

Counting II

JV Practice 3/21/21

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1 General tips for counting

- Draw a picture of the things or way you are counting, so you have a clear idea of what you're working with.
- Figure out whether order matters.
- Divide out by things you can rearrange, multiply in by the number of choices you make.
- Decide whether counting all the things you want is easier, or if counting all the things you *don't* want is easier.
- Try a smaller case to check your logic.
- Make a plan for how to count before you start, in case one of the steps of your plan turns out to be impossible.

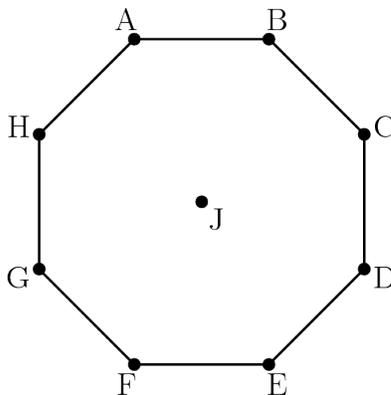
2 Warmups

1. How many positive integers less than or equal to 200 are divisible by 3 or 5?
2. How many different words can you get by rearranging the letters in the word "COMBINATORICS"?

3 Practice 2

1. (AMC 10B 2013) Let S be the set of sides and diagonals of a regular pentagon. A pair of elements of S are selected at random without replacement. What is the probability that the two chosen segments have the same length?
2. (AMC 12A 2015) A box contains 2 red marbles, 2 green marbles, and 2 yellow marbles. Carol takes 2 marbles from the box at random; then Claudia takes 2 of the remaining marbles at random; and then Cheryl takes the last 2 marbles. What is the probability that Cheryl gets 2 marbles of the same color?
3. (AMC 10A/12A 2010) A solid cube has side length 3 inches. A 2-inch by 2-inch square hole is cut into the center of each face. The edges of each cut are parallel to the edges of the cube, and each hole goes all the way through the cube. What is the volume, in cubic inches, of the remaining solid?

4. (AMC 10A 2012) Adam, Benin, Chiang, Deshawn, Esther, and Fiona have internet accounts. Some, but not all, of them are internet friends with each other, and none of them has an internet friend outside this group. Each of them has the same number of internet friends. In how many different ways can this happen?
5. (AMC 10B 2013) The regular octagon $ABCDEFGH$ has its center at J . Each of the vertices and the center are to be associated with one of the digits 1 through 9, with each digit used once, in such a way that the sums of the numbers on the lines AJE , BJF , CJG , and DJH are all equal.



- (a) What are the possible values at J ?
 - (b) In how many ways can this be done?
6. (AMC 12A 2008) A permutation $(a_1, a_2, a_3, a_4, a_5)$ of $(1, 2, 3, 4, 5)$ is heavy-tailed if $a_1 + a_2 < a_4 + a_5$. What is the number of heavy-tailed permutations?
 7. (AMC 10B 2010) The entries in a 3×3 array include all the digits from 1 through 9, arranged so that the entries in every row and column are in increasing order. How many such arrays are there?
 8. (AMC 10A/12A 2010) Bernardo randomly picks 3 distinct numbers from the set $\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ and arranges them in descending order to form a 3-digit number. Silvia randomly picks 3 distinct numbers from the set $\{1, 2, 3, 4, 5, 6, 7, 8\}$ and also arranges them in descending order to form a 3-digit number. What is the probability that Bernardo's number is larger than Silvia's number?
 9. (HMMT 2012) Brian has a 20-sided die with faces numbered from 1 to 20, and George has three 6-sided dice with faces numbered from 1 to 6. Brian and George simultaneously roll all their dice. What is the probability that the number on Brian's die is larger than the sum of the numbers on George's dice?
 10. Compute the number of ordered triples (x, y, z) of nonnegative integers such that $x+y+z = 20$.
 11. (AMC 12A 2015) Eight people are sitting around a circular table, each holding a fair coin. All eight people flip their coins and those who flip heads stand while those who flip tails remain seated. What is the probability that no two adjacent people will stand?
 12. Compute the number of derangements (permutations where no element stays in the original position) of 6 elements.