S_3 and the $L=1$ baryons in the quark model and the chiral quark model

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The S_3 symmetry corresponding to permuting the positions of the quarks within a baryon allows us to study the 70-plet of $L=1$ baryons without an explicit choice for the spatial part of the quark wave functions: given a set of operators with definite transformation properties under the spin-flavor group $SU(3)\times SU(2)$ and under this S_3, the masses of the baryons can be expressed in terms of a small number of unknown parameters which are fit to the observed $L=1$ baryon mass spectrum. This approach is applied to study both the quark model and chiral constituent quark model. The latter theory leads to a set of mass perturbations which more satisfactorily fits the observed $L=1$ baryon mass spectrum (though we can say nothing, within our approach, about the physical reasonableness of the parameters in the fit). Predictions for the mixing angles and the unobserved baryon masses are given for both models as well as a discussion of specific baryons. [S0556-2821(99)01907-4]

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I. INTRODUCTION

The nonrelativistic quark model has been used extensively to study the $L=1$ baryons [1,2]. In this model, the observed mass spectrum of the baryons is generated by a two body Coulombic potential [3], produced by a gluon exchange between two quarks. The quark model leads to a definite form for the $SU(3)\times SU(2)$ spin-flavor breaking interactions but not for the ground state quark wave functions. What is typically done is to use harmonic oscillator wave functions for the spatial wave functions. The picture that then emerges for the baryon masses is surprisingly good given the simplicity of the model; nevertheless, it has several serious shortcomings, most notoriously its inability to explain the lightness of the $\Lambda(1405)$. Extensions of the quark model, such as the inclusion of relativistic effects [4] although leading to a better agreement with the entire spectrum of light mesons and baryons, do not seem to improve much upon quark model's description of the $L=1$ baryons.

Another model has recently emerged to explain the observed baryon spectrum [5]. Its assumption is that the correct effective theory within a baryon is not that of constituent quarks exchanging gluons but rather that of the quasiparticles, constituent quarks and Goldstone bosons, appropriate for energies below the scale of chiral symmetry breaking. The low energy quark potential in this theory, which we refer to as the chiral quark model, is also a two body Coulombic potential with an important difference—the inclusion of flavor matrices at the quark-Goldstone boson vertices. We shall here check the claim that the different flavor structure of the chiral quark model leads to a better fit with the observed $L=1$ spectrum.

The new ingredient in our study of the $L=1$ bosons is the use of the permutation group S_3 to organize the spatial behavior of the quarks and their interactions. Physically, this symmetry corresponds to the fact that the confining potential should treat the light quarks equivalently and should be invariant under permutations of the positions of the three quarks. It allows us to circumvent choosing a definite form for the quarks' spatial wave functions using group theory to keep track of our ignorance of this spatial behavior. What distinguishes one model from another is the $SU(3)\times SU(2)$ spin-flavor and S_3 transformation properties of the operators that produce the mass splittings among the $L=1$ baryons. We begin therefore with a description of this symmetry, how the baryons transform under it and how it fits with the standard $SU(3)\times SU(2)$ spin-flavor structure of the baryons and then proceed to determine how the standard mass splitting operators transform under S_3 in each of the models.

Our approach has a potential disadvantage, in addition to its obvious advantages. Because we have not made dynamical assumptions about the wave functions, but only used symmetry, we cannot say within our approach whether the parameters of the fits we obtain are physically reasonable. Thus our results should not be interpreted, by themselves, as evidence in favor of the chiral quark model picture over the nonrelativistic quark model. However, we believe it is worth noting that difficulties for the nonrelativistic quark model persist even in this very general approach.

II. S_3 AND THE $L=1$ BARYONS

The $L=1$ negative parity baryons form a seventy dimensional representation of the spin-flavor group $SU(6)$. This 70-plet breaks into the representations ${}^4 8, {}^2 8, {}^2 10, {}^2 1$ under separate spin and flavor transformations; here the notation ${}^{2S+1}F$ indicates a multiplet that forms an F dimensional representation of the $SU(3)$ flavor group with spin S. Among the interactions that we shall consider are spin-orbit couplings between quarks so that the baryonic states will be written, and are measured, in terms of the total angular momentum, $J=L+S$. The baryonic states are then represented by linear superpositions of spatial, flavor, spin, and orbital angular momentum wave functions for the three contituent

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quarks. The construction of these states for the $SU(3)\times SU(2)$ spin-flavor part is straightforward in the nonrelativistic limit, but the spatial wave functions requires a specific dynamical model. Some of the earlier studies of the $L=1$ baryons used harmonic oscillator wave functions for the spatial wave functions in terms of the relative positions of the quarks ([1] and [2]). One of the disadvantages of this approach is that if a poor agreement is found for a model with the observed baryon spectrum, it is not immediately clear whether that failure lies in the model itself or in the specific choice of the spatial wave functions.

Fortunately, the spatial interactions of the quarks possess a symmetry that allows us to escape the choice of a specific dynamical model for the spatial wave functions. The quark interactions should be invariant under any permutation of the positions of the quarks. This symmetry then implies that the spatial wave functions should form a representation of the group S_3. We shall find that the S_3 group theory is sufficiently powerful to reduce our ignorance of the spatial behavior to a small set of constants. The baryonic matrix elements will be linear combinations of these S_3 constants whose coefficients depend upon the spin-flavor assignments of the quark interactions and are completely determined by the $SU(3)\times SU(2)$ group theory. One of the advantages of this approach is that it treats different models on the same footing.

The baryonic wave functions that we used are those used in [6] in which each of the three-quark states has only one of the quarks in an orbitally excited ($L=1$) state, $|1,m\rangle$. For example, the $|\Delta^{++};\frac{1}{2};\frac{3}{2},\frac{3}{2}\rangle(|\Delta^{++};S;J,M_J\rangle)$ state would thus be

$$\left|\Delta^{++};\frac{1}{2};\frac{3}{2},\frac{3}{2}\right\rangle=-\frac{\sqrt{2}}{6}uuu\{\psi^1_{11}(\vec{r}_1,\vec{r}_2,\vec{r}_3)(|\uparrow\downarrow\uparrow\rangle+|\uparrow\uparrow\downarrow\rangle-2|\downarrow\uparrow\uparrow\rangle)+\psi^2_{11}(\vec{r}_1,\vec{r}_2,\vec{r}_3)(|\uparrow\uparrow\downarrow\rangle+|\downarrow\uparrow\uparrow\rangle-2|\uparrow\downarrow\uparrow\rangle)+\psi^3_{11}(\vec{r}_1,\vec{r}_2,\vec{r}_3)\times(|\downarrow\uparrow\uparrow\rangle+|\uparrow\downarrow\uparrow\rangle-2|\uparrow\uparrow\downarrow\rangle)\}. \tag{2.1}$$

The notation $\psi^a_{lm}(\vec{r}_1,\vec{r}_2,\vec{r}_3)$ represents a three-quark spatial wave function for which the ath quark is in the $|l,m\rangle$ orbitally excited state and the other two lie in the ground state. Note that this spatial wave function could also have been written solely in terms of the relative coordinates, $\vec{r}_i-\vec{r}_j$. The three positions of the quarks have been included to emphasize that this function depends upon the center of mass coordinate. We shall assume that this dependence cancels when the terms are summed so that the final baryonic state only depends on the quarks' relative coordinates.[1] This assumption is certainly true for a wide class of quark potential models including those of [1–5]. This observation is important for understanding the S_3 transformation properties of the terms on each side of this equation.

[1]In [1] and [2] these coordinates are usually written $\vec{\rho}=(\vec{r}_1-\vec{r}_2)/\sqrt{2}$ and $\vec{\lambda}=(\vec{r}_1+\vec{r}_2-2\vec{r}_3)/\sqrt{6}$. $\vec{r}_i$ is the position of the ith quark.

TABLE I. Character table for S_3.

	e	r	c
S	1	1	1
A	1	−1	1
M	2	0	−1

S_3

Since the permutation group S_3 plays an important role in this analysis of the $L=1$ baryons, we review its basic properties at the same time establishing our notation. The group has three irreducible representations: the trivial representation S which corresponds to the completely symmetric Young tableau $\square\square\square$ a one dimensional representation A that maps reflections to -1 and corresponds to the tableau $\begin{smallmatrix}\square\\\square\\\square\end{smallmatrix}$ and the two dimensional representation M corresponding to $\begin{smallmatrix}\square\square\\\square\end{smallmatrix}$. The character table for the conjugacy classes of S_3—the identity e, the three reflections r and the two cyclic permutations c—is presented in Table I. From this table we can derive the following rules for the tensor products of the irreducible representations:

$$S\otimes S=S,\quad S\otimes A=A,\quad S\otimes M=M,$$
$$A\otimes A=S,\quad A\otimes M=M,$$
$$M\otimes M=S\oplus A\oplus M. \tag{2.2}$$

Any three objects that may be permuted among each other form a three dimensional, defining representation of S_3. The positions of the three quarks, $\{\vec{r}_1,\vec{r}_2,\vec{r}_3\}$, for example, form a **3** of S_3. This representation is not irreducible and can be separated into the center of mass coordinate,

$$\vec{R}=\frac{1}{\sqrt{3}}(\vec{r}_1+\vec{r}_2+\vec{r}_3),$$

and a pair of coordinates for the internal motion,

$$\begin{pmatrix}\vec{r}_+\\ \vec{r}_-\end{pmatrix}=\begin{pmatrix}\frac{1}{\sqrt{6}}(\vec{r}_1+\vec{r}_2-2\vec{r}_3)\\ \frac{1}{\sqrt{2}}(\vec{r}_1-\vec{r}_2)\end{pmatrix}.$$

This basis explicitly realizes the decomposition, $\mathbf{3}=S\oplus M$.

The three-quark wave functions in Eq. (2.1) transform as $\mathbf{3}=S\oplus M$ since they depend on all three positions. The $L=1$ baryons, however, transform as a two-dimensional M representation, both under the S_3 which corresponds to the spin-flavor group $SU(6)$ as well as the S_3 referring to the spatial wave functions. This representation for the $L=1$ baryons, combined with those for the mass splitting opera-

tors introduced in the next section, are the only information we require of the spatial behavior of the quarks. The S_3 group theory is sufficiently restrictive to allow all the baryon matrix elements of mass operators to be reduced to expressions involving a small number of undetermined constants.

III. MASS OPERATORS

The definitions of the ground state baryon wave functions are essentially the same for any nonrelativistic quark model. Only in the spatial wave functions might one model differ from another—but what is important for our approach is only the S_3 transformation properties, not the details of the spatial dependence. The lowest order differences among models appear in the perturbations to the ground state functions. The two theories we compare here—the quark model and the chiral quark model—have a similar set of operators which differ in the appearance of flavor matrices in the interactions of the latter theory.

Our problem is to solve for the masses of the $L=1$ baryons in the nonrelativistic limit, $H|\Psi\rangle=E|\Psi\rangle$. The Hamiltonian, $H=H_0+V$, is assumed to be the sum of an $SU(6)$ symmetric confining term H_0 which does not distinguish the masses of the 70-plet and a perturbative potential V that depends on the model being studied. The first model that we study, both as an illustration of the method and a benchmark against which to compare other models, is the constituent quark model ([3] and [1]). In this model the perturbative potential arises from the first-order term in the expansion of a two-body Coulombic interaction between pairs of quarks:[2]

$$V=V_{ss}+V_{so}+V_q. \tag{3.1}$$

Traditionally [3], these interactions are of the form

$$V_{ss}=\sum_{i<j}\frac{16\pi\alpha_s}{9}\frac{1}{m_i m_j}\vec{s}_i\cdot\vec{s}_j\delta(\vec{r}_{ij}), \tag{3.2}$$

a spin-spin interaction,

$$V_{so}=\sum_{i<j}\frac{\alpha_s}{3r_{ij}^3}\left[\frac{1}{m_i^2}(\vec{r}_{ij}\times\vec{p}_i)\cdot\vec{s}_i-\frac{1}{m_j^2}(\vec{r}_{ij}\times\vec{p}_j)\cdot\vec{s}_j+\frac{2}{m_i m_j}\times(\vec{r}_{ij}\times\vec{p}_i)\cdot\vec{s}_j-\frac{2}{m_i m_j}(\vec{r}_{ij}\times\vec{p}_j)\cdot\vec{s}_i\right], \tag{3.3}$$

a spin-orbit coupling, and

$$V_q=\sum_{i<j}\frac{2\alpha_s}{3r_{ij}^3}\frac{1}{m_i m_j}\left[\frac{3}{r_{ij}^2}(\vec{r}_{ij}\cdot\vec{s}_i)(\vec{r}_{ij}\cdot\vec{s}_j)-(\vec{s}_i\cdot\vec{s}_j)\right], \tag{3.4}$$

a quadrupole (or tensor) interaction. Here, $\vec{r}_{ij}\equiv\vec{r}_i-\vec{r}_j$ and $r_{ij}\equiv|\vec{r}_{ij}|$ where $\vec{r}_i$, $\vec{p}_i$, $\vec{s}_i$, and m_i are the position, momentum, spin, and mass of the ith quark. As with the quark wave functions, our treatment is independent of the radial dependences of these potentials. The only important feature of these operators is their spin, orbital angular momentum, and flavor structure. Therefore, our analysis applies equally well to any set of hyperfine and spin-orbit interactions of the form

$$V_{ss}=\sum_{i<j}f_0(r_{ij})\frac{1}{m_i m_j}\vec{s}_i\cdot\vec{s}_j,$$

$$V_{so}=\sum_{i<j}f_1(r_{ij})\left[\frac{1}{m_i^2}(\vec{r}_{ij}\times\vec{p}_i)\cdot\vec{s}_i-\frac{1}{m_j^2}\times(\vec{r}_{ij}\times\vec{p}_j)\cdot\vec{s}_j+\frac{2}{m_i m_j}(\vec{r}_{ij}\times\vec{p}_i)\cdot\vec{s}_j-\frac{2}{m_i m_j}(\vec{r}_{ij}\times\vec{p}_j)\cdot\vec{s}_i\right],$$

$$V_q=\sum_{i<j}f_2(r_{ij})\frac{1}{m_i m_j}[3(\hat{r}_{ij}\cdot\vec{s}_i)(\hat{r}_{ij}\cdot\vec{s}_j)-(\vec{s}_i\cdot\vec{s}_j)], \tag{3.5}$$

where the $f_a(r_{ij})$ are arbitrary functions of the distances between the interacting quarks. Note that we have included a factor of two in the $1/m_i m_j$ terms of the spin-orbit potential, V_{so}, to match the non-relativistic limit for the potential. We have retained this factor here [and later in Eq. (3.6)] since when $SU(3)$ breaking effects, such as a heavier constituent mass for the strange quark, are included in V_{so}, the fits depend upon the choice of this factor. In the limit that the spin-dependent interactions are taken to be $SU(3)$-symmetric, this dependence disappears and the factors of two can be replaced by an arbitrary coefficient without affecting our results.

The second model we study is motivated by a recent proposal by Glozman and his collaborators [5]. The idea is that as the typical momentum of a quarks within a baryon is below the scale of chiral symmetry breaking, the correct dynamical degrees of freedom are those of the constituent quarks which couple to Goldstone boson fields of the broken symmetry group, $SU(3)_L\times SU(3)_R\to SU(3)_V$. This model, the chiral quark model, modifies the low-energy Coulombic potential since the constituent quark-Goldstone boson vertices carry additional $SU(3)$ flavor matrices, λ_i^a (see Fig. 1).

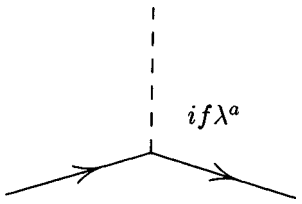

FIG. 1. The constituent quark-Goldstone boson vertex of the chiral quark model. λ^a is a Gell-Mann flavor matrix.

This vertex produces perturbative potentials of the same form as in the quark model except for the inclusion of a flavor factor, $\vec{\lambda}_i\cdot\vec{\lambda}_j\equiv\Sigma_{a=1}^{8}\lambda_i^a\lambda_j^a$:

$$V_{ss}=\sum_{i<j}g_0(r_{ij})\left\{\frac{1}{m_i m_j},\vec{\lambda}_i\cdot\vec{\lambda}_j\right\}\vec{s}_i\cdot\vec{s}_j, \tag{3.6}$$

[2]Sometimes ([1] and [4]) the spin-spin and the quadrupole operator are grouped together and called the hyperfine interaction, $V_{\rm hyp}=V_{ss}+V_q$.

$$V_{so}=\sum_{i<j} g_1(r_{ij})\left[\left\{\frac{1}{m_i^2},\vec{\lambda}_i\cdot\vec{\lambda}_j\right\}(\vec{r}_{ij}\times\vec{p}_i)\cdot\vec{s}_i - \left\{\frac{1}{m_j^2},\vec{\lambda}_i\cdot\vec{\lambda}_j\right\}(\vec{r}_{ij}\times\vec{p}_j)\cdot\vec{s}_j + \left\{\frac{2}{m_i m_j},\vec{\lambda}_i\cdot\vec{\lambda}_j\right\}(\vec{r}_{ij}\times\vec{p}_i)\cdot\vec{s}_j - \left\{\frac{2}{m_i m_j},\vec{\lambda}_i\cdot\vec{\lambda}_j\right\}(\vec{r}_{ij}\times\vec{p}_j)\cdot\vec{s}_i\right],$$

$$V_q=\sum_{i<j} g_2(r_{ij})\left\{\frac{1}{m_i m_j},\vec{\lambda}_i\cdot\vec{\lambda}_j\right\}[3(\hat{r}_{ij}\cdot\vec{s}_i)\times(\hat{r}_{ij}\cdot\vec{s}_j)-(\vec{s}_i\cdot\vec{s}_j)],$$

where we have written the arbitrary functions as $g_a(r_{ij})$ to emphasize that they need not be the same as those in the quark model. The anticommutators ensure that the operators are Hermitian.

S_3 transformation properties of the mass operators

The fact that the states and the potentials in Eqs. (3.5) and (3.6) transform as definite representations of S_3 allows us to constrain greatly the number of unknown parameters in the theory. We therefore first present a method for counting the number of independent constants before writing them in a more concrete form: as matrix elements of specific operators between three-quark states. Both pieces of the hyperfine interaction, the spin-spin and the quadrupole operators, transform as three dimensional representations of S_3; both are manifestly symmetric under exchanging the interacting quarks. In terms of the irreducible representations of S_3, we saw that the **3** could be decomposed as

$$\mathbf{3}=S\oplus M, \tag{3.7}$$

that is, both the spin-spin operator and the quadrupole operator have a piece transforming as the trivial representation and a piece transforming as the two-dimensional representation. The matrix elements $\langle\mathbf{70}|V_{ss}|\mathbf{70}\rangle$ or $\langle\mathbf{70}|V_q|\mathbf{70}\rangle$, which produce the perturbations to the baryon mass spectrum, contain the following tensor product of S_3 representations:

$$M\otimes(S\oplus M)\otimes M=\underbrace{S\oplus S}_{\text{trivial}}\oplus A\oplus A\oplus M\oplus M\oplus M\oplus M. \tag{3.8}$$

The trivial representation appears twice in the matrix element. From the Wigner-Eckart theorem, we can conclude that the matrix elements of V_{ss} and V_q are each completely determined up to two unknown constants. In specific models, these constants correspond physically to spatial integrals.

The spin-orbit term, V_{so}, is slightly more complicated. It transforms as a six dimensional fundamental representation of S_3. The decomposition of the **6** into its irreducible components is

$$\mathbf{6}=S\oplus A\oplus M\oplus M. \tag{3.9}$$

Counting the number of unknown constants, we learn that the matrix elements of V_{so} between **70** states,

$$\langle\mathbf{70}\,|\,V_{so}\,|\,\mathbf{70}\rangle\rightarrow M\otimes\mathbf{6}\otimes M=\underbrace{S\oplus S\oplus S\oplus S}_{\text{trivial}}\oplus A\oplus A\oplus A\oplus A\oplus\overbrace{M\oplus\cdots\oplus M}^{8\text{ copies}}, \tag{3.10}$$

depend upon four undetermined constants since the trivial representation appears four times.

It is helpful to have a specific form for these independent integrals which can be calculated for a particular model for the quark wave functions. The spin-orbit operator being the most complicated, we begin with it. It can be written in the form[3]

$$V_{so}=\sum_{i<j}\left[\frac{1}{m_i^2}\vec{\mathcal{L}}_{ij}\cdot\vec{s}_i+\frac{2}{m_i m_j}\vec{\mathcal{L}}_{ji}\cdot\vec{s}_i+\frac{2}{m_i m_j}\vec{\mathcal{L}}_{ij}\cdot\vec{s}_j+\frac{1}{m_j^2}\vec{\mathcal{L}}_{ji}\cdot\vec{s}_j\right], \tag{3.11}$$

where

$$\vec{\mathcal{L}}_{ij}\equiv f_1(r_{ij})[\vec{L}_i-(\vec{r}_j\times\vec{p}_i)], \tag{3.12}$$

$\vec{L}_i$ being the orbital angular momentum of the ith quark. In the nonrelativistic limit, the spin and the flavor structures are completely calculable—what is relevant for the S_3 group theory is the spatially dependent operator $\vec{\mathcal{L}}_{ij}$. In the matrix

[3]When the masses of the quarks are equal in V_{so}, the factors of two for the $1/m_i m_j$ terms can be replaced by an arbitrary coefficient without altering the results that follow. The expression in the chiral quark model is analogous.

TABLE II. Best-fit values of the S_3 constants for the quark model and chiral quark model. (*) All of the parameters are in units of MeV except δm which is dimensionless.

Perturbation (MeV)*	Quark model	Chiral quark model
$\mathcal{D}_+$	196.9	−87.78
$\mathcal{D}_-$	−19.50	−52.11
$\mathcal{SO}_1$	30.85	−1.106
$\mathcal{SO}_2$	47.99	15.93
$\mathcal{SO}_3$	−90.50	17.40
$\mathcal{Q}_1$	12.63	−14.10
$\mathcal{Q}_2$	7.762	−15.38
δm	0.269	0.286
M_0	1613	1477
χ^2	123.6	24.76

element $\langle \mathbf{70}|V_{so}|\mathbf{70}\rangle$ appear sums of matrix elements of this operator between $L=1$ three-quark states,

$$\langle \psi^a_{1m}(\vec{r}_1,\vec{r}_2,\vec{r}_3)|\vec{\mathcal{L}}_{12}|\psi^b_{1m'}(\vec{r}_1,\vec{r}_2,\vec{r}_3)\rangle = \mathcal{M}^{ab}\langle 1,m|\vec{L}|1,m'\rangle, \tag{3.13}$$

where $\vec{L}$ is the total orbital angular momentum of the three-quark state. The matrix elements for other choices of i and j for $\vec{\mathcal{L}}_{ij}$ are similarly defined but are simply a permutation of the entries of the elements of $\mathcal{M}^{ab}$. The matrix $\mathcal{M}^{ab}$ is a 3×3 Hermitian matrix of spatial integrals differing only as to which of initial or final the quarks is excited. We know from the Wigner-Eckart theorem that the matrix elements of the 70-plet baryons cannot depend separately upon all of the elements of the matrix $\mathcal{M}$, but only upon four linear combinations; these combinations are

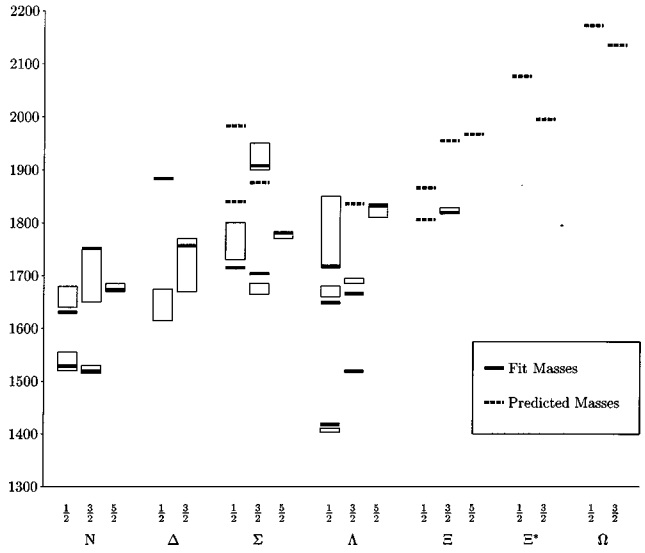

FIG. 2. Masses of the $L=1$ baryons in the quark model; $\chi^2=123.6432$, $N_{fit}=18$.

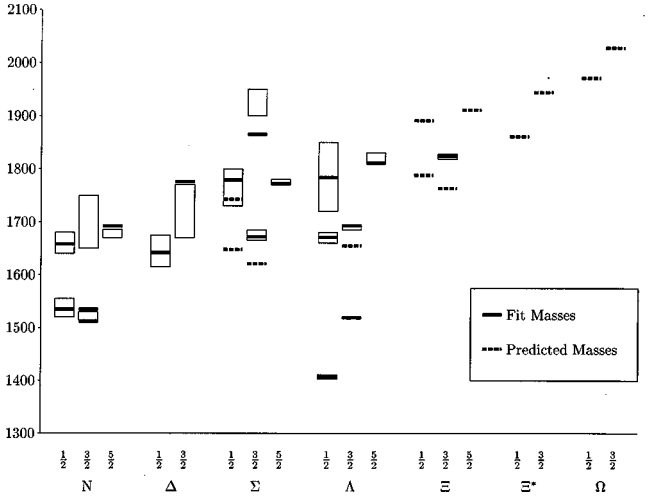

FIG. 3. Masses of the $L=1$ baryons in the chiral quark model; $\chi^2=24.7568$, $N_{fit}=18$.

$$\mathcal{SO}_1 \equiv \mathcal{M}^{11}+\mathcal{M}^{22}-\mathcal{M}^{12}-\mathcal{M}^{21},$$

$$\mathcal{SO}_2 \equiv \mathcal{M}^{11}-\mathcal{M}^{22}-\mathcal{M}^{13}+\mathcal{M}^{23}-\mathcal{M}^{31}+\mathcal{M}^{32},$$

$$\mathcal{SO}_3 \equiv \mathcal{M}^{11}+\mathcal{M}^{22}+4\mathcal{M}^{33}+\mathcal{M}^{12}+\mathcal{M}^{21} -2\mathcal{M}^{13}-2\mathcal{M}^{23}-2\mathcal{M}^{31}-2\mathcal{M}^{32},$$

$$\mathcal{SO}_4 \equiv (\mathcal{M}^{12}-\mathcal{M}^{21})-(\mathcal{M}^{13}-\mathcal{M}^{31}) +(\mathcal{M}^{23}-\mathcal{M}^{32}).$$

The matrix $\mathcal{M}$ being Hermitian, $\mathcal{SO}_4$ must be purely imaginary or zero.

Time reversal provides an additional constraint which imposes $\mathcal{SO}_4\equiv 0$. Under time reversal, we assume that the three quark wave functions transform in the usual way: that

$$\Theta\,\psi^a_{lm}(\vec{r}_1,\vec{r}_2,\vec{r}_3) = (-1)^{l+m}\psi^a_{lm}(\vec{r}_1,\vec{r}_2,\vec{r}_3), \tag{3.14}$$

where Θ represents the time reversal operator, and that $\vec{\mathcal{L}}_{ij}$ transforms as $\Theta\vec{\mathcal{L}}_{ij}\Theta^{-1}=-\vec{\mathcal{L}}_{ij}$. It follows that $\mathcal{M}^{ab}=\mathcal{M}^{ba}$ is a symmetric matrix. Since $\mathcal{M}^{ab}=(\mathcal{M}^{ba})^*$, $\mathcal{M}^{ab}$ is therefore a real symmetric matrix and the linear combination $\mathcal{SO}_4$ must vanish. Thus, for both the quark model and the chiral quark model, the spin-orbit matrix elements of the barons are completely determined by only three constants $\mathcal{SO}_1$, $\mathcal{SO}_2$ and $\mathcal{SO}_3$, which can be calculated in a specific model.

We describe the hyperfine interactions more briefly. As we know in advance from the S_3 group theory that there are only two independent constants in either case, we shall define fewer three quark matrix elements than was done for the spin-orbit operator. Let us define the spatial integrals

$$\langle \psi^1_{1m}(\vec{r}_1,\vec{r}_2,\vec{r}_3)|f_0(r_{12})|\psi^1_{1m'}(\vec{r}_1,\vec{r}_2,\vec{r}_3)\rangle = \mathcal{A}_1\delta_{mm'},$$

$$\langle \psi^1_{1m}(\vec{r}_1,\vec{r}_2,\vec{r}_3)|f_0(r_{12})|\psi^2_{1m'}(\vec{r}_1,\vec{r}_2,\vec{r}_3)\rangle = \mathcal{A}_2\delta_{mm'},$$

TABLE III. Masses and mixing angles of the $L=1$ baryons in the quark model. ** indicates a baryon which has not been observed in experiments, while * indicates that mixing with this spin-flavor representation is forbidden.

Baryon		Mass (Expt.)	Mass (Fit)	$^4 8$	$^2 8$	$^2 10$	$^2 1$
N	$J=\frac{1}{2}$	1535^{+20}_{-15}	1529	−0.3537	−0.9354	*	*
		1650^{+30}_{-10}	1631	−0.9354	0.3537	*	*
N	$J=\frac{3}{2}$	1520^{+10}_{-5}	1520	−0.3283	−0.9446	*	*
		1700^{+50}_{-50}	1752	−0.9446	0.3283	*	*
N	$J=\frac{5}{2}$	1675^{+10}_{-5}	1674	1.0000	*	*	*
Δ	$J=\frac{1}{2}$	1620^{+55}_{-5}	1884	*	*	1.0000	*
Δ	$J=\frac{3}{2}$	1700^{+70}_{-30}	1757	*	*	1.0000	*
Σ	$J=\frac{1}{2}$	1750^{+50}_{-20}	1715	−0.1874	−0.9773	0.0988	*
		**	1840	0.9819	−0.1892	−0.0094	*
		**	1983	0.0279	0.0952	0.9951	*
Σ	$J=\frac{3}{2}$	1670^{+15}_{-5}	1704	0.3113	0.9480	−0.0658	*
		**	1876	0.0803	0.0428	0.9959	*
		1940^{+10}_{-40}	1908	0.9469	−0.3153	−0.0628	*
Σ	$J=\frac{5}{2}$	1775^{+5}_{-5}	1781	1.0000	*	*	*
Λ	$J=\frac{1}{2}$	1407^{+4}_{-4}	1418	0.01780	0.0082	*	−0.9998
		1670^{+10}_{-10}	1649	0.6359	0.7716	*	0.01767
		1800^{+50}_{-80}	1717	0.7716	−0.6361	*	0.0085
Λ	$J=\frac{3}{2}$	1519.5^{+1}_{-1}	1519	0.0503	0.2395	*	−0.9696
		1690^{+5}_{-5}	1666	0.3513	0.9045	*	0.2416
		**	1836	0.9349	−0.3528	*	−0.0386
Λ	$J=\frac{5}{2}$	1830^{+0}_{-20}	1834	1.0000	*	*	*
Ξ	$J=\frac{1}{2}$	**	1806	0.7132	0.7010	*	*
		**	1860	−0.7010	0.7132	*	*
Ξ	$J=\frac{3}{2}$	1823^{+5}_{-5}	1819	−0.3514	−0.9362	*	*
		**	1955	−0.9362	0.3514	*	*
Ξ	$J=\frac{5}{2}$	**	1967	1.0000	*	*	*
Ξ^*	$J=\frac{1}{2}$	**	2076	*	*	1.0000	*
Ξ^*	$J=\frac{3}{2}$	**	1995	*	*	1.0000	*
Ω	$J=\frac{1}{2}$	**	2172	*	*	1.0000	*
Ω	$J=\frac{3}{2}$	**	2113	*	*	1.0000	*

$$\langle \psi^3_{1m}(\vec{r}_1,\vec{r}_2,\vec{r}_3)|f_0(r_{12})|\psi^3_{1m'}(\vec{r}_1,\vec{r}_2,\vec{r}_3)\rangle = \mathcal{A}_3\delta_{mm'} \tag{3.15}$$

for the spin-spin operator. Then only the following linear combinations appear in the matrix elements for the baryons:

$$\mathcal{D}_+ = \mathcal{A}_1 + \mathcal{A}_2 + 2\mathcal{A}_3,$$

$$\mathcal{D}_- = \mathcal{A}_1 - \mathcal{A}_2. \tag{3.16}$$

A sufficient basis of spatial integrals for writing all of the baryon matrix elements of the quadrupole operator is provided simply by

$$\langle \psi^1_{1m}(\vec{r}_1,\vec{r}_2,\vec{r}_3)|Q^{\alpha\beta}|\psi^1_{1m'}(\vec{r}_1,\vec{r}_2,\vec{r}_3)\rangle = \mathcal{Q}_1\langle 1,m|L^\alpha L^\beta - \tfrac{1}{3}\,\delta^{\alpha\beta}L^2|1,m'\rangle,$$

$$\langle \psi^1_{1m}(\vec{r}_1,\vec{r}_2,\vec{r}_3)|Q^{\alpha\beta}|\psi^2_{1m'}(\vec{r}_1,\vec{r}_2,\vec{r}_3)\rangle = \mathcal{Q}_2\langle 1,m|L^\alpha L^\beta - \tfrac{1}{2}\,\delta^{\alpha\beta}L^2|1,m'\rangle, \tag{3.17}$$

TABLE IV. Masses and mixing angles of the $L=1$ baryons in the chiral quark model.

Baryon		Mass (Expt.)	Mass (Fit)	$^4 8$	$^2 8$	$^2 10$	$^2 1$
N	$J=\frac{1}{2}$	1535^{+20}_{-15}	1536	−0.4849	−0.8746	*	*
		1650^{+30}_{-10}	1658	−0.8746	0.4849	*	*
N	$J=\frac{3}{2}$	1520^{+10}_{-5}	1512	0.4915	−0.8709	*	*
		1700^{+50}_{-50}	1535	−0.8709	−0.4915	*	*
N	$J=\frac{5}{2}$	1675^{+10}_{-5}	1692	1.0000	*	*	*
Δ	$J=\frac{1}{2}$	1620^{+55}_{-5}	1643	*	*	1.0000	*
Δ	$J=\frac{3}{2}$	1700^{+70}_{-30}	1776	*	*	1.0000	*
Σ	$J=\frac{1}{2}$	(1620)	1648	−0.5882	−0.7248	−0.3588	*
		**	1743	0.7688	−0.3636	−0.5260	*
		1750^{+50}_{-20}	1779	−0.2508	0.5853	−0.7711	*
Σ	$J=\frac{3}{2}$	(1580)	1621	0.9320	−0.3575	0.0604	*
		1670^{+15}_{-5}	1672	−0.3625	−0.9192	0.1538	*
		1940^{+10}_{-40}	1865	−0.0005	0.1652	0.9863	*
Σ	$J=\frac{5}{2}$	1775^{+5}_{-5}	1772	1.0000	*	*	*
Λ	$J=\frac{1}{2}$	1407^{+4}_{-4}	1408	−0.0395	−0.2071	*	−0.9775
		1670^{+10}_{-10}	1671	0.3384	0.9177	*	−0.2081
		1800^{+50}_{-80}	1784	0.9402	−0.3391	*	0.0339
Λ	$J=\frac{3}{2}$	1519.5^{+1}_{-1}	1519	0.0243	0.4288	*	−0.9031
		**	1655	−0.0240	0.9033	*	0.4283
		1690^{+5}_{-5}	1693	0.9994	0.0113	*	0.0322
Λ	$J=\frac{5}{2}$	1830^{+0}_{-20}	1811	1.0000	*	*	*
Ξ	$J=\frac{1}{2}$	**	1788	−0.2884	−0.9575	*	*
		**	1891	−0.9575	0.2884	*	*
Ξ	$J=\frac{3}{2}$	**	1763	−0.0477	−0.9989	*	*
		1823^{+5}_{-5}	1824	−0.9989	0.0477	*	*
Ξ	$J=\frac{5}{2}$	**	1911	1.0000	*	*	*
Ξ^*	$J=\frac{1}{2}$	**	1861	*	*	1.0000	*
Ξ^*	$J=\frac{3}{2}$	**	1944	*	*	1.0000	*
Ω	$J=\frac{1}{2}$	**	1971	*	*	1.0000	*
Ω	$J=\frac{3}{2}$	**	2028	*	*	1.0000	*

where

$$Q^{\alpha\beta} \equiv f_2(r_{12})(3\hat{r}_{12}^{\alpha}\hat{r}_{12}^{\beta} - \delta^{\alpha\beta}). \tag{3.18}$$

Neither of these cases is further constrained by time-reversal symmetry other than to say that the above constants are all real.

It is now possible to express the masses of the $L=1$ baryons in terms of the seven unknown spatial integrals $(\mathcal{D}_{\pm}, \mathcal{SO}_{1,2,3}, \mathcal{Q}_{1,2})$ and the common 70-plet zero-order mass $\langle \mathbf{70}|H_0|\mathbf{70}\rangle \equiv M_0$. We further shall explicitly break $SU(3)$, while keeping isospin symmetry, by giving the strange quark a larger mass, $m_s > m_u = m_d$. These masses correspond to the constituent masses so that the strange-up mass difference can be assumed to be small,

$$\delta m \equiv \frac{m_s - m_u}{m_u} \ll 1. \tag{3.19}$$

Since $SU(3)$ is only weakly broken, we shall work only to

TABLE V. A comparison of the compositions of the baryons obtained from the $SU(6)w$ model fit to **70**$\rightarrow$**56**$+\cdots$ decays [12] with those obtained in our fits for both the quark model (qm) and the chiral quark model (χqm).

Quark model	Chiral quark model
$_{\text{decays}}\langle N_{J=\frac{1}{2}}\vert N_{J=\frac{1}{2}}\rangle_{qm}=-0.98$	$_{\text{decays}}\langle N_{J=\frac{1}{2}}\vert N_{J=\frac{1}{2}}\rangle_{\chi qm}=-1.00$
$_{\text{decays}}\langle N_{J=\frac{3}{2}}\vert N_{J=\frac{3}{2}}\rangle_{qm}=-0.87$	$_{\text{decays}}\langle N_{J=\frac{3}{2}}\vert N_{J=\frac{3}{2}}\rangle\chi qm=-0.94$
$_{\text{decays}}\langle\Lambda(1405)\vert\Lambda(1405)\rangle_{qm}=-0.84$	$_{\text{decays}}\langle\Lambda(1405)\vert\Lambda(1405)\rangle_{\chi qm}=-0.94$
$_{\text{decays}}\langle\Lambda(1670)\vert\Lambda(1670)\rangle_{qm}=-0.63$	$_{\text{decays}}\langle\Lambda(1670)\vert\Lambda(1670)\rangle_{\chi qm}=-0.42$
$_{\text{decays}}\langle\Lambda(1800)\vert\lambda(1800)\rangle_{qm}=-0.70$	$_{\text{decays}}\langle\Lambda(1800)\vert\Lambda(1800)\rangle_{\chi qm}=-0.48$

first order in $\delta m\approx 0.27-0.29$. The mass factors appear explicitly in the $1/m_i$ coefficients of the perturbative potentials.

We should here pause to remark on the power of the S_3 argument. The group theory allows us to calculate the perturbations to a model with a specific spin, orbital angular momentum and flavor structure but with other details left arbitrary. This feature allows us to test the plausibility of a model's ability to explain the observed $L=1$ baryon spectrum by adjusting the independent constants to fit these masses. If the model fails to fit the data to a reasonable confidence level, then regardless of the dynamical model for the quark wave functions used, it will still fail adequately to generate the observed $L=1$ mass splittings. The converse, however, is not true. Even should a model fit the data well, realistic choices for the quark wave functions may not achieve the best fit attainable in the full parameter space.

As an example, the mass perturbation to the $|\Delta;\frac{1}{2};J=\frac{3}{2}\rangle$ states in the quark model is

$$\Delta m_{\Delta,3/2}=M_0+\frac{1}{2}\mathcal{D}_1-\frac{9}{2}\mathcal{D}_2+\mathcal{SO}_2+\mathcal{SO}_3, \quad (3.20)$$

while for the chiral quark model, the perturbation becomes

$$\Delta m_{\Delta,3/2}=M_0+\frac{2}{3}\mathcal{D}_1-6\mathcal{D}_2+\frac{4}{3}\mathcal{SO}_2+\frac{4}{3}\mathcal{SO}_3. \quad (3.21)$$

The calculation of these matrix elements, as well as those for the rest of the baryons was accomplished with the Maple symbolic manipulation program. The mass splitting operators will in general mix baryons which have equivalent total angular momentum, isospin and strangeness.

IV. THE COMPARISON WITH EXPERIMENT

Among the baryons observed to date, eighteen have been reliably identified with the $L=1$ baryons [7]. Our program then is to obtain the best possible fit with the nine unknown quantities—the seven spatial integrals for the interactions, a parameter for the $SU(3)$ breaking, and zero-order baryon mass—to the masses of these eighteen baryons. Fits were made for each of the two models. The actual fitting routine applied a Levenberg-Marquardt algorithm [8] which chose its initial conditions randomly within this nine-dimensional parameter space. For those baryons within an incompletely measured set with the same *J, I,* and strangeness, specifically the $\Sigma_{J=\frac{3}{2}}$, the $\Sigma_{J=\frac{1}{2}}$, the $\Lambda_{J=\frac{3}{2}}$, and the $\Xi_{J=\frac{3}{2}}$, all the possible assignments of elements of these sets to the measured masses were sampled; those which produced the best fit to the mass spectrum are displayed in this section. The values for the parameters for the two pictures of the low energy physics within a baryon are shown in Table II, with the χ^2 for each of the fits.

These values produce the mass spectra displayed in Fig. 2 for the quark model and in Fig. 3 for the chiral quark model. In these figures, the baryons used to fit the experimentally observed masses are shown in unbroken lines while the remaining baryons masses, shown in dashed lines, represent predictions. The composition of the baryons in terms of spin-flavor multiplets is summarized in Tables III and IV.

The eighteen baryons chosen for the fits are those listed in the Baryon Summary Table of [7]; we should mention that the existence two other $L=1$ states, the $\Sigma_{J=\frac{1}{2}}(1620)$ and the $\Sigma_{J=\frac{3}{2}}(1580)$, has been fairly well established. As we shall see, the quark model is unable to fit the baryon spectrum with a satisfactory χ^2 even with the omission of these two baryons. But as the chiral quark model successfully fit the baryons listed in the Baryon Summary Table, we have included these two in Table IV. Although they were not included in the fitting routine, each baryon has a ''predicted'' state within 20–40 MeV so that we do not expect that our conclusions, nor the χ^2 of the fit, would alter greatly had they been included in the fitting procedure.

A. The quark model

The standard quark model fares rather poorly with a best χ^2 value of 123.6 for only eighteen fit parameters. The masses of the $\Lambda(1405)[J=\frac{1}{2}]$ and the $\Lambda(1520)[J=\frac{3}{2}]$ baryons have been been measured to within ± 4 MeV and ± 1 MeV, respectively, and tend to drive the fit parameters to produce a precise fit for these states—at the expense of others. An accurate fit of the Λ's tends to produce a poor fit for the decuplet states, most glaringly, giving a predicted mass of 1884 MeV for the $\Delta(1620)$. More generally, the quark model predicts higher masses for decuplet states with lower total angular momentum in contrast with the general trend for the $l=1$ baryons, in particular the observed reversed ordering of the $J=\frac{1}{2}$ and $J=(\frac{3}{2})\Delta$ masses. The original study of the 70-plet baryons by Isgur and Karl [1] succeeded in obtaining a better fit for the decuplet states but only at the expense of a predicted mass of 1490 MeV for the $\Lambda(1405)$ and a consequently poorer χ^2. It seems difficult for the constituent quark model to account for both the lightness of the

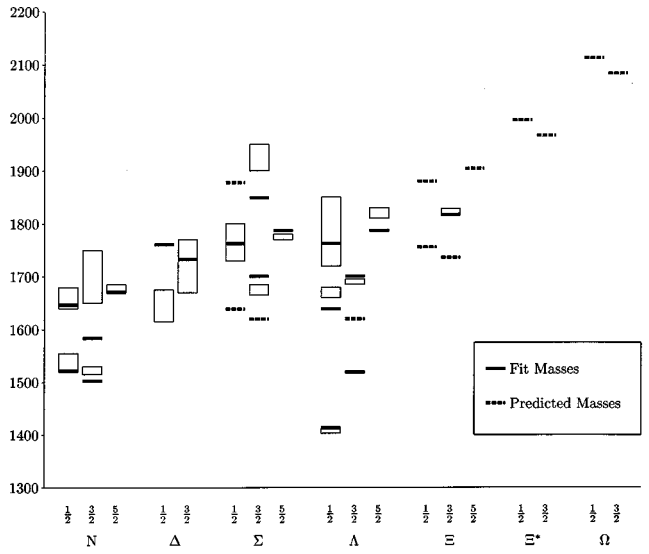

FIG. 4. Masses of the $L=1$ baryons in the quark model with $SU(3)$ symmetric interactions; $\chi^2=79.2456$, $N_{fit}=18$.

Λ states and the decuplet mass spectrum. This failure is often described in terms of the size of the spin-orbit coupling: a weak coupling is needed to fit the majority of the baryons but a strong coupling is required to generate the observed $\Lambda(1520)$-$\Lambda(1405)$ mass splitting [9].

B. The chiral quark model

The chiral quark model is able to reconcile successfully these two features and produce an acceptable fit for the detected baryon spectrum: a χ^2 of only 24.76. Its worst failure among the observed baryons is that the model does not generate a sufficient splitting in the $J=(\frac{3}{2})N$ states, only about 20 MeV compared with an experimental splitting of almost 200 MeV. At present, the large experimental error in the $N(1700)$ allows a ''good'' fit to be achieved but it may be difficult to accommodate the actual splitting as more precise data are obtained.

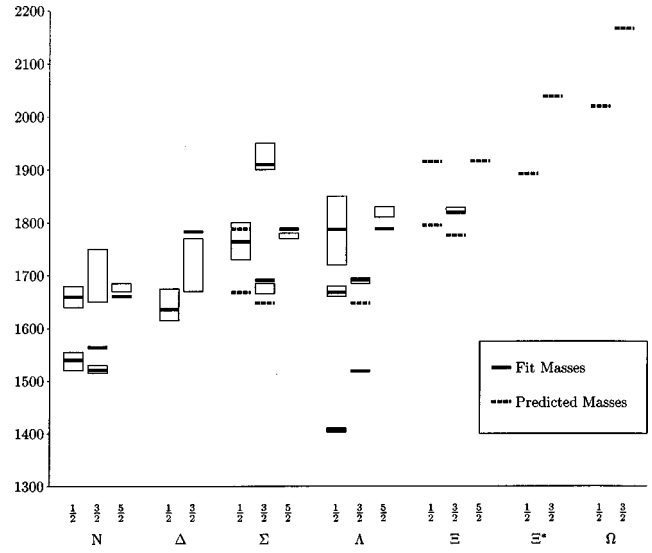

FIG. 5. Masses of the $L=1$ baryons in the chiral quark model with $SU(3)$ symmetric interactions; $\chi^2=33.2254$, $N_{fit}=18$.

As mentioned, the quark model analysis of Isgur and Karl [1] found that, aside from large $\Lambda(1520)$-$\Lambda(1405)$ mass splitting, the splitting among the baryon multiplets required an extremely small spin-orbit contribution to the masses. The difficulty in justifying this small spin-orbit coupling has been called the ''spin-orbit puzzle.'' In the chiral quark model, while the spin-spin interactions dominate with a strength roughly five times that of the other terms, the spin-orbit interactions are comparable to the quadrupole interactions, with only two of the three independent integrals responsible for essentially all of the spin-orbit contribution. Thus the spin-orbit puzzle does not seem to occur in the chiral quark model.

C. The $\Lambda(1405)$

The low mass of the $\Lambda(1405)$ has marked it as something of a conundrum among the $L=1$ baryons. At one extreme, it would seem reasonable to regard it as a $\bar{K}N$ bound state since its mass lies 30 MeV below the $\bar{K}N$ threshold. Alternatively, the nonrelativistic quark model treats the $\Lambda(1405)$ as an ordinary $L=1$ baryon composed of some mixture of $SU(3)$ singlet and octet states. Traditionally, the quark model [1] predicts that the $\Lambda(1405)$ mainly is composed of the singlet state with a small admixture of the octet states and such a behavior is seen in the best-fit results for the quark model (gluon exchange):

$$\Lambda(1405)_{qm}=-0.9998|^2 1;\tfrac{1}{2};\rangle+0.0082|^2 8;\tfrac{1}{2};\tfrac{1}{2}\rangle + 0.0178|^4 8;\tfrac{3}{2};\tfrac{1}{2}\rangle. \quad (4.1)$$

The chiral quark model (Goldstone boson exchange) gives a similar result for the composition of the $\Lambda(1405)$ except that the spin-$\frac{1}{2}$ octet contribution is slightly enhanced:

$$\Lambda(1405)_{\chi qm}=-0.9775|^2 1;\tfrac{1}{2};\tfrac{1}{2}\rangle-0.2071|^2 8;\tfrac{1}{2};\tfrac{1}{2}\rangle - 0.0395|^4 8;\tfrac{3}{2};\tfrac{1}{2}\rangle. \quad (4.2)$$

Since the chiral quark model succeeds in fitting the observed baryon masses well, it is instructive to probe the model further by comparing the consequences of this predicted composition with some of the other phenomenological properties of this baryon.

Nathan Isgur [10] has recently proposed that the $\Lambda(1405)$ can be studied in heavy quark effective theory limit. In this picture, the $\Lambda(1405)$ is a *uds* quark bound state where the strange quark mass is taken to be heavy compared to the up and the down quark masses. Singling out the *s* quark breaks the $SU(3)$ flavor symmetry and its spin and orbital angular momentum completely determine that of the $\Lambda(1405)$. The *ud* quarks form an inert $S=0$, $L=0$ pair. Such a state no longer can be described in terms of pure $SU(3)$ states; however it does contain equal amounts of the singlet and spin$\frac{1}{2}$ octet states. Such a composition contrasts with that emerging

TABLE VI. Masses and mixing angles of the $L=1$ baryons in the quark model with $SU(3)$ symmetric interactions.

Baryon		Mass (Expt.)	Mass (Fit)	$^4 8$	$^2 8$	$^2 10$	$^2 1$
N	$J=\frac{1}{2}$	1535^{+20}_{-15}	1522	−0.2404	−0.9707	*	*
		1650^{+30}_{-10}	1647	−0.9707	0.2404	*	*
N	$J=\frac{3}{2}$	1520^{+10}_{-5}	1503	0.8903	0.4554	*	*
		1700^{+50}_{-50}	1584	−0.4554	0.8903	*	*
N	$J=\frac{5}{2}$	1675^{+10}_{-5}	1671	1.0000	*	*	*
Δ	$J=\frac{1}{2}$	1620^{+55}_{-5}	1761	*	*	1.0000	*
Δ	$J=\frac{3}{2}$	1700^{+70}_{-30}	1732	*	*	1.0000	*
Σ	$J=\frac{1}{2}$	**	1639	−0.2404	−0.9707	0.0000	*
		1750^{+50}_{-20}	1764	−0.9707	0.2404	0.0000	*
		**	1878	0.0000	0.0000	1.0000	*
Σ	$J=\frac{3}{2}$	**	1620	0.8903	0.4554	0.0000	*
		1670^{+15}_{-5}	1701	−0.4554	0.8903	0.9959	*
		1940^{+10}_{-40}	1849	0.0000	0.0000	1.0000	*
Σ	$J=\frac{5}{2}$	1775^{+5}_{-5}	1788	1.0000	*	*	*
Λ	$J=\frac{1}{2}$	1407^{+4}_{-4}	1414	0.0000	0.0000	*	1.0000
		1670^{+10}_{-10}	1639	−0.2404	−0.9707	*	0.0000
		1800^{+50}_{-80}	1763	−0.9707	0.2404	*	0.0000
Λ	$J=\frac{3}{2}$	1519.5^{+1}_{-1}	1519	0.0000	0.0000	*	1.0000
		**	1620	0.8903	0.4554	*	0.0000
		1690^{+5}_{-5}	1701	−0.4554	0.8903	*	0.0000
Λ	$J=\frac{5}{2}$	1830^{+0}_{-20}	1788	1.0000	*	*	*
Ξ	$J=\frac{1}{2}$	**	1756	−0.2404	−0.9707	*	*
		**	1880	−0.9707	0.2404	*	*
Ξ	$J=\frac{3}{2}$	**	1736	0.8903	0.4554	*	*
		1823^{+5}_{-5}	1818	−0.4554	0.8903	*	*
Ξ	$J=\frac{5}{2}$	**	1904	1.0000	*	*	*
Ξ^*	$J=\frac{1}{2}$	**	1995	*	*	1.0000	*
Ξ^*	$J=\frac{3}{2}$	**	1966	*	*	1.0000	*
Ω	$J=\frac{1}{2}$	**	2112	*	*	1.0000	*
Ω	$J=\frac{3}{2}$	**	2083	*	*	1.0000	*

in either the quark model or the chiral quark model. In both cases, the $\Lambda(1405)$ remains essentially a singlet state although the chiral quark model does match the heavy quark theory's predictions marginally better:

$$\langle \Lambda(1405)_{\chi qm} | \Lambda(1405)_{HQET} \rangle = 0.6079 \qquad (4.3)$$

compared to an overlap of

$$\langle \Lambda(1405)_{qm} | \Lambda(1405)_{HQET} \rangle = 0.5089 \qquad (4.4)$$

for the quark model.

TABLE VII. Masses and mixing angles of the $L=1$ baryons in the chiral quark model with $SU(3)$ symmetric interactions.

Baryon		Mass (Expt.)	Mass (Fit)	$^4 8$	$^2 8$	$^2 10$	$^2 1$
N	$J=\frac{1}{2}$	1535^{+20}_{-15}	1540	0.6313	−0.7755	*	*
		1650^{+30}_{-10}	1660	−0.7755	−0.6313	*	*
N	$J=\frac{3}{2}$	1520^{+10}_{-5}	1521	−0.1808	−0.9835	*	*
		1700^{+50}_{-50}	1564	−0.9835	0.1808	*	*
N	$J=\frac{5}{2}$	1675^{+10}_{-5}	1661	1.0000	*	*	*
Δ	$J=\frac{1}{2}$	1620^{+55}_{-5}	1636	*	*	1.0000	*
Δ	$J=\frac{3}{2}$	1700^{+70}_{-30}	1783	*	*	1.0000	*
Σ	$J=\frac{1}{2}$	(1620)	1668	0.6313	−0.7755	0.0000	*
		1750^{+50}_{-20}	1764	0.0000	0.0000	1.0000	*
		**	1788	−0.7755	−0.6313	0.0000	*
Σ	$J=\frac{3}{2}$	(1580)	1648	−0.1808	−0.9835	0.0000	*
		1670^{+15}_{-5}	1692	−0.9835	0.1808	0.0000	*
		1940^{+10}_{-40}	1911	0.0000	0.0000	1.0000	*
Σ	$J=\frac{5}{2}$	1775^{+5}_{-5}	1788	1.0000	*	*	*
Λ	$J=\frac{1}{2}$	1407^{+4}_{-4}	1407	0.0000	0.0000	*	1.0000
		1670^{+10}_{-10}	1668	0.6313	−0.7755	*	0.0000
		1800^{+50}_{-80}	1788	−0.7755	−0.6313	*	0.0000
Λ	$J=\frac{3}{2}$	1519.5^{+1}_{-1}	1519	0.0000	0.0000	*	1.0000
		**	1649	−0.1808	−0.9835	*	0.0000
		1690^{+5}_{-5}	1692	−0.9835	0.1808	*	0.0000
Λ	$J=\frac{5}{2}$	1830^{+0}_{-20}	1788	1.0000	*	*	*
Ξ	$J=\frac{1}{2}$	**	1795	0.6313	−0.7755	*	*
		**	1915	−0.7755	−0.6313	*	*
Ξ	$J=\frac{3}{2}$	**	1776	−0.1808	−0.9835	*	*
		1823^{+5}_{-5}	1819	−0.9835	0.1808	*	*
Ξ	$J=\frac{5}{2}$	**	1916	1.0000	*	*	*
Ξ^*	$J=\frac{1}{2}$	**	1892	*	*	1.0000	*
Ξ^*	$J=\frac{3}{2}$	**	2038	*	*	1.0000	*
Ω	$J=\frac{1}{2}$	**	2019	*	*	1.0000	*
Ω	$J=\frac{3}{2}$	**	2166	*	*	1.0000	*

D. Mixing angles from $L=1$ decays

The decays of the **70** states into **56** states provide another estimate of the observed baryons' compositions in terms of $SU(3)\times SU(2)$ eigenstates. While fits to the decay amplitudes have been performed for the quark model [11], they do not include estimates for the mixing angles; however, the mixing angles have been extracted for the $SU(6)_W$ model [12], which has the same algebraic structure for the decays as the standard quark model $SU(6)$. A comparison of the compositions of the states fit to **70** decays [12] with those of the two models fit here to the mass spectrum are shown in Table V. In this table we have only included the states from mixed J-multiplets that have been completely observed—the N states and the three $\Lambda_{J=\frac{1}{2}}$ states.

Both models agree extremely well with the decay estimates for the N state compositions, but they begin to disagree for the Λ states. The decays of the $L=1$ baryons suggest that the $\Lambda(1405)$ and the $\Lambda(1520)$ are principally singlet states, which is in accord with our fits. However, the $\Lambda(1800)$ in the chiral quark model is predominantly an $S=\frac{3}{2}$ octet state whereas the fits to the decay amplitudes suggest it is principally an $S=\frac{1}{2}$ octet state. While not conclusive, these disagreements suggest it may be a challenge for the chiral quark model to fit simultaneously the mass spectrum and the observed decay amplitudes.

E. $SU(3)$ symmetric perturbations

In evaluating the matrix elements of the mass operators of Eqs. (3.5) and (3.6), we explicitly broke flavor symmetry by giving the strange quark a heavier mass. We shall now examine what happens to the fits when $SU(3)$ is preserved in these spin operators. The rationale for taking this limit is that if both the spin splitting and the flavor breaking effects are small, terms that simultaneously break $SU(3)$ and $SU(2)$ can be regarded as higher order effects.

The results for the fits to the $L=1$ baryon spectrum due to an explicit $SU(3)$ breaking term plus flavor-symmetric versions of the operators in Eqs. (3.5) and (3.6) are shown in

TABLE VIII. Best-fit values of the S_3 constants for the quark model and chiral quark model with $SU(3)$ symmetric spin operators. (*) All of the parameters are in units of MeV except δm which is dimensionless.

Perturbation (MeV)*	Quark model	Chiral quark model
$\mathcal{D}_+$	123.9	−79.73
$\mathcal{D}_-$	−21.14	−57.05
$\mathcal{SO}_1$	−14.32	0.744
$\mathcal{SO}_2$	−77.90	16.24
$\mathcal{SO}_3$	68.34	20.38
$\mathcal{Q}_1$	−18.20	2.699
$\mathcal{Q}_2$	−10.12	−0.982
δm	0.221	0.265
M_0	1585	1445
χ^2	79.24	33.23

Figs. 4 and 5. The mixing angles are included in Tables VI and VII while Table VIII displays the best fit values of the S_3 constants in units of MeV [except for the dimensionless $SU(3)$ breaking parameter]. Surprisingly, the χ^2 improves for the quark model fit, from 123 to 79, although the general pattern remains as before. Some of the assignments of the states being fit, among the $\Sigma_{J=\frac{1}{2}}$, $\Sigma_{J=\frac{3}{2}}$ and $\Lambda_{J=\frac{3}{2}}$ states, have changed. The ordering of the multiplets of decuplet remains unaltered. One feature that Fig. 4 does not convey is that many other arrangements of the baryons in the incompletely observed multiplets also lead to a better fit than that of Fig. 2, the best fit obtained for the $SU(3)$-breaking spin operators.

The value of χ^2 for the chiral quark model predictably worsened when $SU(3)$ was imposed on the spin-splitting operators. The pattern of masses otherwise did not change significantly. Comparing the the masses in the $SU(3)$ symmetric and the $SU(3)$-broken limits (Tables VII and IV) provides an estimate for the theoretical errors associated with our fits—most of the fit baryon masses agreed to within 10–20 MeV. Most of the sizes of the fit parameters did not differ much between the two fits with the exception of the quadrupole interaction which is substantially smaller in the $SU(3)$ symmetric limit.

V. CONCLUSION

The S_3 permutation group provides a new tool for the study of the physics within baryons. In addition to freeing us from a specific choice for the quark-quark potential, this approach allows a comparison of theories differing in the flavor structure of their interactions. When applied to the traditional quark model and the more recent chiral quark model, our approach places the two theories on an equal footing with a one-to-one mapping of the unknown S_3 parameters between the two theories. The results of this comparison were somewhat surprising—the chiral quark model shows a clearly better fit with the observed $L=1$ baryon spectrum.

Since the chiral quark model provided a good fit and seems to be able to avoid the spin-orbit problem, we should mention some of the challenges that it still faces. As stated earlier, the fitting routine ranged over the entire available parameter space and it remains to show that the best-fit set of parameters can be realized by a physical potential for the quark-quark interactions. It would also be interesting to see whether this superiority over the traditional quark model fit persists when we attempt to fit simultaneously the $L=0$, the negative and positive parity $L=1$, and lowest excited states of the $N=2$ band. This program was carried out by Capstick and Isgur [4] for a relativized quark model with harmonic oscillator wave functions. Finally, if the model is to provide a believable explanation of the low-energy physics within a baryon it must not only describe the mass spectrum, but also accommodate the excited state decays.

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