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DECISION MAKING OVER TIME AND UNDER UNCERTAINTY: A COMMON APPROACH*

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This paper considers a number of parallels between risky and intertemporal choice. We begin by demonstrating a one-to-one correspondence between the behavioral violations of the respective normative theories for the two domains (i.e., expected utility and discounted utility models). We argue that such violations (or preference reversals) are broadly consistent with three propositions about the weight that an attribute receives in both types of multiattribute choice. Specifically, it appears that: (1) if we add a constant to all values of an attribute, then that attribute becomes *less* important; (2) if we proportionately increase all values of an attribute, or if we change the sign of an attribute, from positive to negative, then that attribute becomes *more* important. The generality of these propositions, as well as the constraints they would impose on separable representations of multiattribute preferences, is discussed.
(TIME PREFERENCE; TIME DISCOUNTING; INTERTEMPORAL CHOICE; DECISION MAKING UNDER UNCERTAINTY; CHOICE ANOMALIES)

1. Introduction

The expected utility (EU) and discounted utility (DU) models provide easy access to the interconnected domains of time and uncertainty. Parallel in structure, both models view decision makers as selecting between alternatives based on a weighted sum of utilities, with the weights being either probabilities (for EU) or discount factors based on time delays (for DU). EU and DU have been applied to such diverse topics as labor supply, saving behavior, crime, and fertility, and continue to be broadly utilized as both descriptive and normative models of choice despite growing criticism of their assumptions and implications and an onslaught of alternative formulations.

The initial appeal of the two theories was due, no doubt, to their formal simplicity, and their close similarity to the familiar financial calculations of actuarial value and present value. Subsequent axiomatic derivations—by von Neumann and Morgenstern (1953) for EU, and Koopmans (1960) for DU—have clarified the logic behind these calculations, and have brought to the surface the fundamental assumptions that justify them. These turn out to be surprisingly few in number, and relatively easy to accept, at least when presented in a transparent formal context.

In parallel with the axiomatic development, however, researchers have uncovered a growing number of "anomalous" examples, indicating that people reliably violate the EU and DU axioms when confronted with certain special configurations of choices. The EU anomalies, and especially the Allais paradox, have stimulated the development of theoretical alternatives to expected utility (e.g., Kahneman and Tversky 1979, Machina 1982). In contrast, until recently (Loewenstein and Prelec forthcoming), the discounted utility violations have not been documented or subjected to the same intense discussion and exposure.

In this paper we draw attention to some remarkable parallels between EU and DU violations, and, more importantly, propose that the existence of such "matching" violations is not coincidental, but rather reflects certain fundamental psychological properties

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of multidimensional prospect evaluation. These properties concern the weight that attributes receive in decision making as a function of their magnitude. Our analysis is restricted to simple prospects consisting of a single nonzero outcome which is to be received either with a given probability, or at a specified point in time. In subsequent papers (Loewenstein and Prelec 1989, 1991) we detail a variety of "sequential choice anomalies" applicable to temporal prospects with multiple outcomes and extend our theoretical analysis of simple prospects to explain them.

EU and DU are at once excessively restrictive and unnecessarily vague in their specification of attribute weighting. They are too restrictive regarding the nonmoney attribute (probability and time), and not sufficiently restrictive toward the money attribute. For example, DU makes the extreme assumption that people are sensitive to only absolute time delays—that a given time delay has the same impact on choice regardless of when it occurs. The empirical observation that people tend to view a given absolute time delay as more significant if it occurs earlier is the source of one important DU anomaly. EU's similarly rigid assumption about probability weighting—namely that only relative probabilities matter—leads to an analogous EU anomaly. In their specification of the money attribute, however, both models are quite vague. Beyond the assumption of decreasing marginal utility, which is in any case not integral to either model, virtually any pattern of weighting is permissible.

By treating the money and nonmoney dimensions commensurably—by relaxing the restrictions on the nonmoney dimension, while simultaneously adding new restrictions to the money dimension—we show that it is possible to explain the basic patterns of anomalous choice in both domains. With one notable exception, the weighting of money, time, and probability each conform to three simple patterns that we label "decreasing absolute sensitivity," "increasing relative sensitivity" and "loss amplification."

The paper starts with a brief discussion of the central implications of the normative theories in §2. §3 summarizes, and draws connections between the anomalies and paradoxes that have been uncovered so far in both domains. The theoretical core of the paper, in §4, presents a diagnosis of the sources of these violations, in the form of three properties of attribute weighting that can explain most of the basic choice patterns in decision making under uncertainty and intertemporal choice. §5 develops a multiplicative choice model that embodies the three properties, and §6 discusses evidence supporting and challenging the model.

2. The Normative Models

Encoding or "Framing" Assumptions

The common starting point for any normative theory is the establishment of a particular canonical format for describing the choice objects distinguished by the theory. In expected utility theory, it is traditional to "integrate" outcomes into final wealth positions so that the ultimate objects of choice are lotteries over aggregate levels of wealth. For example, a decision maker with wealth w would frame an option defined by

$$X \equiv (x, p) \equiv (x_1, p_1; \ldots; x_n, p_n) \quad \text{as} \quad (w + x, p) \equiv (w + x_1, p_1; \ldots; w + x_n, p_n).$$

Discounted utility also typically assumes integration. In a standard application, individuals face choices between *temporal prospects*, which are (finite) collections of *dated* consumption outcomes. A temporal prospect

$$X \equiv (x, t) \equiv (x_1, t_1; \ldots; x_n, t_n)$$

indicates that outcome x_i will be obtained at time t_i. Often the time intervals are presumed to be equally spaced (i.e., they form a "consumption program" over successive periods $t_i = it_1$), in which case one omits the t_i and expresses the prospects as an outcome sequence $(x_1, \ldots, x_n)$. Integration, here, involves aggregating new consumption options with existing plans. If the decision maker's existing plans call for $(c_1, \ldots, c_n)$, then by accepting the prospect $(x_1, \ldots, x_n)$, the decision maker reveals a preference for $(c_1 + x_1, \ldots, c_n + x_n)$ over $(c_1, \ldots, c_n)$.

Both models have also been derived under the assumption that the final objects of choice are outcomes evaluated without regard for existing consumption or wealth (see, e.g., Fishburn and Rubinstein 1982, for time, and Fischer, Kamlet, Fienberg and Schkade 1986, for uncertainty). However, without the assumption of asset integration such models can easily lead to suboptimal decisions if a person is confronted by repeated choices. Without asset integration, for example, it is possible that a person who preferred \$300 in one year to \$200 today, if the decision were divided into two subsidiary choices, would repeatedly prefer \$100 today over \$150 in one year. Since a decision maker with such preferences could be manipulated into violating dominance by a skillful arbitrageur, it is not clear that such models should be considered "normative" in the strict sense of the term.

In addition to these book-keeping assumptions, the normative models are each committed to a specific numerical representation for preferences. For risky prospects, the representation is additively separable across outcomes and linear in probabilities, so that prospect $(x_1, p_1, \ldots, x_n, p_n)$ is preferred to $(y_1, q_1, \ldots, y_n, q_n)$ at base wealth level w, if and only if it has higher value according to the expected utility formula

$$\sum p_i u(w + x_i) \geq \sum q_i u(w + y_i), \tag{1}$$

where $u(x + w)$ is the utility level for a wealth level equal to $x + w$. For temporal prospects, the representation is also additively separable across outcomes, but exponential in time, so that prospect $(x_1, \ldots, x_n)$ is preferred to $(y_1, \ldots, y_n)$ at base consumption level $(c_1, \ldots, c_n)$, iff

$$\sum (1 + r)^i u(c_i + x_i) \geq \sum (1 + r)^i u(c_i + y_i). \tag{2}$$

Both representations make strong claims about how preference should change if the time or probability scales are transformed in certain ways. The expected utility representation implies that only relative probability ratios of nonzero outcomes are significant, so that the operation of reducing proportionately the probabilities of nonzero outcomes in two prospects by the same factor should not affect preference. For instance, a person who is indifferent between two lotteries should still be indifferent if there is only a 10% chance that either lottery will be played.

The discounted utility representation implies, in contrast, that only absolute time intervals matter. Contemplating the same choice between (x) and (y) from an earlier vantage point shifts all time intervals by a constant amount (τ), which, according to (2), reduces both sides of the equation by $(1 + r)^\tau$. This is the stationarity property, as originally formulated by Koopmans (1960) in a discrete-time context.

3. Anomalies

Even though EU and DU have been widely applied in economic research, recently both models have been challenged by a proliferation of anomalies—common patterns of behavior inconsistent with the models' predictions. In this section, we review the most significant EU and DU anomalies and show that there is a virtual one-to-one correspondence between them.

The Common Ratio/Common Difference Effects

The independence axiom implies that preferences between risky prospects should not be affected by multiplying the probabilities of nonzero outcomes by a common factor.

TABLE 1
DU and EU Anomalies

DU Anomalies		EU Anomalies	
Name	Description	Name	Description
Common difference effect	$(x, t) \approx (y, t')$ but $(x, t + k) < (y, t' + k)$	Common ratio effect	$(x; p) \approx (y; q)$ but $(x; pk) < (y; qk)$
Immediacy effect	Overweighting of immediate consump.	Certainty effect	Overweighting of certain outcomes
Gain/loss asymmetry	Lower devaluation of losses	Reflection effect	Risk seeking toward losses
Magnitude effect	Lower devaluation of large than small gains/losses	"Peanuts effect"	Risk taking for small gains, risk aversion for small losses.
Framing effects	Sensitivity to different descriptions of objectively equivalent prospects	Framing effects	Sensitivity to different descriptions of objectively equivalent prospects

The common ratio effect (Kahneman and Tversky 1979) refers to a violation of this prediction defined by: $(x; p) \sim (y; q)$ implies $(x; \alpha p) \prec: (y; \alpha q)$, for $y > x$; $0 < \alpha < 1$. A person who is indifferent, say, between a 25% chance of \$2500 and a 50% chance of \$1000 would probably prefer a 2.5% chance of winning \$2500 to a 5% chance of winning \$1000.

The corresponding anomaly in intertemporal choice is the *common difference effect*. DU implies that a person's preference between two single-outcome temporal prospects should depend on the absolute time interval between delivery of the objects. However, both intuition, and experimental evidence (Thaler 1981, Benzion, Rapoport and Yagil 1989) indicate that the impact of a constant time difference between two outcomes becomes less significant as both outcomes are made more remote. Formally, $(x, t) \sim (y, d)$ implies $(x, t + \epsilon) \prec (y, d + \epsilon)$, for $y > x$, $\epsilon > 0$. A person who is indifferent, say, between \$20 today and \$25 in one month will most likely prefer \$25 in eleven months to \$20 in ten.

The common-ratio and common-difference anomalies imply that preference is generally sensitive to both ratios and differences along a dimension. Consider, first, the common-ratio effect. Sensitivity to differences can explain why shrinking all probabilities proportionately reduces the significance of the probability dimension, thereby shifting preferences in favor of the prospect that contains the larger money outcome.[1] Likewise, *increasing* the outcome dates in a pair of temporal prospects by a constant interval, decreases the ratio between the time delays. Sensitivity to time ratios implies a reduction in the relative importance of time, again shifting preferences in favor of the prospect with the larger money outcome.

The Immediacy/Certainty Effects

Several researchers have argued that, in addition to the common ratio and difference effects, a virtual discontinuity of preference occurs when the dimensions of time and uncertainty approach their point of maximal significance (i.e., $t = 0$, and $p = 1$). The *certainty effect* (Kahneman and Tversky 1979) refers to the tendency to value disproportionately outcomes that are certain to occur. If $\alpha p = 1$, then $(x; p) \sim (y; q)$ implies

[1] See Rubinstein (1987) for an explicit model of the similarity relations.

$(x;\ \alpha p) \succ (y;\ \alpha q)$, for $y > x$; $\alpha > 1$. The immediacy effect refers to the enhanced significance decision makers attach to outcomes that are experienced immediately—i.e., $(x, t) \sim (y, d)$ implies $(x, t + \epsilon) \prec (y, d + \epsilon)$, for $t = 0$, and $y > x$, $\epsilon > 0$. It can be detected in the extremely high discount rates estimated for short time delays in several empirical studies of discounting (Thaler 1981, Benzion, Rapoport and Yagil 1989), and can also be observed in intertemporal decision making that does not involve monetary payoffs (Christensen-Szalanski 1984).[2] The immediacy and certainty effects are special cases of violations of stationarity and proportionality, and can therefore be formally subsumed under the common ratio and common difference effects. Many researchers feel, however, that these phenomena are qualitatively distinct, and warrant separate treatment (see, e.g., Cohen and Jaffray 1988, Benzion, Rapoport and Yagil 1989).

Sign Effects

The "reflection effect" in decision making under uncertainty (Kahneman and Tversky 1979) refers to the finding that modal preferences shift from risk aversion to risk seeking when positive domain gambles are "reflected" by making all payoffs negative—$(x;\ p) \sim (y;\ q)$ implies $(-x;\ p) \succ (-y;\ q)$ for $p < q$ (although see Hershey and Schoemaker 1980).

A similar discrepancy in treatment of losses and gains also can be observed in intertemporal choice defined by $(x, t_1) \sim (y, t_2)$ implies $(-x, t_1) \succ (-y, t_2)$, for $y > x > 0$ and $t_2 > t_1$. The attractiveness of gains is reduced faster than the aversiveness of losses. Loewenstein (1988b) reports evidence of gain/loss asymmetries on the order of 15–20% for small and moderate outcomes delayed by one year. Thaler (1981) reports even greater discrepancies.

A necessary condition for sharp sign effects is that people segregate new choices from their existing assets and "frame" the outcomes of such choices as losses and gains (Markowitz 1952, Kahneman and Tversky 1979) which they treat differently. EU, which usually assumes a utility function denominated in total assets, cannot account for the reflection effect, because sign shifts for small amounts have a negligible effect on total wealth (ruling out wealth effects as an explanation for the behavior) but a major impact on risk preferences (Kahneman and Tversky 1979). Nor can a model that assumes that utility is a function of asset levels account for the observation that people at all income levels gamble and buy insurance (Markowitz 1952; see also Harvey 1988).

The same logic applies equally well to intertemporal choice. People display the loss/gain asymmetry at a wide range of consumption levels. Only by assuming that new outcomes are segregated, and encoded as gains or losses, is it possible to explain why people who consume at such disparate baseline levels display the loss/gain asymmetry.

Magnitude Effects

Although not as well established as other EU anomalies, there is also evidence of a relationship between outcome magnitude and risk taking that is incompatible with EU (Markowitz 1952). The general pattern is one of (a) risk seeking for very small gains turning progressively to risk aversion as outcome magnitudes increase, and (b) risk aversion for very small losses turning to risk seeking as the absolute magnitude of negative outcomes increase—i.e., $(x; p) \sim (y; q)$ implies $(\alpha x; p) \prec (\alpha y; q)$, and $(-x; p) \sim (-y; q)$ implies $(-\alpha x; p) \prec (-\alpha y; q)$ for $\alpha > 1$, $x > y > 0$, and $p < q$.

[2] Christensen-Szalanski interviewed expectant women at different times during their pregnancy concerning their preference for using anaesthesia during childbirth. At all times prior to labor the great majority expressed a preference for not using anaesthesia. Following the onset of labor, however, preferences abruptly shifted, with a large fraction opting for relief. Apparently the cost of desisting from anaesthesia (the pain of labor) and the benefits (e.g., later satisfaction from having given birth "naturally") were balanced differently when both outcomes were in the future and when one (the pain) was experienced in the present.

Magnitude effects can also be observed in intertemporal choice. Numerous survey studies (Thaler 1981, Loewenstein 1988b, Benzion, Rapoport and Yagil 1989) have revealed a common pattern: a negative relationship between the absolute magnitude of an outcome and the rate at which it loses value when delayed. For example, Thaler's (1981) subjects were indifferent between \$15 immediately and \$60 in a year (\$15/\$60 = 1/4) and between \$250 immediately and \$350 in a year (\$250/\$350 = 5/7). For losses, the comparable figures were (\$15/\$20 = 3/4) and (\$250/\$270 = 25/27). This pattern can be formalized as $(x, t_1) \sim (y, t_2)$ implies $(\alpha x, t_1) \prec (\alpha y, t_2)$, for $\alpha > 1$, $y > x > 0$ and $t_2 > t$, and $(-x, t_1) \sim (-y, t_2)$ implies $(-\alpha x, t_1) \succ (-\alpha y, t_2)$. Like sign effects, the magnitude effects observed in intertemporal choice are incompatible with the assumption of integration (see Loewenstein and Prelec forthcoming).

Framing Effects

Framing effects, even more than conventional sign and magnitude effects, strike at the heart of the representational assumptions because they demonstrate that changing the way in which a prospect is represented, holding constant its "objective" characteristics, can have a significant impact on choice. Framing effects are generally induced by shifting the implicit point of reference in a choice (Payne, Laughhunn and Crum 1980, Tversky and Kahneman 1981). Such reference point shifts can be viewed as linear transformations of attributes. For example, Kahneman and Tversky were able to obtain preference shifts by reframing simple gambles into two stages, one common to both prospects, and one different. If the common stages tend to be ignored by subjects, as they hypothesized, then the new frame can be interpreted as a scalar transformation of probabilities. Other illustrations of framing express prospects as losses or gains by adding or subtracting a constant amount to the payoffs of two prospects, while compensating for this manipulation with a lump sum payment.

Framing effects have also been documented in intertemporal choice (Loewenstein 1988a, Loewenstein and Prelec forthcoming). In one study, for example, subjects were given a choice between two vacation options, one offering 37 vacation days two years in a row, the other offering 44 days in the first year and 30 in the second. In one experimental condition, the 30 days common to all options were segregated from each outcome; in the other they were integrated. Choices in the two conditions differed significantly, with the option offering greater up-front rewards preferred more in the segregated condition. A second study found a significant shift of choice when payment options for a TV were expressed in terms of gains (a rebate) or losses (payments).

4. Theoretical Integration

The goal of this section is to distill from the behavioral clues just enumerated a minimal set of principles that apply to multiattribute choice generally, and that are consistent with the violations of the normative models in both the time and probability domains.

Representation of Objects of Choice

The starting point for any model of choice is its assumptions concerning the nature of choice objects. Although often regarded as unproblematic or self-evident, the so-called "representational" assumptions are psychologically pivotal. Both EU and DU typically assume that people integrate new alternatives, either with existing wealth (EU), or with planned consumption (DU). But evidence reviewed in the previous section—sign, absolute magnitude, and framing effects—suggests that people in fact often segregate new alternatives from current wealth and from future consumption plans. Even when people do take wealth or consumption plans into account, they probably do so in a much cruder way than required by asset integration. The assumption of asset integration is undermined by the fact that typical decision makers may have only imprecise knowledge of their current wealth, and only vague or sketchy future consumption plans.

Our starting assumption is that in a multiattribute prospect, the value of each attribute is encoded or "framed" as a pair of values, the first being the absolute magnitude of the attribute, and the second being the *polarity* of the attribute, which is positive if larger values along the attribute improve the prospect, and negative otherwise. Thus, for example, a temporal prospect of $1,000 due in 6 months would be encoded as: (1,000, 6) with polarities $(+, -)$, where the negative polarity for the time attribute indicates that we are dealing with delays, which diminish the overall prospect value.

The central idea behind our explanations of the anomalies is that certain transformations of values along an attribute, as well as changes in polarity, lead to systematic changes in the weight or "psychological salience" that is attached to that attribute.[3] In stating our conditions, we will denote generic two-attribute prospects by (a, b), where both a and b are formally elements of real intervals A and B. The two special cases of concern to us, of course, are preferences over simple lotteries (x, p), interpreted as a p-chance of receiving $\$x$, and delayed cash payments (x, t) received at time t. The letters x, y, will always stand for values along the money attribute, while p, q, and t, s, for values along the probability and time attributes, respectively. It is understood that $0 \leq p$, $q \leq 1$, and t, $s \geq 0$.

We now review the assumptions needed to secure a real-valued representation of preferences (these are adapted from Fishburn and Rubinstein's 1982 axioms for preferences over single, dated cash flows):

A1. ORDERING. *$\succsim$ is a weak order on (A, B).*

A2. CONTINUITY. *The sets weakly preferred to a given (a, b) are closed.*

A3. MONOTONICITY. *If $a \succsim b$ then $(a, c) \succsim (a, c)$.*

A4. ZERO OUTCOME. *If $c = 0$ then $(c, a) \sim (c, b)$.*

Taken together, A1–A4 guarantee that preferences can be represented by a continuous real-valued function $u(a, b)$, which is increasing in a and satisfying $u(0, b) = 0$ (Fishburn and Rubinstein 1982, Theorem 1).

We now give a more precise definition of a transformation to increase the weight of an attribute.

DEFINITION 1. Let (a, b) be a generic two attribute prospect, and $f(a)$ a transformation of the first attribute. We say that the transformation f *increases* the weight of an attribute iff whenever two prospects are indifferent originally, $(a_1, b_1) \sim (a_2, b_2)$, then the transformation has the effect of making the transformed attribute decisive:

$$(f(a_1), b_1) \prec (f(a_2), b_2) \quad \text{iff} \quad (f(a_1), b_2) \prec (f(a_2), b_1).$$[4]

The definition for *decreasing* the weight on an attribute is derived by reversing exactly one of the preferential inequalities.

Our definition of attribute weighting is illustrated in Figure 1. The inside indifference curve connects the two indifferent prospects (a_1, b_1) and (a_2, b_2); the two indifference curves lying to the northeast pass through the prospects after the first attribute has been

[3] There is a close affinity between our approach and that taken by Tversky, Sattath and Slovic (1988) to explain the preference reversal phenomenon. Their basic claim is that the weight of a dimension is enhanced if the response scale is expressed in units of that dimension (i.e., certainty equivalents for gambles will be relatively more sensitive to dollars than probabilities). Our hypothesis is that numerical transformations of a dimension also have a predictable impact on its weight.

[4] The definition easily generalizes to prospects with three or more attributes by letting the second attribute, B, stand for several distinct attributes.

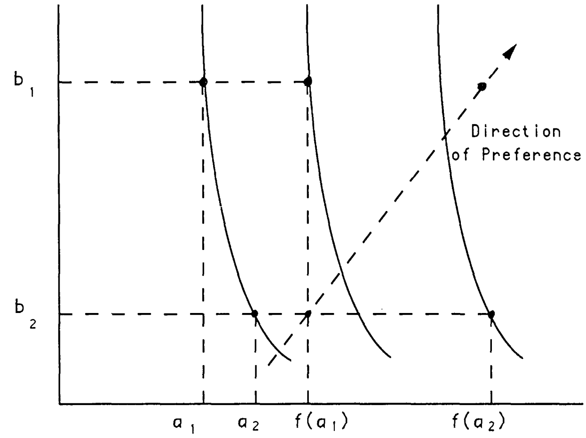

FIGURE 1

subjected to the transformation $f(\cdot)$. Since $(f(a_2), b_1) \succ (f(a_1), b_2)$, preference increases toward the northeast, and the prospect $(f(a_2), b_2)$ is preferred to $(f(a_1), b_1)$. Intuitively, after applying a transformation $f(\cdot)$ that increases the weight on the first attribute, the prospect that was initially stronger on that attribute becomes decisive.

We now state three assumptions about the effect of linear transformations on attribute weight. The first two formalize the intuition that preferences, generally, will be invariant neither with respect to transformations that preserve ratios nor those that preserve differences along an attribute.

A5. DECREASING ABSOLUTE SENSITIVITY (DAS). *Increasing the absolute magnitude of all values of an attribute by a common additive constant decreases the weight of the attribute. Specifically, if* $(a_1, b_1) \sim (a_2, b_2)$, *and* $\epsilon a_1 > 0$, *then*

$$(a_1 + \epsilon, b_1) \succ (a_2 + \epsilon, b_2) \quad \textit{iff} \quad (a_1 + \epsilon, b_2) \prec (a_2 + \epsilon, b_1).$$

In the temporal case, with attributes A and B standing for time and money (T and X) this is inconsistent with stationarity, which preserves the original indifference judgment. Had we introduced, for example, the Fishburn and Rubinstein version of stationarity,

A5*. STATIONARITY. *If* $(x, t) \sim (y, s)$, *and* $\epsilon > 0$, *then* $(x, t + \epsilon) \sim (y, s + \epsilon)$,

then the representation of intertemporal preference would conform to the discounted utility formula $(1 + r)^t u(x)$ (Fishburn and Rubinstein 1982, Theorem 2).

In a similar spirit, we state

A6. INCREASING PROPORTIONAL SENSITIVITY (IPS). *Increasing the absolute magnitude of all values of an attribute by a common multiplicative constant increases the weight of the attribute. Specifically, if* $(a_1, b_1) \sim (a_2, b_2)$, *and* $\alpha a_1 > 0$, $|\alpha| > 1$, *then*

$$(\alpha a_1, b_1) \prec (\alpha a_2, b_2) \quad \textit{iff} \quad (\alpha a_1, b_2) \prec (\alpha a_2, b_1).$$

For risky choice, with attributes A and B standing for probability and money (P and X), this is inconsistent with the independence axiom, which states that if a given lottery is indifferent to a second lottery, then indifference should be preserved if both lotteries are "mixed" with a common third lottery. In the domain of single-outcome lotteries, the only possible mixtures are those involving the lottery and the certain zero outcome. Restricted to this domain, the independence axiom may be expressed as

A6*. PROPORTIONALITY. *If* $(x, p) \sim (y, q)$, *then for any* $0 < \alpha < 1$, $(x, \alpha p) \sim (y, \alpha q)$, which yields a numerical representation of the form $p^c u(x)$, where c is an arbitrary positive constant.[5]

Increasing proportional sensitivity is a generalization of the subproportionality condition, proposed by Kahneman and Tversky (1979), to the more general setting of non-separable preferences (see further discussion below). For unbounded dimensions, like time and money, IPS will clearly fail at sufficiently large levels: Doubling an award of 10 billion dollars clearly matters less than doubling an award of $500,000; likewise, the difference between losing one or two million dollars is less important than that between losing one or two thousand, to most people at least.[6]

Our final condition states that losses (outcomes below a zero reference point) are treated just like somewhat larger gains.

A7. LOSS AMPLIFICATION. *The two operations* (i) *increasing the absolute magnitude of all values along an attribute by a common multiplicative constant, and* (ii) *changing the polarity of an attribute, from positive to negative, have the same effect on the weight of that attribute.*[7] *Likewise, changing the polarity from negative to positive will have the same effect as shrinking values by a common multiplicative constant.*

In conjunction with the IPS condition, loss amplification implies that changing values from gains to losses of equal absolute magnitude increases the weight of the money dimension. By yoking together changes in polarity with scale changes, we explicitly avoid the claim that losses always have greater weight. We are not convinced that this stronger condition is true for very large amounts, in that the perceived differences between two huge losses may in fact be less salient than the difference between two equivalent gains. Indeed, A7 implies that if increasing proportional sensitivity were violated at some level of an attribute, then changing the sign of the attribute at that level would not increase the weighting of that attribute.

5. Restrictions on Separable Models

Previous work on descriptive models of risky and intertemporal choice has often concentrated on separable representations, which arise when preferences satisfy the double cancellation condition (Krantz, Luce, Suppes and Tversky 1971):

A8. DOUBLE CANCELLATION. *If* $(x, a) \sim (y, b)$ *and* $(y, c) \sim (z, a)$, *then* $(x, c) \sim (z, b)$.

For example, A1, A2, A3 and A8 imply that preferences can be represented by the multiplicative formula $f(a)g(b)$ (Fishburn and Rubinstein 1982, Theorem 3). The prospect theory model of risky choice, usually written as $\pi(p)v(x)$, has this form, as do the intertemporal models of Fishburn and Rubinstein (1982) and Loewenstein and Prelec (1989). How do our three conditions (A5, A6 and A7) restrict the shape of the component functions, f and g? Considering decreasing absolute sensitivity first, we see that in the context of a separable representation, $f(a_1)g(a_1) = f(a_2)g(a_2)$ implies

$$f(a_1 + \epsilon)g(a_1) < f(a_2 + \epsilon)g(a_2) \qquad \text{iff} \qquad f(a_1 + \epsilon)g(a_2) > f(a_2 + \epsilon)g(a_1).$$

After eliminating the ratio, $g(a_1)/g(a_2)$, from the three expressions, DAS may be stated as

[5] To derive the representation, apply the transformation, $p \rightarrow \ln(1/p)$, and invoke the Fishburn-Rubinstein axioms, with $\ln(1/p)$ being interpreted as time. A proof of this result for the discrete time model is given in Harvey (1986).

[6] We thank Amos Tversky for bringing this problem with IPS to our attention.

[7] Loss amplification only applies to the money attribute since the time and probability attributes cannot be reversed in sign.

$$f(a_1+\epsilon)/f(a_2+\epsilon) < f(a_1)/f(a_2) \quad \text{iff} \quad f(a_1+\epsilon)/f(a_2+\epsilon) > f(a_2)/f(a_1),$$

or more compactly as

$$\frac{f(a_2)}{f(a_1)} \lesseqgtr \frac{f(a_1+\epsilon)}{f(a_2+\epsilon)} \lesseqgtr \frac{f(a_1)}{f(a_2)} \qquad (\text{if } \epsilon a_1 > 0).$$

Increasing the absolute magnitude of a_1 and a_2 by a common constant moves the ratio, $f(a_1)/f(a_2)$, closer to one (i.e., it increases that ratio if $f(a_1) < f(a_2)$, and decreases it if $f(a_1) > f(a_2)$). Equivalently, this means that the slope of the function $\log|f(a)|$ becomes less and less steep as one moves further away from the $a = 0$ point, in either the positive or negative direction.

The same argument applied to the increasing proportional sensitivity condition shows that

$$\frac{f(a_2)}{f(a_1)} \lesseqgtr \frac{f(a_1/\alpha)}{f(a_2/\alpha)} \lesseqgtr \frac{f(a_1)}{f(a_2)} \qquad (\text{if } \alpha a_1 > 0 \text{ and } |\alpha| > 1).$$

A reduction of the absolute magnitude of a_1 and a_2 by a common proportional constant moves the ratio, $f(a_1)/f(a_2)$, closer to one. The implication is that the slope of $\log|f(a)|$ becomes shallower as $\log|a|$ *decreases*, or, equivalently, that $\log|f(a)|$ is a convex function of $\log|a|$. This is *subproportionality*, as Kahneman and Tversky (1979) defined it for the probability weighting function.

Finally, loss amplification, together with IPS, means that the scaled ratio of two positive values will be closer to one than the scaled ratio of two negative values of the same absolute magnitude:

$$\frac{f(-|a_1|)}{f(-|a_2|)} > \frac{f(|a_1|)}{f(|a_2|)} \qquad \text{if} \qquad |a_1| > |a_2|.$$

Equivalently, this means that $\log|f(a)|$ is a steeper function of $\log|x|$ for negative than for positive values of a.

This property is distinct from "loss aversion"—the requirement that $f(a)$ is steeper in the negative domain, which has been postulated to hold for decision making under uncertainty (Kahneman and Tversky 1979). The behavioral implications of loss aversion are that a gain cannot compensate for a loss of equal absolute magnitude (an even gamble to win or lose some amount should be rejected, for example). Sign-sensitivity, on the other hand, does not compare gains against losses directly; rather, it compares the probability-money tradeoff across the two domains, of gains and losses; in that sense, it identifies a second-order gain-loss asymmetry.[8]

Although only one model of intertemporal choice incorporates a value function that satisfied the three attribute sensitivity properties (Loewenstein and Prelec forthcoming), many models have been proposed that include probability weighting or discount functions that embody some of the properties discussed above.

Prelec (1989) has proposed a two-parameter family of discount functions, that spans the range from the exponential to the power function—i.e., the range between constant absolute and constant proportional sensitivity. Such a family of functions is generated by the following axiom:

[8] The influence of reference points on the shape of indifference curves has been given a comprehensive treatment in a recent paper by Tversky and Kahneman (1989). By *loss aversion*, they refer to a local distortion in a person's global preference structure, produced by a reference, or "status quo" point. *Constant* loss aversion, a special case, occurs when the distortion can be expressed by a simple proportional rescaling of values below and above the reference point on each dimension. In other words, the loss of x units along a dimension is assessed as if it were a shift in the negative direction of αx units along that same dimension (with α being a constant specific to that dimension, but always greater than one).

A8. SECOND-ORDER STATIONARITY. *If* $(x, t) \sim (y, s)$ *and* $(x, t') \sim (y, s')$, *then for any* $0 \leq \epsilon \leq 1$,

$$(x, \epsilon t + (1 - \epsilon)t') \sim (y, \epsilon s + (1 - \epsilon)s').$$

In other words, if the two intervals, $t - s$ and $t' - s'$, are subjectively equivalent, then they will also be equivalent to any weighted average. Stationary time preferences are included here, since the premise in the axiom will only hold when the two intervals are equal: $t - s = t' - s'$. Included also are so-called relative timing preferences (Harvey 1986), for which the initial premise will hold if the ratios, t/s and t'/s', are equal. The most general discount function consistent with A8 is the general hyperbola (Prelec 1989, Theorem 5)

$$f(t) = (1 + \alpha t)^{-\beta/\alpha},$$

which reduces to the exponential function as the α-parameter goes to zero and the power function as α becomes infinitely large. A special case of this function was proposed by Mazur (1987, with $\alpha = \beta$), to account for animal preferences over delayed rewards.

The limiting case where time preferences are invariant under proportional changes in time intervals has been studied by Harvey (1986), in the context of discrete time. Harvey referred to this property as *relative timing preferences*, and proved that the discount function must be a decreasing power function,

$$f(t) = t^{\beta}, \qquad \beta < 0.$$

Harvey pointed out that if time relative to the present is split into some constant delay s, and an additional delay t', then the power discount function becomes

$$f(t) = (s + t')^{\beta}, \qquad t' = t - s,$$

which is formally equivalent to the hyperbolic form stated above. Harvey referred to s as a *time lag*, which might correspond, for example, to a period during which no contemplated policy alternative can have an effect on society.

Many probability weighting functions also satisfy DAS and IRS. For example, as noted earlier, the probability weighting function in Kahneman and Tversky's Prospect Theory satisfies subproportionality. Similarly, Karmarkar's (1978, 1979) probability weighting function

$$\pi(p) = \frac{p^s}{p^s + (1 - p)^s},$$

has the requisite s-shape, for $s < 1$, though it is not subproportional over the full probability range. Rachlin, Raineri and Cross (1989) proposed the form

$$\pi(p) = \frac{p}{p + k(1 - p)},$$

which is subproportional everywhere, for $k > 1$, but which does not overweigh small probabilities. Prelec (1990) derived a log-hyperbolic form

$$\pi(p) = (1 - \alpha \log \{p\})^{-\alpha/\beta},$$

from a "compounding" axiom, which states that the pair of indifferences, $(x, p) \sim (y, q)$ and $(x, rp) \sim (y, sq)$ imply $(x, r^c p) \sim (y, s^c q)$, for any $c > 1$. The common ratio effect, in other words, can be replicated at lower probability levels, by compounding the reduction of the two probabilities (p and q) at different rates (r and s). The function is subproportional everywhere, and traces out the s-shape predicted by Kahneman and Tversky.

6. Predictions of the Model

Single outcome prospects (temporal and risky) provide four separate attributes on which to test the predictions of the model. Only the money dimension can be reversed in sign, however, so that the three conditions identified in the previous section yield a total of 10 predictions of choice behavior, which are summarized in Table 2. We consider first the predictions for temporal prospects, shown in the two right-hand columns of the table.

Intertemporal Choice

The common-difference effect—perhaps the fundamental intertemporal choice anomaly—illustrates decreasing absolute sensitivity for the time dimension. Adding a constant to both delays diminishes the importance of time, thus shifting preference in favor of the prospect with the larger money outcome. The increasing proportional sensitivity condition has not been systematically tested, but we have little reason to doubt that it holds: For example, if one is indifferent between a large prize in two years time and a smaller prize in one year, then one is likely to prefer the larger prize when months, or days are substituted for years in the comparison.

Turning to the money attribute, decreasing absolute sensitivity has again not been tested, perhaps because it follows from utility function concavity, which is widely assumed. For example, a person who is indifferent between $1000 now and $2000 two years from now, is likely to prefer $2000 now over $3000 at the later date. Increasing relative sensitivity corresponds to the magnitude effect discussed in §3, and has been tested extensively. While robust in the range of small to intermediate dollar outcome, as noted above, it is likely that the effect will reverse at the very high end: A person indifferent between $1000 now and $2000 three years from now, may well prefer ten million now to twenty in three years (assuming again that he cannot draw on the funds now).

The third condition, loss amplification, is consistent with the observed lower discounting of losses than gains discussed in §3. Consider a person who is indifferent between $20 in one year or $10 now. According to increasing relative sensitivity, the same person would prefer $200 in one year to $100 now, since magnifying both dollar outcomes by a common constant increases the weight placed on the money attribute. If, instead, we change the sign to create a choice between −$20 in one year or −$10 now, loss amplification states that it should have the same effect as increasing the outcome magnitudes—i.e., to increase the weight on the dollar attribute relative to the time delay attribute. Thus, following a sign shift, preference should shift in favor of the earlier smaller loss. It should be noted that loss amplification does not preclude the operation of other reasons for underdiscounting losses, such as "wanting to get it over with," dread, etc. (Loewenstein 1987).

Decision Making Under Uncertainty

Increasing sensitivity on the probability dimension is illustrated by the common-ratio effect (Kahneman and Tversky 1979). *Shrinking* (multiplicatively) the nonzero prob-

TABLE 2

Model Predictions

	Decision Making Under Uncertainty		Intertemporal Choice	
Choice Properties	Probability	Money	Time	Money
Decreasing Absolute Sensitivity	Plausible within a limited range	Follows from risk aversion	Common difference effect	Follows from *U* fn. concavity
Increasing Relative Sensitivity	Common ratio effect	Implausible	Unverified but plausible	Magnitude effect
Loss Amplification		Reflection effect		Sign effect

abilities in two risky prospects by a common factor decreases the perceived difference between the probabilities, which, in accordance with IPS, reduces the weight of the probability dimension. This causes preference to shift in favor of the lower-probability higher-money outcome prospect.

The situation with the decreasing absolute sensitivity condition is a bit more ambiguous. For small probabilities, the condition is likely to hold. For example, a person who is indifferent between, say, ($1000; 0.1) and ($2000; 0.05), will most certainly prefer ($2000; 0.5) to ($1000, 0.55). Indeed, risk-aversion or risk neutrality are sufficient to ensure DAS, in the context of expected utility theory. However, if we make the probabilities very high, then the condition is no longer self-evident. A person may very well be indifferent between an 80% chance at $2 million and a 90% chance at $1 million, and at the same time prefer a 99% chance at a million to a 89% chance of two million.

Possibly, the violation of DAS suggested by this example is due to an intrinsic ambiguity in how probabilities are framed. With the almost certain windfall in the example, one is inclined to think in terms of the probabilities of not winning, rather than winning the prize. In that frame, the relative impact of 11% vs. 1% is greater than the relative impact of 20% vs. 10%.

Turning to the money attribute, decreasing absolute sensitivity is commensurate with risk aversion and seems perfectly compelling. Adding a constant to the outcomes of risky alternatives increases the relative weighting of probability, shifting preferences in favor of the smaller, safer, prospect. A person who is indifferent, say, between $100 for sure and a 50% chance at $300, will surely prefer $200 to a 50% chance at $400, or $500 to a 50% chance at $700. The opposite preference pattern would go against robust expected value increases, and would only be justified by extreme risk-proneness at the higher dollar amounts. In the context of prospect theory, for example, DAS is satisfied provided that the logarithm of the value function is concave in money (which is a weaker requirement than the traditionally assumed concavity of the value function itself).

Unfortunately for our model, risky choice typically does *not* conform to increasing proportional sensitivity. Magnifying the "stakes" of risky prospects increases the importance of the probability dimension, resulting in more risk-averse behavior. What can account for this one exception? We believe that the discrepancy results from interactions between attributes, which are different for risky and intertemporal choice.

In decision making under uncertainty probabilistic outcomes are mutually exclusive—one and only one outcome occurs. When evaluating a gamble in isolation, all possible comparisons take place between the outcome that occurs and those that do not. Furthermore, the effect of such comparisons is generally taken to be contrastive. As a result of disappointment (Bell 1985), losing a gamble is more painful the greater the potential gain. Likewise, the utility of winning is heightened by the elation that results from pondering the negative outcome that could have occurred, although the disappointment effect is generally assumed to dominate the elation effect (Bell 1985). We believe that anticipation of disappointment is responsible for the violation of increasing proportional sensitivity for risky choice.

Consider what happens as one magnifies the stakes of a simple prospect consisting, say, of a 0.5 chance at $100 and a 0.5 chance at $0. Although the gamble increases in expected value, its subjective value does not increase commensurately even after allowing for risk aversion. This is because, due to disappointment, the zero outcome becomes progressively worse in utility terms as the positive outcome increases. Disappointment and elation also have the combined effect of amplifying the utility difference between the positive and negative outcomes as the positive outcome is improved. Thus, as the value of the positive outcome increases, the decision maker becomes increasingly preoccupied with the probability of obtaining the better of the two outcomes, increasing the weighting of the probability dimension. The probability dimension becomes relatively more important because, as the gamble is magnified, its overall value does not increase commensurately but the decision maker's concern with obtaining the more desirable outcome increases.

Interaction effects in intertemporal choice are different from those in decision making under uncertainty, and the difference can explain why increasing proportional sensitivity is generally observed for temporal prospects but not for risky ones. For simple, one-outcome prospects, the most important interaction effect is the "endowment" effect (Elster 1985, Tversky and Griffin 1990). People derive positive utility from "savoring" a desirable outcome or from dreading an undesirable one (Loewenstein 1987).

Magnifying the outcome in a simple intertemporal prospect enhances utility in every period, due to savoring (and possibly memory) so that overall utility increases even more than what would be predicted on the basis of simple discounted utility. Note that this is precisely the opposite of what occurs in decision making under uncertainty, where the dominant interaction effect—disappointment—attenuates the gamble's increase in desirability as the value of the positive outcome is magnified. On the other hand, magnifying the value of the outcome in a simple intertemporal prospect does not have a strong impact on the importance of time delay. Although one might be more concerned about experiencing a highly desirable outcome earlier, possibilities for savoring, and hence willingness to delay may also increase as the delayed outcome improves. Again, this is in marked contrast to the case of decision making under uncertainty where magnifying the magnitude of the positive outcome increases preoccupation with the probability of obtaining it.

Loss amplification does seem plausible when applied to decision making under uncertainty, though it has not been tested systematically. For example, a person who is indifferent between $10 or a 50% chance at $20 would probably prefer $100 to a 50% chance at $200, reflecting an increase in the weight placed on probability when outcomes are magnified. If changing the sign of the money dimension from positive to negative has the same impact on attribute weighting as does magnifying outcomes, then we would expect the weight on probability to be increased if we changed the signs of the payoffs in the original prospects. Loss amplification therefore predicts that the same decision maker would prefer a 50% chance of losing $20 to a sure loss of $10, a plausible pattern of choice.

7. Discussion

In this paper we have outlined several significant parallels between intertemporal choice and decision making under uncertainty. Actual patterns of behavior in both domains are surprisingly similar, as revealed by the parallelism between the DU and EU *anomalies* discussed in §3.[9] As a result, with the sole exception of the weighing of value in decision making under uncertainty, behavior in both domains can be expressed in terms of three

[9] Our argument for a close correspondence between patterns of choice in the domain of time and uncertainty is bolstered by recent research demonstrating that preference reversals analogous to those observed in decision making under uncertainty also occur in intertemporal choice (Tversky, Slovic and Kahneman 1990). There are further parallels that we do not explore here. Recently a number of models of preferences over multiple outcome lotteries have been developed in which a nonlinear transformation is applied to the cumulative probability dimension. In these *rank order* models extreme outcomes are weighted disproportionately (e.g., Quiggin 1982, Yaari 1987). We have found a similar pattern with preferences toward sequences of outcomes; the outcomes that appear at the beginning and end of the sequence receive more weight than called for by conventional discounting schemes (Loewenstein and Prelec 1990, 1991).

simple principles laid out in §4: increasing relative and decreasing absolute sensitivity, and loss amplification.

The discounted and expected utility models both make extreme assumptions about the sensitivity of preferences to transformations of the nonvalue attribute; in both cases actual behavior lies between these extremes. At the same time both theories are relatively agnostic about how transformations of the value attribute will affect preference—yet it is possible to specify such effects with some precision.

Our analysis reiterates the importance, already evident in research on decision making under uncertainty, of preferential interactions. The most common practice in attempts to produce alternative models to EU has been to modify EU to take into account various violations of independence. Our analysis suggests however that EU and DU may provide skewed foundations for such efforts; the pattern of choice described by intermediate sensitivity appears to provide a superior base on which to add interaction effects.

In its conclusion that preferences are sensitive both to absolute and relative transformations of attributes, our findings are related to several earlier contributions. MacCrimmon and Messick (1976), Lurie (1987), and Loewenstein, Thompson and Bazerman (1989) have demonstrated that interpersonal comparisons of material outcomes reflect a concern both for absolute and relative differences between one's own and other persons' payoffs. Even more closely related is Harvey's (1988) discussion of two approaches to discounting in public policy which he terms absolute and relative timing preference. These are equivalent to what we would call constant absolute and constant relative sensitivity for time.

Our conclusion that patterns of weighting are relatively similar for different attributes is also reminiscent of earlier work on decision making under uncertainty which has treated the probability and time dimensions symmetrically—as diverse attributes are typically handled in generic multiattribute choice problems (e.g., Slovic and Lichtenstein 1968, Payne and Braunstein 1971). Most existing choice models, including EU and DU, assume that qualitatively different patterns of weighting apply to money and nonmoney (probability or time) attributes. However, such a discrepancy in treatment is not evident from actual choice behavior. Rather than employing separate rules for different choice attributes, people seem to apply certain fundamental and relatively simple rules to evaluating different types of attributes.

An interesting question is whether the similarity between intertemporal choice and decision making under uncertainty applies more broadly to other types of multi-attribute choice. Time and uncertainty are closely related in a number of ways so it is possible that the observed parallelism of choice behavior is unique to these two domains. Both are attributes that pertain to choice objects' delivery—its time or probability of occurring—rather than to characteristics of the objects themselves. Furthermore, time and uncertainty are typically correlated with one another in the real world, perhaps also contributing to their comparable treatment. Anything that is delayed is almost by definition uncertain. And since uncertainty takes time to resolve, uncertain outcomes are also typically delayed. Indeed some researchers have argued that the two attributes really measure the same thing, although there is disagreement regarding which is the more fundamental (Rachlin, Raineri and Cross 1989, Benzion, Rapoport and Yagil 1989). All of these connections raise the possibility that the similarity of choice behavior is peculiar to time and uncertainty—that it reflects their intimate connection rather than fundamental processes underlying all multiattribute choices. The answer to this question awaits further research.[10]

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