Artificial Intelligence Methods for Social Good

M1-3 [Optimization]:

Combinatorial Optimization and Robust Optimization

08-537 (9-unit) and 08-737 (12-unit)

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Learning Objectives

- Understand the concept of
  - Maximin model
  - Minimax regret
- Briefly describe
  - Branch and bound algorithm
- Optional
  - Dual problem of a linear program

Robust Optimization

- How do we deal with uncertainty?
- Exp1: Choose location for a party
- When you know the probability of raining (whether forecast): Compute expected utility
- When you don't know: robust optimization
  - Maximin: Maximize the worst case utility (Conservative)
  - Minimax regret (Less conservative)
    - Minimize maximum regret
    - Regret: Gap between my current utility and the best possible utility given the whether condition

Robust Optimization

- Solve Minimax Regret when there are many actions and possibilities
  - Constraint sampling/constraint generation
  - Sample a subset of actions and possibilities
  - Solve the restricted problem
  - Find most "critical" combination of action and possibilities
  - Resolve the restricted problem
  - Repeat until some stopping criteria

Quiz 1

Effort Spend

Difficulty of Exam

	Low	Medium	High
Easy	9-0=9	9-1=8	10-3=7
Medium	5-0=5	7.5-1=6.5	9-3=6
Hard	2-0=3	5-1=4	8-3=5

- What is the value of minimax regret?
  - 1
  - 2
  - 0.5
  - 3

Branch and Bound

- Binary Program with linear objective and constraints

- Key idea:
  - Incrementally build a binary search tree
  - Prune the tree using lower bound and upper bound
  - Expand the most promising leaf node

Branch and Bound

- Exp 2: Knapsack (W=10)

Items	1	2	3	4	5
Weight	5	4	2	6	7
Value	4	3	6	9	5

- Upper bound: LP relaxation (poly-time computable)

- Lower bound: Any feasible integer solution

Solving (Mixed) Integer Program with Branch and Bound

- Recall MILP: An optimization problem whose optimization objective is a linear function and feasible region is defined by linear constraints and integer constraints

- Key Ideas of Branch and Bound for MILP
  - If value of all integer variables are determined, then become an LP (can be efficiently solved using e.g., Simplex algorithm!)
  - Substitute the integer variables with binary variables
  - Build a binary search tree for the binary variables
  - Prune the tree using lower bound and upper bound

Recall: Linear Program

- An optimization problem whose optimization objective is a linear function and feasible region is defined by linear constraints
- Dual problem of an LP: also a linear program
- Strong duality theorem: LP and its dual have the same value

Write the Dual of an LP

- Exp 3: Maximize Profit

	Price	Labor	Machine
Product 1	$30	0.2 hour	4 hour
Product 2	$30	0.5 hour	2 hour
Total		<=90	<=800

Write the Dual of an LP (Non-Standard form)

Maximize	Minimize
ith constraint ≤	ith variable ≥ 0
ith constraint ≥	ith variable ≤ 0
ith constraint =	ith variable unrestricted
jth variable ≥ 0	jth constraint ≥
jth variable ≤ 0	jth constraint ≤
jth variable unrestricted	jth constraint =

Other properties of LP Duality

- Complementary slackness
  - Primal constraint is not tight → dual variable=0
  - Primal variable is not zero → dual constraint is tight
  - Dual constraint is not tight → primal variable=0
  - Dual variable is not zero → primal constraint is tight

- Optimality Conditions: If x, y are feasible solutions to the primal and dual problems, respectively, then they are optimal solutions to these problems if, and only if, the complementary-slackness conditions hold for both the primal and the dual problems

Reference and Related Work

- *Combinatorial Optimization: Algorithms and Complexity, Chapters 3*
  - Christos H. Papadimitriou, Kenneth Steiglitz
- *Branch-and-price: Column generation for solving huge integer programs*
  - Cynthia Barnhart, Ellis L. Johnson, George L. Nemhauser, Martin W. P. Savelsbergh, Pamela H. Vance
- *Robust protection of fisheries with COmPASS*
  - William Haskell, Debarun Kar, Fei Fang, Milind Tambe, Sam Cheung, Elizabeth Denicola

Paper Discussion

- (PRA5) *Keeping Pace with Criminals: An Extended Study of Designing Patrol Allocation against Adaptive Opportunistic Criminals*
  - Chao Zhang, Shahrzad Gholami, Debarun Kar, Arunesh Sinha, Manish Jain, Ripple Goyal, Milind Tambe
- Summary
  - Societal challenge
  - AI method
  - Contributions
- Questions
- Brainstorming Ideas
  - Improvement / future direction / other valid discussions
  - Societal challenge and AI method that can potentially be used to tackle it (not necessarily relevant to the paper)