain wall moving in an AdS-Schwarzschild (AdSS) bulk geometry, which, by Birkhoff's theorem in five dimensions, is the unique spherically symmetric solution to the bulk Einstein equations with a bulk cosmological constant Λ.

While the RS2 alternative to compactification is extremely interesting from many points of view, such as the AdS/CFT (conformal field theory) correspondence [11], it could be argued that from a particle physics perspective RS1 geometries [1] are more useful, allowing as they do for solutions to the hierarchy problem. The cosmology of RS1 geometries has been studied extensively in the literature [12–16], and a number of interesting results have been found. The main difference between RS1 and RS2 cosmologies is that the existence of *two* branes (the TeV and Planck branes) in the RS1 case allows the interbrane separation to become a dynamical degree of freedom, the so-called *radion*. Depending on whether or not the radion has been stabilized in some way, such as with the Goldberger-Wise mechanism (GW) [17] or in some other way, the cosmological consequences and even consistency of RS1 models can be quite different.

Our purpose in this work is to treat the cosmology of RS1 models using the Kraus approach. That is to say, we consider the two branes to be moving in an AdSS bulk geometry and ask once again what is happening to the intrinsic brane geometry as specified through the Israel conditions. There are (at least) two reasons for doing this. One is that this will allow us to understand some of the previously known results from a different perspective. In particular, in this approach, there is *no* radion mode in the bulk metric. The radion appears as the geodesic relative distance between the branes. This naturally leads us to ask how the equation of motion for the radion gets generated, and in particular, how the GW mechanism gets implemented. There should also be some differences between the so-called Gaussian normal coordinates used in most of the works on RS1 cosmology and this method, since in the latter, the bulk is *static*, at least to a first approximation; all the cosmological evolution occurs on the branes. This approach also allows the role of the GW mechanism in producing an acceptable cosmology to be seen in more detail than for the effective action used in the Gaussian normal formulation. Finally, we can make contact with the ideas of holography [18] by placing a horizon in the bulk geometry and seeing how that influences the cosmology and stabilization mechanisms.

Section II provides the setup for our calculation. We describe the bulk geometry and use it to compute the geodesic distance between the branes. Doing this, it becomes clear that there is an issue in how to define what we mean by the hierarchy of scales between the branes, which was the point of the RS1 scenario [1]. We discuss this in some detail and argue that when the bulk geometry has a horizon, the interbrane separation does *not* set the hierarchy, at least not in as direct a manner as when the horizon is absent.

In Sec. III we solve the junction conditions and find the cosmology on the branes. We write down the bulk and boundary actions, the equations of motion and the junction conditions, from which the cosmological equations on both branes is derived. We find as in [12,16], that if the radion is not stabilized, there is a constraint between the energy densities on the branes leading to one of them being negative.

The cosmology observed on a brane depends solely on its motion through the bulk and since the two branes evolve independently there is no reason to expect that for general densities on the branes they should both move in such a way that the hierarchy is maintained. In Sec. IV we show how the presence of the Goldberger-Wise mechanism alters the picture. In addition to fixing the geodesic distance between the branes, the GW scalar field introduces small time-dependent corrections to the bulk metric which communicate the field content of each brane to the other. These features can be seen explicitly since the GW field can be treated as a small per-

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turbation to the AdS background. We then consider an AdSS metric which contains a horizon and find that the leading correction to the purely AdS cosmology is suppressed compared with the corresponding result with only one brane [3,4].

Section V contains our conclusions.

II. RS1 IN AdSS COORDINATES

Birkhoff's theorem in five dimensions tells us that the most general spherically symmetric solution to the Einstein equations with a bulk cosmological constant is given by the metric

$$ds^2 = -u(r)dt^2 + \frac{dr^2}{u(r)} + r^2 d\Sigma_k^2, \tag{2.1}$$

where r is the bulk coordinate, $d\Sigma_k^2$ is the line element for the fixed r 3-spatial hypersurfaces, which are homogeneous and isotropic, and $k=0,\pm 1$ is the 3-spatial curvature parameter. Writing the bulk cosmological constant as $\Lambda = -6/l^2$, the metric coefficient $u(r)$ is given by

$$u(r) \equiv \frac{r^2}{l^2} + k - \frac{\mu}{r^2}, \tag{2.2}$$

where l is the AdS radius of curvature and μ is proportional to the mass of the black hole in AdS [19]. In the RS2 picture this interpretation can be made exact since there are asymptotic regions in r; in RS1, the compactness in the r direction prevents such a direct interpretation. However, for nonzero μ, the solutions have horizons at $r=r_h$ where

$$r_h^2 = \frac{l^2}{2}(-k + \sqrt{k^2 + 4\mu/l^2}). \tag{2.3}$$

In the RS2 case there is an explicit coordinate transformation [20,21] that maps the above space-time to the cosmological metric for RS2 as written in Ref. [12].

A. Geodesic interbrane separation

The first question we need to ask is: where is the radion mode? It certainly is *not* explicitly present in Eq. (2.1), in sharp contrast to what happens when the RS1 scenario is worked out in Gaussian normal coordinates [12,17,22]. We can find this mode by realizing that it is supposed to describe the interbrane separation. Given Eq. (2.1), we can calculate this explicitly, by computing the geodesic distance between points on the two branes.

Consider two branes placed at

$$(t,r,x^i) = (T_0(\tau_0), R_0(\tau_0), x^i),$$
$$(t,r,x^i) = (T_1(\tau_1), R_1(\tau_1), x^i), \tag{2.4}$$

where the $\tau_{0,1}$ are the proper times on the branes. We shall take $R_0 > R_1$ so that the brane at R_1 will be the TeV or IR brane while the brane at R_0 is the Planck or UV brane. The symmetry of the bulk metric allows us to consider a geodesic between these branes that only propagates in the t and r directions:

$$X^a(\lambda) = (t(\lambda), r(\lambda), 0,0,0), \tag{2.5}$$

where λ is the affine parameter on the geodesic and we take $\lambda = 0,1$ to correspond to the positions of the UV and IR brane, respectively.

The geodesic equations read

$$\frac{d}{d\lambda}\left(u(r)\frac{dt}{d\lambda}\right) = 0$$

$$\frac{d}{d\lambda}\left(\frac{1}{u(r)}\frac{dr}{d\lambda}\right) + \frac{1}{2}\frac{\partial u}{\partial r}\left(\frac{1}{u(r)}\frac{dr}{d\lambda}\right)^2 = -\frac{1}{2}\frac{\partial u}{\partial r}\left(\frac{dt}{d\lambda}\right)^2. \tag{2.6}$$

In the sequel, we shall frequently take $k=0$, i.e., we take the fixed r hypersurfaces to have flat 3-spatial sections. Note that in this case, the horizon is located at $r_h = (\mu l^2)^{1/4}$.

The geodesic equations above have first integrals,

$$\frac{dt}{d\lambda} = \frac{l\tilde{L}}{u(r(\lambda))}$$

$$\frac{dr}{d\lambda} = l\sqrt{\tilde{L}^2 + \varepsilon^2 u(r(\lambda))}. \tag{2.7}$$

We have scaled all the dimensionful parameters by the AdS radius of curvature l so that $\tilde{L}$ and ε are dimensionless.

We can solve this set of equations and then impose the boundary conditions from Eq. (2.4). However, we can save some time by noting that the relevant distance is the one measured at the *same* bulk time on both branes. This means that the geodesic we want is the one for which $\tilde{L}=0$. Integrating the $dr/d\lambda$ equation, we find

$$r^2(\lambda) = r_h^2 \cosh(2\varepsilon\lambda + \theta), \tag{2.8}$$

with

$$\theta = \cosh^{-1}\left(\frac{R_0^2}{r_h^2}\right), \quad 2\varepsilon + \theta = \cosh^{-1}\left(\frac{R_1^2}{r_h^2}\right). \tag{2.9}$$

Note that this assumes that both R_0, R_1 are larger than r_h. The equation for $r(\lambda)$ in Eq. (2.7) shows that, with $\tilde{L}=0$, the smallest value of r allowed on this geodesic is r_h.

From Eq. (2.7) we can compute the geodesic distance between the branes:

$$\Delta s = \int_0^1 d\lambda \sqrt{\frac{\dot{r}(\lambda)^2}{u(r(\lambda))}} = |l\varepsilon|$$
$$= \frac{l}{2}\left[\cosh^{-1}\left(\frac{R_0^2}{r_h^2}\right) - \cosh^{-1}\left(\frac{R_1^2}{r_h^2}\right)\right]. \tag{2.10}$$

In particular, in the $\mu \to 0$ $(r_h \to 0)$ limit, we find

$$\Delta s = -l \ln\frac{R_1}{R_0}, \quad \text{or} \quad \frac{R_1}{R_0} = e^{-\Delta s/l}. \tag{2.11}$$

B. Hierarchy of scales in AdSS coordinates

We now ask how we see the fact that the separation between the branes gives rise to a hierarchy of mass scales between the branes.

This is easy when $\mu=0$, since $u(r)=r^2/l^2$ and the bulk line element can be written as

$$ds^2 = \frac{r^2}{l^2}(-dt^2 + d\vec{x}^2) + \frac{l^2}{r^2}dr^2, \tag{2.12}$$

where $d\vec{x}^2 \equiv l^2 d\Sigma^2_{k=0}$. If we now ask what sets the scale on each brane, the fact that the branes (at least at a given instant of bulk time) are located at a fixed value of r allows us to set $r=R, dr=0$ which tells us that the relation between distance measurements on the two branes is given by the ratio R_1^2/R_0^2. As we saw above, this is just $e^{-2\Delta s/l}$, which is exactly the standard RS result [1].

The $\mu \neq 0$ case contains some extra subtleties. In this case, for $k=0$ the line element becomes

$$ds^2 = -\left(\frac{r^2}{l^2} - \frac{\mu}{r^2}\right)dt^2 + \frac{r^2}{l^2}d\vec{x}^2 + \frac{dr^2}{\frac{r^2}{l^2} - \frac{\mu}{r^2}}. \tag{2.13}$$

At fixed r this metric will *not* take the Minkowski form. This makes the comparison between branes somewhat less obvious, but we can ask how distance scales on each brane compared to the case where all quantities are *static* in bulk time t. In this situation it is again the ratio R_1^2/R_0^2 that sets the relative scales between the branes. It is important to note that this is *not* related to the interbrane separation in any simple way as can be seen from Eq. (2.10).

This last point makes the issue of stabilization of the hierarchy somewhat murkier in this formalism. For the no-horizon case, stabilizing the interbrane distance stabilizes the hierarchy; with a horizon, there are two variables to deal with: namely the ratio R_1^2/R_0^2 as well as one of the positions R_0, say. In the limit that $R_i^2 \gg \sqrt{\mu l^2} = r_h^2$, we recover the no-horizon results.

III. RS1 COSMOLOGY WITH AN AdSS BULK

The gravitational action for the RS1 scenario will be taken as the sum of the bulk Einstein-Hilbert action with a cosmological constant Λ,

$$S_{\text{bulk}} = \frac{1}{16\pi G_5}\int d^5x\sqrt{-g}[-2\Lambda + R], \tag{3.1}$$

and a boundary action of the form

$$\begin{aligned}S_{\text{brane}} = &\frac{1}{16\pi G_5}\int_0 d^4x\sqrt{-h_0}[-2\sigma_0 + 16\pi G_5\mathcal{L}_0]\\ &+\frac{1}{8\pi G_5}\int_0 d^4x\sqrt{-h_0}K_0\\ &+\frac{1}{16\pi G_5}\int_1 d^4x\sqrt{-h_1}[-2\sigma_1 + 16\pi G_5\mathcal{L}_1]\\ &+\frac{1}{8\pi G_5}\int_1 d^4x\sqrt{-h_1}K_1.\end{aligned} \tag{3.2}$$

A subscript 0 (1) refers to the UV (IR) brane. Here G_5 denotes the bulk Newton constant, $h_{0,1}$, $K_{0,1}$ and $\sigma_{0,1}$ are the determinant of the induced metric, the trace of the extrinsic curvature and brane surface tension respectively for the appropriate brane. If we define a unit normal orthogonal to the tangent space of the branes by $[n_{0,1}]_a$, then the induced metric and the extrinsic curvature are given in the bulk coordinates, for example on the UV brane, by

$$\begin{aligned}[h_0]_{ab} &= g_{ab} - [n_0]_a[n_0]_b\\ [K_0]_{ab} &= [h_0]_a^c[h_0]_b^d\nabla_c[n_0]_d.\end{aligned} \tag{3.3}$$

The field content on each is summarized by $\mathcal{L}_{0,1}$; we shall generally study the case where the fields on each brane produce the energy momentum tensor of a perfect fluid.

Following [3,4], we allow the positions of the branes within the bulk to evolve so as to give rise to cosmological evolution on each brane. The position of the brane in the direction transverse to the brane, given in Eq. (2.4), will be related to the cosmological scale factor on the given brane. Now choose the normal to the brane to be of the form

$$[n_0]_a = \pm(-\dot{R}_0, \dot{T}_0, 0,0,0), \tag{3.4}$$

where the dot denotes a derivative with respect to the proper time on the appropriate brane. Noting that the condition $g^{ab}n_a n_b = 1$ relates $\dot{T}_0$ to $\dot{R}_0$,

$$\dot{T}_0 = \frac{[\dot{R}_0^2 + u(R_0)]^{1/2}}{u(R_0)}, \tag{3.5}$$

we can write the induced metric in terms of brane coordinates (τ_0, x^i):

$$\begin{aligned}ds_0^2 &= -u(R_0)[\dot{T}_0^2 - (u(R_0))^{-2}\dot{R}_0^2]d\tau_0^2 + R_0^2(\tau_0)\,d\Sigma_k^2\\ &= -d\tau_0^2 + R_0^2(\tau_0)d\Sigma_k^2\\ &\equiv [h_0]_{\mu\nu}dx^\mu dx^\nu\end{aligned} \tag{3.6}$$

where μ, ν run over brane coordinates. At the UV brane, the extrinsic curvature becomes

$$[K_0]_{\mu\nu}dx^\mu dx^\nu = \pm \frac{1}{u(R_0(\tau_0))\dot{T}_0}\left[\ddot{R}_0 + \frac{1}{2}\frac{\partial u(R_0)}{\partial R_0}\right]d\tau_0^2 \mp u(R_0)\dot{T}_0 R_0 d\Sigma_k^2. \tag{3.7}$$

The signs that appear in the definition of the normal and consequently in the extrinsic curvature determine how the space is sliced. We shall use the upper sign in Eq. (3.4) which corresponds to keeping the space $r < R_0(\tau_0)$. The form of the extrinsic curvature at the IR brane is exactly the same, once we replace the subscripts, except that the lower signs should be used to retain the region $r > R_1(\tau_1)$.

Let p_0 and ρ_0 be the pressure and density of the perfect fluid on the UV brane so that the UV brane stress energy is

$$[T_0]_\mu^\nu = \mathrm{diag}(-\rho_0, p_0, p_0, p_0). \tag{3.8}$$

The Israel condition [10] relating the discontinuity in the extrinsic curvature to the presence of the brane is given by

$$\Delta[K_0]_{\mu\nu} = \frac{1}{3}\sigma_0[h_0]_{\mu\nu} + 8\pi G_5\left[[T_0]_{\mu\nu} - \frac{1}{3}[T_0]_\lambda^\lambda[h_0]_{\mu\nu}\right]. \tag{3.9}$$

We shall assume that the two 3-branes reside at the fixed points of an $S^1/\mathbb{Z}_2$ orbifold so that the jump in the extrinsic curvature is $\Delta[K_0]_{\mu\nu} = 2[K_0]_{\mu\nu}$. Although Eq. (3.9) appears to yield *two* constraints from the temporal and the spatial components, the $\Delta[K_0]_{\tau\tau}$ constraint follows from the $\Delta[K_0]_{ij}$ constraint [3],

$$\sqrt{\dot{R}_0^2 + u(R_0)} = \frac{1}{6}R_0\sigma_0 + \frac{4\pi G_5}{3}R_0\rho_0, \tag{3.10}$$

provided energy and momentum are conserved on the brane

$$\frac{d}{d\tau_0}(\rho_0 R_0^3) = -p_0\frac{d}{d\tau_0}R_0^3. \tag{3.11}$$

The main advantage of letting the brane positions within the bulk evolve in time is that the bulk metric is unaffected by the behavior of the branes other than by their specifying which slice of the AdS-Schwarzschild metric Eq. (2.1) is relevant. Thus, for the UV brane we have

$$\sqrt{\dot{R}_0^2 + u(R_0)} = \frac{1}{6}R_0\sigma_0 + \frac{4\pi G_5}{3}R_0\rho_0, \tag{3.12}$$

while on the IR brane,

$$-\sqrt{\dot{R}_1^2 + u(R_1)} = \frac{1}{6}R_1\sigma_1 + \frac{4\pi G_5}{3}R_1\rho_1. \tag{3.13}$$

The sign change corresponds to the fact that the normals of the two branes have opposite orientations. As in Ref. [3], the evolution on the UV brane approaches a standard Robertson-Walker cosmology when we make the fine tuning $\sigma_0 = 6/l$ required in the original RS1 scenario and consider the limit $R_0 \gg l$. On the IR brane, after squaring both sides of Eq. (3.13), we see that we also need to impose $\sigma_1 = \pm 6/l$ in order for the large terms in the $R_1 \gg l$ limit to cancel,

$$\dot{R}_1^2 + \frac{R_1^2}{l^2} + k - \frac{\mu}{R_1^2} = \frac{1}{36}R_1^2\sigma_1^2 + \frac{4\pi G_5}{9}\sigma_1\rho_1 R_1^2 + \left(\frac{4\pi G_5}{3}R_1\rho_1\right)^2. \tag{3.14}$$

In fact, we must choose $\sigma_1 = -6/l$ so as to satisfy the Israel condition, Eq. (3.13), which, in turn, leads to a universe in which the energy density ρ_1 must be negative to obtain standard FRW cosmological evolution on the IR brane,

$$\dot{R}_1^2 + k = -\frac{8\pi G_5}{3l}\rho_1 R_1^2 + \frac{\mu}{R_1^2} + \left(\frac{4\pi G_5}{3}\right)^2\rho_1^2 R_1^2. \tag{3.15}$$

If the interbrane separation is kept fixed at Δz in the original Gaussian normal coordinates of the RS model *without* a stabilization mechanism at play, then it was shown in Ref. [12] that the energy densities on the branes have to satisfy

$$\rho_0 = -\rho_1 e^{-2\Delta z/l}. \tag{3.16}$$

How does this constraint emerge from the AdSS bulk coordinate approach? In order to compare with Ref. [12] we set $\mu = 0$, $k = 0$, and impose the fine tuning above: $\sigma_0 = -\sigma_1 = 6/l$. Now assume that the geodesic distance between the branes has been fixed so that $\Delta s = -l\ln(R_1/R_0)$ is constant. From this it follows that

$$\begin{aligned}\frac{1}{R_1}\frac{dR_1}{d\tau_1} &= \frac{1}{R_0}\frac{dR_0}{d\tau_1} = \frac{d\tau_0}{d\tau_1}\frac{1}{R_0}\frac{dR_0}{d\tau_0} \\ &= \frac{\sqrt{\dfrac{R_0^2}{l^2} - \dfrac{l^2}{R_0^2}\dfrac{dR_0}{dt}}}{\sqrt{\dfrac{R_1^2}{l^2} - \dfrac{l^2}{R_1^2}\dfrac{dR_1}{dt}}}\frac{1}{R_0}\frac{dR_0}{d\tau_0} \\ &\approx \frac{R_0}{R_1}\frac{1}{R_0}\frac{dR_0}{d\tau_0} = e^{\Delta s/l}\frac{1}{R_0}\frac{dR_0}{d\tau_0}.\end{aligned} \tag{3.17}$$

Using Eq. (3.17) in Eqs. (3.12) and (3.13), then

$$\frac{\dot{R}_0^2}{R_0^2} = \frac{8\pi G_5}{3l}\rho_0 + \cdots, \quad \frac{\dot{R}_1^2}{R_1^2} = -\frac{8\pi G_5}{3l}\rho_1 + \cdots, \tag{3.18}$$

where the brane tensions satisfy the fine-tuning condition and we assume that the energy densities are small compared to the tension. In this limit, we arrive at $\rho_0 \approx -\rho_1 e^{-2\Delta s/l}$.

IV. AdSS COSMOLOGY WITH A STABILIZATION MECHANISM

When the bulk space-time only contains a cosmological constant, Birkhoff's theorem allows us to write the metric in the conventional, time-independent form given in Eq. (2.1). The motion of the branes through this bulk depends on the local environment of the branes, as expressed by the Israel junction condition, so that without any means of interacting each brane evolves independently. However, the existence of a fixed hierarchy requires that the motions of the branes must be carefully correlated. From this perspective then, the appearance of an unphysical energy density is not altogether surprising. We have simply imposed a constraint upon two independently moving branes without any mechanism to enforce it.

To evade Birkhoff's theorem, the mechanism that stabilizes the hierarchy must distort the background away from a pure AdSS space-time. These distortions will then appear in the extrinsic curvature term of the Israel equation which thus allows the motions of the branes to be naturally correlated with real, positive, independent densities on each brane. Because of this correlated motion of the branes, we generally expect that the cosmology observed on either of the branes now should depend on the field content of both, as was seen in [12]. Note that this distortion can be small compared with the scale of the bulk cosmological constant—thus allowing a perturbative treatment—since the terms that yielded an FRW cosmology on the branes were subleading and only became important once the tension was finely tuned to cancel the leading bulk effect.

The Goldberger-Wise stabilization mechanism [17] adds a free massive scalar field to the bulk with a potential on each brane to fix the boundary values of the scalar field,

$$
\begin{aligned}
S_{\rm GW}=&\frac{1}{8\pi G_5}\int d^5x\sqrt{-g}\left[-\frac{1}{2}\nabla_a\phi\nabla^a\phi-\frac{1}{2}m^2\phi^2\right]\\
&+\frac{1}{8\pi G_5}\int_0 d^4x\sqrt{-h_0}[-\lambda_0(\phi^2-v_0^2)^2]\\
&+\frac{1}{8\pi G_5}\int_1 d^4x\sqrt{-h_1}[-\lambda_1(\phi^2-v_1^2)^2]. \qquad (4.1)
\end{aligned}
$$

We have normalized the fields to extract the factor $(8\pi G_5)^{-1}$ to simplify the form of some of the later equations. Varying the total action produces the usual Einstein and scalar field equations in the bulk,

$$
\begin{aligned}
&R_{ab}-\tfrac{1}{2}g_{ab}R=-\Lambda g_{ab}+\mathcal{T}_{ab}\\
&\nabla^2\phi-m^2\phi=0, \qquad (4.2)
\end{aligned}
$$

while at the UV brane the equations of motion are

$$
\begin{aligned}
&\Delta[K_0]_{ab}=\tfrac{1}{3}\sigma[h_0]_{ab}+8\pi G_5[[T_0]_{ab}-\tfrac{1}{3}[T_0]^c_c[h_0]_{ab}]\\
&[n_0]^a\partial_a\phi=2\lambda_0(\phi^2-v_0^2)\phi \qquad (4.3)
\end{aligned}
$$

with an analogous equation at the IR brane. Here, $[T_0]_{ab}$ and $\mathcal{T}_{ab}$ are the energy-momentum tensors associated with the fields confined to the brane and with ϕ, respectively.

The presence of the scalar field alters the bulk geometry, but in the limit where it is small compared to the cosmological constant, $ml\ll 1$, we can treat its effect as a perturbation to the AdSS background,

$$
\begin{aligned}
ds^2=&-u(r)[1+\chi_{tt}(t,r)]dt^2+\frac{1}{u(r)}[1+\chi_{rr}(t,r)]dr^2\\
&+r^2d\Sigma^2_{k=0}, \qquad (4.4)
\end{aligned}
$$

where throughout this section we set $k=0$. In this metric we have still assumed that the three large spatial dimensions are isotropic. Since the scalar field is responsible for the perturbations, χ_{tt} and χ_{rr} will be of the same order as $\mathcal{T}_{ab}$.

As the induced metric at the brane suggests, the scale R_1 ($<R_0$) is associated with the scale factor of a Friedmann-Robertson-Walker (FRW) universe while l is naturally of the order of the Planck length. Therefore, for an acceptable cosmology, we can assume that $l/r\ll 1$ throughout the bulk. In this limit the form of the metric naturally suppresses terms with time derivatives relative to those with r derivatives. This feature greatly simplifies the analysis of the back reaction of the scalar field on the bulk geometry since Eq. (4.2) will effectively only constrain the r dependence of χ_{tt}, χ_{rr} and ϕ. This behavior can be seen, for example, by considering $\mathcal{T}_{tt}$ which is given up to χ_{tt}, χ_{rr} corrections by

$$
\mathcal{T}_{tt}=\tfrac{1}{2}(\partial_t\phi)^2+\tfrac{1}{2}u^2(r)(\phi')^2+\tfrac{1}{2}u(r)m^2\phi^2+\cdots \qquad (4.5)
$$

with $\phi'\equiv\partial_r\phi$. If neither brane is near the black hole horizon, then the r-derivative term is enhanced by a factor of $u^2(r)\approx r^4/l^4$ relative to the time derivative term. The same feature appears in the Einstein equations (4.2). Thus, as long as the time derivatives are not excessively large, as we shall later show, the bulk field equations only constrain the radial dependence.

Expanding the field equations to first order in χ_{tt}, χ_{rr} and ϕ^2, and substituting in the zeroth order solution for an AdS (or AdSS) background, we find

$$
\begin{aligned}
&\frac{12}{l^2}\chi_{rr}+\frac{3u}{r}\chi'_{rr}=u\phi'^2+m^2\phi^2\\
&\frac{12}{l^2}\chi_{rr}-\frac{3u}{r}\chi'_{tt}=-u\phi'^2+m^2\phi^2\\
&\frac{3}{r}\partial_t\chi_{rr}=2\phi'\partial_t\phi\\
&\frac{12}{l^2}\chi_{rr}+\left[2\frac{r^2}{l^2}+u\right]\frac{\chi'_{rr}}{r}-\left[6\frac{r^2}{l^2}-u\right]\frac{\chi'_{tt}}{r}-u\chi''_{tt}\\
&\quad=u\phi'^2+m^2\phi^2 \qquad (4.6)
\end{aligned}
$$

from the Einstein equations and

$$u\phi''+u'\phi'+\frac{3u}{r}\phi'-m^2\phi=0 \tag{4.7}$$

from the scalar field equation (4.2). The Bianchi identity relates these equations so that, for example, the last equation in Eq. (4.6) is not independent.

A. A stabilized AdS cosmology

For a purely AdS space, i.e. $\mu=0$, and in the limit that we suppress the contributions of time derivatives, we can solve the scalar field solution to find, as in Ref. [17],

$$\phi(r)=\frac{1}{r^2}[a(t)r^{\nu}+b(t)r^{-\nu}], \tag{4.8}$$

where

$$\nu\equiv\sqrt{4+m^2l^2}. \tag{4.9}$$

Substituting this solution into Eq. (4.6) and solving for the metric perturbations yields

$$\chi_{tt}(t,r)=\frac{1}{3}\frac{a(t)b(t)(\nu^2-4)-3c(t)}{r^4}+d(t)$$

$$\chi_{rr}(t,r)=\frac{1}{3}a^2(t)(\nu-2)r^{-2(2-\nu)}-\frac{1}{3}b^2(t)(\nu+2)r^{-2(2+\nu)}+\frac{c(t)}{r^4}. \tag{4.10}$$

The functions $a(t)$, $b(t)$, $c(t)$, and $d(t)$ are constants of integration with respect to the r derivatives and are only fixed to this order in l/r by the third equation of Eq. (4.6) which requires that

$$\partial_t c=-\tfrac{2}{3}(\nu+2)b\partial_t a+\tfrac{2}{3}(\nu-2)a\partial_t b. \tag{4.11}$$

The scalar field must satisfy the jump conditions described by Eq. (4.3). However, if as in [17] we assume that the potentials on the branes are sufficiently rigid, $\lambda_0,\lambda_1\to\infty$, then the scalar field value is forced to the minimum value of the potential on each brane,

$$\phi(T_0,R_0)=v_0=a(T_0)R_0^{-2+\nu}+b(T_0)R_0^{-2-\nu}$$

$$\phi(T_1,R_1)=v_1=a(T_1)R_1^{-2+\nu}+b(T_1)R_1^{-2-\nu}. \tag{4.12}$$

Now, we saw in Sec. II that the stabilized hierarchy in these coordinates corresponds to fixing the geodesic distance between the branes along a *constant time* geodesic, so that $T_0=T_1$. Setting $a=a(T_0)=a(T_1)$ and $b=b(T_0)=b(T_1)$, we thus find

$$a=\frac{v_0-v_1(R_1/R_0)^{2+\nu}}{1-(R_1/R_0)^{2\nu}}R_0^{2-\nu}$$

$$b=\frac{v_1(R_1/R_0)^{2-\nu}-v_0}{1-(R_1/R_0)^{2\nu}}\left(\frac{R_1}{R_0}\right)^{2\nu}R_0^{2+\nu}. \tag{4.13}$$

when evaluated at either brane. This result is exactly that of Ref. [17] except for the implicit time dependence in R_0 and R_1.

When the scalar field satisfies Eq. (4.12), the scalar potentials vanish and the only contribution to the energy-momentum tensor on the branes is due to the fields confined to the branes which we take to be a perfect fluid stress tensor, as in Eq. (3.8). For fluids satisfying the conservation law of Eq. (3.11), we only need to solve for the ij component since the $\tau\tau$ component of the Israel condition does not give rise to an independent constraint. Retaining only the corrections linear in the perturbations and unsuppressed by powers of l/r in

$$[K_0]_{ij}=\frac{\sqrt{u(R_0)+\dot R_0^2}}{R_0}\times[1-\tfrac{1}{2}\chi_{rr}(T_0,R_0)+\cdots][h_0]_{ij}, \tag{4.14}$$

the Israel condition on the UV brane becomes

$$2\frac{R_0^2}{l^2}+\dot R_0^2=\frac{R_0^2}{3l}\sigma_0+\frac{8\pi G_5}{3l}\rho_0R_0^2+\frac{R_0^2}{l^2}\chi_{rr}(T_0,R_0). \tag{4.15}$$

At the IR brane, since we have different signs from the opposite orientation for the normal, we arrive at

$$2\frac{R_1^2}{l^2}+\dot R_1^2=-\frac{R_1^2}{3l}\sigma_1-\frac{8\pi G_5}{3l}\rho_1R_1^2+\frac{R_1^2}{l^2}\chi_{rr}(T_1,R_1). \tag{4.16}$$

In both of these equations, we have neglected $\mathcal{O}(\rho^2)$ corrections.

The small change in the bulk background produced by the Goldberger-Wise field will in general require a corresponding shift in the usual choice of the brane tensions to cancel the cosmological constant in the effective cosmology on the brane. We therefore set the brane tensions to

$$\sigma_0=\frac{6}{l}\left(1+\frac{1}{2}\delta\sigma_0\right)\quad\text{and}\quad\sigma_1=-\frac{6}{l}\left(1+\frac{1}{2}\delta\sigma_1\right). \tag{4.17}$$

Substituting these into Eqs. (4.15) and (4.16) and using our solution for the metric perturbation in Eq. (4.10) yields

$$\dot{R}_0^2 = \frac{8\pi G_5}{3l}\rho_0 R_0^2 + \frac{c(T_0)}{l^2}\frac{1}{R_0^2} + \left[\delta\sigma_0 - \frac{4(v_1-v_0)^2}{3}\frac{R_1^8}{R_0^8}\right]\frac{R_0^2}{l^2} + \cdots$$

$$\dot{R}_1^2 = -\frac{8\pi G_5}{3l}\rho_1 R_1^2 + \frac{c(T_1)}{l^2}\frac{1}{R_1^2} + \left[\delta\sigma_1 - \frac{4(v_1-v_0)^2}{3}\right]\frac{R_1^2}{l^2} + \cdots. \tag{4.18}$$

Since we wish to establish that the leading behavior mimics a FRW cosmology, we have neglected $\mathcal{O}(ml)$ corrections in Eq. (4.18) by setting $\nu=2$.

The final function $c(t)$ is determined on the branes by the requirement that the hierarchy should be stabilized. As in [17], we can determine the preferred separation between the branes for a given set of parameters (m,v_0,v_1) by integrating the scalar action over the radial dimension to define an effective potential for R_1/R_0:

$$\int d^4x\sqrt{-g_{\rm eff}}[-V_{\rm eff}(R_0,R_1)] \equiv \int_{R_1}^{R_0} dr \int d^4x\sqrt{-g}[-\tfrac{1}{2}\nabla_a\phi\nabla^a\phi - \tfrac{1}{2}m^2\phi^2]. \tag{4.19}$$

Substituting Eq. (4.8) into Eq. (4.19) and integrating over a constant time hypersurface so that we can use Eq. (4.13), we discover that the dependence on geodesic distance R_1/R_0 in the effective potential factorizes

$$V_{\rm eff}(R_0,R_1) = \frac{R_0^4}{l^2}\hat{V}_{\rm eff}\left(\frac{R_1}{R_0}\right) \tag{4.20}$$

with

$$\hat{V}_{\rm eff}(x) = \frac{(\nu-2)[v_0 - v_1 x^{2+\nu}]^2}{1-x^{2\nu}} + \frac{(\nu+2)[v_0 - v_1 x^{2-\nu}]^2 x^{2\nu}}{1-x^{2\nu}}. \tag{4.21}$$

In the $ml \ll 1$ limit, $\hat{V}_{\rm eff}(x)$ is minimized by

$$x = \frac{R_1}{R_0} \approx \left(\frac{v_1}{v_0}\right)^{4/m^2l^2}. \tag{4.22}$$

Note that by adding an $\mathcal{O}(m^2l^2)$ constant to the right side of Eq. (4.19), we can arrange the minimum to occur at $\hat{V}_{\rm eff}=0$, thus keeping the R_0^4 coefficient in Eq. (4.20) from driving $R_0 \to 0$ or $R_0 \to \infty$.

Once the relative brane separation has settled to the value determined by Eq. (4.22), the motion of the branes is correlated so that, as in Eq. (3.17),

$$\dot{R}_0 = \frac{dR_0}{d\tau_0} \approx \dot{R}_1 = \frac{dR_1}{d\tau_1}. \tag{4.23}$$

In the absence of a stabilizing mechanism, this condition was arbitrarily imposed as a constraint on the two brane system which required a negative energy density on one of the branes. We now see that Eq. (4.23) arises as a natural consequence of minimizing the radion effective potential and instead of constraining the field densities on the branes, it fixes the remaining integration function $c(t)$ evaluated on the branes at equal times, $c \equiv c(T_0) = c(T_1)$,

$$c = l^2 R_1^2\left(1 - \frac{R_1^2}{R_0^2}\right)^{-1}\left[\frac{8\pi G_5}{3l}(\rho_0 R_0^2 + \rho_1 R_1^2) + \frac{1}{l^2}(\delta\sigma_0 R_0^2 - \delta\sigma_1 R_1^2) + \frac{4}{3l^2}(v_1-v_0)^2 R_1^2\right]. \tag{4.24}$$

When Eq. (4.24) is substituted into the equation for the evolution of the IR brane, the leading ρ_1 term cancels the density term in Eq. (4.18) with the undesired sign. The next to leading term in ρ_1, where the expansion is in powers of the exponential hierarchy, has the required sign for a well-behaved FRW cosmology:

$$\dot{R}_1^2 = \frac{8\pi G_5}{3l}\frac{R_1^2}{R_0^2}\left[\rho_1 + \rho_0\frac{R_0^4}{R_1^4}\right]R_1^2 + \left[\delta\sigma_0\frac{R_0^4}{R_1^4} - \delta\sigma_1 + \frac{4}{3}(v_1-v_0)^2\right]\frac{R_1^2}{R_0^2}\frac{R_1^2}{l^2} + \cdots. \tag{4.25}$$

Choosing the tensions such that the second term vanishes, we see that the presence of a stabilized radion has led to a realistic effective cosmology on the branes,

$$\frac{\dot{R}_1^2}{R_1^2} = \frac{8\pi G_5}{3l}e^{-2\Delta s/l}[\rho_1 + \rho_0 e^{4\Delta s/l}] + \cdots. \tag{4.26}$$

The fixed hierarchy implies that the equation for the evolution of the UV brane is exactly the same up to a rescaling by $e^{-2\Delta s/l}$,

$$\frac{\dot{R}_0^2}{R_0^2} = \frac{8\pi G_5}{3l}[\rho_0 + \rho_1 e^{-4\Delta s/l}] + \cdots. \tag{4.27}$$

We might have naively expected that once the hierarchy between the two brane system is stabilized, the cosmological evolution should depend on the field content of both branes, weighted by possible exponential factors. The approach where the brane cosmology arises from the motion of the branes through a nearly static bulk explicitly shows the origin of this dependence. By distorting the bulk from pure anti–de Sitter space, the scalar field both communicates the energy density from one brane to the other as in Eq. (4.24) and cancels the unphysical term in the IR brane motion. As noted in [12], the presence of matter on the UV brane can easily overwhelm the effect of matter on the IR brane and so drive the cosmology unless it is exponentially smaller than that of the IR brane. Also, Eq. (4.26) suggests that we should define the effective Newton constant for an observer on the IR brane to be

$$G_N \equiv \frac{G_5}{l} e^{-2\Delta s/l}; \tag{4.28}$$

this result also agrees with the effective Newton constant appropriate for two masses confined to the IR brane, as derived in [12].

We can also discuss some of the corrections to Eq. (4.26) which would allow us to distinguish a brane-induced cosmology from a standard FRW cosmology. As in Eq. (3.15), the presence of ρ_1^2 effects reflects a general feature of brane-induced cosmologies. To find the detailed form of such corrections would require solving the equations of motion to second order in the metric perturbations since, from Eqs. (4.24), (4.10), χ_{rr} has terms that depend linearly on the field densities of the branes. The leading $\mathcal{O}(ml)$ corrections are proportional to R_1^2, or R_0^2, and can be eliminated by appropriately choosing the brane tensions. If we do not fine-tune the brane tensions, then the cosmology will contain a term resembling an effective cosmological constant.

The solutions to the perturbed metric were found in the limit in which the time derivatives are suppressed in the equations of motion. We can determine when these terms would be important, thus requiring that the full r and t dependent equations be solved, by comparing the typical size of such terms in the solutions presented above. For example, the scalar potentials in Eq. (4.1) hold the value of the field constant at the branes so that as the brane moves, the r and t derivatives are related via

$$\dot{T}_0 \partial_t \phi + \dot{R}_0 \phi' = \frac{d\phi}{d\tau_0} = 0. \tag{4.29}$$

Thus, using Eq. (3.5) and neglecting $\dot{R}_0^2$ compared to $u(R_0) = R_0^2/l^2$,

$$\partial_t \phi \approx -\frac{R_0}{l} \dot{R}_0 \phi'. \tag{4.30}$$

As we saw earlier in Eq. (4.5), the ϕ' terms are enhanced by a factor $u^2(r) = r^4/l^4$ over $\partial_t \phi$ terms,

$$\begin{aligned} \mathcal{T}_{tt} &= \tfrac{1}{2}(\partial_t \phi)^2 + \tfrac{1}{2} u^2(R_0)(\phi')^2 + \cdots \\ &= \frac{1}{2} \frac{R_0^4}{l^4} \left[\frac{l^2}{R_0^2} \dot{R}_0^2 (\phi')^2 + (\phi')^2 \right] + \cdots, \end{aligned} \tag{4.31}$$

so that despite Eq. (4.30), the first term is still negligible compared with the second.

B. A stabilized AdSS cosmology

The meaning of a stabilized hierarchy becomes less clear when a black hole horizon is introduced into the bulk metric, even if the horizon does not actually appear in the bulk. The parameter μ in the AdSS metric breaks the conformal flatness of the metric in the large $3+1$ dimensions, which is needed to define an unambiguous hierarchy. Yet when this parameter is sufficiently small

$$\frac{r^2}{l^2} - \frac{\mu}{r^2} \gg 1 \quad \text{for} \quad R_1 \leqslant r \leqslant R_0 \tag{4.32}$$

so that we can still neglect the time derivatives, the quantity that sets the hierarchy and therefore must be fixed by the stabilization mechanism is still $R_1/R_0 \equiv e^{-\Delta s/l}$, up to μ-dependent corrections. In fact, if these corrections are to be larger than those due to the time derivatives that we are neglecting, then the black hole mass should not be too small,

$$1 \gg \frac{l\sqrt{\mu}}{r^2}, \frac{l}{\sqrt{\mu}}. \tag{4.33}$$

Neglecting time derivatives again, the important terms in the bulk Einstein equations (4.6) and bulk scalar field equation (4.7) are

$$\begin{aligned} 12\chi_{rr} + \frac{3}{r} \frac{r^4 - \mu l^2}{r^2} \chi'_{rr} &= \frac{r^4 - \mu l^2}{r^2} \phi'^2 + m^2 l^2 \phi^2 \\ 12\chi_{rr} - \frac{3}{r} \frac{r^4 - \mu l^2}{r^2} \chi'_{tt} &= -\frac{r^4 - \mu l^2}{r^2} \phi'^2 + m^2 l^2 \phi^2 \end{aligned} \tag{4.34}$$

and

$$\frac{r^4 - \mu l^2}{r^2} \phi'' + \frac{5r^4 - \mu l^2}{r^3} \phi' - m^2 l^2 \phi + \cdots = 0. \tag{4.35}$$

While we require $ml > 0$ in order to determine the effect of the presence of the horizon on the Goldberger-Wise mechanism, once the hierarchy has been stabilized, $\mathcal{O}(ml)$ terms will be unimportant for the leading description of the brane cosmology. Setting $ml \to 0$ in Eqs. (4.34)–(4.35) yields the following leading behavior for the scalar field,

$$\phi(r) = a(t) - \frac{b(t)}{\mu l^2} \ln\left[1 - \frac{\mu l^2}{r^4}\right] + \mathcal{O}(ml) \tag{4.36}$$

while for corrections to the background metric we find

$$\begin{aligned} \chi_{tt}(t,r) = d(t) + \frac{-1}{r^4 - \mu l^2} \Bigg[c(t) + \frac{8}{3} \frac{b^2(t)}{\mu l^2} \\ + \frac{8}{3} \frac{b^2(t) r^4}{\mu^2 l^4} \left(1 - \frac{1}{2} \frac{\mu l^2}{r^4}\right) \ln\left[1 - \frac{\mu l^2}{r^4}\right] \Bigg] \end{aligned}$$

$$\chi_{rr}(t,r)=\frac{4}{3}\frac{b^2(t)}{\mu l^2(r^4-\mu l^2)}\ln\left[1-\frac{\mu l^2}{r^4}\right]+\frac{c(t)}{r^4-\mu l^2}, \tag{4.37}$$

up to $\mathcal{O}(ml)$ terms.

Determining the boundary conditions as before and retaining only the leading μ-dependent corrections yields the following cosmology on the IR brane

$$\dot{R}_1^2=\frac{8\pi G_5}{3l}\frac{R_1^2}{R_0^2}\left[\rho_1+\frac{R_0^4}{R_1^4}\rho_0\right]R_1^2+\mathcal{O}(\mu^2)+\left[\frac{R_0^4}{R_1^4}\delta\sigma_0-\delta\sigma_1+\frac{4}{3}(v_1-v_0)^2\right]\frac{R_1^2}{R_0^2}\frac{R_1^2}{l^2}. \tag{4.38}$$

Upon fine-tuning the brane tensions,

$$\frac{\dot{R}_1^2}{R_1^2}=\frac{8\pi G_N}{3}[\rho_1+\rho_0 e^{4\Delta s/l}]+\mathcal{O}(\mu^2)+\cdots, \tag{4.39}$$

where $e^{-\Delta s/l}\equiv R_1/R_0$ as before and the effective Newton constant is defined in Eq. (4.28). Notice that unlike the scenario with a single brane [3,4], where the leading terms in the brane cosmology are of the form

$$\frac{\dot{R}^2}{R^2}=\frac{8\pi G_5}{3l}\rho+\frac{\mu}{R^4}+\cdots, \tag{4.40}$$

such a term, linear in μ, does not appear in the two brane scenario. In the one brane scenario, such a term behaves, with its R^{-4} dependence, like a radiation fluid but is canceled here by the stabilization mechanism. Therefore, the distance between the IR brane and the horizon can be much smaller than in the single brane model and still produce an acceptable cosmology, at least until terms quadratic in $\rho_{0,1}$ and μ become important.

V. CONCLUSIONS

There were two main points we wanted to make in this work. The first was that, while some of the features of RS1 models are less obvious in the AdSS formulation presented above, and despite the fact that the bulk is static to leading order in this formulation, when the black hole horizon vanishes, the brane cosmologies have the same behavior here as in the Gaussian normal formulation. This was not *a priori* obvious, especially because the bulk geometry appears quite different in the two different formulations.

The second point was that introducing a horizon into the geometry can modify the cosmological behavior on the IR brane. While we were only able to see this in a perturbative expansion in r_h/r, we were able to see that the stabilization mechanism cancels the leading effect so that the "dark radiation" contribution that generically appears in the one brane scenario is absent here.

It would be interesting to go beyond a perturbative solution in the $\mu\neq 0$ case; this is technically complicated and may involve some conceptual issues such as how the horizon affects the boundary conditions on bulk fields, the GW scalar in particular. We hope to return to this in a future publication.

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