A Better Algorithm for Societal Tradeoffs

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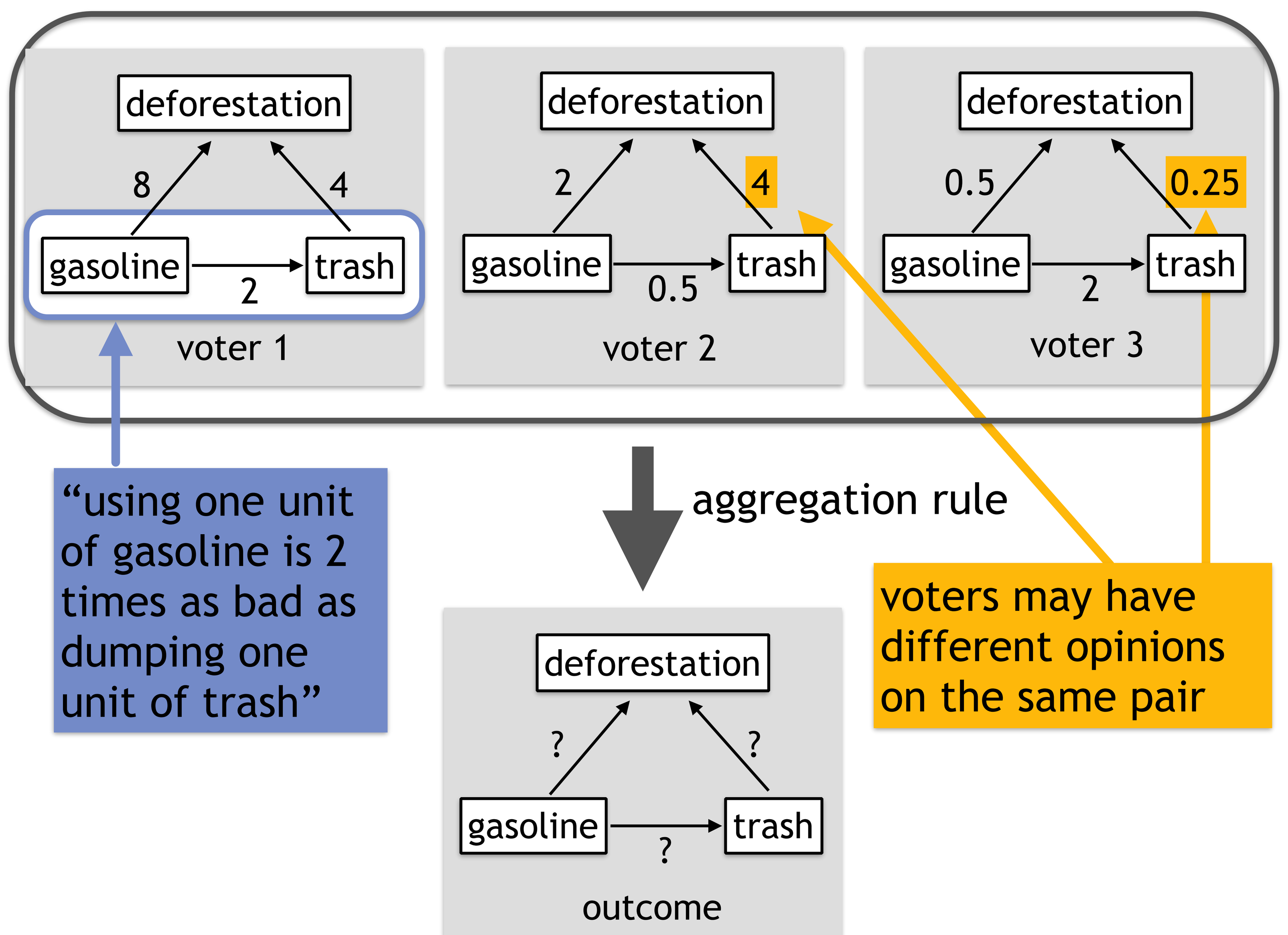
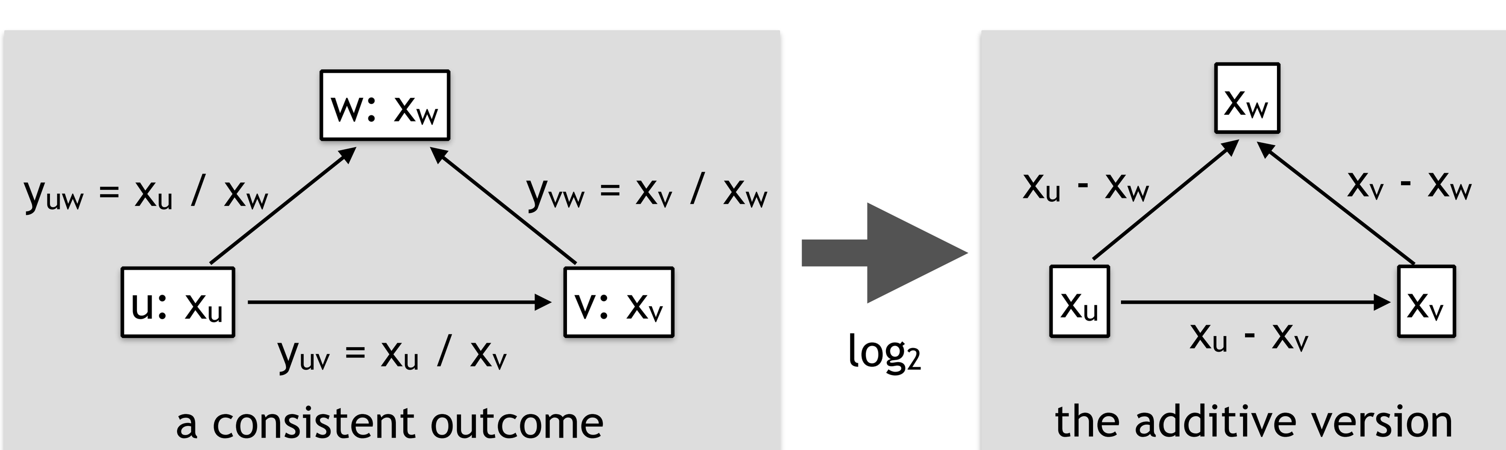
Quantitative Judgement Aggregation



- Voters have different opinions on how bad one activity is compared to another ("using one unit of gasoline is 2 times as bad as dumping one unit of trash")
- But all three voters are consistent (e.g. 2 × 4 = 8)

Consistency of the Outcome



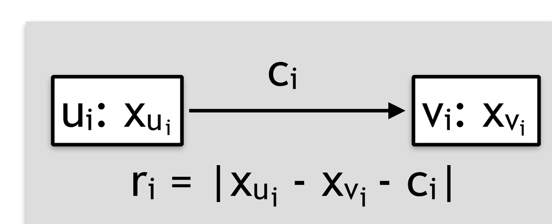
- Consistency: the tradeoffs $\{y_{uv}\}$ satisfy $y_{uv} * y_{vu} = y_{uw}$
- Or equivalently: there exist $\{x_u\}$ such that $y_{uv} = x_u / x_v$
- Furthermore: taking $\log_2$ makes everything additive, so:

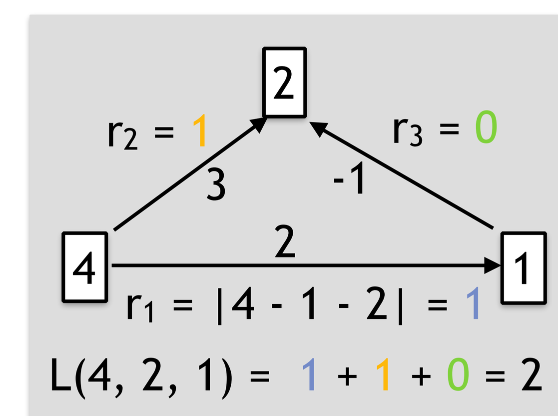
The problem becomes finding "good" potentials $\{x_u\}$

The Societal Tradeoffs Rule

Societal Tradeoffs [2]:

- Suppose potentials $\{x_u\}$ are assigned
- For the i-th vote (u_i, v_i, c_i) with weight w_i, suggesting the difference between u_i and v_i should be c_i, the residual of $\{x_u\}$ on this vote is $r_i = w_i |x_{u_i} - x_{v_i} - c_i|$
- The total loss $L(\{x_u\})$ of $\{x_u\}$ is the ***sum of residuals $\{r_i\}$***, and Societal Tradeoffs rule chooses $\{x_u\}$ to ***minimize $L(\{x_u\})$***





- Conitzer et al. [2]: Societal Tradeoffs is the ***unique*** rule enjoying a number of good properties, and
- In particular, it chooses the ***median tradeoff*** when there is only one pair of activities

LP for Societal Tradeoffs

LP for Societal Tradeoffs:

- minimize $\sum_i r_i$
- s.t. $r_i \geq w_i(x_{u_i} - x_{v_i} - c_i)$ and $r_i \geq w_i(x_{v_i} - x_{u_i} + c_i)$ for vote i

Naive LP solution is poly-time, but doesn't scale ideally

Why not take the dual since it's free?

Dual LP:

- maximize $\sum_i c_i f_i$ (maximize total cost)
- s.t. $\sum_{i:\, u = u_i} f_i = \sum_{i:\, u = v_i} f_i$ for activity u (flow conservation)
- s.t. $-w_i \leq f_i \leq w_i$ for vote i (capacity constraint)

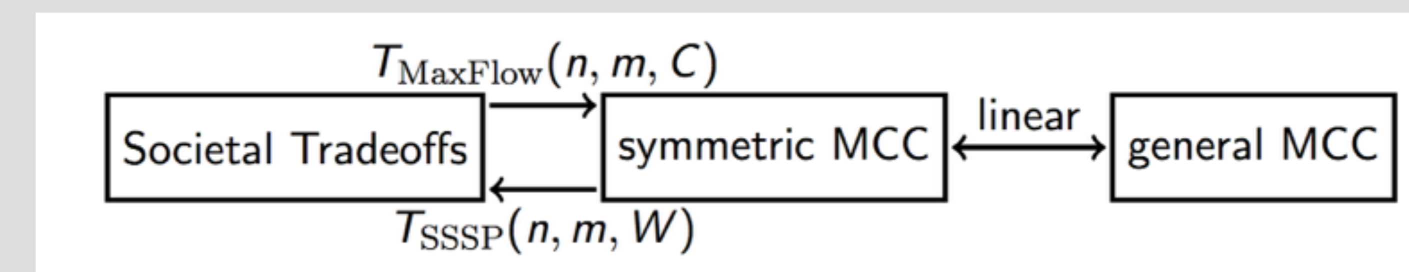
This is precisely the ***max-cost circulation*** problem with symmetric capacity, which ***we know how to solve***!

Equivalence between Societal Tradeoffs and General Max-Cost Circulation

Strong duality gives immediately the equality between the optimal ***objective values*** of ST and symmetric MCC, but:

- What about ***solutions***? Given optimal dual flow assignment, how do we compute optimal primal potentials?
- Can we say something about general asymmetric MCC?

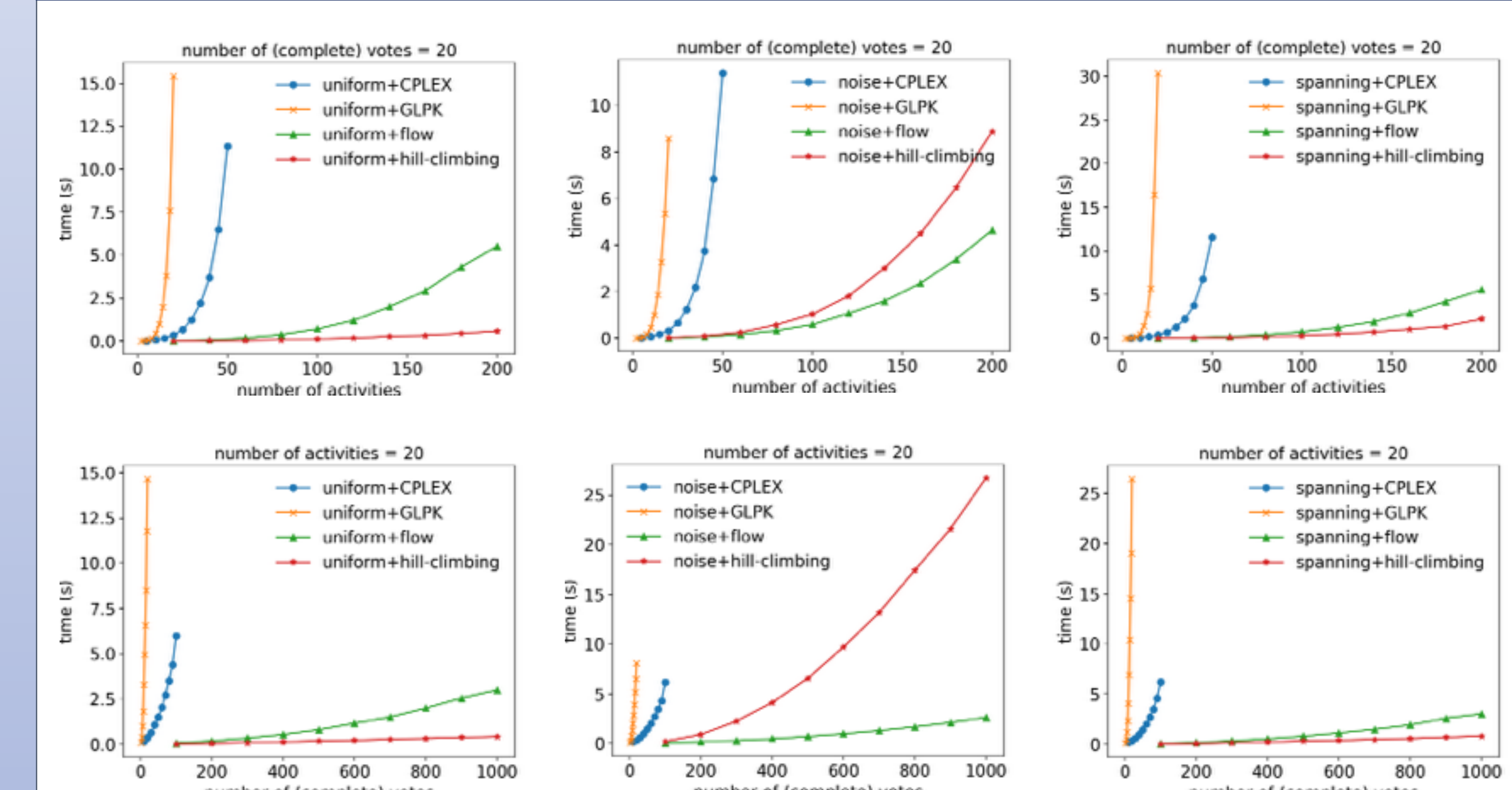
Theorem:



So using [1, 3] as subroutines, we obtain:

Theorem: the Societal Tradeoffs outcome can be computed in time $\tilde{O}(m^{10/7})$ / $\tilde{O}(m^{3/2})$ with unit / general weights, and we ***cannot do better*** without finding better flow algorithms

Experimental Evaluation



References

1. Michael B. Cohen, Aleksander Madry, Piotr Sankowski, and Adrian Vladu. Negative-Weight Shortest Paths and Unit Capacity Minimum Cost Flow in $\tilde{O}(m^{10/7} \log W)$ Time.
2. Vincent Conitzer, Rupert Freeman, Markus Brill, and Yuqian Li. Rules for Choosing Societal Tradeoffs.
3. Harold N. Gabow and Robert E. Tarjan. Faster Scaling Algorithms for Network Problems.