

Geometry Review

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1. In a triangle ABC the median AM is half of side BC . The angle between AM and the height of the triangle, AH is 40° . Find the measure of BAC .
2. Let $CHOPIN$ be a regular hexagon, and let $OPERA$ be a regular pentagon. Find all possible values of measure of $\angle PIE$.
3. Right $\triangle ABC$ has $\overline{AB} = 3$, $\overline{BC} = 4$, and $\overline{AC} = 5$. Square $XYZW$ is inscribed in $\triangle ABC$ with X and Y on \overline{AC} , W on \overline{AB} , and Z on \overline{BC} . What is the side length of the square?
4. Let ABC be a triangle with $\angle BAC = 117^\circ$. The angle bisector of $\angle ABC$ intersects side AC at D . Suppose $\angle ABD = \angle ACB$, and $\angle BDA = \angle CBA$. Compute the measure of $\angle ABC$, in degrees.
5. An equilateral triangle lies inside a square of side length 2. Find the maximum possible side length of the triangle.
6. Triangle ABC has $BC = 2 \cdot AC$ and $\angle A = 3\angle B$. Compute $\angle A$.
7. Circle C with radius 2 has diameter \overline{AB} . Circle D is internally tangent to circle C at A . Circle E is internally tangent to circle C , externally tangent to circle D , and tangent to \overline{AB} . The radius of circle D is three times the radius of circle E and can be written in the form $\sqrt{m} - n$, where m and n are positive integers. Find $m + n$.
8. Let ABC be a triangle. The internal bisector of $\angle B$ meets AC in P . Let I be the incenter of ABC . If $AP + AB = CB$ and $AI = 10$, compute AP . *Hint: let P' be a point in the extension of ray BA such that $P'A = PA$.*
9. Point A lies on the circumference of a circle Ω with radius 78. Point B is placed such that AB is tangent to the circle and $AB = 65$, while point C is located on Ω such that $BC = 25$. Compute the length of \overline{AC} .
10. $\triangle ABC$ has side lengths $AB = 15$, $BC = 34$, and $CA = 35$. Let the circumcenter of ABC be O . Let D be the foot of the perpendicular from C to AB . Let R be the foot of the perpendicular from D to AC , and let W be the perpendicular foot from D to BC . Find the area of quadrilateral $CROW$.