Additions to Figures

JV Practice

C.J. Argue

1 Warm-Up

1. (ARML 1994) Semicircles are drawn on sides AB and AD of square $ABCD$, each lying outside the square. E is the center of the square. QAP is a line segment with Q and P on the two semicircles, $QA = 7$ and $AP = 23$. Compute AE.

2. (ARML 2008) Hexagon $ABCDEF$ is inscribed in circle O and $AB = CD = EF = 2BC = 2DE = 2AF$. If $AD = 8$, compute the perimeter of $ABCDEF$.

3. (AMC 10 2005) Let AB be a diameter of a circle and let C be a point on AB with $2 \cdot AC = BC$. Let D and E be points on the circle such that $DC \perp AB$ and DE is a second diameter. What is the ratio of the area of $\triangle DCE$ to the area of $\triangle ABD$?

2 Problems

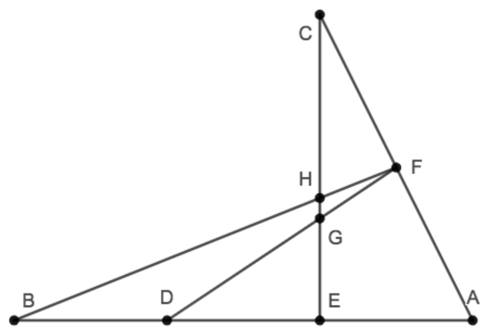
1. (AIME 2007) Square $ABCD$ has side length 13, and points E and F are exterior to the square such that $BE = DF = 5$ and $AE = CF = 12$. Find EF^2.

2. (ARML 2011) In triangle ABC, C is a right angle and M is on AC. A circle with radius r is centered at M, is tangent to AB, and is tangent to BC at C. If $AC = 5$ and $BC = 12$, compute r.

3. (AMC10 2002) In trapezoid $ABCD$ with bases AB and CD, we have $AB = 52$, $BC = 12$, $CD = 39$, and $DA = 5$. Find the area of $ABCD$. There are many different constructions that can help solve this problem, try to find at least 2 different solutions.

4. (ARML 2013) The square $ARML$ is contained in the xy-plane with $A = (0, 0)$ and $M = (1, 1)$. Compute the length of the shortest path from the point $(2/7, 3/7)$ to itself that touches three of the four sides of square ARML.

5. Given noncollinear points A, B, C, segment AB is trisected by points D and E, and F is the midpoint of segment AC. DF and BF intersect CE at G and H, respectively. If triangle DEG has area 18, compute the area of triangle FGH.



6. (AIME 2001) Triangle ABC has $AB = 21$, $AC = 22$ and $BC = 20$. Points D and E are located on $\overline{AB}$ and $\overline{AC}$, respectively, such that $\overline{DE}$ is parallel to $\overline{BC}$ and contains the center of the inscribed circle of triangle ABC. Then $DE = m/n$, where m and n are relatively prime positive integers. Find $m + n$.

3 Nice problems that are somewhat unrelated

1. (UMD 2016) In the triangle ABC consider the point M on BC with $BM < CM$. From M we draw lines parallel to AB and AC. Suppose the area of the resulting parallelogram is $5/18$ of the area of ABC. What is the ratio BM/CM?

2. (ARML 2013) Regular hexagon $ABCDEF$ and regular hexagon $GHIJKL$ both have side length 24. The hexagons overlap, so that G is on AB, B is on GH, K is on DE, and D is on JK. If the area of $GBCDKL$ is 12 times the area of $ABCDEF$, compute LF.

4 Challenge

1. (UMD 2017) In an isosceles triangle ABC, we know $AB = AC$. Point D on side AC is selected so that BD is the angle bisector of B. Suppose $BC = AD + BD$. What is the angle A, in degrees?

2. (UMD 2014) In triangle ABD, point C is on AD such that BC is perpendicular to AC. If $AB = CD = 1$ and $\angle CBD = 30°$, compute AC.

Varsity Practice

Annie Xu

5 Warm-Up Problems

1. (AMC10 2002) In trapezoid $ABCD$ with bases AB and CD, we have $AB = 52$, $BC = 12$, $CD = 39$, and $DA = 5$. Find the area of $ABCD$. There are many different constructions that can help solve this problem, try to find at least 2 different solutions.
2. (AIME 2007) Square $ABCD$ has side length 13, and points E and F are exterior to the square such that $BE = DF = 5$ and $AE = CF = 12$. Find EF^2.
3. (AIME 1999) Point P is located inside triangle ABC so that angles PAB, PBC, and PCA are all congruent. The sides of the triangle have lengths $AB = 13, BC = 14$, and $CA = 15$, and the tangent of angle PAB is m/n, where m and n are relatively prime positive integers. Find $m + n$.
4. (AHSME 1995) Two parallel chords in a circle have lengths 10 and 14, and the distance between them is 6. The chord parallel to these chords and midway between them is of length $\sqrt{a}$. Find a.

6 Problems

1. (AMC10 2005) An equiangular octagon has four sides of length 1 and four sides of length $\frac{\sqrt{2}}{2}$, arranged so that no two consecutive sides have the same length. What is the area of the octagon?
2. (ARML 1994) Rectangle $PQRS$ is inscribed in rectangle $ABCD$, as shown. If $DR = 3$, $RP = 13$, and $PA = 8$, compute the area of rectangle $ABCD$.
3. (AIME 2003) Triangle ABC is isosceles with $AC = BC$ and $\angle ACB = 106°$. Point M is in the interior of the triangle so that $\angle MAC = 7°$ and $\angle MCA = 23°$. Find the number of degrees in $\angle CMB$.
4. (AMC10 2006) Circles with centers O and P have radii 2 and 4, respectively, and are externally tangent. Points A and B on the circle with center O and points C and D on the circle with center P are such that AD and BC are common external tangents to the circles. What is the area of the concave hexagon $AOBCPD$?
5. (AIME 2001) Triangle ABC has $AB = 21$, $AC = 22$ and $BC = 20$. Points D and E are located on $\overline{AB}$ and $\overline{AC}$, respectively, such that $\overline{DE}$ is parallel to $\overline{BC}$ and contains the center of the inscribed circle of triangle ABC. Then $DE = m/n$, where m and n are relatively prime positive integers. Find $m + n$.
6. (AIME 2000) In triangle ABC, it is given that angles B and C are congruent. Points P and Q lie on $\overline{AC}$ and $\overline{AB}$, respectively, so that $AP = PQ = QB = BC$. Angle ACB is r times as large as angle APQ, where r is a positive real number. Find the greatest integer that does not exceed $1000r$.

7. (AIME 2000) A circle is inscribed in quadrilateral $ABCD$, tangent to $\overline{AB}$ at P and to $\overline{CD}$ at Q. Given that $AP = 19$, $PB = 26$, $CQ = 37$, and $QD = 23$, find the square of the radius of the circle.

8. (IMO 2001) ABC is a triangle. X lies on BC and AX bisects angle A. Y lies on CA and BY bisects angle B. Angle A is $60°$. $AB + BX = AY + YB$. Find all possible values for angle B.

7 Challenge Problems

1. (AIME 1994) Given a point P on a triangular piece of paper $ABC,$ consider the creases that are formed in the paper when $A, B,$ and C are folded onto $P.$ Let us call P a fold point of $\triangle ABC$ if these creases, which number three unless P is one of the vertices, do not intersect. Suppose that $AB = 36, AC = 72,$ and $\angle B = 90°.$ Then the area of the set of all fold points of $\triangle ABC$ can be written in the form $q\pi - r\sqrt{s},$ where $q, r,$ and s are positive integers and s is not divisible by the square of any prime. What is $q + r + s$?

2. (IMO Shortlist 2001) Let A_1 be the center of the square inscribed in acute triangle ABC with two vertices of the square on side BC. Thus one of the two remaining vertices of the square is on side AB and the other is on AC. Points $B_1,\ C_1$ are defined in a similar way for inscribed squares with two vertices on sides AC and AB, respectively. Prove that lines $AA_1,\ BB_1,\ CC_1$ are concurrent.