Playback Delay in On-Demand Streaming Communication with Feedback

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***Abstract*—We consider a streaming communication system where the source packets must be played back sequentially at the destination and study the associated average playback delay. We assume that all the source packets are available before the start of transmission at the transmitter and consider the case of an i.i.d. erasure channel with perfect feedback. We first consider the case when the receiver buffer can be arbitrarily large, and show that the average playback delay remains bounded in the length of the stream provided that the channel bandwidth is greater than a critical threshold. Our analysis involves the application of martingale theory to study the transient behaviour of a one dimensional random walk with drift. Conversely when the channel bandwidth is smaller than the above threshold, the average playback delay increases linearly with the stream length. We also consider the finite buffer case and analyse the playback delay of a greedy dynamic bandwidth scheme. We further show through simulations that the achievable delay with a finite receiver buffer is close to the infinite buffer case for moderately large buffer values.**

I. Introduction

In streaming communication, a sequence of source packets must be delivered to the destination in-order and under strict delay constraints. Unlike classical block transmission, the study of fundamental limits of streaming communication remains a fertile area of research. In this paper we are interested in a point-to-point streaming setup when the entire stream is available at the source at the start of the communication. The source packets are labelled sequentially and must be played in the same order. In each time step, only one packet can be played, and the receiver is subject to a playback interruption until the current packet becomes available. We consider an i.i.d. packet erasure channel with ideal feedback and study the achievable delay.

In [1] a similar setup is considered for real-time sources where source packets are revealed to the transmitter in a causal fashion. A delay metric called the *total playback delay* is introduced (see also [2]) and shown to increase logarithmically with the length of the stream, with or without feedback, when the channel bandwidth is larger than a certain critical threshold. In contrast in the present paper we show that in the same setup, the expected playback delay remains bounded when all the source packets are available non-causally at the transmitter. Intuitively this gain arises from the fact that in the non-causal setting, the receiver can fill its buffer with as many packets as the channel allows the transmitter to send successfully, without any limitations due to the unavailability of new packets. This induces a positive drift on the buffer size, and the expected time to get back to the empty buffer state tends to infinity. Our formal analysis is based on the martingale theory and involves analysis of the expected time spent in the transient state of a one dimensional random walk with drift. We also consider the case when the buffer at the receiver is finite, and show via simulations that the achievable delay approaches the infinite buffer case for moderately large buffer values. In contrast the analysis technique in [1] is very different. In the real-time setup the receiver experiences a sequence of renewal processes. Using the Generalized Ballot theorem [3] the probability distribution of the length of renewal processes is derived and it is shown that the introduced delay metric always grows logarithmically with the length of the stream. In related works, broadcast extensions have been studied in [4], [5], while streaming of causal sources in bursty adversarial channels and without feedback has been studied in [6].

II. Problem Setup

The source consists of a stream of k information packets, $s_1, \cdots, s_k$, to be transmitted to the destination. Each source packet is of unit size. Throughout this work we will interchangeably refer to the order of the packets with their age, as if they have been created with that order, i.e. the packet s_j will be said to be older than the packet $s_{i+j},\ i>0$. The transmitter transmits encoded packets x_i at time step $i \geq 1$, based on a transmission scheme known by the receiver. Each encoded packet is of size B for some integer[1] $B>0$ and packet x_i is transmitted at time step i over the channel. The link between the source and the receiver is assumed to be an i.i.d. packet erasure channel. We will denote the probability of erasure in the channel by ϵ. Hence, the receiver will receive y_i in time step i which is equal to x_i with probability $1-\epsilon$ or is an erasure indicator with probability ϵ independently for all $i \geq 1$. We assume that the transmitter will receive an instantaneous and error-free feedback message about the transmitted packets. As a result, the transmitter produces packet x_i using an encoder function f_i as $x_i = f_i(s_1, \cdots, s_k, y_1, \cdots, y_{i-1}),\ i>0$.

The receiver-end application plays the decoded packets strictly in-order, at the rate of one packet per time step. We

[1] Although we consider the integer case for simplicity in this work the results are extendible to the case of non-integer B as well.

assume that all packets decoded until time step i are available for playback in the same time step. At the receiver side, correctly received packets will be collected and the receiver uses recovery functions $\hat{s}_{j,i} = g_{j,i}(y_1, \cdots, y_i)$ to recover the information packet s_j at time step i, which has not been recovered before that time step. We assume $\hat{s}_{j,i}$ is either equal to s_j or is equal to a failure symbol.

Since the playback is strictly in-order, any out-of-order decoded packets are added to a playback buffer. Let the buffer size be m. If the number of packets that are decoded but not played exceeds m, the extra packets are dropped and marked erased in the feedback sent to the source. We will denote the first time step a specific source packet s_j is correctly decoded at the receiver and used or saved in the buffer by t_j. We denote the time step at which a source packet s_j is used at the receiver by d_j. Therefore, for the first source packet s_1 we have $d_1 = t_1$, while for any other source packet $s_j,\ j > 1$ we have $d_j = \max\{d_{j-1} + 1, t_j\}$. In Section III we first consider infinite buffer size m, and study the general case of finite buffer in Section IV.

Definition 1 (Total Playback Delay)**.** *Assuming that the receiver uses the last information packet at time step t_k we will refer to the quantity $D_k = d_k - k$, as the total playback delay for the stream.*

Remark 1. *Note that for the ideal channel case, clearly $d_j = j$ for $j \in \{1, \cdots, k\}$, and therefore we must have that $d_j \geq j$ in general. The difference $d_j - j$ represents the delay at the receiver for using source packet s_j compared to the ideal playback. Moreover, since $d_j = \max\{d_{j-1} + 1, t_j\}$, then d_j is indeed a non-decreasing function of the source packet index. D_k then is referring to the maximum of the individual packet delays in the stream consisting of k packets.*

Remark 2. *Having instantaneous and error-free feedback available at the transmitter and only one receiver, it is easy to see that the simple ARQ scheme which transmits the oldest B packets at every time step is the optimal strategy in terms of reducing the total playback delay D_k. Hence, throughout this work we limit our discussion to this transmission strategy and its dynamic bandwidth usage variations.*

III. Bandwidth-Delay trade-off with infinite buffer

In this section we assume that the receiver buffer is infinite, while the finite buffer case will be studied in section IV. Our main result is summarized below.

Theorem 1. *If $B(1-\epsilon) > 1$, then the expected total playback delay, $\mathbb{E}[D_k]$ for a stream of length k, is upper bounded by a constant independent of k. Moreover, if $B(1-\epsilon) < 1$, then the expected total playback delay, $\mathbb{E}[D_k]$ for a stream of length k, grows linearly with k.*

The key tool used in establishing the first half of Theoem 1 is an analytical upper bound on the number of visits at a transient state in a general one dimensional random walk.

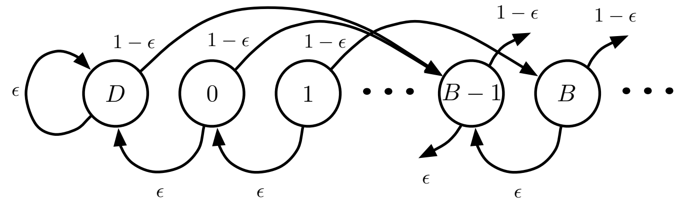

Fig. 1. The one dimensional random walk defined on the set of states $\mathcal{S}$ with infinite buffer size, fixed bandwidth usage B, and memoryless transition probabilities as depicted in the figure.

Lemma 1. *Consider a discrete time, one dimensional random walk defined on the set of states $\mathcal{S} = \{D, 0, 1, 2, \cdots\}$ as depicted in Fig. 1 for a fixed positive integer B and $0 < \epsilon < 1$. State D transitions to itself with probability ϵ and to state $B - 1$ otherwise. Also for any other state $i \in \{0, 1, \cdots\}$, the state will change to $i - 1$ with probability ϵ and to $i + B - 1$ otherwise.*

Let the number transitions from state 0 to state D, be denoted by N_D. Then starting from state 0, if $B(1-\epsilon) > 1$ then the expected time spent at state D will be upper bounded by

$$\frac{\mathbb{E}[N_D]}{1-\epsilon} \leq \frac{\epsilon B}{(1-\epsilon)((1-\epsilon)B - 1)}. \tag{1}$$

The proof of this Lemma is provided in the Appendix. In what follows the proof of Theorem 1 is provided.

Proof of Theorem 1: The first part of Theorem 1 is a direct consequence of Lemma 1. Let us model the receiver buffer with a one dimensional random walk with states $\mathcal{S} = \{D, 0, 1, 2, \cdots\}$, where state D is the buffer starvation state where the receiver experiences an interruption in the playback. Hence the playback delay which is the number of interruptions in the playback is equal to the number of visits to state D. Every other state refers to the case that the receiver has played back the required packet and the number of remaining packets in the buffer is denoted by the state name. Hence the random walk describing the buffer state of the receiver is isomorphic to the random walk introduced in the description in Lemma 1, and we can directly apply Eq. (1) to first part of this proof:

$$\mathbb{E}[D_k] \leq \mathbb{E}[D_\infty] = \frac{\mathbb{E}[N_D]}{1-\epsilon}.$$

Since spending a time step in state D represents experiencing a delay in the playback when $B(1-\epsilon) > 1$, the expected total playback delay is upper bounded by (1).

For the second part of the proof, if $B(1-\epsilon) < 1$, lets consider the same one dimensional random walk as used above, but this time assign the numerical value -1 to the state D. We will upper bound the expected time between two entrances to state D, as a renewal. Then showing this renewal process has a finite renewal duration, using the law of large numbers for the renewals [3] we conclude the number of entrances to state D and hence the total playback delay grows linearly with the stream length k. Let X_j denotes the change in the number of packets stored in the receiver buffer at time step j. We define $S_0 = B - 1$ since whenever the receiver buffer gets out of the state D it restart at state $B-1$. Also let $S_i = S_0 + \sum_{j=1}^{i} X_j\ i \geq 1$, and also $Y_i = S_i + i(1 - B(1-\epsilon))$

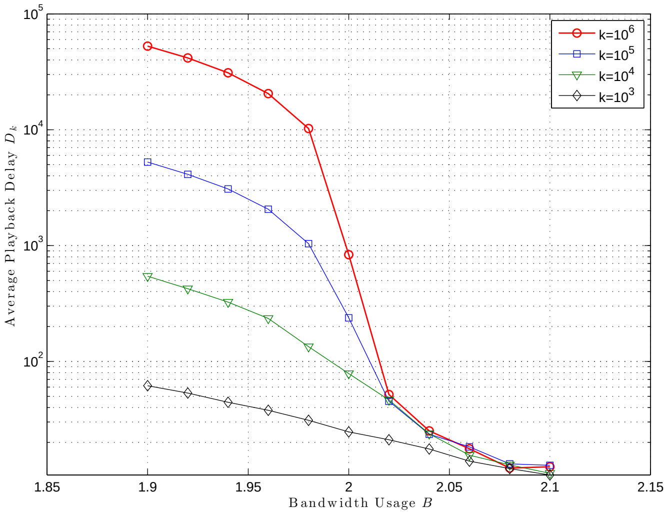

Fig. 2. The bandwidth-delay trade-off for different values of stream length k, and $\epsilon = 0.5$. This figure shows the transition in the behaviour of playback delay at $B = \frac{1}{(1-\epsilon)} = 2$.

for $i \geq 0$. Hence, S_i is not a martingale since it has a negative drift, but Y_i for $i \in \{1, \cdots\}$ is a martingale with respect to $\mathcal{F}_i = \sigma(X_1, \cdots, X_i)$ for $i \in \{1, \cdots\}$, and $\mathcal{F}_0 = \emptyset$ as the drift in the mean value of S_i is removed in Y_i. Now starting in state $B-1$, we have

$$Y_0 = B - 1. \tag{2}$$

We define $T = \inf\{t > 0 \text{ s.t. } S_t = -1\}$. Then T is a stopping time and at T we have

$$Y_T = S_T + \mathbb{E}[T](1 - B(1-\epsilon)) = -1 + \mathbb{E}[T](1 - B(1-\epsilon)). \tag{3}$$

Now using the optional stopping time theorem [7] from (2) and (3) we have

$$\begin{aligned} &\mathbb{E}[Y_T|\mathcal{F}_0] = \mathbb{E}[Y_0] \\ \Rightarrow &-1 + \mathbb{E}[T](1 - B(1-\epsilon)) = B - 1 \\ \Rightarrow &\mathbb{E}[T] = \frac{B}{1 - B(1-\epsilon)}. \end{aligned} \tag{4}$$

Note that the expected time before the first interruption is upper bounded by this value since at the beginning we start in state 0 rather than state $B-1$. This means that the expected time between two consecutive interruptions in the playback at the receiver side would be upper bounded by (4). Since (4) is a constant, then the number of interruptions before k packets are recovered at the receiver would grow linearly with k, and we will have

$$\mathbb{E}[N_D] \geq \frac{k(1 - B(1-\epsilon))}{B} \Rightarrow \mathbb{E}[D_k] \geq \frac{k(1 - B(1-\epsilon))}{B(1-\epsilon)}. \tag{5}$$

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Figure 2 shows the transition in the behaviour of the average total playback delay as a function of the bandwidth usage B, for different values of the stream length k from $k = 10^3$ packets to $k = 10^6$ packets. Here, $\epsilon = 0.5$, and as depicted in the figure, when $B < \frac{1}{(1-\epsilon)}$ the average playback delay increases linearly with the size of the stream, unlike the $B > \frac{1}{(1-\epsilon)}$ where it converges to a constant as k grows.

IV. Finite receiver buffer

In this section we will consider the case that the receiver buffer is limited. As a result at some time steps, depending on the available free space in the receiver's buffer, the transmitter might not be able to transmit B packets. Therefore the transmission scheme would be different in the sense that the bandwidth usage at any given time step t would be adaptively chosen based on the state of the receiver and the transmitter would then transmit B_t packets. However, as will be shown in this section, the average bandwidth usage in this case will always be smaller than the minimum required average bandwidth usage for having constant expected total playback delay. In other words, it would not be possible to achieve $\mathbb{E}[B_t] > (1-\epsilon)^{-1}$, when $\mathbb{E}[B_t]$ denotes the expected value of B_t over the duration of transmission. As a result, the expected total playback delay in this case will always be a linear function of the the stream length k. However simulations show that in practice for a fixed length of the stream and for moderately large receiver buffers, we can achieve a playback delay very close to the infinite buffer case.

First we propose a dynamic bandwidth scheme where the source transmits just enough packets to refill the playback buffer after each slot. This transmission scheme hence achieves the best possible expected total playback delay as it maximizes the packet transmission at any time step, according to the limitation of the receiver's buffer. Here we assume that the receiver buffer has just enough capacity to keep maximum of m source packets.

Definition 2 (Buffer Refill Scheme)**.** *The source transmits just enough packets to refill the playback buffer. Thus the number of packets transmitted in slot t is given by*

$$B_t = m - N_{t-1} + 1$$

where N_{t-1} is the number of packets in the receiver buffer at the end of slot $t-1$.

Thus starting at time zero, the scheme transmits $m+1$ packets in each slot. The first successful slot will result in a full playback buffer. Since one packet is played in each slot, the source has to transmit at least 1 to replenish the buffer. In the following we provide the expected bandwidth usage and the expected total playback delay for this scheme. Note that the expected total playback delay for this scheme is the lower bound for the expected total playback delay for any transmission scheme with finite receiver buffer.

A. Bandwidth Usage

The bandwidth usage in slot t can be expressed as

$$B_t = 1 + \min(m, E_{t-1})$$

where E_{t-1} is the length of the continuous burst of erasures ending in slot $t-1$. It represents the amount of empty space in the receiver buffer and follows the geometric distribution with parameter ϵ.

The expected bandwidth usage $\mathbb{E}[B_t]$ for $t > m$ is given

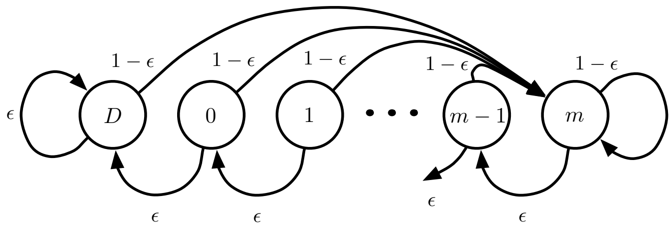

Fig. 3. The one dimensional random walk defined for the Buffer Refill scheme with finite buffer size and memoryless transition probabilities as depicted in the figure.

by,

$$\begin{aligned}\mathbb{E}[B_t] &= 1 + \sum_{i=1}^{m-1} i\epsilon^i(1-\epsilon) + m\epsilon^m \\ &= 1 + \epsilon + \epsilon^2 + \cdots \epsilon^m \\ &= \frac{1-\epsilon^{m+1}}{1-\epsilon} \end{aligned} \tag{6}$$

In the analysis of the fixed bandwidth scheme, we saw that when the buffer size m is infinity, we require $B > 1/(1-\epsilon)$. In (6) we observe that as $m \to \infty$, the bandwidth usage goes to the same limit $1/(1-\epsilon)$. This implies that the the expected bandwidth usage of any scheme with finite receiver buffer will always be below the required bandwidth for the finite expected total playback delay. In the following the expected total playback delay of the Buffer Refill scheme is provided.

B. Playback Delay

The expected playback delay is equal to the expected total playback time, $\mathbb{E}[T_k]$, times the steady state probability π_D of being in state D (the buffer starvation state).

The Markov chain for the buffer state N_t is as illustrated in Fig. 3. We can evaluate the steady-state probabilities by solving the following state transition equations.

$$\begin{aligned}(1-\epsilon)\pi_D &= \epsilon\pi_0, \\ \pi_i &= \epsilon\pi_{i+1} \text{ for } 0 \le i \le m-1, \\ \pi_m &= \frac{1-\epsilon}{\epsilon}\left(\pi_D + \pi_0 + \cdots \pi_{m-1}\right), \\ 1 &= \pi_D + \sum_{i=0}^{m} \pi_i. \end{aligned}$$

Solving, we get

$$\pi_m = 1-\epsilon, \quad \pi_D = \epsilon^{m+1}. \tag{7}$$

Moreover, the expected total playback time could be calculated as

$$\mathbb{E}[T_k] \ge \frac{k}{1-\pi_D} \;\Rightarrow\; \mathbb{E}[D_K] \ge \frac{\epsilon^{m+1}k}{1-\epsilon^{m+1}}.$$

Therefore $\mathbb{E}[D_K]$ grows linearly with k. However, as depicted in the figure 4, using a more practical transmission scheme, the delay-bandwidth trade-off having a practical buffer size will be very similar to the case of infinite receiver buffer. In these simulations the transmitter sets $B_t = \min\{m - N_{t-1} + 1, B\}$ for some fixed B. Hence the size of the transmission packet is clamped to the available buffer space whenever necessary and remains constant otherwise to address the practical limits on the transmission packet size.

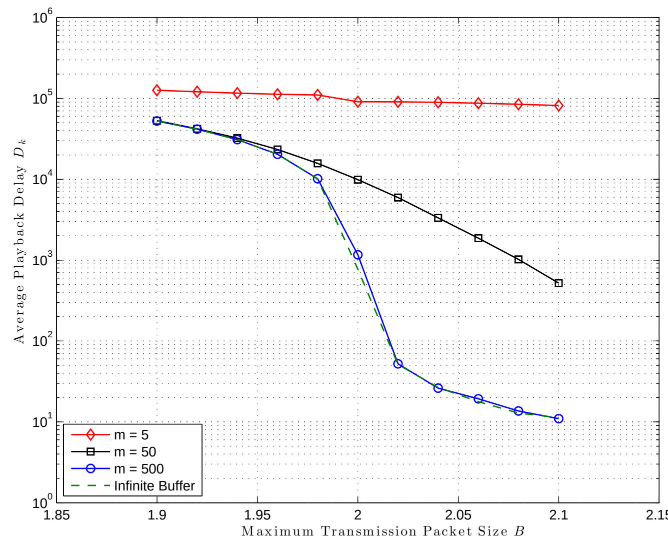

Fig. 4. Effect of buffer size m, on bandwidth-delay trade-off for Buffer Refill scheme with maximum transmission packet size B, $k = 10^6$, $\epsilon = 0.5$.

V. Conclusion

We studied the achievable playback delay for perfect feedback over an i.i.d. erasure channel. Assuming all the packets are initially available at the transmitter, we formulated the problem based on a random walk describing the state of receiver buffer. Our analysis is based on introducing a new theorem to describe the transient behaviour of such a random walk with drift. We showed that when the bandwidth usage is above the inverse of the channel capacity, the expected playback delay remains constant while otherwise it grows linearly with the stream length. Both the result and the analysis technique are different from the case when the source packets are generated in real-time at the encoder. We also studied the finite buffer limitation and dynamic bandwidth schemes both analytically and using simulations. Study of delayed feedback, without feedback, and broadcast cases remain as some interesting follow-ups.

Appendix A
Proof of Lemma 1

Proof: Starting from state zero, lets denote the probability that the random walk would eventually enter the state D some time in the future by P_0. Similarly, starting from state $B-1$, lets denote the probability of entering state D sometime in the future by P_{B-1}. Then denoting the expected number of times, random walk enters the state D, starting from state zero by $\mathbb{E}[N_D]$, we have

$$\mathbb{E}[N_D] = \sum_{j=1}^{\infty} j\mathrm{P}_0(1-\mathrm{P}_{B-1})(\mathrm{P}_{B-1})^{(j-1)} = \frac{\mathrm{P}_0}{1-\mathrm{P}_{B-1}}.$$

In order to find an upper bound for this quantity, we will now derive an upper bound for P_0. Also, since according to the definition of the random walk, $\mathrm{P}_{B-1} \le \mathrm{P}_0$, then

$$\mathbb{E}[N_D] \le \frac{\mathrm{P}_0^*}{1-\mathrm{P}_0^*}. \tag{8}$$

To derive P_0^*, we first derive the bound for the first n time steps, and then we let n to tend to infinity.

Let X_i for $i \in \{1, 2, \cdots, n\}$ be a random variable which takes value 0 with probability ϵ and value B with probability $1-\epsilon$. The random variable X_i corresponds to the jump at time step i in the random walk, such that X_i is equal to the size of the jump plus one. Also let $M_i = \frac{1}{i}\sum_{j=1}^{i} X_j$.

Note that according to the definitions, whenever $M_j < 1$, for any $j \in \{1, \cdots, n\}$, the sum of all the jumps towards right in the random walk up to time step j is less than the sum of the jumps towards left, and hence the state D of the random walk should have been visited.

Now we also define

$$Z_{-i} = \frac{B - M_i}{B - 1}.$$

Then Z_{-i} is a reverse martingale with respect to the filtration $\mathcal{F}_{-i} = \sigma(M_i, M_{i+1}, \cdots, M_n)$.

Note that to emphasise the reverse order considered for the martingale Z_{-i} and its corresponding information filtration $\mathcal{F}_{-i}$ we are using negative indexes for them. We skip the proof that Z_{-i} is a martingale with respect to the above information filtration in the interest of space in this paper, but one could find similar proofs in [7] (e.g. example 5.6.1).

We now define a stopping time T in the reverse time order from n to 1 as follows. Let $T = \max\{j \le n \text{ s.t. } M_j < 1\}$, and set $T = 1$ if the set is empty. We will refer to this event that the set $\{j \ge -n \text{ s.t. } M_{-j} < 1\} = \emptyset$ as the event E. Note that in order for T to be a stopping time on the event E, T could not take any value larger than 1 since we need to wait until the end of the reverse order of time from n to 1 to realize such event has happened. In the E^c however, starting from time n coming backwards to 1, T refers to the first time that the empirical mean M_T drops below one.

Also note that the event E is the event that the random walk never hits state D as in E the empirical mean M_i is always above one. Then clearly $\mathrm{P}_0 = \mathrm{P}[E^c]$. Now we claim that on E, since we have $T = 1$ by the definition, then $M_T = B$. To show this, note that on E, the random walk never goes to state D, then at the first time step $i = 1$, the random walk should have jumped to $B - 1$, and hence we have $X_1 = B$. Then $M_T = M_1 = X_1 = B$. This in turn implies,

$$Z_{-T} = 0 \quad \text{on} \quad E. \tag{9}$$

Lets now partition the event E^c into the disjoint events $E_j^c = \{T = j\}$ for $j \in \{1, \cdots, n\}$. Since j is the last time $M_j < 1$ on E_j^c, then we can conclude that on E_j^c, $M_j = M_{T \le (1-j)/j}$. As a result we have,

$$M_T \le \frac{j-1}{j} \Rightarrow Z_{-T} \ge 1 + \frac{1}{j(B-1)} \quad \text{on} \quad E_j^c, \tag{10}$$

Now note that, due to the definition of the partition on E^c, the events E_i^c and E_j^c are disjoint for any $i \neq j$, and they are also disjoint given the information $\mathcal{F}_{-n} = \sigma(M_n)$. Therefore we have

$$\mathrm{P}[E^c|M_n] = \sum_{j=1}^{n} \mathrm{P}[E_j^c|\mathcal{F}_{-n}]. \tag{11}$$

Moreover, having (9), and (10), by definition we have

$$\mathbb{E}[Z_{-T}|\mathcal{F}_{-n}] = \sum_{j=1}^{n} \mathrm{P}[E_j^c|\mathcal{F}_{-n}] \left(1 + \frac{1}{j(B-1)}\right). \tag{12}$$

Therefore, since $1 \le 1 + 1/(j(B-1))$ for any $j \in \{1, \cdots, n\}$, then from (11) and (12) we have

$$\mathrm{P}[E^c|M_n] \le \mathbb{E}[Z_{-T}|\mathcal{F}_{-n}].$$

However, using the optional stopping time theorem [7], we can also see that

$$\mathbb{E}[Z_{-T}|\mathcal{F}_{-n}] = \mathbb{E}[Z_{-n}|\mathcal{F}_{-n}] = Z_{-n} = \frac{B - M_n}{B - 1},$$

and as a result we have

$$\mathrm{P}[E^c|M_n] \le \frac{B - M_n}{B - 1}.$$

Now note that the probability of visiting state D, starting from state zero, is equal to $\mathrm{P}[E^c]$, and also note that this is an upper bound on the probability of visiting state D starting from any state $i > 0$, due to the definition of the random walk. Hence, when n goes to infinity, the probability of entering state D starting from any state is upper bounded by

$$\begin{aligned} \mathrm{P}_0^* &= \lim_{n\to\infty} \mathbb{E}_{M_n}\left[\frac{B - M_n}{B - 1}\right] \\ &= \lim_{n\to\infty} \frac{B - \mathbb{E}_{M_n}[M_n]}{B - 1} \\ &= \lim_{n\to\infty} \frac{B - (1-\epsilon)B}{B - 1} \\ &= \frac{\epsilon B}{B - 1}. \end{aligned}$$

Now substituting P_0^* in (8) we have

$$\mathbb{E}[N_D] \le \frac{\mathrm{P}_0^*}{1 - \mathrm{P}_0^*} = \frac{\epsilon B}{(1-\epsilon)B - 1}.$$

In order to complete the proof, please note that according to the definition of the random walk, once we enter the state D, the random variable indicating the waiting time for exiting that state is a Geometric random variable with mean $(1-\epsilon)^{(-1)}$. As a result, the expected total time residing in state D is given by

$$\frac{\mathbb{E}[N_D]}{1-\epsilon} = \frac{\epsilon B}{(1-\epsilon)\left((1-\epsilon)B - 1\right)}.$$

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