Egon Balas: Pioneer in Integer Programming
(June 7, 1922 - March 18, 2019)

by Gérard Cornuéjols[1]

Egon Balas' life reads like an adventure tale. Born in Cluj, Romania, in 1922 into a Hungarian-Jewish family, he was involved in underground activities during World War II, hiding, captured, tortured, and escaped. After the war he became a Romanian diplomat in London, and held other high positions in post-war Communist Romania. Then it was prison again, repeated interrogation by the dreaded Securitate, solitary confinement for more than two years, release, and expulsion from the Communist party. He only started a career in Mathematics at age 37 in the late 1950s [4].

At 37, Egon Balas joined the Forestry Institute in Bucharest, which planned and scheduled timber harvest in Romania. To develop appropriate logistics tools, Egon Balas became an autodidact, studying mathematics and operations research on his own from books he could get his hands on. Peter Hammer, who was to become a prominent Operations Researcher himself, was also working at the Forestry Institute at the time. Together, they developed tools for transportation planning based on network flow theory and linear programming. They published a dozen papers together (Peter Hammer was using the name Ivanescu at the time) mostly in Romanian.

In 1962 Egon Balas confronted an interesting variation on the wood harvesting problem: In one area of the forest, a network of roads had to be built in order to access various plots. Decisions about which plots to harvest and where to build roads were intricately connected. It involved logical decisions: If road segment A were built, then road segment B would also have to be built to reach A. Egon Balas formulated the problem as a linear program in 0,1 variables, recognizing the versatility of this model for a wide range of applications.

There was no computer code to solve 0,1 programs at the time, so Egon Balas designed his own algorithm, simple enough that instances with about 30 binary variables could be solved by hand. The algorithm performed an implicit enumeration of the solution set, relied on a series of logical tests that explored the implications of fixing certain variables to 0 or to 1, and the only operations it needed were additions and comparisons. Egon Balas' algorithm can be viewed as a precursor of constraint propagation in Constraint Programming.

The work was presented to the research community in the West at the International Symposium in Mathematical Programming in London in 1964. The Romanian government would not allow Egon Balas to travel to England to present his paper, so his talk was read by a colleague at the conference. This was one of the many constraints that lead to Egon Balas' disillusionment with Communist Romania.

The same year a short version of his paper was published in *Comptes Rendus de l'Académie des Sciences*, Paris, France. The full-length paper "An Additive Algorithm for Solving Linear Programs with Zero-One Variables" appeared in *Operations Research* in 1965 [1]. It was extremely influential in the development of integer programming, establishing implicit enumeration and branch-and-bound as a simple and powerful solution methodology. In fact, this paper was identified as the most cited paper in Operations Research in the early 80s (Citation Classic in *Current Contents 1982*).

[1]Tepper School of Business, Carnegie Mellon University, Pittsburgh PA 15213. gc0v@andrew.cmu.edu

By the early 1960s, the oppressive conditions and lack of freedom in Romania had become intolerable to Egon Balas and he applied to emigrate. His application was denied. After several further rejected applications, each time with increased hardship on the family, he and his wife and two daughters were granted permission to emigrate in 1966. By that time, Egon Balas already had visibility in the West thanks to his work on the additive algorithm. He first went to Rome, Italy, where he spent six months as a Research Fellow at the International Computation Centre, headed by Claude Berge. During this period, Egon Balas also managed to earn two doctorates, one in Economics from the University of Brussels, and the other in Mathematics from the University of Paris. The next year, in 1967, Egon Balas was offered a professorship at Carnegie Mellon University. Bill Cooper, one of founders of the Graduate School of Industrial Administration, and a faculty member there, was key in this brilliant recruitment decision. He was very familiar with Egon Balas' recent research accomplishments as he had been the Associate Editor of *Operations Research* in charge of handling Egon Balas' paper on the additive algorithm. Egon Balas was forever grateful to Carnegie Mellon for the stability that this position provided to his family.

Egon Balas' most significant contribution is undoubtedly his extensive work on disjunctive programming, starting with intersection cuts (*Operations Research 1971*). These ideas were novel and the Operations Research community was slow to accept them.

Cutting planes had been introduced by Dantzig, Fulkerson and Johnson (1954) and Gomory (1958). But by the late 60s and throughout the 70s and 80s, the general sentiment regarding the practicality of general cutting planes had become rather negative, in great part due to the success of branch-and-bound algorithms such as the additive algorithm.

Egon Balas understood that enumeration was inherently exponential in complexity and that it could only tackle small or medium-size instances. He felt that the real potential was in convexifying the feasible set, potentially reducing an integer program to a simpler linear program. He also felt that theory was lacking. Using tools of convex analysis, he showed how to derive rich families of cutting planes from any feasible basis of a linear relaxation, and any convex set S whose interior contains the basic solution but no feasible integer point. These cuts are Balas' intersection cuts. When the convex set is the region between two parallel hyperplanes, one recovers Gomory's mixed integer cut as a special case. Egon Balas observed that intersection cuts derived from polyhedral sets S can be understood using an alternate point of view: if the polyhedron S is defined by linear inequalities, then the requirement that no feasible point is in the interior of S can be described through a disjunction of the reverse inequalities. The feasible region can then be regarded as a union of polyhedra.

This new viewpoint gives rise to important generalizations. It motivated Egon Balas to introduce disjunctive programming, defined as optimization over a union of polyhedra. He proved two fundamental results on disjunctive programs that have far-reaching consequences for the solution of linear 0,1 programs. First, there is a compact representation of the convex hull of the union of polyhedra in a higher-dimensional space. Projecting back onto the original space gives a full description of the convex hull. As a result, one can compute a deepest disjunctive cut by solving a linear program. The number of variables and constraints in the higher-dimensional representation only grows linearly in the number of polyhedra, which makes this a practical tool when the number of disjunctions is not too large. Second, for a large class of disjunctive programs, called facial, the convexification process described above can be performed sequentially. For example, 0,1 programs are facial disjunctive programs. This means that if there are n 0,1 variables, the convex hull of the solution set can be obtained in n steps, imposing the 0,1 conditions one at a time. This distinguishes 0,1 programs from more general integer linear programs. These theorems were proved in a technical report [3] in 1974. Unfortunately, the significance of the results was not recognized by the referees at the time and the paper was rejected for publication. Twenty-four years later the importance of Egon Balas' pioneering work had finally become clear, and the technical report was eventually published as an invited paper in 1998.

In the Spring of 1988, Laci Lovász gave a beautiful talk at Oberwolfach about his on-going work with Lex Schrijver on cones of matrices. Sebastian Ceria, who was just starting his PhD at Carnegie Mellon, and I decided to investigate what would happen if one performed the Lovász-Schrijver operation sequentially, one variable at a time, with the idea of making it more practical for implementation. We were very excited to realize that, as in the full Lovász-Schrijver procedure, our simplified lift-and-project procedure still generated the convex hull in n steps for a problem with n 0,1 variables. When we showed this result to Egon, his reaction was immediate: "There is nothing new under the sun. This is the sequential convexification theorem!" Egon was right of course. There was a nice connection between our streamlined version of the Lovász-Schrijver procedure and disjunctive programming.

This connection was very fruitful, providing a perfect framework for cut generation. Sebastian, Egon and I had much fun collaborating on this project [6]. We developed the solver MIPO (Mixed Integer Program Optimizer) which incorporated lift-and-project cuts within a branch-and-cut framework. The success of lift-and-project cuts motivated us to try other general-purpose cuts, such as the Gomory mixed integer cuts. These cuts had a bad reputation, with repeated claims in the literature that they have a poor performance in practice. So we were very surprised to discover that they also worked very well [7]. The sentiment about general cutting planes changed overnight in the Integer Programming community. By the late 1990s, all commercial solvers for mixed integer linear programs were using a whole battery of general-purpose cuts, resulting in a very significant improvement in the size of instances that could be solved optimally.

Egon Balas' research contributions span a broad range of topics. On of subject of disjunctive cuts itself, I left out several important aspects, such as monoidal cut strengthening (Balas and Jeroslow [8]), the efficient generation of the cuts (Balas and Perregaard [9]), and other recent directions such as generalized intersection cuts. At age 96, Egon Balas wrote his first (and only) textbook *Disjunctive Programming* [5] presenting the advances made in this area over nearly five decades.

Egon Balas also made noteworthy research contributions on the knapsack and set-covering problems, and in the area of scheduling: machine scheduling via disjunctive graphs, the shifting bottleneck procedure for job shop scheduling (with Joe Adams and Dan Zawack), choosing the overall size of the US strategic petroleum reserve, the prize-collecting traveling salesman problem, an application to scheduling rolling mills in the steel industry (with Red Martin), and several others.

Egon Balas' contributions have been recognized through numerous prizes and honors.

- Prizes
  - 1995: John von Neumann Theory Prize of INFORMS,
  - 2001: EURO Gold Medal of the European Association of Operational Research Societies,
  - 2017: Larnder Prize of the Canadian Operations Research Society.
- Honors

- 2002: elected an INFORMS Fellow,
- 2004: elected an external member of the Hungarian Academy of Sciences,
- 2006: inducted into the IFORS Hall of Fame,
- 2006: elected a member of the National Academy of Engineering,
- 2011: elected a corresponding member of the Academy of Sciences of Bologna, Italy,
- 2016: elected a SIAM Fellow.

• Honorary doctorates

- 2002: University of Elche, Spain,
- 2005: University of Waterloo, Canada,
- 2008: University of Liege, Belgium.

Egon Balas was a good friend and colleague, as well as a great tennis partner. He was still a formidable player in December 2018 a few months before his death.

References

[1] E. Balas, An Additive Algorithm for Solving Linear Programs with Zero-one Variables, *Operations Research* 13 (1965) 517-546.

[2] E. Balas, Intersection Cuts – A New Type of Cutting Planes for Integer Programming, *Operations Research* 19 (1971) 19-39.

[3] E. Balas, Disjunctive Programming: Properties of the Convex Hull of Feasible Points. MSRR No. 348, Carnegie Mellon University, July 1974. Published as an invited paper in *Discrete Applied Mathematics* 89 (1998) 3-44.

[4] E. Balas, *Will to Freedom: A Perilous Journey Through Fascism and Communism*, Syracuse University Press (2000) 496p.

[5] E. Balas, *Disjunctive Programming*, Springer (2018) 243p.

[6] E. Balas, S. Ceria, G. Cornuéjols, A Lift-and-Project Cutting Plane Algorithm for Mixed 0-1 Programs, *Mathematical Programming* 58 (1993) 295-324.

[7] E. Balas, S. Ceria, G. Cornuéjols, N. Natraj, Gomory Cuts Revisited, *Operations Research Letters* 19 (1996) 1-9.

[8] E. Balas, R.G. Jeroslow, Strengthening Cuts for Mixed Integer Programs, *European Journal of Operations Research* 4 (1980) 224-234.

[9] E. Balas, M. Perregaard, A Precise Correspondence Between Lift-and-Project Cuts, Simple Disjunctive Cuts, and Mixed Integer Gomory Cuts for 0-1 Programming, *Mathematical Programming B* 94 (2003) 221-245.