Matter in a warped and oscillating background

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We examine the role of matter in an oscillating background with a warped, compact extra dimension. This background is compatible with an $S^1/\mathbb{Z}_2$ orbifold structure which allows chiral fermions to be included in the scenario. When the background oscillates rapidly, the leading coupling of these oscillations is to gauge fields rather than fermions. If the decay of these oscillations were to occur today, it could provide an alternative mechanism for generating the ultrahigh energy cosmic rays.

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I. INTRODUCTION

A remarkable property of actions that generalize the Einstein-Hilbert action for gravity is that they admit metrics with a periodic dependence on one or more of the coordinates. The simplest such example occurs for an action composed of a general set of curvature invariants, with up to four derivatives of the metric, and a scalar field. In $4+1$ dimensions a metric periodic in one of the spatial dimensions provides a naturally compact space—by identifying the compactification length with the period—without any singularities or discontinuities in the metric [1]. Moreover, these periodic solutions exist without the need for finely tuning any of the parameters in the action. The coefficients of the four-derivative terms determine an essentially unique compactification size.

Similarly, we can search for metrics that oscillate both in the fifth dimension and in time [2]. Although these metrics do not describe the most general isotropic vacuum solutions for the action, they could have intriguing consequences for the early and the late evolution of the universe. In the very early universe, we shall see that the rapid decay of the oscillating background into gauge fields provides an example of how a universe with a Planck-scale time dependence can relax into one that can admit a realistic, slow evolution. Moreover, if some slowly relaxing, but rapid, oscillation persisted or arose recently, then it could contribute a flux of ultrahigh energy cosmic rays. To be successful, the theory would still require a mechanism to maintain these oscillations at a satisfactory rate of decay today.

In this article we shall explore the effect of including fermion and gauge fields in these oscillating backgrounds. Even for a purely static background, introducing chiral fermions requires that the compact extra dimension should have the topology of an $S^1/\mathbb{Z}_2$ orbifold. The metric and the scalar field that supports the compact geometry must be, respectively, even and odd under this $\mathbb{Z}_2$. When the metric additionally oscillates rapidly in time, the coupling of these fields with gravity leads to a potential decay channel for the oscillations. We shall show by expanding a fermion and an Abelian gauge field in Kaluza-Klein modes that to leading order the fermion zero mass mode does not couple to the oscillating component of the metric. The preferred channel for the relaxation of the metric is therefore into gauge fields.

In the following section we review the features of oscillating metrics in $4+1$ dimensions. Section III describes how to place the theory on an orbifold to allow for chiral fermions in a warped, static background. Section IV examines the coupling of the time-dependent components of the metric with the Kaluza-Klein modes of a fermion and an Abelian gauge field placed in an oscillating, compact background. Finally, in Sec. V, we study the phenomenological signature in ultrahigh energy cosmic rays of a very small, but rapid, oscillation today.

II. OSCILLATING METRICS

At distances approaching the Planck length, the usual Einstein-Hilbert action should be supplemented by higher derivative curvature invariants. For example, an action with up to four derivatives of the metric and a scalar field has the form[1]

$$S=M_5^3\int d^4x\,dy\sqrt{-g}[-\Lambda+R+aR^2+bR_{ab}R^{ab}+cR_{abcd}R^{abcd}]+M_5^3\int d^4x\,dy\sqrt{-g}\times[-\tfrac{1}{2}\nabla_a\Phi\nabla^a\phi+\Delta\mathcal{L}_\phi]+\cdots. \tag{2.1}$$

M_5 and Λ denote the Planck mass and cosmological constant, respectively. This action admits smooth, nonsingular metrics that are compact in the fifth dimension,

[1]Our convention for the signature of the metric is $(-,+,+,+,+)$ while the Riemann curvature tensor is defined by $-R^a{}_{bcd}\equiv\partial_d\Gamma^a_{bc}-\partial_c\Gamma^a_{bd}+\Gamma^a_{ed}\Gamma^e_{bc}-\Gamma^a_{ec}\Gamma^e_{bd}$. Here the indices $a,b,\ldots=0,1,2,3,y$ range over all coordinates while $\mu,\nu,\ldots=0,1,2,3$ label the ordinary space-time coordinates.

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$$ds^2 = g_{ab} dx^a dx^b = e^{A(y)} \eta_{\mu\nu} dx^\mu dx^\nu + dy^2, \qquad (2.2)$$

where the exponent $A(y)$ is a periodic function, when $\Delta\mathcal{L}_\phi$ is either an interaction, such as $(\nabla_a \phi \nabla^a \phi)^2$ [1], or a Casimir term [3] with differing values for its components in the large ordinary x^μ and the compact y directions. In either case, the existence of such solutions does not require finely tuning any of the parameters in the action. The effects of the R^2 terms can be conveniently parametrized by

$$\mu = 16a + 5b + 4c, \quad \lambda = 5a + b + \tfrac{1}{2} c,$$
$$\nu = 3a + b + c. \qquad (2.3)$$

λ and ν, in particular, represent the coefficients of a Gauss-Bonnet and a squared Weyl tensor, respectively.

While the static metrics are adequate for studying a theory with a flat $(3+1)$-dimensional long distance limit, models that are to include a realistic cosmology should evolve in time as well,

$$ds^2 = -e^{A(y)} dt^2 + e^{A(y)} e^{B(t)} \delta_{ij} dx^i dx^j + e^{C(t)} dy^2; \qquad (2.4)$$

here we have still assumed an isotropy in the three large spatial dimensions but have allowed a different evolution in the compact dimension.

We shall principally consider a universe that oscillates rapidly so that the time dependence and the y dependence, which fixes the size of the compact dimension, are on a similar footing. The fourth-order, two-variable differential equations that result from varying the action (2.1) are more difficult to solve than the static case (2.2), which can be solved numerically, but fortunately it is possible to build a solution order by order in a small amplitude expansion. To first order in $\epsilon_y, \epsilon_t \ll 1$, we find [2]

$$A(y) = \epsilon_y \cos(\omega_y y) + \cdots,$$
$$B(t) = \epsilon_t \cos(\omega_t t) + \cdots,$$
$$C(t) = -3\epsilon_t \cos(\omega_t t) + \cdots, \qquad (2.5)$$

with

$$\omega_y = \sqrt{-3/\mu} \quad \text{and} \quad \omega_t = \frac{1}{\sqrt{3\mu - 16\nu}}, \qquad (2.6)$$

so that a periodic solution exists when $\mu < 0$ and $\nu < \frac{3}{16}\mu$—no fine-tuning is needed. In Eq. (2.6) we see explicitly that the higher derivative terms in the action determine the size of the extra dimension and the oscillation frequency. Proceeding to the next order in the small ϵ_y, ϵ_t expansion [2] introduces some corrections to each of $A(y)$, $B(t)$, and $C(t)$, which depend on *both* t and y, and more importantly relates the small amplitudes to the size of the cosmological constant:

$$\Lambda = \tfrac{3}{4}(\omega_y^2 \epsilon_y^2 + \omega_t^2 \epsilon_t^2). \qquad (2.7)$$

The relative sizes of ϵ_y and ϵ_t in this solution are undetermined. This relation allows us to state more precisely the regime in which the small amplitude expansion exists. When μ and ν have a natural size $\mathcal{O}(M_5^{-2})$, then a small amplitude translates to a small cosmological constant, $M_5 \Lambda^{-1/2} \gg 1$. Note, however, that the existence of periodic solutions—beyond the small amplitude regime—does not require a small cosmological constant as was shown numerically in [1].

In the large $3+1$ dimensions, once we have substituted the leading effects from the y-dependent scalar field, we discover that the density ρ and the pressure p for the noncompact spatial dimensions are

$$\rho = \tfrac{3}{4}\omega_t^2 \epsilon_t^2 + \tfrac{1}{4}\dot{\phi}^2 + \cdots \approx \tfrac{3}{2}\omega_t^2 \epsilon_t^2 > 0,$$
$$p = -\tfrac{3}{4}\omega_t^2 \epsilon_t^2 + \tfrac{1}{4}\dot{\phi}^2 + \cdots \approx 0, \qquad (2.8)$$

so that $\frac{3}{4}\omega_t^2 \epsilon_t^2$ resembles an effective cosmological constant. The fundamental cosmological constant that appears in the action effects the warping of the extra dimension, as the pressure in that dimension indicates, $p_y = -2\Lambda + \frac{3}{2}\omega_t^2 \epsilon_t^2$.

We find that the small amplitude expansion described above cannot be extended to the case of a de Sitter expansion $B(t) = kt + \cdots$ that coexists with the oscillations, if the rate of expansion is to be comparable to the leading oscillating terms in the metric, i.e., $k \sim \mathcal{O}(M_5 \epsilon_t)$. In general, the dynamics that links the short scale oscillations with the large scale evolution of the universe is complicated and deserves further study.

If the amplitude of the oscillations starts with a value large compared to any expansion rate, the amplitude must eventually decrease sufficiently so as to allow a realistic cosmology. Given that the vacuum energy density in Eq. (2.8) is positive, a natural mechanism for its relaxation is through the decay of the rapid oscillations into energetic particles, analogous to the decay of the inflation in inflation. In Sec. IV, we shall study how fermions and gauge fields couple to the oscillating functions $B(t)$ and $C(t)$, thus providing an explicit decay mechanism.

Note that the presence of the extra dimension is necessary for this decay. In $3+1$ dimensions the metric would not contain a $C(t)$ term and would therefore be conformally flat; then the oscillation could not decay into conformally coupled fields.

III. CHIRAL FERMIONS

In order to recover the standard model fields, the theory must contain chiral fermions. A difficulty arises for the usual Kaluza-Klein compactification with only one extra dimension since the Lorentz symmetry group SO(4,1) has only one, *nonchiral*, spin-$\frac{1}{2}$ representation. Chiral spinor representations do arise when we break the full SO(4,1) symmetry group, for example by placing the theory on an $S^1/\mathbb{Z}_2$ orbifold in the fifth dimension [4,5].

We can similarly introduce an orbifold into the warped Kaluza-Klein picture with a static metric (2.2). The field equations for the action (2.1) that determine $A(y)$ and $\phi(y)$

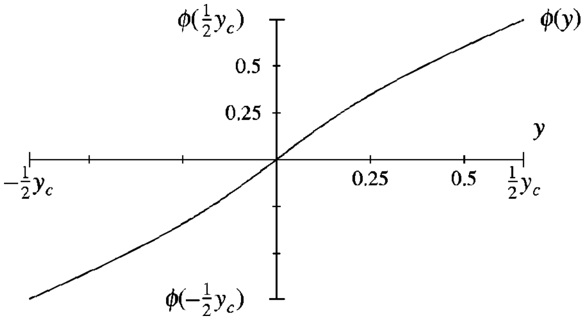

FIG. 1. The behavior of $\phi(y)$ on the orbifold for $\Lambda=1$, $\mu=0.1$, $\lambda=0$, and $\Delta\mathcal{L}_\phi=-(1/4)(\nabla_a\phi\nabla^a\phi)^2$. The initial condition is $A''(0)=23.77364592$. Here $y\in[-0.6578,0.6578]$ and $\phi\in[-0.744,0.744]$. The end points of each dimension are identified so that both are compact.

do not depend explicitly on y so we are free to translate an extremum of $A(y)$ to $y=0$. If the period of $A(y)$ is y_c then we can compactify the space by restricting $y\in[-\frac{1}{2}y_c,\frac{1}{2}y_c]$ and identifying the end points. The solutions for $A(y)$ found numerically in [1] and analytically in a small amplitude expansion in [2] are manifestly even under $y\to -y$ so the background metric respects an $SO(3,1)\times\mathbb{Z}_2$ symmetry.

This $\mathbb{Z}_2$ invariance allows us to introduce an orbifold geometry in the extra dimension by identifying y with $-y$. In order to define a theory consistently on this orbifold, the fields must be odd or even under this discrete $\mathbb{Z}_2$. As we have seen, the background metric is even and this orbifold geometry will further constrain the allowed gravitational excitations of this background.

The scalar field, in contrast, must be odd, $\phi(-y)=-\phi(y)$. The reason is that the field equations for (2.1) relate $\phi'(y)$ to an expression that depends on $A(y)$ only through terms with even numbers of derivatives. Therefore, $\phi'(y)$ oscillates with the same period as $A(y)$ and is also even under $\phi'(y)=\phi'(-y)$. For the solutions found in [1–3], $\phi'(y)$ is everywhere positive so that after integrating we obtain a $\phi(y)$ that increases monotonically. Choosing the constant of integration so that $\phi(0)=0$ produces a $\phi(y)$ that is odd. To accommodate the boundary values, we must further impose that $\phi(y)$ itself assumes values only in a compact space by identifying $\phi(y_c/2)$ and $\phi(-y_c/2)$. In Fig. 1, we show explicitly an example with this geometry. The form for $\phi(y)$ was found numerically for $\Delta\mathcal{L}_\phi=k(\nabla_a\phi\nabla^a\phi)^2$ but with the parameters chosen arbitrarily within the region of the $\{\Lambda,\mu,\lambda,k\}$ space with periodic solutions in y.

In order to be compatible with this $\mathbb{Z}_2$ orbifold structure, a separate scalar field should be included if we wish also to allow the rapid time oscillations. This requirement follows from the leading order behavior $\dot{\phi}^2=3\omega_t^2\epsilon_t^2$ [2]; upon integration we obtain a contribution that is even under the $\mathbb{Z}_2$ symmetry which is incompatible with the symmetry of the scalar field that supports the compact extra dimension.

We now obtain chiral fermions through the standard construction [4–6]. If we choose the following boundary conditions on a five-dimensional fermion:

$$\psi(x^\lambda,-y)=\gamma^5\psi(x^\lambda,y), \tag{3.1}$$

then expanding in a tower of Kaluza-Klein modes, we have a chiral zero mass mode,

$$\psi_L^{(0)}(x^\lambda,y)=0,$$
$$\psi_R^{(0)}(x^\lambda,y)=e^{-A(y)}\psi_R^{(0)}(x^\lambda), \tag{3.2}$$

as well as a series of paired massive left and right modes $\psi_{L,R}^{(n)}(x^\lambda,y)$ for $n>0$. Unlike the flat Kaluza-Klein expansion, the zero mass mode does depend on the fifth coordinate through the $e^{-A(y)}$ factor.

IV. FERMIONS AND GAUGE FIELDS IN AN OSCILLATING BACKGROUND

The presence of ordinary fermion or gauge fields affects the evolution of the universe with rapid time oscillations. The interaction of such fields with the oscillating metric offers a route for the relaxation of these oscillations. However, we shall see that the zero mass mode of the Kaluza-Klein expansion of a massless five-dimensional fermion does not couple directly to the oscillating components of the metric, to leading order. A decay into gauge fields (or scalar fields) becomes the dominant channel for the relaxation of the metric.

Returning to a metric with a dependence on both time and the extra dimension (2.4), when $B(t)\neq C(t)$ this metric is not conformally flat. This feature becomes more apparent if we define new coordinates

$$\eta(t)\equiv\int^t e^{-B(t')/2}dt'$$
$$r(y)\equiv\int^y e^{-A(y')/2}dy', \tag{4.1}$$

in terms of which the metric (2.4) becomes

$$ds^2=e^{A(r)}e^{B(\eta)}[-d\eta^2+\delta_{ij}dx^idx^j+e^{C(\eta)-B(\eta)}dr^2], \tag{4.2}$$

where $A(r)$, $B(\eta)$, and $C(\eta)$ are understood to be $A(y(r))$, $B(t(\eta))$, and $C(t(\eta))$.

We now introduce a fermion field ψ and an Abelian gauge field A_a through the action[2]

$$S_{\psi,A}=\int d^4x\,dr\sqrt{-g}\{e_A^a\bar{\psi}\Gamma^A(D_a-igA_a)\psi-\tfrac{1}{4}g^{ab}g^{cd}F_{ac}F_{bd}\}, \tag{4.3}$$

[2]Because of our choice for the metric's signature, we have not included the usual factor of i in the fermion action. The γ matrices are defined by $\Gamma^A=\{i\gamma^\mu,\gamma_5\}$, where γ^μ and γ_5 are the standard γ matrices, so that $\{\Gamma^A,\Gamma^B\}=2\eta^{AB}$ for $\eta^{AB}=\mathrm{diag}[-1,1,1,1,1]$. Refer to [7] or [8] for further details on fermions in a curved background.

where F_{ab} is the gauge field strength. We work in the limit in which these fields do not significantly affect the background geometry. In this action, we have not assumed an orbifold structure in the extra dimension, but as in the previous section one can be readily introduced. The e_A^a is a vierbein which connects the curved coordinates with a locally Lorentzian frame:

$$e_A^a e_B^b g_{ab} = \eta_{AB}. \tag{4.4}$$

The covariant derivative of a fermion

$$D_a = \partial_a + \tfrac{1}{2}\omega_{aBC}\sigma^{BC} \tag{4.5}$$

involves the spin connection defined by

$$\begin{aligned}\omega_{aAB} = {} & \tfrac{1}{2} e_A^b [\partial_a e_{Bb} - \partial_b e_{Ba}] \\ & - \tfrac{1}{2} e_B^b [\partial_a e_{Ab} - \partial_b e_{Aa}] \\ & - \tfrac{1}{2} e_A^c e_B^d [\partial_c e_{Dd} - \partial_d e_{Dc}] e_a^D \end{aligned} \tag{4.6}$$

and $\sigma^{BC} = \frac{1}{4}[\Gamma^B, \Gamma^C]$.

The contributions from the spin connection are canceled by introducing a rescaled fermion field defined by

$$\psi(x^\lambda, r) = e^{-A(r)} e^{-(3/4)B(\eta)} e^{-(1/4)C(\eta)} \Psi(x^\lambda, r). \tag{4.7}$$

The fermion-gauge field action then becomes

$$\begin{aligned} S_{\psi,A} = {} & \int d^4x\, dr \{ i\bar{\Psi}\gamma^\mu(\partial_\mu - igA_\mu)\Psi \\ & + e^{(B-C)/2}\bar{\Psi}\gamma^5(\partial_r - igA_r)\Psi \} \\ & + \int d^4x\, dr e^{(A+C)/2} \{ -\tfrac{1}{4}\eta^{\mu\nu}\eta^{\lambda\rho}F_{\mu\lambda}F_{\nu\rho} \\ & - \tfrac{1}{4} e^{(B-C)}\eta^{\mu\nu}F_{\mu r}F_{\nu r} \}; \end{aligned} \tag{4.8}$$

note that the first term no longer contains $A(r)$, $B(\eta)$, or $C(\eta)$.

A. The Kaluza-Klein expansion

To investigate the effect of the oscillating metric in the long distance limit, we expand the fermion and gauge fields in a tower of Kaluza-Klein modes. To simplify, we work in the axial gauge $A_r = 0$. The fermions are expanded separately in left- and right-handed fields,

$$\Psi_{L,R}(x^\lambda, r) \equiv \tfrac{1}{2}(1 \mp \gamma_5)\Psi = \sum_{n=0}^{\infty} \Psi_{L,R}^{(n)}(x^\lambda) f_{L,R}^{(n)}(r) \tag{4.9}$$

where $f_{L,R}^{(n)}(r)$ satisfy

$$\partial_r f_L^{(n)} = m_n f_R^{(n)}, \quad \partial_r f_R^{(n)} = -m_n f_L^{(n)} \tag{4.10}$$

and obey the following orthogonality condition:

$$\begin{aligned} & \int_{-r_c/2}^{r_c/2} dr\, f_L^{(m)*}(r) f_L^{(n)}(r) \\ & = \int_{-r_c/2}^{r_c/2} dr\, f_R^{(m)*}(r) f_R^{(n)}(r) = \delta^{mn}, \end{aligned} \tag{4.11}$$

where r_c is the volume of the extra dimension. Analogously, we expand the gauge field

$$A_\mu(x^\lambda, r) = \sum_{n=0}^{\infty} A_\mu^{(n)}(x^\lambda) h^{(n)}(r), \tag{4.12}$$

defining the masses of the modes through

$$\partial_r [e^{A/2}\partial_r h^{(n)}] = -M_n^2 e^{A/2} h^{(n)} \tag{4.13}$$

and normalizing the states through

$$\int_{-r_c/2}^{r_c/2} dr\, e^{A(r)/2} h^{(m)}(r) h^{(n)}(r) = \delta^{mn}. \tag{4.14}$$

The fermion-gauge interaction will induce couplings among the various Kaluza-Klein modes,

$$G_{L,R}^{mnp} \equiv g \int_{-r_c/2}^{r_c/2} dr\, f_{L,R}^{(m)*}(r) f_{L,R}^{(n)}(r) h^{(p)}(r). \tag{4.15}$$

The effective action that appears in four dimensions as a result of Eq. (4.8) is thus

$$\begin{aligned} S_{\psi,A} = {} & \int d^4x \Bigg\{ \sum_{m,n} i\bar{\Psi}_L^{(m)}\gamma^\mu \bigg(\delta^{mn}\partial_\mu - i\sum_p G_L^{mnp} A_\mu^{(p)} \bigg) \Psi_L^{(n)} \\ & + \sum_{m,n} i\bar{\Psi}_R^{(m)}\gamma^\mu \bigg(\delta^{mn}\partial_\mu - i\sum_p G_R^{mnp} A_\mu^{(p)} \bigg) \Psi_R^{(n)} \\ & + e^{(B-C)/2} \sum_n m_n (\bar{\Psi}_L^{(n)}\Psi_R^{(n)} + \bar{\Psi}_R^{(n)}\Psi_L^{(n)}) \Bigg\} \\ & + \int d^4x\, e^{C/2} \sum_n \{ -\tfrac{1}{4}\eta^{\mu\nu}\eta^{\lambda\rho}F_{\mu\lambda}^{(n)}F_{\nu\rho}^{(n)} \\ & - \tfrac{1}{2} e^{(B-C)} M_n^2 \eta^{\mu\nu} A_\mu^{(n)} A_\nu^{(n)} \}. \end{aligned} \tag{4.16}$$

At low energies, well below the Planck scale, only the dynamics of the massless modes will be important in the effective theory. The r-dependent parts of the lowest lying fermions are

$$f_L^{(0)} = f_R^{(0)} = r_c^{-1/2}$$

along with a factor of $e^{-A(r)}$ from Eq. (4.7). The couplings among the massless modes simplify to

$$g_0 \equiv G_L^{000} = G_R^{000} = \frac{g}{r_c}\int_{-r_c/2}^{r_c/2} dr\, h^{(0)}(r). \tag{4.17}$$

The low energy effective action is thus

$$S_{\mathrm{eff}}=\int d^4x\{i\bar{\Psi}^{(0)}\gamma^\mu(\partial_\mu-ig_0A_\mu^{(0)})\Psi^{(0)} -\tfrac{1}{4}e^{C/2}F_{\mu\nu}^{(0)}F^{(0)\mu\nu}+\cdots\}. \tag{4.18}$$

In this effective action, the gravitational oscillations do not couple directly to the fermions, but rather to the gauge fields. In the small amplitude limit that we have assumed, the leading interaction is

$$\mathcal{L}_{\mathrm{interaction}}=-\tfrac{1}{8}C(t)F_{\mu\nu}^{(0)}F^{(0)\mu\nu}. \tag{4.19}$$

This coupling offers a natural channel for the decay of the oscillating component of the metric. Similarly, any fundamental scalar fields in the theory would also allow the decay of the oscillating gravitational field. The kinetic energy term for the zero mass Kaluza-Klein mode of a massless five-dimensional scalar field has a prefactor of $e^{B+C/2}$.

If the fermion in Eq. (4.18) had a small realistic mass m, either through a fermion mass in the five-dimensional action or if a massless fermion subsequently develops a dynamical mass, a mass term would produce a coupling between the fermion and the gravitational oscillations. However, this term would be suppressed by m/M_5 relative to Eq. (4.19).

V. ULTRAHIGH ENERGY COSMIC RAYS

If some slowly decaying residual oscillations were to exist today, the gauge field products of this decay would provide a possible source for the ultrahigh energy cosmic rays. Here we shall only focus on demonstrating that a realistic cosmic ray spectrum can arise and then establish a rough limit on the rate of the decay. We shall not attempt to develop a detailed model that produces this rate. The dominant signal would likely result from a decay of the oscillating background into energetic gluons. The natural size for ω_t when μ and ν are not finely tuned in Eq. (2.6) is $M_5\sim M_{\mathrm{Pl}}$, where M_{Pl} is the effective four-dimensional Planck scale. Moreover, since the oscillating metric in Eq. (2.4) is spatially isotropic, the interaction (4.19) superficially resembles the decay of a very massive particle at rest. An initial gluon of energy $E_0\sim M_{\mathrm{Pl}}$, produced in any of the two, three, or four gluon channels in Eq. (4.19), will fragment into a high multiplicity jet of particles with a wide range of energies.[3] For an initial photon pair the secondaries are only those produced as the photons scatter from the intergalactic radio background, and thus should not provide as strong a constraint as gluon production. Additionally, the presence of any gauge interactions beyond the standard model could provide other decay channels.

The observed flux of ultrahigh energy cosmic rays, assumed here to be protons, can then set a bound on rate of production, or equivalently on the rate of decay of the effective vacuum energy density ρ_{vac}. We estimate the flux of protons above some energy $E_>$ by

$$J_>\approx\frac{1}{4\pi}\dot{\rho}_{\mathrm{vac}}E_0^{-1}\ell N_>. \tag{5.1}$$

$N_>$ is the number of protons with energy $E>E_>$ in a jet produced by the initial gluon of energy $E_0\sim M_{\mathrm{Pl}}$. ℓ is the attenuation length of these protons due to scattering from the cosmic microwave background radiation [9]. When $E_>\approx10^{11}$ GeV then ℓ is approximately a few tens of Mpc [10]. This energy is just above the expected but not seen Greisen-Zatsepin-Kuzmin (GZK) cutoff [9], which is exceeded here by high energy protons originating within a distance ℓ of us.

We shall obtain an estimate of $N_>$ from the perturbative analysis of multiparticle production in jets based on the modified leading logarithm approximation (e.g., [11]).[4] The resulting "limiting spectrum" is known but since the initial gluon energy is so high, $\tau\equiv\ln(E_0/\Lambda_{\mathrm{QCD}})\gg1$, we may simplify further with a Gaussian representation. We extract the following results from [11] where $\xi\equiv\ln(E_0/E)$:

$$\frac{dN}{d\xi}\propto\exp\left(-\frac{1}{2}\frac{(\xi-\bar{\xi})^2}{\sigma^2}\right), \tag{5.2}$$

$$\sigma=\frac{\tau}{\sqrt{3z}}\left(1-\frac{3}{4z}\right),\quad \bar{\xi}=\tau(\tfrac{1}{2}+\sqrt{C/\tau}). \tag{5.3}$$

Here $z=\sqrt{16N_c\tau/b}$, $C=a^2/(16N_cb)$, $3b=11N_c-2n_f$, and $3a=11N_c+2n_f/N_c^2$. We also note that a Gaussian spectrum resembles the results from Monte Carlo simulations [12]. For the total multiplicity $\mathcal{N}=\int_0^\infty(dN/d\xi)d\xi$ we use the full "limiting spectrum" result,

$$\mathcal{N}=\Gamma(B)\left(\frac{z}{2}\right)^{1-B}I_{B+1}(z), \tag{5.4}$$

where $B=a/b$. This gives $\mathcal{N}\approx7\times10^5$. Assuming that 5% of the energy from each initial gluon emerges as protons, these results imply that $N_>\approx3000$ for $E_>=10^{11}$ GeV and $n_f=6$. This value for $N_>$ triples when the $\mathcal{O}(\tau^{-1/2})$ corrections in Eq. (5.3) are absent. Another indication of the sensitivity of the result to the approximation is that no single value of n_f is actually correct over the range of energies involved, and $N_>$ ranges from 1400 to 6000 as n_f ranges from 3 to 8. This last result also shows how $N_>$ can be strongly affected by new physics beyond the standard model.

The observed integrated flux of cosmic rays with $E>10^{11}$ GeV is approximately $2\times10^{-20}\ \mathrm{cm}^{-2}\,\mathrm{s}^{-1}\,\mathrm{sr}^{-1}$ [13], and so Eq. (5.1) and $N_>\approx3000$ imply that

$$\dot{\rho}_{\mathrm{vac}}<3\times10^{-53}\ \mathrm{g\,cm}^{-3}\,\mathrm{s}^{-1}. \tag{5.5}$$

Thus the current vacuum energy density of the universe, assumed to be about two-thirds of the critical density, has a decay rate

[3] For simplicity we shall treat the initial gluons as monoenergetic, rather than integrating over the energy distributions in the three- and four-gluon cases, since the strongest constraint comes from the most energetic gluons.

[4] In this approximation the main difference at leading order between a quark and a gluon jet is that the multiplicity of particles in the gluon jet is enhanced by a factor of 9/4 relative to the quark jet for the same initial jet energy. The mean particle energy of products will correspondingly be slightly lower in the gluon jet. In the following we have treated a gluon jet the same as a quark jet.

$$\frac{\dot{\rho}_{\rm vac}}{\rho_{\rm vac}} < 10^{-6}\frac{1}{t_{\rm universe}}, \tag{5.6}$$

where $t_{\rm universe} \approx 14$ Gyr is the age of the universe.

If this limit is saturated then we have a model for the ultrahigh energy cosmic rays. The spectrum in Eq. (5.2) implies a hard energy spectrum $dN/dE \sim 10^{-\alpha}$ with $\alpha \approx 1.2$ at the relevant energies. The observed flux spectrum dJ/dE is modified by the rapid decrease of the attenuation length by almost a factor of 100 in the range $4\times 10^{10} < E < 10^{11}$ GeV (the GZK cutoff). The frequently plotted $E^3 dJ/dE$ thus rises for $E < 4\times 10^{10}$ GeV, drops for $4\times 10^{10} < E < 10^{11}$ GeV, and continues to rise for $E > 10^{11}$ GeV. This behavior is a simple consequence of a hard initial spectrum of protons produced uniformly throughout space, and is quite consistent with the data above 10^{10} GeV. Below 10^{10} GeV the observed flux rises much faster with decreasing energy, and some other source for cosmic rays must dominate.

This picture is similar to models involving decay of supermassive long lived particles [14–16]. The primary difference is that the supermassive particles tend to congregate in the galactic halo, and this produces a galactic component to the signal in addition to a possible extragalactic one. But the galactic component would not produce a dip in the spectrum, and it would produce some amount of large scale anisotropy [15,16]. In our picture the absence of a galactic component is natural, although there remains the question of just how spatially uniform the decay mechanism would be.

The bound on the decay of the oscillating background in Eq. (5.5) provides a constraint only on the *cosmologically recent* decay rate. Yet, just as in models in which the ultrahigh energy cosmic rays originate from the decay of very massive particles, the low energy diffuse γ-ray spectrum provides an additional constraint on how the decay rate itself changes during the evolution of the universe. For example, assuming that the decay rate in Eq. (5.5) evolves as t^{-4+p} [17], then the observed flux of γ rays with energies 10 MeV–100 GeV generally allows models with $p \geqslant 2$ [18]. For smaller values of p, the observed spectrum does not allow sources that produce initial gluons with energies greater than about 10^{16} GeV [18,19]. Therefore, decay of the background either should not proceed too efficiently, which accords with Eq. (5.6), or should have only begun recently. As an example of the former, a mechanism that produced a slow exponential decay at the rate given in Eq. (5.6) would effectively decay with $p \approx 4$.

The production of cosmic gluon jets also ties in with another mechanism for producing air showers above the GZK cutoff, this time involving jets produced outside the ℓ^3 volume. This is because gluon jets contain neutrinos, and these energetic cosmic neutrinos have some probability for producing Z bursts within the ℓ^3 volume, as they travel toward us [20]. In particular, neutrinos with energy within $\delta E/E_R = \Gamma_Z/M_Z \approx 3\%$ of the Z resonance energy $E_R = 4\,({\rm eV}/m_\nu)\times 10^{12}$ GeV may annihilate with an enhanced cross section on the nonrelativistic relic antineutrinos (and vice versa) to produce the Z. We estimate (with the same uncertainties as before) that the number of neutrinos in this energy band, produced per gluon jet, ranges from 300 to 1100 as E_R ranges from 10^{13} to 10^{11} GeV. The probability for such a neutrino to produce a Z burst within the ℓ^3 volume is in the range 0.025%–1%, depending on the relic neutrino clustering [20]. On the other hand, there is an enhancement factor of about 100 for the neutrino flux relative to the direct proton flux since the former originates from the whole Hubble volume. Each Z burst produces two protons with typical energies $E_p \approx E_R/30$, and thus this mechanism produces protons peaked in a fairly narrow energy range.

VI. CONCLUSIONS

A warped, compact background space-time with a compact scalar field, introduced to address the cosmological constant problem, does not present any obstructions to including chiral fermions. Since the compactness of the extra dimension resulted from a periodic solution to the field equations, rather than adding brane boundaries, the most appropriate approach is to give the universe an $S^1/\mathbb{Z}_2$ orbifold structure in this dimension.

A Kaluza-Klein expansion of massless fermion and gauge fields in a warped and oscillating background reveals that the zero mass modes of the fermion do not couple at leading order with the oscillating terms in the metric. Thus the gauge fields would most readily facilitate the decay of the oscillating gravitational field.

The decay of the oscillations in the metric could proceed rapidly in the early universe, at least until the amplitude is sufficiently small that the oscillatory effects are comparable to any de Sitter expansion or to effects from any matter and radiation present, in which case the simple picture developed in Sec. II breaks down and the subsequent evolution is more complex. Yet it would be useful to determine when such oscillations could coexist with a familiar cosmology in the $(3+1)$-dimensional effective theory since we have seen that a small, slowly relaxing oscillation today could provide a source for the observed ultrahigh energy cosmic rays.

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