# A Geographical Approach to Fair Redistricting

Margot Stewart (erstewar)

November 2, 2018

## 1 Project Overview

### 1.1 Members

I will be carrying out this project under the supervision of Dr. Ariel Procaccia in the Department of Computer Science. Dr. Procaccia does a wide array of work in CS theory, encompassing such sub-domains as computational game theory, social choice mechanisms, fair division and resource allocation, algorithms, and complexity. I will also be working directly alongside one of his PhD students, Gerdus Benad, whose work lies in the theoretical side of operations research.

### 1.2 Justification

A commonly studied application of fair division is that of distracting regions into congressional districts for voting; the appeal of this work in a democratic society is not hard to see. Much fair redistricting research seeks to address the problem of partisan unfairness in districting, more colloquially known as gerrymandering. Gerrymandering describes a situation in which a districting is designed in such a way as to present an unfair advantage to one pair in the election process.

What constitutes an unfair advantage depends on our definitions of fairness. Intuitively, for instance, we would like to be able to expect that the ratio of districts a political party wins in a state is roughly equivalent to the ratio of voters in the state who support said party. Whatever exact model of fairness we hold our districting processes to, they must be rigorous and provable, or else our methods cannot be supported.

A variety of districting methodologies have been proposed to fulfill a number of different fairness specifications. For instance, last saw a paper by Dr. Procaccia and others propose a novel I-Cut-You-Freeze mechanism. This mechanism guaranteed a $B$-target property that prevented parties from monopolizing or corralling the other party into certain regions of the state. This mechanism is notable in that it can be carried out without the intervention of an independent arbitrator.

A common challenge with fair redistricting work, however, is that it is difficult to guarantee particularly rigorous fairness constraints while also respecting geographic or geometric constraints. Fair division work likes to model the resources being divided as a featureless interval $[0, 1]$, or perhaps a disk, with some valuation measures over them for each player. However, states in the real world are bizarrely shaped, and there may be certain districting configurations that are illegal. At the very least, we would like that our districts be contiguous, and ideally that they satisfy some rudimentary compactness guarantees.

### 1.3 Our Project

Over the summer, I joined Gerdus in developing a districting mechanism to be carried out by an independent arbitrator that would be able to satisfy reasonable target guarantees for regions of any geography. The target fairness specification that interested us was tied to the best and worst possible outcomes either party could expect to see in any districting. Suppose we have two parties A and B, where without loss of generality party A is the minority party. If we have some region $P$ to district and a set $\mathcal{D}$ of districtings $d$ that partition $P$ into $n$ districts, we can notate the number of districts A wins in a given districting $d$ as $a(d)$ and the number of districts $b$ wins as $d(b)$. Then our best and worst districtings for party A are

$$d_A^+ = \text{argmax}_d a(d)$$

$$d_A^- = \text{argmin}_d a(d)$$

and likewise for party B. We wanted to guarantee each party a target $t_A$ or $t_B$ that is the average of their best and worst possible scores, that is

$$t_A = \frac{a\left(d_A^+\right) + a\left(d_A^-\right)}{2}$$

We were in fact able to satisfy this fairness property, giving an algorithm over a plane $P = [0, 1]$ with a set of voters or census nodes $V$ loyal to either party that would produce a districting in which party A is guaranteed to win at least $\lfloor t_A \rfloor$ districts and party B is guaranteed to win at least $\lfloor t_B \rfloor$ districts. Our algorithm operated by freezing half of the nodes won by party A (the minority party without loss of generality), giving them their target, and partitioning the rest of the space into districtings with the same ratio of voters as the overall state, giving party B (the majority party) their target.

We have already submitted our findings and algorithm. However, we have yet to construct an implementation using it. This will be my task in this project. We have already collected voter data from at least three states; I will use this data to produce proposed districtings for these states via our methodology in order to see if they are indeed reasonable. I will most likely use a linear programming approach to implementing our algorithm. I will then analyze the costs of our mechanisms, as well as if it is possible to optimize for other desirable objectives such as district compactness and competitiveness.

### 1.4 Project Website

Information and resources related to my project will eventually be available at `www.andrew.cmu.edu/user/erstewar/districting/`.

## 2 Project Goals

The algorithm and the districting examples for extant states are the primary goals of this project, whereas the augmented versions of the algorithms for additional constraints will be extra. Of these additional constraints, district compactness will be the highest priority.

### 2.1 75% Goal

We would like to at least have state districtings to show for our work, even if they are not necessarily produced by a consistent implementation of algorithm. As long as they are produced using the essential steps of our algorithm and satisfy all of our guarantees, they will be sufficient.

### 2.2 100% Goal

The ideal complete outcome is at least three state districtings, all produced by the same implementation of our mechanism. The complexity and efficiency of this implementation will be analyzed based on relevant parameters.

### 2.3 125% Goal

If extra time is available, we will produce additional versions of the algorithm and additional districtings optimized for further constraints. District compactness will be our first new constraint to address.

## 3 Project Milestones

### 3.1 15-300 End Of Semester Milestone

By the end of the semester, I would like to have a rough implementation of our algorithm that satisfies our guarantees at least for simple arbitrary test cases. This implementation may still need to be improved before it can be used to produce actual state districtings.

### 3.2 15-400 Milestones

Some loose milestones for the coming semester are as follows:

- Second Week: We should have an implementation that works for our real state data.
- Fourth Week: We should have at least one workable state districting.
- Sixth Week: We should have all of our state districtings complete.
- Eighth Week: Our core implementation should be complete and its parametric complexity should be analyzed.
- Tenth Week: We should have attempted optimization with respect to district compactness.
- Twelfth Week: We should have attempted optimization with respect to some other additional constraint.
- Fourteenth Week: We should have attempted optimization with respect to some other additional constraint.