

# The Expected Cost of Default\*

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## Abstract

The sample of observed defaults significantly understates the average firm's true expected cost of default due to a sample selection bias. I use a dynamic capital structure model to estimate firm-specific expected default costs and quantify the selection bias. The average firm expects to lose 45% of firm value in default, a cost higher than existing estimates. However, the average cost among defaulted firms in the estimated model is only 25%, a value consistent with existing empirical estimates from observed defaults. This substantial selection bias helps to reconcile the levels of leverage and default costs observed in the data.

**Keywords:** Default costs; Structural estimation; Costs of financial distress; Dynamic capital structure

**JEL Classification:** G33; G32; G30

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# 1 Introduction

The cost of default is an essential component to understanding the joint behavior of default rates, credit spreads, and firms' optimal financing decisions. A common view in the finance literature, supported by empirical studies of defaulted firms, maintains that the average firm's cost of default is relatively low.<sup>1</sup> This conclusion plays a central role in the challenge faced by existing models to simultaneously explain the levels of leverage, credit spreads, and default rates observed in the data.

I show that estimates of default costs drawn from the sample of defaulted firms are subject to a significant selection bias. This selection bias is the result of firms and credit markets internalizing default costs when choosing leverage and pricing debt, respectively. All else equal, firms with a higher cost of default will choose a lower level of leverage, making default less likely. Therefore, the firms that default *ex post* are disproportionately those with a low cost of default. Consequently, existing estimates of default costs, drawn from the sample of observed defaults, significantly understate the cost that the average firm expects to incur in default.

In this paper, I estimate firm-specific, expected default costs from a structural model. These costs, which are not subject to the selection bias, are the costs used *ex ante* by firms in setting their leverage and by credit markets in pricing debt. In my sample of 2,505 U.S. public firms, the mean estimated cost of default is 45% of firm value (with a median of 37%), which is significantly higher than existing estimates obtained from the empirical sample of defaulted firms. However, this value does not have a direct empirical counterpart and, given the selection bias, one should expect this value to be larger than what is obtained from a sample of defaulted firms.

The striking result is that the estimated model produces an average default cost for the

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<sup>1</sup>Davydenko, Strebulaev, and Zhao (2012) estimate an average default cost of 21.7% of the market value of assets from a sample of 175 defaulted firms. This measure is intended to capture both direct and indirect costs. Estimates of direct bankruptcy costs are much smaller. Warner (1977), Weiss (1990), and Altman (1984) all find small direct costs of bankruptcy of 5.3%, 3.1%, and 6% of pre-bankruptcy firm value, respectively.

subset of defaulted firms of only 25%. This value, which is the model counterpart to the empirical sample of defaulted firms, is significant for two reasons. First, it implies a large selection bias — the average firm expects a cost of default nearly twice as large as the average inferred from the sample of defaulted firms. Second, and perhaps more importantly, this value is closely in line with existing estimates of average default costs from the empirical sample of defaulted firms.<sup>2</sup>

A number of existing conclusions relating to leverage, credit spreads, and the importance of default costs rely on the assumption that the low observed default costs accurately reflect the costs faced by the broader population of firms. A central message of this paper is that many of these conclusions should be revisited. In particular, I show that accounting for heterogeneous default costs, and the sample selection bias that they induce, can significantly help in explaining some of the low leverage ratios observed in the data. In addition, the sample selection bias has significant implications for a wide class of credit risk models, not just the framework used in this paper.

Using the values for default costs reported in Andrade and Kaplan (1998) and tax benefits to debt estimated by Graham (2000), previous work has concluded that default costs are too low for a tradeoff model of leverage to explain the low levels of leverage seen for many firms in the data. I show that, due to the sample selection bias, low observed default costs can be reconciled with low observed leverage ratios in a tradeoff model of leverage. The estimated model is not only consistent with observed default rates and credit spreads, but is also able to match the cross-section of leverage, including firms with low leverage, while still replicating the low observed default costs seen in the data.

In a broad sense, my work is related to a growing body of literature that considers the interactions of corporate financing decisions and asset prices. My approach to estimating firm-specific default costs and cash flow parameters is related to other recent papers estimating structural models.<sup>3</sup> A novel aspect of this paper is that I am able to estimate

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<sup>2</sup>See, for example, Davydenko, Strebulaev, and Zhao (2012) and Andrade and Kaplan (1998).

<sup>3</sup>Recent examples include Hennessy and Whited (2007), Morellec, Nikolov, and Schuerhoff (2010), and Nikolov and Whited (2010). The recent survey article of Strebulaev and Whited (2012) provides a very nice review of the corporate finance literature on dynamic models and structural estimation.

firm-specific parameters. In contrast, most related work estimates the parameters of a single representative firm.

More specifically, my work is related to a strand of empirical literature that seeks to measure the cost of distress or default. The existing literature has generally found the average default costs observed in the data to be relatively low. Andrade and Kaplan (1998) estimate distress costs of 10-23% of firm value for a sample of 31 highly leveraged transactions. Davydenko, Strebulaev, and Zhao (2012) estimate an average default cost of 21.7% of the market value of assets from a sample of 175 defaulted firms. Using a natural experiment resulting from asbestos litigation, Taillard (2010) isolates financial from economic distress and finds little of evidence of significant costs of financial distress.<sup>4</sup>

The relatively small average default costs observed in the data has led to the conclusion that many firms are too conservative in their choice of leverage. Miller (1977) notes that default and distress costs appear far too small, given estimated tax benefits to debt, to explain empirical leverage ratios. Graham (2000) estimates the tax benefits of debt up to 5% of firm value and concludes that from a tradeoff model of leverage many firms appear, on average, under levered.

Almeida and Philippon (2007) note that default is more likely to occur in bad states when marginal utility is high. Using risk-neutral probabilities and the estimates of Andrade and Kaplan (1998), they conclude that firms are not, on average, under levered. Elkamhi, Ericsson, and Parsons (2010) note that this calculation does not filter out economic shocks, which are unrelated to leverage, that drive the firm to default or distress. They argue that once the economic shocks are accounted for separately, the default cost estimates of Andrade and Kaplan (1998) are too low to account for the observed leverage ratios. The structural model that I use avoids this issue.

Using the marginal tax benefit estimates of Graham (2000), van Binsbergen, Graham, and Yang (2010) estimate firm-specific costs of debt under the assumption that firms are

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<sup>4</sup>Additional examples of work studying distress or default costs include Pulvino (1998), Franks and Torous (1989), Opler and Titman (1994), Gilson (1997), Ofek (1993), Asquith, Gertner, and Scharfstein (1994), Bris, Welch, and Zhu (2006), and Acharya, Bharath, and Srinivasan (2007).

optimally levered. They conclude that approximately half of their estimated cost of debt is due to default or distress costs. My results suggest that, due to the sample selection bias, default costs account for a significantly larger component of a firm's total cost of debt.

Korteweg (2010) estimates the net benefits to leverage and, consistent with previous work, concludes that many firms are under levered. Using firms at or near distress, he estimates distress costs of 15-30%. These firms at or near distress, however, are likely to be disproportionately those for which default costs were relatively low. George and Hwang (2010) also note that firms with high distress costs can be expected to choose low leverage to avoid distress. They argue that this provides an explanation for the distress risk and leverage puzzles observed in equity returns.

The model that I estimate is based on a class of structural models of capital structure and credit risk that build upon the seminal papers of Merton (1974) and Leland (1994). Strebulaev (2007) develops a structural model of this form and shows that the model is able to produce simulated capital structure dynamics consistent with several documented empirical patterns. The model in this paper is closest to the models of Chen (2010), Bhamra, Kuehn, and Strebulaev (2010a,b), and Hackbarth, Miao, and Morellec (2006).<sup>5</sup> These models are primarily concerned with matching aggregate facts regarding credit spreads, default frequency, and leverage. Chen (2010) seeks to explain the observed credit spreads and leverage ratios while Bhamra, Kuehn, and Strebulaev (2010b) focus on a levered equity premium. Bhamra, Kuehn, and Strebulaev (2010a) focus on the dynamics of leverage in an economy with macroeconomic risk. In their model, all firms are identical ex ante, but differ ex post due to idiosyncratic shocks. In contrast, I focus on computing a firm-specific measure of the cost of default and quantifying the magnitude of the sample selection bias. To that end, the economy I consider features a cross-section of firms which are ex ante heterogeneous, differing in the parameters of their cash flow process as well as default costs.

The remainder of the paper is organized as follows. In Section 2 I introduce the model framework. In Section 3 I estimate firm-specific expected default costs and cash flow pa-

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<sup>5</sup>Other similar models include Chen, Collin-Dufresne, and Goldstein (2009), Fischer, Heinkel, and Zechner (1989), Goldstein, Ju, and Leland (2001).

rameters from the model. In Section 4 I simulate the model again under the estimated joint cross-sectional distribution of the firm-specific parameters, which gives an estimate for the sample selection bias in observed default costs. Section 5 examines how these firm-specific estimates relate to firm characteristics, industry, and credit rating. Section 6 concludes.

## 2 Model

I construct a partial equilibrium model featuring a cross-section of ex ante heterogeneous firms and time-varying macroeconomic conditions. The model setup is similar to the models of Chen (2010) and Bhamra, Kuehn, and Strebulaev (2010a,b), with a key difference being that I allow for ex ante heterogeneity in multiple dimensions at the firm level. These papers show that including time-varying macroeconomic conditions and a countercyclical price of risk is important for this class of model to match observed average leverage and credit spreads.<sup>6</sup>

Illustrating the existence of a selection bias does not depend on the inclusion of the time-varying macroeconomic conditions. One should expect a selection bias in observed default costs to arise in a wide array of model environments. As long as the cost of default influences a firm's choice of leverage and a firm's default probability is a function of its leverage, a selection bias would emerge. Thus, the existence of a selection bias is a general point that could be illustrated qualitatively in other models. However, in this paper I am interested in estimating unobservable, expected default costs at the firm level as well as the magnitude of the selection bias they generate. The size and distribution of the firm-level default costs I estimate obviously depends on the structural model used in the estimation. To that end, I follow the framework of existing models that have been shown to perform well quantitatively.

I use a structural tradeoff model of the firm's dynamic capital structure decision in

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<sup>6</sup>Hackbarth, Miao, and Morellec (2006) and Chen, Collin-Dufresne, and Goldstein (2009) also consider time-varying macroeconomic conditions in models of credit risk, though in slightly different frameworks than the one used in this paper.

which the cash flows are specified exogenously. Firms are exposed to both systematic and idiosyncratic cash flow shocks in an environment with time-varying macroeconomic risk. Firms choose leverage ratios by weighing the tax benefits of debt against the deadweight losses incurred in default. Leverage, credit spreads, and firms' optimal default decisions are determined endogenously in the model with equityholders choosing optimal leverage to maximize firm value. Conditional on not defaulting, a firm can restructure upwards by issuing additional debt at any point in time.<sup>7</sup> Restructuring is assumed to entail a cost, however, which results in firms choosing to restructure only once their cash flows exceed an optimally chosen restructuring boundary. In trading off the benefits of a tax shield with the costs of default, the model gives optimal leverage choices endogenously.<sup>8</sup>

Time is continuous and firms' investment policies are fixed. The state of the economy is determined by the state variable  $\nu_t$ , which evolves according to a 2-state time-homogeneous Markov chain. That is,  $\nu_t \in \{H, L\}$ , where the switching between regimes follows a Poisson arrival process. Changes in the aggregate state are assumed to be observable by all agents in the economy and given  $\nu_t$  the state-dependent parameters are known constants.

The aggregate earnings of the economy, denoted by  $X_{A,t}$ , evolve according to a Markov-modulated geometric Brownian motion:

$$\frac{dX_{A,t}}{X_{A,t}} = \mu_A(\nu_t)dt + \sigma_A(\nu_t)dW_t^A \quad (1)$$

where  $W_t^A$  is a standard Brownian motion. As indicated by the notation, the expected growth rate,  $\mu_A(\nu_t)$ , and volatility,  $\sigma_A(\nu_t)$ , of aggregate earnings depend on the aggregate state of the economy,  $\nu_t$ .

In the model, a firm's earnings growth depends on aggregate earnings shocks as well as idiosyncratic shocks specific to the firm. Firms' earnings are taxed at rate  $\tau_c$  and full loss

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<sup>7</sup>The option to restructure downwards is excluded for tractability. While perhaps limiting, this assumption is common to other dynamic capital structure models, such as, Goldstein, Ju, and Leland (2001), Chen (2010), and Bhamra, Kuehn, and Strebulaev (2010a,b).

<sup>8</sup>Note, however, that due to fluctuations in firm cash flows and economic conditions and the assumed cost of restructuring, the firm's actual leverage will drift away from its optimal target. In the model, the firm is at its optimally chosen leverage ratio only at time 0 and subsequent restructuring dates.

offset is assumed. Firm  $i$ 's before-tax earnings,  $X_{i,t}$ , evolve according to

$$\frac{dX_{i,t}}{X_{i,t}} = (\mu_i + \mu_A(\nu_t))dt + \beta_i\sigma_A(\nu_t)dW_t^A + \sigma_{i,F}dW_t^{i,F} \quad (2)$$

This implies that firm  $i$ 's expected earnings growth in state  $\nu_t$  is given by  $(\mu_i + \mu_A(\nu_t))$ , where  $\mu_i$  represents a state-invariant, firm-specific component and  $\mu_A(\nu_t)$  is the state-dependent expected growth rate of aggregate earnings. Thus, the expected earnings growth rate for all firms is assumed to depend, in part, on the aggregate state of the economy. Additionally,  $\beta_i$  parameterizes firm  $i$ 's exposure to the aggregate earnings shocks generated by the Brownian motion  $W_t^A$ . Note that the volatility of aggregate earnings shocks,  $\sigma_A(\nu_t)$ , is assumed to be state-dependent, but a firm's exposure to these shocks,  $\beta_i$ , is constant. Finally, firm  $i$  is exposed to idiosyncratic earnings shocks with volatility  $\sigma_{i,F}$  generated by the firm-specific Brownian motion  $W_t^{i,F}$ . By assumption,  $W_t^{i,F}$  is independent of  $W_t^A$  for all firms  $i$ . Thus, firms are exposed to three types of shocks: aggregate earnings shocks generated by  $W_t^A$ , idiosyncratic earnings shocks generated by  $W_t^{i,F}$ , and changes in the aggregate state of the economy,  $\nu_t$ .

## 2.1 Pricing Kernel, Risk Neutral Measure

I assume markets are complete and that there exists a default-risk-free asset that pays a state-dependent interest rate,  $r(\nu_t)$ . The model is partial equilibrium and I take the pricing kernel as exogenous. Specifically, the pricing kernel is assumed to evolve according to

$$\frac{d\pi_t}{\pi_t} = -r(\nu_t)dt - \varphi(\nu_t)dW_t^A. \quad (3)$$

In this economy,  $\varphi(\nu_t)$  is the state-dependent market Sharpe ratio and the risk premium for firm  $i$ 's cash flows in state  $\nu_t$  is given by  $\beta_i\sigma_A(\nu_t)\varphi(\nu_t)$ . Given the specification for the pricing kernel, I can derive the risk-neutral probability measure,  $\mathcal{Q}$ , which will be used for pricing assets.<sup>9</sup> Under the risk-neutral measure, firm  $i$ 's cash flow process evolves according

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<sup>9</sup>Details of the derivation of the risk-neutral measure and risk-neutral cash flow dynamics are provided in Appendix A.



to

$$\frac{dX_{i,t}}{X_{i,t}} = \widehat{\mu}_i(\nu_t)dt + \sigma_{i,X}(\nu_t)d\widehat{W}_t^i. \quad (4)$$

where  $\widehat{\mu}_i(\nu_t)$  represents the cash flow growth under the risk-neutral measure,  $\widehat{W}_t^i$  is a  $\mathcal{Q}$ -Brownian motion, and

$$\sigma_{i,X}(\nu_t) = \sqrt{(\beta_i\sigma_A(\nu_t))^2 + (\sigma_{i,F})^2}. \quad (5)$$

represents the total earnings volatility for firm  $i$ .

## 2.2 Unlevered Firm Value

The unlevered value of the firm is the value if the firm were to never issue any debt, which is simply the value of a claim to the firm's perpetual cash flow stream.<sup>10</sup> The firm's earnings are taxed at rate  $\tau_c$  and full loss offset is assumed. At time  $t$  in state  $\nu_t$ , the value before taxes of unlevered firm  $i$  is given by

$$V_i^U(X_{it}, \nu_t) = E_t \left[ \int_t^\infty \frac{\pi_s}{\pi_t} X_{i,s} ds \mid \nu_t \right] \quad (6)$$

That is, the value of the unlevered firm is simply a claim to its perpetual stream of cash flows. Note that this value is state-conditional but time-independent. Alternatively, the before-tax unlevered value of firm  $i$  in state  $\nu_t$  at time  $t$  can be expressed as

$$V_i^U(X_{i,t}, \nu_t) = \frac{X_{i,t}}{r_i^U(\nu_t)} \quad (7)$$

where  $r_i^U(\nu_t)$  is the discount rate applied to firm  $i$ 's unlevered cash flows in state  $\nu_t$ . For current state  $H$ ,

$$r_i^U(H) = \frac{[\lambda_{HL} + r^f(H) - \widehat{\mu}_i(H)][\lambda_{LH} + r^f(L) - \widehat{\mu}_i(L)] - \lambda_{HL}\lambda_{LH}}{r^f(L) - \widehat{\mu}_i(L) + \lambda_{HL} + \lambda_{LH}} \quad (8)$$

where  $r^f(H)$  is the instantaneous risk-free rate in state  $H$ ,  $\widehat{\mu}_i(H)$  is firm  $i$ 's risk-neutral cash flow growth rate in state  $H$ , and  $\lambda_{HL}$  is the probability of switching from state  $H$  to  $L$ . This expression shows that the discount rate applied to the firm's cash flows accounts for

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<sup>10</sup>Details of the derivation of unlevered firm value are provided in Appendix B.

the possibility of a change in the aggregate state. An analogous expression holds for  $r_i^U(L)$ . With no regime-switching, the expression for unlevered firm value collapses to a familiar Gordon growth formula:

$$V_i^U(X_{i,t}, \nu_t) = \frac{X_{i,t}}{r^f - \hat{\mu}_i}. \quad (9)$$

The value in state  $\nu_t$  of a consol bond,  $b(\nu_t)$ , that has no default risk and pays a constant coupon rate of 1 can be computed similarly and is given by

$$b(\nu_t) = \frac{1}{r^P(\nu_t)} \quad (10)$$

where  $r^P(\nu_t)$ , is the interest rate in state  $\nu_t$  on a default-risk-free perpetuity. For current state  $H$ ,

$$r^P(H) = r^f(H) + \frac{\lambda_{HL}(r^f(L) - r^f(H))}{\lambda_{HL} + \lambda_{LH} + r^f(L)}. \quad (11)$$

with an analogous expression holding for  $r^P(L)$ .

## 2.3 Financing Decision

Firms make their leverage and default decisions by balancing the benefit of the interest tax shield against the cost of default, with the objective of maximizing the value of equity. Firm  $i$  issues debt in the form of a perpetuity that pays a constant coupon rate of  $C_i$ . This rate is chosen at issuance and paid to bondholders until equityholders choose to default or restructure by issuing additional debt. In the case of restructuring, a firm calls its outstanding debt and issues a new perpetuity with a new coupon rate.

The firm is assumed to distribute all earnings after the coupon payment and corporate taxes to equity holders in the form of a dividend, which is taxed at rate  $\tau_d$ . In the event that current earnings are less than the coupon payment owed,  $X_{i,t} - C_i < 0$ , the firm can issue additional equity. Due to limited liability, equity holders are not obligated to inject additional funds to pay the bondholders. However, failure to do so results in default at which point the bondholders receive ownership rights to the firm. Consequently, equity holders will optimally choose to raise additional funds only in the event that the value of equity in the

current state is positive. Thus, under the assumption that the absolute priority rule holds, the equity holders' optimal default decision satisfies the usual smooth-pasting condition.

## 2.4 Default Event and the Cost of Default

In the event of default, debtholders take over the firm with equityholders receiving nothing. Firms incur a cost in the event of default, which reduces debtholders' recovery rate. In particular, if firm  $i$  defaults at time  $t$ , bondholders receive  $(1-\alpha_i)V_i^U(X_t, \nu_t)$  where  $V_i^U(X_t, \nu_t)$  is the unlevered value of firm  $i$  given in equation (7). Thus,  $\alpha_i$  represents the fraction of firm  $i$ 's unlevered value that is lost in the event of default. As indicated by the notation, these costs are assumed to vary across firms but are constant across aggregate states. While I do not specifically model the nature of this loss, it may be due to a variety of factors such as reputation costs, asset fire sales, loss of customer or supplier relationships, legal and accounting fees, and costs of changing management.

## 2.5 Overview of the Firm's Problem

In order to solve for a firm's optimal capital structure, the values of debt and equity must first be computed. Given the specified cash flow process and pricing kernel, I use a contingent claims approach to solve for the values of these securities and then find the optimal coupon that maximizes initial firm value for each firm.<sup>11</sup> The solution procedure is as follows. First, I solve for the unlevered value of the firm. Then, I solve for the values of debt and equity for arbitrary coupon rate and set of restructuring thresholds with the optimally chosen default thresholds determined by the smooth-pasting conditions. Given these security values, I solve for the optimal default thresholds chosen by the equity holders as a function of an arbitrary coupon. Finally, I solve for the optimal coupon rate and set of upward restructuring thresholds subject to the smooth pasting conditions for the default thresholds.

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<sup>11</sup>The solution technique follows that of Chen (2010), which is based on a method of pricing options on securities with Markov-modulated dynamics presented in Jobert and Rogers (2006).

## 2.6 Valuing Debt and Equity

Debt and equity are contingent claims on a firm's cash flows that pay a continuous dividend rate while the firm is solvent and a lump sum payment in the event of default. As time-homogeneous contingent claims, the values of these two securities at time  $t$  depend only on the present cash flows,  $X_{i,t}$ , and the current state,  $\nu_t$ . Thus, the debt and equity values can be solved for in a manner analogous to the technique used to solve for the unlevered firm value. In particular, the values for debt and equity can be characterized as systems of ordinary differential equations.

Once a firm has issued debt, default becomes a possibility and the firm must choose a cash flow threshold at which it defaults. Since the value of a firm's cash flows (as well as a contingent claim on the cash flows) is different in the two states, a firm will have a different default threshold for each state. I denote the threshold at which firm  $i$  defaults in state  $\nu_t$  as  $X_{D_i, \nu_t}$ . Since equity holders receive nothing in default, the default threshold for a given state will always be less than the coupon payment.

Debt is a contingent claim on firm  $i$ 's cash flows that pays the constant coupon payment  $C_i$  while the firm is solvent and pays  $(1 - \alpha_i)V_i^U(X_{i,t}, \nu_t)$  in the event of default by firm  $i$  at time  $t$  in state  $\nu_t$ . That is, debt holders receive a fraction  $(1 - \alpha_i)$  of the unlevered firm value in the event of default, where the size of the fraction as well as the unlevered firm value depend on the state.

In what follows, I suppress the firm-specific subscript for notational convenience. The values presented apply to a given firm, but are not fixed to be constant across firms. In the event of restructuring, the debt is called and the bondholders receive  $D(X_0; \nu_0)$ . When default occurs at time  $t$  in state  $\nu_t$ , the bondholders receive a payment of  $(1 - \alpha)V(X_t, \nu_t)$ . For current cash flow  $X_t$ , debt issued when the state was  $\nu_0$  has current value given by

$$D(X_t; \nu_0) = \sum_{j=1}^k w_{k,j}^D(\nu_0) \mathbf{g}_{k,j} X_t^{\psi_{k,j}} + \xi_k^D(\nu_0) X_t + \zeta_k^D(\nu_0), \quad X_t \in \mathcal{R}_k, \quad k = 1, 2, 3 \quad (12)$$

where  $\mathcal{R}_k$  represents the current cash flow region. The  $\psi$ 's are the eigenvalues and  $g$  represent the eigenvectors of the firm's eigenvalue problem presented in the Appendix. The terms

$\xi_k^D(\nu_0)$  and  $\zeta_k^D(\nu_0)$  represent solutions to the inhomogeneous equation.

Similarly, equity is a contingent claim that pays a dividend  $(X_{i,t} - C_i)$  until default or restructuring occurs. In the event of restructuring, the equity holders have a claim to the newly levered firm value. As previously mentioned, in default, equity holders receive nothing. Thus, for current cash flow  $X_t$  and initial debt issuance occurring in state  $\nu_0$ , the value of equity is given by

$$E(X_t; \nu_0) = \sum_{j=1}^k w_{k,j}^E(\nu_0) \mathbf{g}_{k,j} X_t^{\psi_{k,j}} + \xi_k^E(\nu_0) X_t + \zeta_k^E(\nu_0), \quad X \in \mathcal{R}_k, \quad k = 1, 2, 3 \quad (13)$$

With these expressions, we can solve for the firm's optimal capital structure, which consists of choosing a coupon rate and default and restructuring boundaries.

## 2.7 Firm's Problem

The firm faces a dynamic capital structure decision at time  $t = 0$ . In choosing its capital structure, the firm balances the tax benefits of debt against the expected cost of default. The debt issued is a perpetuity and the firm is able to restructure upwards in the future by issuing additional debt, subject to a proportional cost of debt issuance,  $\phi_D$ . This proportional issuance cost is paid on the total amount of debt outstanding. As such, the firm faces effectively a fixed cost component on its current outstanding debt and consequently it chooses not to issue debt continuously. Instead, it will choose thresholds for the level of earnings at which point the firm finds it optimal to issue additional debt. Given the initial state,  $\nu_0$ , the firm chooses the coupon rate and two state-dependent default and upward restructuring boundaries,  $\{\mathbf{X}_D(\nu_0), \mathbf{X}_U(\nu_0)\}$ , to maximize the initial value of equity. At time 0, the initial value of the firm for initial cash flow level  $X_0$  and initial state  $\nu_0$  is given by

$$E(X_0, \nu_0; \nu_0) + (1 - \phi_D)D(X_0, \nu_0; \nu_0) \quad (14)$$

where  $\phi_D$  is a proportional cost of debt issuance. Note that even at a later date,  $t$ , the equity and debt value depend on the initial state,  $\nu_0$ , and well as the current state,  $\nu_t$ . Similarly,

the coupon rate and thresholds chosen will depend on the initial state. The firm's problem is given by

$$\max_{\mathbf{C}(\nu_0), \mathbf{X}_{\mathbf{U}}(\nu_0)} E(X_0, \nu_0; \nu_0) + (1 - \phi_D)D(X_0, \nu_0; \nu_0) \quad s.t. \quad (15)$$

$$\left. \frac{\partial}{\partial X} E(X, k, \mathbf{C}; \nu_0) \right|_{X \downarrow X_D^k(\nu_0)} = 0, \quad k = 1, 2 \quad (16)$$

where  $\mathbf{X}_{\mathbf{U}}(\nu_0) = \{X_U^1(\nu_0), X_U^2(\nu_0)\}$ .

The initial optimal leverage ratio is given by

$$L_0(X_0, \nu_0) = \frac{D(X_0, \nu_0; C^*(\nu_0))}{E(X_0, \nu_0; C^*(\nu_0)) + D(X_0, \nu_0; C^*(\nu_0))} \quad (17)$$

where  $C^*(\nu_0)$  is the optimally chosen coupon rate for initial state  $\nu_0$ .

## 2.8 Distress and Default Costs

In the model, firms do not incur distress costs prior to declaring default, at which point the equity holders no longer have a claim to the firm. In reality, firms typically incur distress costs prior to the event of default and some firms may incur distress costs without ever declaring default. Moreover, the costs of distress outside of default are borne directly by equity holders (though debtholders may suffer losses as a result), whereas default in the model occurs when equity value is zero, with the subsequent default costs coming out of the bondholders' recovery.

Despite this simplification, the effect on a firm's capital structure decision is similar to a model with explicit distress costs. In the model, equityholders do not explicitly incur distress costs, but they behave as if they did insofar as the costs borne by the debtholders in default are internalized by the equityholders. In the model, a higher cost of default results in a lower recovery rate for debtholders, all else equal. Recognizing this, the debtholders demand a higher credit spread for a given level of leverage and default probability. This leads a firm with larger default costs to issue less debt and thus have a lower default probability than an otherwise identical firm with smaller default costs. While the default costs are not directly

borne by the equityholders, they internalize these costs as they adjust their optimal level of leverage. Thus, even without explicit distress costs incurred prior to default, equityholders still behave similarly to a case with such explicit costs. This underscores the importance of using a modeling framework in which leverage choices and debt prices are determined jointly and endogenously. The equilibrium pricing of the firm’s debt ensures that default costs, which are borne directly by debtholders, are internalized by the equityholders when choosing leverage.

### 3 Model Estimation

I now turn to the model calibration and estimation of firm-specific parameters, including default costs. Before estimating the firm-specific parameters, I first calibrate the aggregate parameters in the model.

#### 3.1 Aggregate Parameter Calibration

I estimate the parameters of the regime-switching aggregate earnings process using quarterly aggregate earnings data from NIPA Table 1.14 provided by the BEA.<sup>12</sup> In the model, aggregate earnings is assumed to evolve according to a Markov-modulated geometric Brownian motion given in equation (1). By Itô’s Lemma, the quarterly log earnings growth rate,  $x_{t+1}$ , can be written as

$$x_{t+1}^A \equiv \Delta \log(X_{t+1}^A) = \mu^A(\nu_t) - \frac{1}{2}\sigma^A(\nu_t)^2 + \varepsilon_{t+1}^A \quad (18)$$

where  $\varepsilon_{t+1}^A \sim \mathcal{N}(0, (\sigma^A(\nu_t))^2)$ . The identifying assumption for the two regimes is a negative earnings growth rate in the low state. I estimate the six parameters of the aggregate earnings process,  $\{\mu_1^A, \mu_2^A, \sigma_1^A, \sigma_2^A, \lambda_{12}, \lambda_{21}\}$ , via maximum likelihood. The estimates for the aggregate earnings process and the generator matrix,  $\Lambda$ , are presented in Table I. Note that the low state, which is identified by the negative earnings growth, also has higher volatility. For details on the estimation procedure, see Appendix F.

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<sup>12</sup>Additional details can be found in Appendix E.

The cost of debt issuance,  $\phi_D$ , comes from the estimates found in Altinkilic and Hansen (2000) and is in line with values used in the prior literature. The tax rate on equity distributions,  $\tau_d$ , and interest income,  $\tau_i$ , are set to 12% and 29.6%, respectively, which are the values computed in Graham (2000). The tax rate on corporate profits is set to 35%, the current top U.S. federal corporate marginal tax rate. This parameterization follows the values used in Chen (2010).

In unreported results, I estimate the model for an alternative case where  $\tau_\pi = 30\%$  and all other parameters are kept at their original values. I find that this has the expected effect of reducing the cross-sectional mean of the estimated firm-level expected default costs from 44.5% to 37.3%. However, the magnitude of the selection bias is essentially unchanged, with a value of 18.4% points in the  $\tau_\pi = 30\%$  case, compared to a bias of 19.9% points in the benchmark parameterization.

### 3.2 Firm-Level Estimation Overview

I estimate firm-specific default costs and cash flow parameters using a simulated method of moments procedure. I construct a sample of firms from the Compustat Fundamentals Quarterly file merged with equity data from CRSP. Details and variable definitions are provided in Appendix E. The dataset consists of firm-specific moments for 2,505 firms. The aggregate parameters are set to their calibrated values given in Table I.

The method of moments estimator selects the vector of parameters for each firm that minimizes the distance between a firm’s moments in the data and moments from simulated data produced by the model. Intuitively, it selects the set of model parameters for each firm that “best” explain that firm’s data moments. Recall that in the model firm  $i$ ’s cash flows evolve according to

$$\frac{dX_{i,t}}{X_{i,t}} = (\mu_i + \beta_i \mu_A(\nu_t))dt + \beta_i \sigma_A(\nu_t) dW_t^A + \sigma_{i,F} dW_t^{i,F} \quad (19)$$

This gives three firm-specific cash flow parameters  $(\mu_i, \beta_i, \sigma_i^F)$ , in addition to the cost of default parameter,  $\alpha_i$ , to be estimated for each of the 2,505 firms in my sample. For each



firm  $i$  in the sample, I estimate a firm-specific vector of parameters,  $\theta_i$ , where

$$\theta_i = [\alpha_i \quad \mu_i \quad \beta_i \quad \sigma_{i,F}] \quad (20)$$

Let  $M^i$  denote the  $K \times 1$  vector of data moments for firm  $i$ . Given a parameter vector  $\theta$ , for each simulation  $s = 1, \dots, S$ , I simulate a time series of length  $T$  and compute a vector of moments from the simulated data,  $\widetilde{\mathcal{M}}_s(\theta)$ , that serves as an analog to the data moments,  $M^i$ . The method of moments estimator for the parameters of firm  $i$  is defined as

$$\widehat{\theta}_i = \underset{\theta}{\operatorname{argmin}} \left( M^i - \frac{1}{S} \sum_{s=1}^S \widetilde{\mathcal{M}}_s(\theta) \right)' W_i \left( M^i - \frac{1}{S} \sum_{s=1}^S \widetilde{\mathcal{M}}_s(\theta) \right) \quad (21)$$

where  $W_i$  is a positive semidefinite weighting matrix for firm  $i$ . Following Duffie and Singleton (1993), I choose  $W_i = \Sigma_{0,i}^{-1}$ , where

$$\Sigma_{0,i} = \sum_{j=-\infty}^{\infty} \mathbb{E}_t \left( [m_{i,t} - \mathbb{E}_t(m_{i,t})][m_{i,t-j} - \mathbb{E}_t(m_{i,t-j})]' \right) \quad (22)$$

with  $\Sigma_{0,i}$  approximated using the estimator of Newey and West (1987). Note that  $m_{i,t}$  is the observation at date  $t$  for firm  $i$  from the data, meaning  $\Sigma_{0,i}$  depends only on firm  $i$ 's empirical data, not the simulated data. Define  $u_{i,t} = (m_{i,t} - \sum_{t=1}^{T_i} m_{i,t})$ , where  $T_i$  is the empirical sample length for firm  $i$ . I approximate the spectral density matrix for firm  $i$  using

$$\widehat{\Sigma}_i = \sum_{j=-k}^k \left( \frac{k - |j|}{k} \right) \frac{1}{T_i} \sum_{t=1}^{T_i} (u_{i,t} u'_{i,t-j}) \quad (23)$$

where I select  $k = 2$ . Duffie and Singleton (1993) show that under the appropriate conditions,

$$\sqrt{T}(\widehat{\theta}_{T,i} - \theta_{0,i}) \rightarrow \mathcal{N}[0, (1 + 1/S)(H'_{0,i} \Sigma_{0,i}^{-1} H_{0,i})^{-1}] \quad (24)$$

where  $S$  denotes the number of simulations of length  $T_i$  and

$$H_{0,i} = E \left[ \frac{\partial \widetilde{\mathcal{M}}_s(\theta_{0,i})}{\partial \theta} \right]. \quad (25)$$

I repeat the SMM procedure for each firm in the sample, obtaining a  $1 \times 4$  vector of parameter estimates,  $\widehat{\theta}_i$ , and standard errors for each firm  $i = 1, \dots, N$ .

To implement the simulations used in the SMM, I follow the approach used in the literature on simulating structural models of the firm.<sup>13</sup> For each firm-level estimate, I simulate a time series of aggregate shocks to the economy. Fixing this aggregate time series, I simulate 2,000 firm sample paths. I then repeat this procedure 1,000 times ( $S = 1000$ ). I do this for each of the 2,505 firms in my sample. In each of these simulations, I simulate a times series length of 540 quarters, discarding the first 400 quarters of simulated data. In doing this, I effectively begin each simulation path from a stationary distribution that is independent of the initial condition.

### 3.3 Selection of Moments

The selection of moments used in the firm-specific estimation is important to ensure that the four parameters are identified. I select a set of seven firm-specific moments that are informative in that they are sensitive to the parameter values. The set of moments chosen includes moments that are informative about both prices and quantities. Specifically, the moments used in the estimation are the firm's mean book leverage, mean excess equity return, mean price-earnings ratio, mean earnings growth rate, volatility of earnings growth, mean of quasi-market leverage, and volatility of quasi-market leverage.<sup>14</sup> I briefly discuss the moments identifying each of the parameters.

The cost of default parameter,  $\alpha_i$  is identified primarily by the book and quasi-market leverage measures. In the model, the firm's optimal leverage choice is sensitive to the value of default costs. Additionally, since quasi-market leverage contains the market value of equity, it contains information independent of book leverage. As a result, both are informative with respect to the default cost parameter  $\alpha_i$ .

As one would expect, the firm-specific component to expected earnings growth,  $\mu_i$ , is pinned down primarily by the earnings growth rate, however, other moments are informative as well. The price-earnings ratio, for example, is increasing in the rate of earnings growth,

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<sup>13</sup>See, for example, Gomes (2001), Hennessy and Whited (2007), and Strebulaev (2007).

<sup>14</sup>The quasi-market leverage measure is the ratio of the book value of debt to the sum of the book value of debt and market value of equity.

all else equal. Intuitively, controlling for the discount rate, a firm with a higher expected earnings growth has a larger value of equity and thus a higher price-earnings ratio.

The firm's risk exposure, which is parameterized by  $\beta_i$ , impacts the mean excess equity return, price-earnings ratio, and quasi-market leverage values. A larger value of  $\beta_i$  implies greater exposure to systematic risk, which translates to a higher expected return. Similarly, this higher expected return results in a lower present value of equity, which all else equal, means a lower price-earnings ratio. While an increase in  $\beta_i$  does increase the volatility of quasi-market leverage and earnings, the impact on these moments is substantially smaller. This is because most of the variation in the volatility measures is driven by differences in the idiosyncratic volatility, not differences in exposure to aggregate shocks. Additionally, an increase in  $\beta_i$  increases the mean quasi-market leverage ratio in that it reduces the market value of equity, all else equal. However, again, this affect is small compared to the impact of other parameters on the quasi-market leverage ratio.

Finally, the idiosyncratic volatility,  $\sigma_{i,F}$  is determined primarily by the earnings growth and quasi-market leverage volatilities. Again, this is straightforward as these volatility measures are monotonically increasing in  $\sigma_{i,F}$ . However, the volatility also impacts the levels of book and quasi-market leverage as a higher volatility, all else equal, implies a greater default probability. At the same time, this effect is somewhat mitigated by the fact that, all else equal, higher idiosyncratic volatility increases the equityholders' option to delay default.

### 3.4 Estimation Results

The results from the firm-level estimation are presented in Figure 1, which shows the cross-sectional distribution for each of the four firm-specific parameters. Panel B of Figure 1 shows the cross-sectional distribution of the estimated default cost parameter,  $\alpha_i$ . Note that the estimated values of  $\alpha_i$  show considerable cross-sectional dispersion, with a standard deviation of 27%. This suggests that applying a single cost of default to the entire cross-section of firms is likely to give misleading results.

In Table II, I present summary statistics and correlations for the estimated parameters.

As indicated in the first row of Panel A of the table, the mean estimated default cost in the sample of 2,505 firms is 44.5% of firm value, with a median value of 36.8%. Additionally, the estimated values of  $\alpha_i$  display significant heterogeneity with a cross-sectional standard deviation of 27%. The remaining rows of Panel A display statistics for the estimated firm-specific cash flow parameters. These parameters also display considerable cross-sectional standard deviation.

In Panel B of Table II, I report the correlation matrix for the estimated firm-specific parameters for the 2,505 firms in my sample. It is interesting to note that the estimated default cost,  $\alpha$ , has nontrivial correlation with the estimated cash flow parameters. Specifically, the estimated default cost is negatively correlated with a firm's idiosyncratic volatility,  $\sigma_i^F$ , and positively correlated with its systematic risk exposure,  $\beta_i$ , and expected earnings growth,  $\mu_i$ . This correlation structure displayed in Panel B, along with the significant heterogeneity in the estimated cash flow parameters, underscores the importance of jointly estimating the firm-specific default cost and cash flow parameters. Finally, Panel C of Table II displays Spearman rank correlations for the estimated parameters and data moments.

For each firm  $i$  in my sample of 2,505 firms, I estimate a  $4 \times 1$  vector,  $\theta_i$ , of the firm-specific parameters. In addition, I compute standard errors for each of these estimated parameters for each firm. Table III displays summary statistics for the cross-sectional distribution of the standard errors for each of the firm-specific parameter estimates. The first row of the table displays the mean standard error for the firm-specific parameter estimates. For example, the mean standard error on the estimated  $\alpha_i$  across the 2,505 firms is 0.073. As with the parameter estimates themselves, there is heterogeneity in the magnitude of the standard errors across firms. However, the size of the standard errors for most firms indicates that the parameters of the model are relatively precisely estimated. Most importantly, the magnitude of the standard errors indicates that the cross-sectional variation in estimated default costs and the other firm-level parameters is both economically and statistically significant.

## 4 Estimating the Selection Bias in Default Costs

To estimate the selection bias in default costs, I use the firm-specific parameters estimated in the SMM of Section 3 to simulate the model again, but under the estimated joint cross-sectional distribution of the firm-specific parameters. Aggregating the firm-specific estimates obtained from the SMM, I have an estimated four-dimensional joint distribution over the cross-section. I then simulate the model under this joint distribution and estimate the selection bias in the cost of default. I simulate a panel of 5,000 firms at a quarterly frequency for 540 quarters, discarding the first 400 quarters of data, and repeat the simulation 5,000 times. In each simulation, I collect the firms that defaulted in the sample period and compute the average  $\alpha$  for this conditional sample of simulated defaults. Thus, I obtain 5,000 mean values for  $\alpha$ .

Figure 2 displays the distribution of these conditional mean  $\alpha$ 's across simulations. The red vertical line indicates the true unconditional mean  $\alpha$  of the estimated distribution obtained from the SMM. Note that this is also the distribution under which the model is simulated. As indicated by the figure, in none of the 5,000 sample simulations is the conditional average  $\alpha$  computed from the sample of defaulted firms as large as the true unconditional mean.

In Table VIII, I present the estimated selection bias in the average cost of default. The first column of the table reproduces the mean and standard deviation of the distribution of estimated  $\alpha_i$ 's from the SMM. The second column of the table reports the mean and standard deviation of the distribution of default costs for firms that default in the simulated data. Averaging across the 5,000 simulations, the mean cost of default for the defaulted firms is 0.246, or 24.6% of firm value. In contrast, the average default cost among all firms is 44.5%. Thus, the estimated selection bias in default costs is quantitatively large and economically significant.

Using the sample of ex post defaults leads one to conclude that the average default costs are 24.6% of firm value when, in fact, the true mean of the distribution of these costs is 44.5%. In other words, the average firm expects to incur costs in default that are nearly

twice as large as what is inferred by estimating these costs from the sample of defaulted firms. Furthermore, the average default cost among defaulted firms of 24.6% generated by the estimated model is very close to existing empirical estimates from the sample of defaulted firms. For example, Davydenko, Strebulaev, and Zhao (2012) estimate an average default cost of 21.7% from a sample of 175 defaulted firms.

Table VIII also reports the mean and standard deviation of default costs for the sample of simulated defaults conditional on the aggregate economic state in which default occurs. The third column of the table reports the distribution for simulated defaults occurring in the high state of the economy ( $\nu_t = H$ ) and the fourth column reports the distribution for the low state ( $\nu_t = L$ ). Defaults occurring in the high state have an average default cost of 27.6% and those in the low state have an average cost of 20.4%. That is, the selection bias is present in both states of the economy, but more pronounced in the low state.

Recent papers that have considered macroeconomic risk in similar models of capital structure have typically parameterized default costs such that they are larger in a recession state.<sup>15</sup> This is done to match the observed patterns in the time series of default costs. Given the available data, I am not able to estimate expected firm-specific default costs conditional on the aggregate state. However, the simulation evidence presented above suggests that a composition effect does not explain the observed time series pattern in default costs. Put differently, the higher default costs rates observed in recessions are not simply because different types of firms are defaulting. Rather, consistent with the above models' assumption, each firm expects to face a higher cost of default in a recession.

Figure 3 illustrates that the bias affects the entire distribution of default costs, not just the mean value. The figure compares the estimated distribution of expected default costs (Panel A) with the distribution of observed default costs from the sample of defaulted firms generated by simulating the estimated model (Panel B). As shown in the figure, the selection bias has the effect of shifting the entire distribution. While defaults of high default cost firms are observed, these are rare and infrequent. As a result, the observed sample of default costs

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<sup>15</sup>See Hackbarth, Miao, and Morellec (2006), Bhamra, Kuehn, and Strebulaev (2010a,b), and Chen (2010).

is a biased sample that understates the magnitude of default costs.

## 5 Characterizing the Estimated Default Costs

In this section I characterize the estimated firm-level default costs by examining how they relate to firm characteristics, credit ratings, and industry. Using data that was not included in the estimation procedure allows me to check whether the estimates are consistent with previously identified determinants of leverage and default costs. This not only gives an external validation check of the estimates obtained from the structural model, but also provides insights into the determinants of a firm's default costs.

In Table IV, I display summary statistics for the estimated default costs by industry. The industries are grouped according to the Fama-French 17 industry classification, with utilities and financials excluded from the sample.<sup>16</sup> From the table, one can observe some variation in the average cost of default across industries. For example, the Drugs, Perfume, and Tobacco industry classification has relatively high default costs with an industry average of 53%. Given the higher R&D intensity and nature of intangible capital in this industry, the higher estimated default costs are not surprising. In contrast, the Oil and Steel industries have lower average default costs with industry averages of 36% and 37%, respectively. The lower estimated default costs for these industries seems consistent with what one might expect insofar as these industries tend to have higher physical capital intensity.

Table IV also shows significant intra-industry variation in estimated default costs. This suggests that industry alone, at least as defined by the FF 17 industry classification, does not explain most of the variation in estimated default costs. This may be due in part to the industry classification that is used. However, the intraindustry variation in estimated default costs is consistent with the observation that most of the variation in firm investment and financing policies, as well as expected returns, is not explained by industry.

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<sup>16</sup>The Fama-French industry classification is according to Standard Industrial Classification (SIC) codes, which are available for the firms in the Compustat database. Details of the classification are provided on Ken French's website: [http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data\\_library.html](http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html).

Table VI displays the average estimated default costs and cash flow parameters by credit rating. The table shows that the average estimated default costs are increasing in the quality of credit rating. Since firms with high default costs choose leverage such that their probability of default is low, these firms are likely to be those that have a high credit rating, at least at their optimal financing date. This implies that firms which at one point had a high credit rating and later defaulted, the so-called fallen angels, should have higher than average default costs. This prediction is consistent with the findings of Davydenko, Strebulaev, and Zhao (2012), who find empirically that fallen angels have realized default costs significantly higher than those of original-issue junk issuers.<sup>17</sup>

In Table VII, I present regressions of the estimated default costs on various firm characteristics. I present six different specifications in which I compare the results with and without controls for leverage and industry fixed effects. While not all statistically significant, the relationship between default costs and firm characteristics generally appear consistent with intuition and previously identified determinants.

As shown in Table VII, firms with higher market-to-book ratios and investment rates appear to have higher default costs, even after controlling for leverage. These characteristics are often associated with growth firms. Distress or default is likely to be costly for these firms, both because they have less value in physical assets and because such an event would likely result in the firm losing its growth options. Additionally, while not statistically significant, the R&D/Sales ratio is positively correlated with default costs. Similar to firms with growth options, distress or default is likely to be costly for R&D-intensive firms as this intangible capital may be more difficult for the firm to liquidate or transfer. Finally, firms with higher cash to asset ratios appear to be those with higher default costs, which consistent with a hedging motive.

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<sup>17</sup>The original-issue junk issuers are those firms rated speculative grade at the time when the bonds are issued.



## 6 Conclusion

This paper shows that ex ante heterogeneity in firms' expected default costs has important implications for the levels of leverage, credit spreads, and default rates observed in the data. Because firms internalize their expected default costs, those firms with higher costs optimally choose lower levels of leverage, all else equal. As a result, these firms are less likely to default than those firms with lower costs. The estimates of default costs from a sample of defaulted firms, is therefore biased, understating the expected costs faced by the average firm. Since it is the latter that determines a firm's optimal leverage, firms may appear underlevered when, in fact, they simply have high expected default costs.

Using a dynamic capital structure model, I estimate a quantitatively significant selection bias in default costs. My results suggest that many firms may face higher expected default costs than what is indicated by the empirical sample of defaulted firms. Furthermore, the selection bias in default costs that I estimate can help to explain the low leverage ratios adopted by many firms in the data in the context of a tradeoff model of capital structure.

While this paper focuses on default costs, the intuition for the selection bias can be readily applied to other topics in financial economics. Firm-level heterogeneity, combined with firms internalizing this heterogeneity in setting their optimal policies, may produce similar selection biases in other observed outcomes. The dynamic modeling framework and empirical methodology used in this paper could be applied more broadly to study related issues in corporate finance.

Table I:  
**Aggregate Parameters**

This table reports the aggregate parameter values used in simulating and estimating the model. Where applicable, values are quarterly. The aggregate earnings parameters and probability of a regime change are estimated via maximum likelihood using aggregate earnings data. See the appendix for details on the estimation procedure.

<b>Parameter</b>	<b>Symbol</b>	<b>State 1</b>	<b>State 2</b>
Aggregate Earnings Growth Rate	$\mu_A$	0.0192	-0.0076
Aggregate Earnings Volatility	$\sigma_A$	0.0366	0.0770
Market Sharpe Ratio	$\varphi$	0.140	0.238
Instantaneous Risk-free Rate	$r_f$	0.006	0.009
Probability of Regime Change	$\lambda$	0.07	0.10
Tax Rate on Corporate Earnings	$\tau_\pi$	0.35	0.35
Tax Rate on Dividends	$\tau_d$	0.12	0.12
Tax Rate on Interest Income	$\tau_i$	0.296	0.296
Proportional Debt Issuance Cost	$\phi_D$	0.015	0.015

Table II:  
**Cross-sectional Statistics for Firm-Specific Parameter Estimates**

This table reports summary statistics for the firm-specific parameter estimates obtained from the SMM of Section 3. The firm-specific parameters consist of three cash flow parameters ( $\mu_i$ ,  $\beta_i$ , and  $\sigma_i^F$ ) and the cost of default parameter  $\alpha_i$ . Firm  $i$ 's earnings in the model evolve according to

$$\frac{dX_{i,t}}{X_{i,t}} = (\mu_i + \beta_i \mu_A(\nu_t))dt + \beta_i \sigma_A(\nu_t) dW_t^A + \sigma_{i,F} dW_t^{i,F} \quad (26)$$

where  $\mu_i$  represents a firm fixed effect for the expected earnings growth rate,  $\beta_i$  is the loading of the firm's cash flows on the aggregate earnings shock, and  $\sigma_i^F$  is the volatility of the firm's idiosyncratic earnings shocks. The fraction of unlevered firm value lost in default for firm  $i$  is given by  $\alpha_i$ , where the unlevered value is defined in equation (6). Panel A displays cross-sectional moments for the firm-specific parameter estimates. Note that the  $\mu$  row reports statistics for firms' unconditional expected growth rate, not the firm specific component  $\mu_i$ . The cross-sectional correlation of the parameter estimates are shown in Panel B. Panel C displays Spearman rank correlations for the estimated parameters with firm data moments. The sample consists of 2,505 firms from the merged Compustat and CRSP databases. See the Appendix for further details.

**Panel A: Parameter Estimate Summary Statistics**

	Mean	Median	Std
$\alpha_i$	0.445	0.368	0.270
$\sigma_i^F$	0.132	0.147	0.055
$\beta_i$	1.278	1.000	0.577
$\mu$	0.004	0.001	0.007

**Panel B: Correlation of Parameter Estimates**

	$\alpha_i$	$\sigma_i^F$	$\beta_i$	$\mu_i$
$\alpha_i$	1			
$\sigma_i^F$	-0.270	1		
$\beta_i$	0.320	0.023	1	
$\mu_i$	0.358	-0.125	0.567	1

**Panel C: Spearman Rank Correlations**

	$\alpha_i$	$\sigma_i^F$	$\beta_i$	$\mu_i$
Mean Book Lev	-0.862	0.095	-0.357	-0.326
Mean Earnings Growth	0.046	-0.085	0.274	0.315
Std Earnings Growth	-0.144	0.660	0.191	0.159
Mean P/E Ratio	0.288	-0.131	0.286	0.562
Mean Quasi-Market Lev	-0.743	0.292	-0.484	-0.504
Mean Excess Ret	0.135	-0.059	0.207	0.154

Table III:  
**Standard Errors for Firm-level Parameter Estimates**

This table presents distribution statistics for the cross-section of standard errors on the firm-specific parameter estimates obtained in the SMM procedure of Section 3. For each firm, four firm-level parameters are estimated with standard errors. The table displays statistics for the cross-sectional distribution of these standard errors. For more details on the estimation, see Section 3.

	<b>SE(<math>\alpha</math>)</b>	<b>SE(<math>\sigma^F</math>)</b>	<b>SE(<math>\beta</math>)</b>	<b>SE(<math>\mu</math>)</b>
<b>Mean</b>	0.073	0.009	0.478	0.005
<b>Std</b>	0.176	0.006	0.393	0.007
<b>Q1</b>	0.027	0.004	0.179	0.002
<b>Median</b>	0.043	0.008	0.378	0.003
<b>Q3</b>	0.072	0.013	0.655	0.006

Table IV:  
**Default Cost Estimates by Industry**

This table reports summary statistics by industry for the estimated  $\alpha$ 's obtained in the firm-level SMM. Industries correspond to one of the 17 Fama-French industry index, based on SIC codes. Financials and utilities are excluded. N reports the number of firms in each industry classification.

	<b>Mean</b>	<b>Std</b>	<b>N</b>
<b>Food</b>	0.389	0.255	128
<b>Mining and Minerals</b>	0.463	0.275	36
<b>Oil</b>	0.364	0.262	158
<b>Clothing</b>	0.452	0.273	110
<b>Cons Durable</b>	0.422	0.267	113
<b>Chemicals</b>	0.435	0.248	62
<b>Drugs, Perfume, Tobacco</b>	0.532	0.259	88
<b>Construction</b>	0.374	0.206	143
<b>Steel</b>	0.369	0.191	64
<b>Fabricated Products</b>	0.350	0.203	46
<b>Machinery</b>	0.489	0.25	411
<b>Automobiles</b>	0.397	0.205	58
<b>Transportation</b>	0.413	0.181	41
<b>Retail Stores</b>	0.442	0.266	251
<b>Other</b>	0.474	0.302	752

Table V:  
**Parameter Estimates by Industry**

This table reports the mean firm-specific parameter estimates by industry. Estimates are obtained from the SMM procedure described in Section 3. Industries correspond to one of the 17 Fama-French industry index, based on SIC codes. Financials and utilities are excluded.

	$\alpha$	$\sigma^F$	$\beta$	$\mu$
<b>Food</b>	0.389	0.135	1.195	0.003
<b>Mining and Minerals</b>	0.463	0.141	1.319	0.005
<b>Oil</b>	0.364	0.134	1.246	0.005
<b>Clothing</b>	0.452	0.137	1.251	0.001
<b>Cons Durable</b>	0.422	0.140	1.209	0.001
<b>Chemicals</b>	0.435	0.121	1.131	0.002
<b>Drugs, Perfume, Tobacco</b>	0.532	0.112	1.430	0.004
<b>Construction</b>	0.374	0.146	1.146	0.001
<b>Steel</b>	0.369	0.147	1.128	0.001
<b>Fabricated Products</b>	0.350	0.150	1.141	0.001
<b>Machinery</b>	0.489	0.133	1.338	0.005
<b>Automobiles</b>	0.397	0.140	1.162	0.002
<b>Transportation</b>	0.413	0.136	1.085	0.001
<b>Retail Stores</b>	0.442	0.131	1.235	0.003
<b>Other</b>	0.474	0.125	1.363	0.005

Table VI:  
**Parameter Estimates by Credit Rating**

This table reports the mean firm-specific parameter estimates by credit rating for those firms in the sample for which a credit rating is available. The parameter estimates are obtained in the SMM of Section 3.

	$\alpha$	$\sigma^F$	$\beta$	$\mu$
<b>Aaa</b>	0.568	0.094	0.840	0.002
<b>Aa</b>	0.585	0.093	1.128	0.003
<b>A</b>	0.445	0.102	1.031	0.002
<b>Baa</b>	0.419	0.112	1.100	0.003
<b>Ba</b>	0.313	0.125	1.234	0.004
<b>B</b>	0.305	0.137	1.296	0.004
<b>Caa-C</b>	0.189	0.167	1.400	0.008

Table VII:  
**Regressions of Estimated Default Costs on Firm Characteristics**

This table reports regressions of the estimated firm-specific default costs,  $\alpha$ , on firm characteristics. The firm-specific default costs,  $\alpha$ , are estimated using the SMM procedure described in Section 3. Unless indicated otherwise, independent variables are a time series mean of the data available for each firm. In the regressions, all independent variables are normalized by their (cross-sectional) standard deviation, thus the coefficient can be interpreted as the absolute change in  $\alpha$  for a one standard deviation change in the independent variable. Regressions (4), (5), and (6) include industry fixed effects for the 15 Fama-French industries included in the sample. Robust standard errors are in parentheses. For more details on the data construction see the appendix.

VARIABLES	(1) $\hat{\alpha}$	(2) $\hat{\alpha}$	(3) $\hat{\alpha}$	(4) $\hat{\alpha}$	(5) $\hat{\alpha}$	(6) $\hat{\alpha}$
Book Leverage		-0.200*** (0.004)	-0.216*** (0.008)		-0.202*** (0.004)	-0.221*** (0.008)
Cash/Assets			0.008 (0.007)			0.012 (0.007)
R&D/Sales			0.015 (0.009)			0.018* (0.010)
PPE/Assets	-0.039*** (0.006)	0.005 (0.004)	0.019*** (0.007)	-0.041*** (0.007)	0.008* (0.004)	0.016** (0.008)
I/K	0.050*** (0.007)	0.014*** (0.004)	0.026*** (0.008)	0.052*** (0.007)	0.013*** (0.004)	0.021** (0.009)
Earnings/Assets	0.074*** (0.006)	0.030*** (0.004)	0.033*** (0.007)	0.074*** (0.006)	0.030*** (0.004)	0.033*** (0.008)
M/B	0.016** (0.007)	0.024*** (0.005)	0.034*** (0.007)	0.014** (0.007)	0.024*** (0.005)	0.031*** (0.008)
COGS/Sales	-0.024*** (0.006)	-0.007* (0.004)	-0.008 (0.008)	-0.017*** (0.007)	-0.006 (0.004)	-0.003 (0.008)
log(Assets)	-0.010** (0.005)	-0.003 (0.003)	0.000 (0.005)	-0.010** (0.005)	-0.002 (0.003)	0.004 (0.006)
Constant	0.444*** (0.005)	0.443*** (0.003)	0.437*** (0.007)	0.445*** (0.005)	0.442*** (0.003)	0.433*** (0.007)
Observations	2,381	2,381	670	2,340	2,340	653
R-squared	0.206	0.663	0.733	0.185	0.651	0.723
Industry FE	N	N	N	Y	Y	Y

Robust standard errors in parentheses

\*\*\* p<0.01, \*\* p<0.05, \* p<0.1

Regression of Estimated  $\alpha$ 's on Firm Characteristics

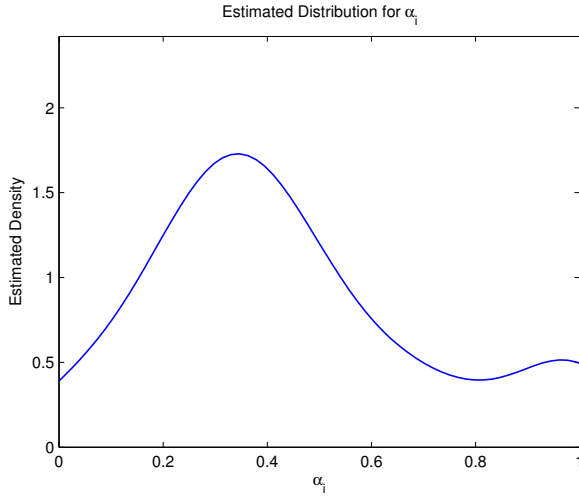
Table VIII:  
**Estimated Bias in Default Costs**

This table presents the inferred mean default costs from the sample of defaulted firms in the simulated data, using the parameter distributions estimated in the SMM of Section 3. A panel of firms is simulated in the model under the estimated joint distribution for the four firm-specific parameters. The panel consists of 5,000 firms simulated at a quarterly frequency for 140 quarters for each simulation and 5,000 simulations are performed. For each simulation, the  $\alpha$ 's for the defaulted firms are collected. A mean and standard deviation of the  $\alpha$ 's of the simulated defaulted firms is computed for all defaults and separately for the defaults occurring in each of the two aggregate states. These means and standard deviations are then averaged across all the simulated economies and reported in the table. The unconditional estimated  $\hat{\alpha}$  refers to the mean value of firm-specific  $\alpha_i$ 's from the SMM estimation of Section 3. "Ex Post  $\hat{\alpha}$ 's" refer to the firms that default in the simulated data under the estimated distribution. The second column reports statistics for the distribution of all simulated defaults and the third and fourth columns report statistics conditional on the aggregate state of the economy in which the default occurred.

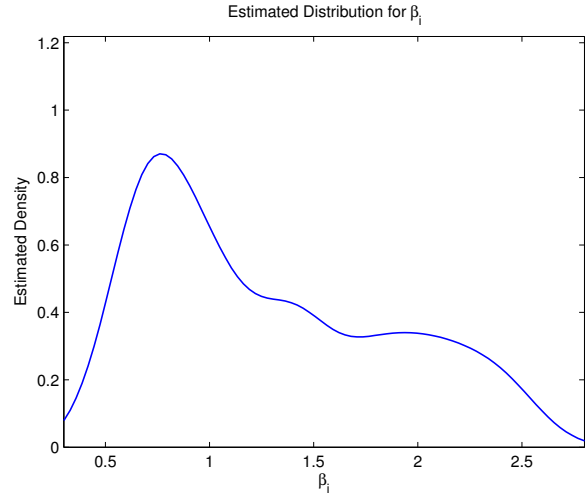
	Unconditional Estimated $\hat{\alpha}$	Ex Post $\hat{\alpha}$ : All Defaults	Ex Post $\hat{\alpha}$ : Defaults in $\nu_t = H$	Ex Post $\hat{\alpha}$ : Defaults in $\nu_t = L$
Mean	0.445	0.246	0.276	0.204
Std Dev	0.270	0.243	0.227	0.264
Bias in Mean		-0.199	-0.169	-0.242



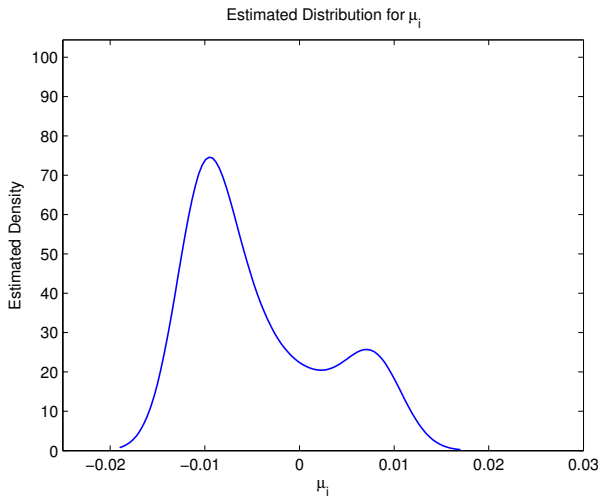
**A. Distribution of Estimated  $\alpha_i$**



**B. Distribution of Estimated  $\beta_i$**



**C. Distribution of Estimated  $\mu_i$**



**D. Distribution of Estimated  $\sigma_i^F$**

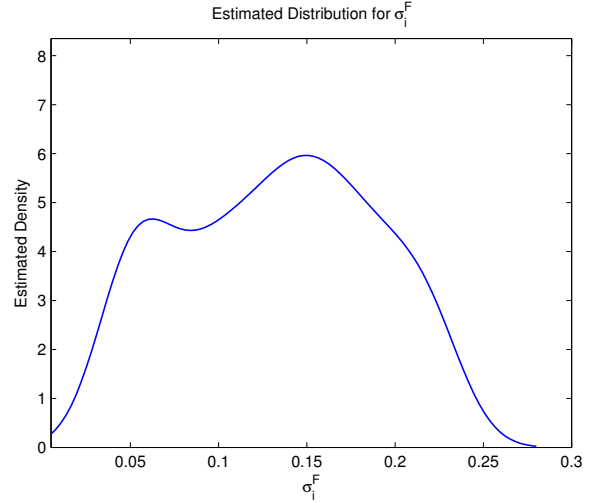


Figure 1: Estimated Cross-Sectional Distributions for the Firm-Specific Parameters. This figure displays the cross-sectional distribution of the firm-specific parameter estimates from the SMM described in Section 3

## Simulated Mean Default Costs from Estimated Parameter Distributions

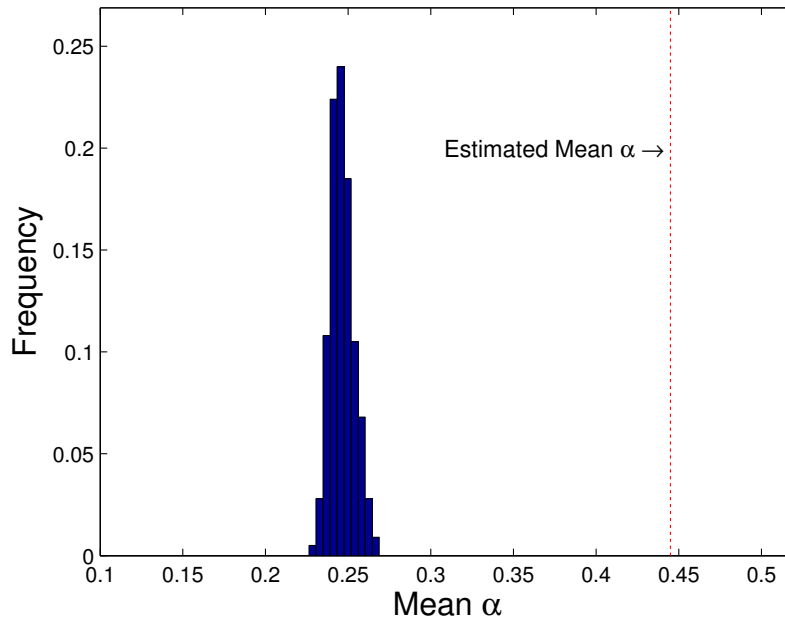
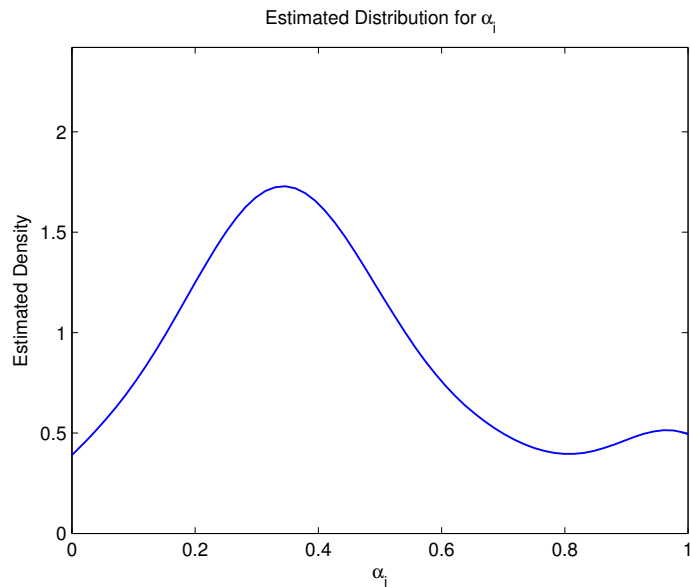


Figure 2: Distribution of Mean Default Costs from the Sample of Defaulted Firms. A panel of 5,000 firms is simulated over 140 quarters and the simulation is repeated 5,000 times. For each simulation, a mean value of  $\alpha$  is computed from the population of firms which defaulted during the simulation. The histogram indicates the distribution of mean  $\alpha$ 's across the 5,000 simulations. The vertical red dashed lines indicates the estimated unconditional mean  $\alpha$  from the SMM procedure of Section 3

### A. Distribution of Estimated $\alpha$



### B. Distribution of $\alpha$ for Simulated Defaults

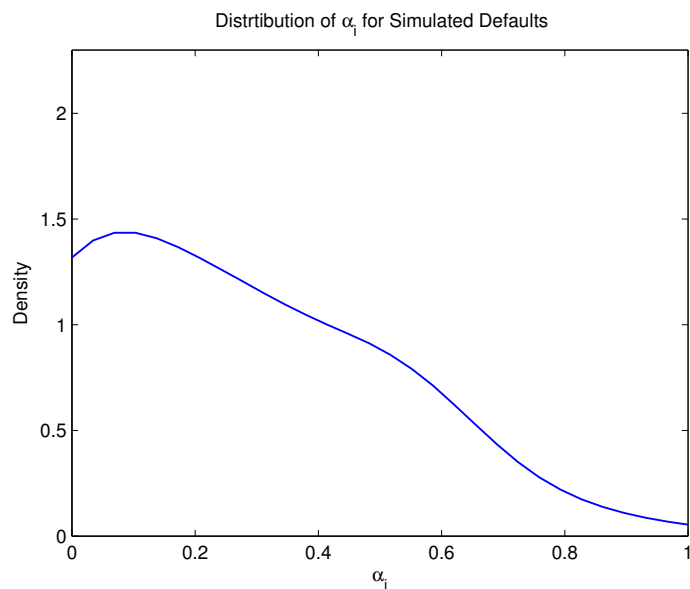


Figure 3: Estimated Distribution of Default Costs vs. Distribution for Simulated Defaults. This figure compares the estimated unconditional distribution of default costs,  $\alpha$ , with the conditional distribution from the sample of simulated defaults. The estimated distribution, which is displayed in Panel A, is obtained from the firm-specific SMM described in Section 3. I simulate 5,000 model economies under the estimated joint cross-sectional distribution of  $\{\alpha_i, \mu_i, \beta_i, \sigma_i^F\}$  and collect the sample of firms which defaulted in each simulation. Panel B plots the distribution of default costs, aggregated over all simulations, of the firms that defaulted.

## References

- Acharya, Viral V., Sreedhar T. Bharath, and Anand Srinivasan. 2007. “Does industry-wide distress affect defaulted firms? Evidence from creditor recoveries.” *Journal of Financial Economics* 85 (3):787–821.
- Almeida, Heitor and Thomas Philippon. 2007. “The risk-adjusted cost of financial distress.” *The Journal of Finance* 62 (6):2557–2586.
- Altinkilic, Oya and Robert S. Hansen. 2000. “Are there economies of scale in underwriting fees? Evidence of rising external financing costs.” *Review of Financial Studies* 13 (1):191.
- Altman, Edward I. 1984. “A further empirical investigation of the bankruptcy cost question.” *Journal of Finance* 39 (4):1067–1089.
- Andrade, Gregor and Steven N. Kaplan. 1998. “How costly is financial (not economic) distress? Evidence from highly leveraged transactions that became distressed.” *The Journal of Finance* 53 (5):1443–1493.
- Asquith, Paul, Robert Gertner, and David Scharfstein. 1994. “Anatomy of financial distress: An examination of junk-bond issuers.” *The Quarterly Journal of Economics* 109 (3):625–658.
- Bhamra, Harjoat S, Lars-Alexander Kuehn, and Ilya A Strebulaev. 2010a. “The aggregate dynamics of capital structure and macroeconomic risk.” *Review of Financial Studies* 23 (12):4187–4241.
- . 2010b. “The levered equity risk premium and credit spreads: A unified framework.” *Review of Financial Studies* 23 (2):645–703.
- van Binsbergen, Jules H., John R. Graham, and Jie Yang. 2010. “The Cost of Debt.” *Journal of Finance* 65 (6):2089–2136.

- Bris, Arturo, Ivo Welch, and Ning Zhu. 2006. “The costs of bankruptcy: Chapter 7 liquidation versus Chapter 11 reorganization.” *Journal of Finance* 61 (3):1253–1303.
- Chen, Hui. 2010. “Macroeconomic Conditions and the Puzzles of Credit Spreads and Capital Structure.” *Journal of Finance* 65 (6):2171–2212.
- Chen, Long, Pierre Collin-Dufresne, and Robert S. Goldstein. 2009. “On the relation between the credit spread puzzle and the equity premium puzzle.” *Review of Financial Studies* 22 (9):3367–3409.
- Davydenko, Sergei A., Ilya A. Strebulaev, and Xiaofei Zhao. 2012. “A Market-Based Study of the Costs of Default.” *Working Paper* .
- Duffie, Darrell. 2001. *Dynamic Asset Pricing Theory*. Princeton University Press.
- Duffie, Darrell and Kenneth J. Singleton. 1993. “Simulated Moments Estimation of Markov Models of Asset Prices.” *Econometrica* 61 (4):929–952.
- Elkamhi, Redouane, Jan Ericsson, and Christopher A. Parsons. 2010. “The cost of financial distress and the timing of default.” *Working paper* .
- Fischer, Edwin O., Robert Heinkel, and Josef Zechner. 1989. “Dynamic capital structure choice: Theory and tests.” *Journal of Finance* 44 (1):19–40.
- Franks, Julian R. and Walter N. Torous. 1989. “An empirical investigation of US firms in reorganization.” *Journal of Finance* 44 (3):747–769.
- George, Thomas J. and Chuan-Yang Hwang. 2010. “A resolution of the distress risk and leverage puzzles in the cross section of stock returns.” *Journal of Financial Economics* 96 (1):56–79.
- Gilson, Stuart C. 1997. “Transactions costs and capital structure choice: Evidence from financially distressed firms.” *Journal of Finance* 52 (1):161–196.

- Goldstein, Robert S., Nengjiu Ju, and Hayne Leland. 2001. "An EBIT-based model of optimal capital structure." *Journal of Business* 74 (4):483–512.
- Gomes, J.F. 2001. "Financing investment." *American Economic Review* :1263–1285.
- Graham, John R. 2000. "How big are the tax benefits of debt?" *Journal of Finance* 55 (5):1901–1941.
- Guo, X. and Q. Zhang. 2004. "Closed-form solutions for perpetual American put options with regime switching." *SIAM Journal on Applied Mathematics* 64 (6):2034–2049.
- Hackbarth, Dirk, Jianjun Miao, and Erwan Morellec. 2006. "Capital structure, credit risk, and macroeconomic conditions." *Journal of Financial Economics* 82 (3):519–550.
- Hamilton, James D. 1989. "A new approach to the economic analysis of nonstationary time series and the business cycle." *Econometrica: Journal of the Econometric Society* 57 (2):357–384.
- . 1994. *Time Series Analysis*. Princeton University Press.
- Harrison, J. Michael and David M. Kreps. 1979. "Martingales and arbitrage in multiperiod securities markets." *Journal of Economic Theory* 20 (3):381–408.
- Hennessy, Christopher A. and Toni M. Whited. 2007. "How costly is external financing? Evidence from a structural estimation." *Journal of Finance* 62 (4):1705.
- Jarrow, Robert A., David Lando, and Stuart M. Turnbull. 1997. "A Markov model for the term structure of credit risk spreads." *Review of Financial Studies* 10 (2):481.
- Jobert, A. and LCG Rogers. 2006. "Option pricing with Markov-modulated dynamics." *SIAM Journal on Control and Optimization* 44 (6):2063–2078.
- Korteweg, Arthur. 2010. "The Net Benefits to Leverage." *Journal of Finance* 65 (6):2137–2170.

- Leland, Hayne E. 1994. “Corporate Debt Value, Bond Covenants, and Optimal Capital Structure.” *Journal of Finance* 49:1213–1213.
- Merton, Robert C. 1974. “On the pricing of corporate debt: The risk structure of interest rates.” *Journal of Finance* 29 (2):449–470.
- Miller, Merton H. 1977. “Debt and taxes.” *Journal of Finance* 32 (2):261–275.
- Morellec, Erwan, Boris Nikolov, and Norman Schuerhoff. 2010. “Dynamic capital structure under managerial entrenchment: Evidence from a structural estimation.” *Working Paper, University of Lausanne* .
- Newey, Whitney K. and Kenneth D. West. 1987. “A Simple, Positive Semi-Definite, Heteroskedasticity and Autocorrelation Consistent Covariance Matrix.” *Econometrica* 55 (3):703–708.
- Nikolov, Boris and Toni M. Whited. 2010. “Agency conflicts and cash: Estimates from a structural model.” .
- Ofek, Eli. 1993. “Capital structure and firm response to poor performance:: An empirical analysis.” *Journal of Financial Economics* 34 (1):3–30.
- Opler, Tim C. and Sheridan Titman. 1994. “Financial distress and corporate performance.” *Journal of Finance* 49 (3):1015–1040.
- Pulvino, Todd C. 1998. “Do Asset Fire Sales Exist? An Empirical Investigation of Commercial Aircraft Transactions.” *The Journal of Finance* 53 (3):939–978.
- Strebulaev, Ilya A. 2007. “Do tests of capital structure theory mean what they say?” *The Journal of Finance* 62 (4):1747–1787.
- Strebulaev, Ilya A and Toni M Whited. 2012. “Dynamic Models and Structural Estimation in Corporate Finance.” *Foundations and Trends® in Finance* 6 (1-2):1–163.

Taillard, Jerome. 2010. “Thriving in the midst of financial distress? An analysis of firms exposed to asbestos litigation.” *Working paper* .

Warner, Jerold B. 1977. “Bankruptcy, absolute priority, and the pricing of risky debt claims.” *Journal of Financial Economics* 4 (3):239–276.

Weiss, Lawrence A. 1990. “Bankruptcy resolution:: Direct costs and violation of priority of claims.” *Journal of Financial Economics* 27 (2):285–314.



# Supplemental Appendices

## (Not intended for publication)

### A Pricing Kernel, Risk-Neutral Measure

Given the exogenously specified process for the pricing kernel, the risk-neutral measure can be derived.<sup>18</sup> The pricing kernel,  $\pi_t$ , evolves according to

$$\frac{d\pi_t}{\pi_t} = -r(\nu_t)dt - \varphi^m(\nu_t)dW_t^m \quad (\text{A-1})$$

Define the density process for the risk-neutral measure by

$$\xi_t = E_t \left[ \frac{dQ}{dP} \right] \quad (\text{A-2})$$

We know this density process and the pricing kernel are related by<sup>19</sup>

$$\xi_t = B_t \pi_t \quad (\text{A-3})$$

where

$$B_t = \exp \left\{ \int_0^t r(\nu_s) ds \right\} \quad (\text{A-4})$$

is the time  $t$  price of a bond paying the riskless rate and  $B_0$  has been normalized to 1.

Applying Itô's Lemma gives

$$d\xi_t = B_t d\pi_t + \pi_t dB_t \quad (\text{A-5})$$

Plugging in the expression for  $d\pi_t$ ,

$$d\xi_t = B_t [-r(\nu_t)\pi_t dt - \varphi^m(\nu_t)\pi_t dW_t^m] + \pi_t dB_t \quad (\text{A-6})$$

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<sup>18</sup>Since the horizon is infinite, the risk-neutral measure,  $\mathcal{Q}$ , that will be used for pricing contingent claims is not an equivalent probability measure to the physical measure,  $\mathcal{P}$ . Still, the risk-neutral measure  $\mathcal{Q}$  will have the necessary properties for risk-neutral pricing. See Duffie (2001), Section 6N, for more details.

<sup>19</sup>See Harrison and Kreps (1979).

Replacing  $\pi_t$  with  $\frac{\xi_t}{B_t}$  and dividing through by  $\xi_t$  gives

$$\frac{d\xi_t}{\xi_t} = -r(\nu_t)dt - \varphi^m(\nu_t)dW_t^m + \frac{1}{B_t}dB_t \quad (\text{A-7})$$

Itô's Lemma implies

$$dB_t = r(\nu_t)dt \quad (\text{A-8})$$

Thus the density process,  $\xi_t$ , evolves according to

$$\frac{d\xi_t}{\xi_t} = -\varphi^m(\nu_t)dW_t^m \quad (\text{A-9})$$

Applying Girsanov's Theorem, we have a new Brownian motion under the risk-neutral measure, given by

$$d\widehat{W}_t^m = dW_t^m + \varphi^m(\nu_t)dt \quad (\text{A-10})$$

Note that the firm-specific Brownian motion,  $W_t^{f,n}$ , that generates the idiosyncratic shocks to firm  $n$ 's cash flows is independent of the Brownian motion,  $W_t^m$  generating systematic shocks to the economy. Thus  $W_t^{f,n}$  is still a Brownian motion under the risk-neutral measure for all firms  $n$ . Thus, under the risk-neutral measure, cash flows for firm  $n$  evolve according to

$$\frac{dX_t^n}{X_t^n} = \widehat{\mu}^n(\nu_t)dt + \sigma_m^n(\nu_t)d\widehat{W}_t^m + \sigma_f^n dW_t^{f,n} \quad (\text{A-11})$$

where  $\widehat{\mu}^n(\nu_t)$  is the drift under the risk-neutral measure,

$$\widehat{\mu}^n(\nu_t) = \mu^n(\nu_t) - \sigma_m^n(\nu_t)\varphi^m(\nu_t). \quad (\text{A-12})$$

The total volatility of the cash flows of firm  $n$  is given by

$$\sigma_X^n(\nu_t) = \sqrt{(\sigma_m^n(\nu_t))^2 + (\sigma_f^n)^2} \quad (\text{A-13})$$

Additionally, the two Brownian motions driving the idiosyncratic and systematic shocks to firm  $n$ 's cash flows under the risk-neutral measure can be aggregated into a single Brownian

motion (under the risk-neutral measure) for firm  $n$  which is given by

$$d\widehat{W}_t^n = \frac{\sigma_m^n(\nu_t)}{\sigma_X^n(\nu_t)} d\widehat{W}_t^m + \frac{\sigma_f^n}{\sigma_X^n(\nu_t)} dW_t^{f,n}. \quad (\text{A-14})$$

So the evolution of firm  $n$ 's cash flows under the risk-neutral measure can be expressed as

$$\frac{dX_t^n}{X_t^n} = \widehat{\mu}^n(\nu_t) dt + \sigma_X^n(\nu_t) d\widehat{W}_t^n \quad (\text{A-15})$$

## B Solving for Unlevered Firm Value

Here I show how to solve for the unlevered firm value.<sup>20</sup> The pair of ODEs characterizing the unlevered firm value has an associated characteristic function given by:

$$g_1(\beta)g_2(\beta) = \lambda_1\lambda_2 \quad (\text{A-16})$$

where

$$g_1(\beta) = \lambda_1 + r - (\mu_1 - \frac{1}{2}\sigma_1^2)\beta - \frac{1}{2}\sigma_1^2\beta^2 \quad (\text{A-17})$$

$$g_2(\beta) = \lambda_2 + r - (\mu_2 - \frac{1}{2}\sigma_2^2)\beta - \frac{1}{2}\sigma_2^2\beta^2 \quad (\text{A-18})$$

This characteristic function has four distinct roots  $\beta_1 < \beta_2 < 0 < \beta_3 < \beta_4$ . The general form of the solution is given by

$$A^1(X) = \phi_1(X) + \sum_{i=1}^4 G_i x^{\beta_i} \quad (\text{A-19})$$

$$A^2(X) = \phi_2(X) + \sum_{i=1}^4 H_i x^{\beta_i} \quad (\text{A-20})$$

$$H_i = l(\beta_i)G_i = \frac{g_1(\beta_i)}{\lambda_1}G_i = \frac{\lambda_2}{g_2(\beta_i)}G_i \quad (\text{A-21})$$

However boundedness conditions on the unlevered firm value need to be imposed. These

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<sup>20</sup>The exposition follows Guo and Zhang (2004). See also Chen (2010) and Jobert and Rogers (2006).

are

$$\lim_{x \rightarrow \infty} \frac{A^i(x)}{x} < \infty \quad \text{and} \quad \lim_{x \rightarrow 0} A^i(x) < \infty \quad (\text{A-22})$$

These two conditions imply  $\beta_i = 0$ ,  $i = 1, \dots, 4$ . Thus the unlevered firm value has the form:

$$A^i(X) = \phi_i(X) \quad (\text{A-23})$$

We conjecture that the unlevered firm value is affine in  $X$ . That is,

$$A^i(X) = c_i X + d_i \quad (\text{A-24})$$

Furthermore,  $d_i = 0$ ,  $i = 1, 2$ , since  $A^i(0) = 0$

Thus the conjecture becomes

$$A^i(X) = c_i X \quad (\text{A-25})$$

Plugging these expressions into the two ODEs characterizing the unlevered firm value and with some rearranging gives a linear system of two equations in two unknowns.

$$\mu_i c_i X - (\lambda_i + r) c_i X + X + \lambda_i c_j X = 0, \quad j \neq i \quad (\text{A-26})$$

Solving these two equations for  $c_1, c_2$  gives the unlevered firm value in state  $i$  as:

$$A^i(X) = \frac{(\lambda_1 + \lambda_2 + r - \mu_j)X}{\lambda_2(r - \mu_1) + (r - \mu_2)(\lambda_1 + r - \mu_1)} \quad (\text{A-27})$$

Note that if  $\mu_1 = \mu_2$  then the unlevered firm value is the same in both states and is given by

$$A(X) = \frac{X}{r - \mu} \quad (\text{A-28})$$

## C Eigenvalue Problem

This section describes the eigenvalue problem for the cash flow region in which neither default nor restructuring are immediate threats. Define the log cash flow process,  $x_t = \log(X_t)$ . By

Itô's Lemma, under the risk-neutral measure, the log cash flow process evolves according to

$$dx_t = \left[ \widehat{\mu}(\nu_t) - \frac{1}{2} \sigma_X(\nu_t)^2 \right] dt + \sigma_X(\nu_t) d\widehat{W}_t \quad (\text{A-29})$$

Under the risk-neutral measure, the price process of any contingent claim on firm cash flows will be a martingale with the cash flows discounted by investors at the risk-free short rate,  $r(\nu_t)$ . Thus, these contingent claims will be martingales of the form:

$$M_t^f = \exp\left(-\int_0^t r(\nu_u) du\right) f(\nu_t, x_t) \quad (\text{A-30})$$

for some function  $f$  that depends on the payoffs of the given security.

Applying Itô's Lemma gives

$$dM_t^f = \exp\left(-\int_0^t r(\nu_u) du\right) \left[ (\Lambda - R)f + \frac{1}{2} \Sigma f_{xx} + \Theta f_x \right] dt \quad (\text{A-31})$$

$R$  is the diagonal matrix of  $r_i$ 's.  $\Sigma$  is the diagonal matrix of  $\sigma_{iX}^2$ 's.  $\Theta$  is the diagonal matrix of the risk-neutral drifts of the log cash flow process.  $\Lambda$  is the generator matrix of the Markov chain,  $\nu_t$ .

Since  $M_t^f$  is a martingale, it has zero drift, implying

$$(\Lambda - R)f + \frac{1}{2} \Sigma f_{xx} + \Theta f_x = 0 \quad (\text{A-32})$$

We seek a separable  $f$  of the form

$$f(\nu_t, x_t) = g(\nu_t) \exp(-\beta x_t) = g(\nu_t) X_t^\beta \quad (\text{A-33})$$

This gives the following equation to be solved in  $\beta$  and  $g$ :

$$(\Lambda - R)g + \frac{1}{2} \beta^2 \Sigma g - \beta \Theta g = 0. \quad (\text{A-34})$$

Premultiplying the above equation by  $2\Sigma^{-1}$  gives

$$2\Sigma^{-1}(\Lambda - R)g + \beta^2 g - 2\beta \Sigma^{-1} \Theta g = 0. \quad (\text{A-35})$$

This gives the following system of equations:

$$\beta g = h \tag{A-36}$$

$$\beta h = 2\Sigma^{-1}\Theta h - 2\Sigma^{-1}(\Lambda - R)g \tag{A-37}$$

This can be written as a standard eigenvalue problem of the form

$$A \begin{pmatrix} g \\ h \end{pmatrix} = \begin{pmatrix} 0 & I \\ -2\Sigma^{-1}(\Lambda - R) & 2\Sigma^{-1}\Theta \end{pmatrix} \begin{pmatrix} g \\ h \end{pmatrix} = \beta \begin{pmatrix} g \\ h \end{pmatrix} \tag{A-38}$$

If  $(g, \beta)$  solve this eigenvalue problem, then

$$M_t^f = \exp\left(-\int_0^t r(\nu_u) du - \beta x_t\right) g(\nu_t) \tag{A-39}$$

is a martingale. The matrix A has exactly 2 eigenvalues with positive real parts and 2 with negative real parts.

## D Solving for the $w$ coefficients

For the case in which there are two aggregate states to the Markov chain, there are a total of 3 relevant cash flow regions and each security has a total of 16  $w$  coefficients (8 for each initial state).

The cash flow regions are:

Region 1:  $X \in [X_D^1, X_D^2)$

Region 2:  $X \in [X_D^2, X_U^{u(1)})$

Region 3:  $X \in [X_U^{u(1)}, X_U^{u(2)})$

Note that for  $X < X_D^1$  the firm is always in default regardless of the state and for  $X > X_U^{u(2)}$  the firm has already restructured upwards for any state.

## Debt

For a given initial state,  $\nu_0$ , the 8 boundary conditions for debt are

$$\lim_{X \uparrow X_D^2} D(X, 1, \nu_0) = \lim_{X \downarrow X_D^2} D(X, 1, \nu_0) \quad (\text{A-40})$$

$$\lim_{X \uparrow X_D^2} D_X(X, 1, \nu_0) = \lim_{X \downarrow X_D^2} D_X(X, 1, \nu_0) \quad (\text{A-41})$$

$$\lim_{X \uparrow X_U^{u(1)}} D(X, u(2), \nu_0) = \lim_{X \downarrow X_U^{u(1)}} D(X, u(2), \nu_0) \quad (\text{A-42})$$

$$\lim_{X \uparrow X_U^{u(1)}} D_X(X, u(2), \nu_0) = \lim_{X \downarrow X_U^{u(1)}} D_X(X, u(2), \nu_0) \quad (\text{A-43})$$

$$D(X_D^1, 1, \nu_0) = (1 - \alpha(1))V^U(X_D^1, 1) \quad (\text{A-44})$$

$$D(X_D^2, 2, \nu_0) = (1 - \alpha(2))V^U(X_D^2, 2) \quad (\text{A-45})$$

$$D(X_U^{u(1)}, u(1), \nu_0) = D(X_0, \nu_0) \quad (\text{A-46})$$

$$D(X_U^{u(2)}, u(2), \nu_0) = D(X_0, \nu_0) \quad (\text{A-47})$$

Equations (A-40) and (A-42) are the value-matching conditions across cash flow regions and equations (A-41) and (A-43) are the smooth-pasting conditions across regions. Equations (A-46) and (A-47) are the value-matching boundary conditions for default and equations (A-44) and (A-45) are the value-matching boundary conditions for upward restructuring.

The initial (par value) of debt at time 0 is given by

$$D(X_0, \nu_0; \nu_0) = w_{2,1}^D(\nu_0)g_{2,1}(\nu_0)\exp\{\beta_{2,1}x_0\} + w_{2,2}^D(\nu_0)g_{2,2}(\nu_0)\exp\{\beta_{2,2}x_0\} + w_{2,3}^D(\nu_0)g_{2,3}(\nu_0)\exp\{\beta_{2,3}x_0\} + w_{2,4}^D(\nu_0)g_{2,4}(\nu_0)\exp\{\beta_{2,4}x_0\} + (1 - \tau_i)C(\nu_0)\mathbf{b}(\nu_0)$$

$$D(X_0, \nu_0; \nu_0) = \sum_{j=1}^4 w_{2,j}^D(\nu_0)g_{2,j}(\nu_0)\exp\{\beta_{2,j}x_0\} + (1 - \tau_i)C(\nu_0)\mathbf{b}(\nu_0) \quad (\text{A-48})$$

Note that  $g_{2,j}(\nu_0)$  is a scalar: it's the  $\nu_0$  element of the  $g_{2,j}$  eigenvector, where  $g_{2,j}$  is the  $j$ th eigenvector for the eigenvalue problem for the 2nd cash flow region. Thus, we have a system of 8 equations to solve for the 8 unknown  $w^D$  coefficients.

$$G(X)_{LHS}W^D + \xi(X)_{LHS} + \zeta_{LHS} = G(X)_{RHS}W^D + \xi(X)_{RHS} + \zeta_{RHS} \quad (\text{A-49})$$

$$[G(X)_{LHS} - G(X)_{RHS}]W^D = \xi(X)_{RHS} + \zeta_{RHS} - \xi(X)_{LHS} - \zeta_{LHS} \quad (\text{A-50})$$

Thus,

$$W^D = [G(X)_{LHS} - G(X)_{RHS}]^{-1} (\xi(X)_{RHS} + \zeta_{RHS} - \xi(X)_{LHS} - \zeta_{LHS}) \quad (\text{A-51})$$

## Equity

For a given initial state,  $\nu_0$ , the 8 boundary conditions for equity are

$$\lim_{X \uparrow X_D^2} E(X, 1, \nu_0) = \lim_{X \downarrow X_D^2} E(X, 1, \nu_0) \quad (\text{A-52})$$

$$\lim_{X \uparrow X_D^2} E_X(X, 1, \nu_0) = \lim_{X \downarrow X_D^2} E_X(X, 1, \nu_0) \quad (\text{A-53})$$

$$\lim_{X \uparrow X_U^{u(1)}} E(X, u(2), \nu_0) = \lim_{X \downarrow X_U^{u(1)}} E(X, u(2), \nu_0) \quad (\text{A-54})$$

$$\lim_{X \uparrow X_U^{u(1)}} E_X(X, u(2), \nu_0) = \lim_{X \downarrow X_U^{u(1)}} E_X(X, u(2), \nu_0) \quad (\text{A-55})$$

$$E(X_D^1, 1, \nu_0) = 0 \quad (\text{A-56})$$

$$E(X_D^2, 2, \nu_0) = 0 \quad (\text{A-57})$$

$$E(X_U^{u(1)}, u(1), \nu_0) = \frac{X_U^{u(1)}}{X_0} [(1-q)D(X_0, u(1); u(1)) + E(X_0, u(1); u(1))] - D(X_0, u(1); \nu_0) \quad (\text{A-58})$$

$$E(X_U^{u(2)}, u(2), \nu_0) = \frac{X_U^{u(2)}}{X_0} [(1-q)D(X_0, u(2); u(2)) + E(X_0, u(2); u(2))] - D(X_0, u(2); \nu_0) \quad (\text{A-59})$$

Note that these conditions hold for an arbitrary coupon rate,  $C(\nu_0)$ . For a given initial state,  $\nu_0$ , the optimal default thresholds (for an arbitrary coupon) satisfy the smooth-pasting conditions for equity such that

$$\left. \frac{\partial}{\partial X} E(X, 1; \nu_0) \right|_{X \downarrow X_D^1(\nu_0)} = 0 \quad (\text{A-60})$$

$$\left. \frac{\partial}{\partial X} E(X, 2; \nu_0) \right|_{X \downarrow X_D^2(\nu_0)} = 0 \quad (\text{A-61})$$



## E Data

### Aggregate Earnings

For the aggregate earnings series, I use the quarterly “Net Operating Surplus” series from NIPA Section 1, Table 1.14, Line 8. The quarterly series is available for the period 1947Q1-2010Q2. I construct the log earnings growth series and present summary statistics for the unconditional moments below (all values are quarterly).

Unconditional Moments: Quarterly Aggregate Earnings Growth	
Mean	0.017
Std Dev	0.053
AC(1)	0.111

### Firm Data

I construct the sample of firms to be estimated from the Compustat Fundamentals Quarterly file merged with equity data from CRSP. I require firms to have at least 20 quarters of data in the Compustat and CRSP files.

Variable definitions:

- Book Leverage:  $\frac{dlccq + dlttq}{atq}$

- Earnings Growth:

$$\tilde{e}_{t+1} = \frac{\sum_{j=0}^K e_{t+1-j}}{\sum_{j=0}^K e_{t-j}} - 1 \quad (\text{A-62})$$

where  $e_t$  is Compustat item ‘oiadpq’ in quarter  $t$ .

- Quasi-Market Leverage:  $\frac{dlccq + dlttq}{dlccq + dlttq + ME}$  where ME is constructed from CRSP as Price\*(Shares Outstanding).

## F Estimating Parameters of the Aggregate Earnings Process

The procedure I use to estimate the parameters of the aggregate earnings growth follows the exposition in Chapter 22 of Hamilton (1994) on estimating Markov chain regime-switching

processes. See also Hamilton (1989). In the model, aggregate earnings is assumed to evolve according to a Markov-modulated geometric Brownian motion:

$$\frac{dX_t^A}{X_t^A} = \mu^A(\nu_t)dt + \sigma^A(\nu_t)dW_t^A. \quad (\text{A-63})$$

By Itô's Lemma, the quarterly log earnings growth rate,  $x_{t+1}$ , can be written as

$$x_{t+1} \equiv \Delta \log(X_{t+1}) = \mu^A(\nu_t) - \frac{1}{2}\sigma^A(\nu_t)^2 + \varepsilon_{t+1}^A \quad (\text{A-64})$$

where  $\varepsilon_{t+1}^A \sim \mathcal{N}(0, (\sigma^A(\nu_t))^2)$ .

This gives six parameters to be estimated:  $\mu_1^A, \mu_2^A, \sigma_1^A, \sigma_2^A, \lambda_{12}$ , and  $\lambda_{21}$ . Stacking these parameters into a vector,  $\Theta$ , the vector of conditional densities for each state can be expressed as

$$\eta_t = \begin{bmatrix} f(x_t | \nu_{t-1} = 1, x_{t-1}; \Theta) \\ f(x_t | \nu_{t-1} = 2, x_{t-1}; \Theta) \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{2\pi}\sigma_1^A} \exp \left\{ \frac{-(x_t - \mu_1^A + \frac{1}{2}(\sigma_1^A)^2)}{2(\sigma_1^A)^2} \right\} \\ \frac{1}{\sqrt{2\pi}\sigma_2^A} \exp \left\{ \frac{-(x_t - \mu_2^A + \frac{1}{2}(\sigma_2^A)^2)}{2(\sigma_2^A)^2} \right\} \end{bmatrix} \quad (\text{A-65})$$

Define the vector of optimal inferences for the current state at date  $t$ , given the vector of observations up to and including date  $t$ ,  $\mathcal{X}_t$ , and the vector of population parameters,  $\Theta$ , as

$$\hat{\xi}_{t|t} = \begin{bmatrix} \mathbb{P}\{\nu_t = 1 | \mathcal{X}_t; \Theta\} \\ \mathbb{P}\{\nu_t = 2 | \mathcal{X}_t; \Theta\} \end{bmatrix} \quad (\text{A-66})$$

Similarly, define the vector of optimal one period ahead forecasts for state  $\nu_{t+1}$  as

$$\hat{\xi}_{t+1|t} = \begin{bmatrix} \mathbb{P}\{\nu_{t+1} = 1 | \mathcal{X}_t; \Theta\} \\ \mathbb{P}\{\nu_{t+1} = 2 | \mathcal{X}_t; \Theta\} \end{bmatrix} \quad (\text{A-67})$$

The optimal inference and forecast can be defined recursively as

$$\hat{\xi}_{t|t} = \frac{\hat{\xi}_{t|t-1} \odot \eta_t}{\mathbf{1}'(\hat{\xi}_{t|t-1} \odot \eta_t)} \quad (\text{A-68})$$

$$\hat{\xi}_{t+1|t} = P' \hat{\xi}_{t|t} \quad (\text{A-69})$$

where  $\odot$  denotes element by element multiplication and  $P$  is the discrete time transition matrix given by

$$P = \begin{bmatrix} p_{11} & 1 - p_{11} \\ 1 - p_{22} & p_{22} \end{bmatrix} \quad (\text{A-70})$$

Starting with an initial guess for  $\xi_{1|0}$  equal to the vector of unconditional probabilities and a vector of parameters,  $\Theta$ , the log likelihood function can be constructed by iterating on equations (A-68) and (A-69).

$$\mathcal{L}(\Theta) = \sum_{t=1}^T \log f(x_t | \mathcal{X}_{t-1}; \Theta) = \sum_{t=1}^T \log(\mathbf{1}'(\hat{\xi}_{t|t-1} \odot \eta_t)) \quad (\text{A-71})$$

To estimate the parameter vector  $\Theta$ , I maximize the log likelihood function given in (A-71) numerically. Finally, given the estimated discrete time transition matrix,  $P$ , the generator matrix,  $\Lambda$ , for the continuous time Markov chain can be computed as<sup>21</sup>

$$\Lambda = \begin{bmatrix} \lambda_{11} & \lambda_{12} \\ \lambda_{21} & \lambda_{22} \end{bmatrix} = \begin{bmatrix} \log(p_{11}) & \frac{(1-p_{11})\log(p_{11})}{p_{11}-1} \\ \frac{(1-p_{22})\log(p_{22})}{p_{22}-1} & \log(p_{22}) \end{bmatrix} \quad (\text{A-72})$$

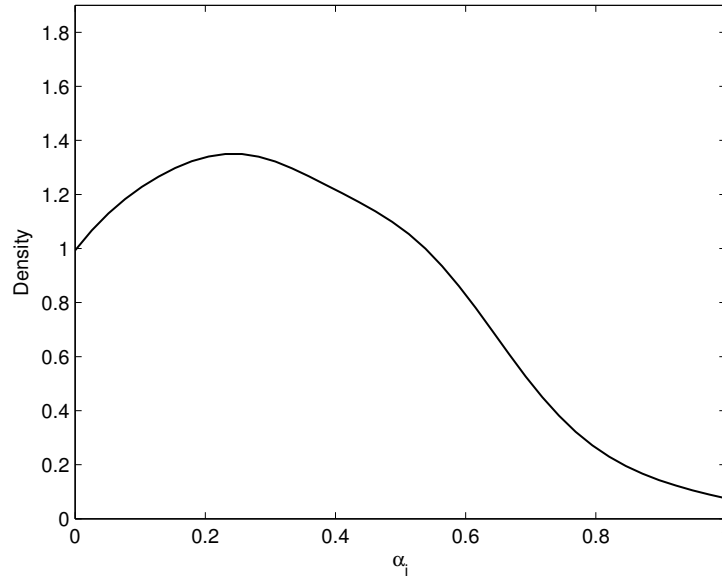
## G Selection Bias Across States

In Figure A-1 I present the distribution of default costs, for simulated defaults in an economy with set of estimated firm-level parameters. I repeat the exercise of computing the selection bias in default costs as in Section 4 of the paper, but display the results conditional on the state of the economy. Panel A of Figure Figure A-1 presents the distribution of default costs for the defaults occurring in the high state of the economy ( $\nu_t = H$ ). Panel B displays the distribution for the low state ( $\nu_t = L$ ).

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<sup>21</sup>Note that this assumes that the probability of switching states more than once in a quarter is zero. See Jarrow, Lando, and Turnbull (1997) for more details.

**A. Distribution of Defaulted  $\alpha_i$ :  $\nu_t = H$**



**B. Distribution of Defaulted  $\alpha_i$ :  $\nu_t = L$**

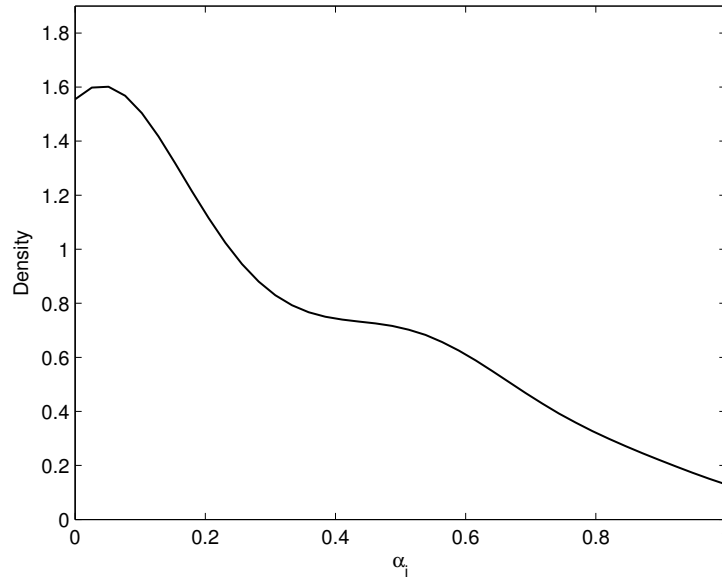


Figure A-1: Distribution of Default Costs for Simulated Defaults, Conditional on State of the Economy. This figure displays the distribution of default costs among the simulated defaults, conditional on the state of the economy. Panel A displays the distribution for defaults occurring in the good state of the economy ( $\nu_t = H$ ) and Panel B displays the distribution for the bad state ( $\nu_t = L$ ).