

Pigeonhole Principle

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1 Varsity

2 Warm-up Problems

1. A point is chosen on the unit circle centered at origin at random. What is the probability that it is at least $\frac{1}{2}$ distance away from X -axis?
2. Alice and Bob play a game where they take turns tossing an unbiased coin. Alice goes first. First player to get a heads wins. What is the probability that Alice wins?
3. (Purple Comet 2014) Five men and Five women stand in a circle in a random order. What is the probability that every man stands next to at least one woman?

3 Probability Problems

1. Anish and Alex are playing tennis. Since Anish has never played tennis before, probability that he wins a game is 0.01. What is the expected number of games they need to play for Anish to win at least once?
2. (Purple Comet 2005) A jar contains 2 yellow candies, 4 red candies, and 6 blue candies. Candies are randomly drawn out of the jar one-by-one and eaten. The probability that the 2 yellow candies are eaten before any of the red candies are eaten is given by the fraction $\frac{m}{n}$ where m, n are relatively prime. Find $m + n$
3. (Purple Comet 2014) Steve needed to address a letter to 2743 Becker Road. He remembered the digis of the address, but he forgot the correct order of digits, so he wrote them down in random order. The probability that Steve got exactly two of the four digits correct is $\frac{m}{n}$ where m and n are relatively prime. Find $m + n$.
4. (Purple Comet 2013) A rectangle has side lengths 6 and 8. There are relatively prime positive integers m and n so that $\frac{m}{n}$ is the probability that a point randomly selected from the inside of the rectangle is closed to a side of the rectangle than to either diagonals of the rectangle. Find $m + n$.
5. (Purple Comet 2011) Let S be a random 4 element subset of $\{1, 2, 3, 4, 5, 6, 7, 8\}$. What is the expected value of largest element of S ?
6. (HMMT 2009) Two jokers are added to a deck of 52 cards and the entire stack of 54 cards is shuffled rmadomly. What is the expected numner of cards that will be strictly between two jokers?
7. (HMMT 2018) Rachel has the number 1000 in their hands. When she puts the numner x in her left pocket, the number changes to $x + 1$. When she puts the number x in her right

pocket, then number changes to x^{-1} . Each minute she put the number into one of the pockets at random. the she takes out the new number from that pocket. If the expected value of the number in Rachel's hands after eights minutes is E , compute $\lfloor \frac{E}{10} \rfloor$