Triangles

JV Practice 3/28/21

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Key Concepts

Given triangle ABC and triangle DEF, $\triangle ABC$ is congruent to $\triangle DEF$ (denoted as $\triangle ABC \cong \triangle DEF$) if and only if any of the following is satisfied:

- $|AB| = |DE|$, $|BC| = |EF|$, $|CA| = |FD|$ (**SSS**)
- $|AB| = |DE|$, $|BC| = |EF|$, $\angle B = \angle E$ (**SAS**)
- $\angle A = \angle D$, $\angle B = \angle E$, $|AB| = |DE|$ (**ASA**)
- $\angle A = \angle D$, $\angle B = \angle E$, $|BC| = |EF|$ (**AAS**)

Triangle ABC is similar to triangle DEF (denoted as $\triangle ABC \sim \triangle DEF$) if and only if one of the following is satisfied:

1. $\angle A = \angle D, \angle B = \angle E$and $\angle C = \angle F$ (**AA**)
2. $\frac{|AB|}{|DE|} = \frac{|BC|}{|EF|} = \frac{|CA|}{|FD|}$ (**SSS**)
3. $\frac{|AB|}{|DE|} = \frac{|BC|}{|EF|}$ and $\angle B = \angle B = \angle E$ (**SAS**)

1 Extended Warm-Up Problems

1. In triangle ABC, if $|AB| = |BC|$, prove that $\angle A = \angle C$.
2. In a right-angled triangle ABC, let M be the midpoint of the hypotenuse BC. Prove that $|MB| = |MC| = |AM|$.
3. Let $ABCD$ be a quadrilateral. Let E be a point on BD such that $EC \perp BD$. Suppose $\angle ADE = \angle DCE$, $\angle EBA = 90$, $|AD| = 24$ and $|DC| = 7$. Find $|AC|$.
4. In triangle ABC, let D be a point on AC such that $|AD| = |AB|$. If $\angle ABC - \angle ACB = 40°$, find $\angle DBC$.
5. Let $ABCD$ be a rectangle with area 300. There exist points E on AB and F on CD such that $DE = EF = FB$. Diagonal AC intersects EF at X. Compute the area of $\triangle AEX$

2 Somewhat-Easy Problems

1. In a right-angled triangle ABC, let D, E, f be points on the hypotenuse AB such that CD is the altitude, CE is the angle-bisector, and CF is the median. Prove that $\angle DCE = \angle ECF$. Is the statement still true if ABC is not right-angled?

2. In a right-angled triangle ABC, let P be a point such that CP is perpendicular to the hypotenuse CA and $|CP| = |CB|$. Let line L_1 be the angle bisector of $\angle A$. Prove that $L)1$ is either perpendicular, or parallel to to BP.

3. In triangle ABC, let D, E be points on AB and AC respectively such that $|DB| = 2, |EC| = 3$ and DE is parallel to BC. If $|AD| = 5$, find $|AC|$.

4. In rectangle $ABCD$, let E be a point on side AB such that $|AE| = 3$ and $|EB| = 6$. Let F be a point on BC such that $|BF| = 2$ and $|FC| = 4$. Let M be the intersection point of EC and AF. Find the area of triangle AMD.

5. Let M be the midpoint of a base CD of the trapezoid $ABCD$. Let F be the point of intersection of AM and the diagonal BD. Extend a line parallel to CD at F so it intersects AD, AC and BC at E, G, H respectively. Prove that $EF = FG = GH$.

6. In rectangle $ABCD$, let M be the midpoint of AB. Let E be a point on AD such that $EM \perp EC$. Prove that MC bisects $\angle ECB$.

3 A-Bit-Harder Problems

1. Let A, K, L, B be four sequential points on a line such that $(AL)^2 = (AK)(AB)$. Let P be another point such that $AP = AL$. Prove that PL bisects $\angle KPB$. Also, prove the converse.

2. In triangle ABC, let AM be the median that also happens to divide $\angle BAC$ into a ratio of $1 : 2$. Extend the line AM to D such that $BD \perp AB$. Prove that $|AD| = 2|AC|$.

3. Let M be the point of intersection of the three altitudes of a triangle ABC. If AB=CM, then determine the angle of ACB.

4. Let $ABCD$ be a parallelogram. For each side of the parallelogram, draw a square incident to that side (of that length) outside of the parallelogram. You should have four non-overlapping squares with a parallelogram between them. Let P, Q, R, s be the centers of the four squares. First, prove that $PQRS$ is a square. Next, show that the center of square $PQRS$ is the same as the center of parallelogram $ABCD$.

5. Let $ABCD$ be a trapezoid where the long base AB is 90 unit long. Let E, F be the midpoints of the diagonals and suppose EF is 4 unit long. What is the measurement of the shorter base?

6. In $\triangle ABC$, AD is the angle bisector of $\angle A$ with D lying on BC. Given that $\angle B = 2\angle C$, $AB = 3$ and $BD = 1$, find AD. (Hint: BA tcelfer dna yrtemmys esu)