# Field- and Shear-Driven Collective Phenomena in Suspensions

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**Support**

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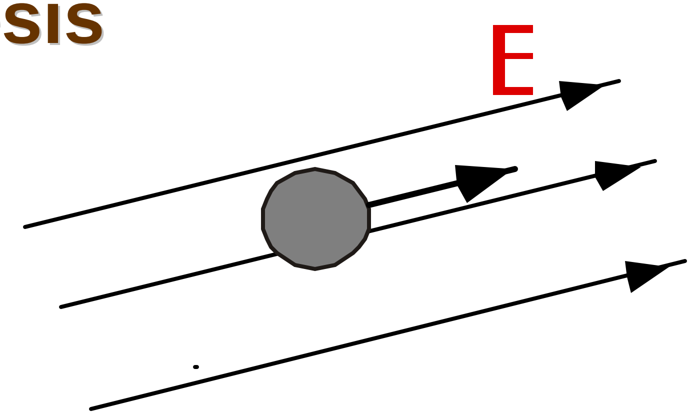
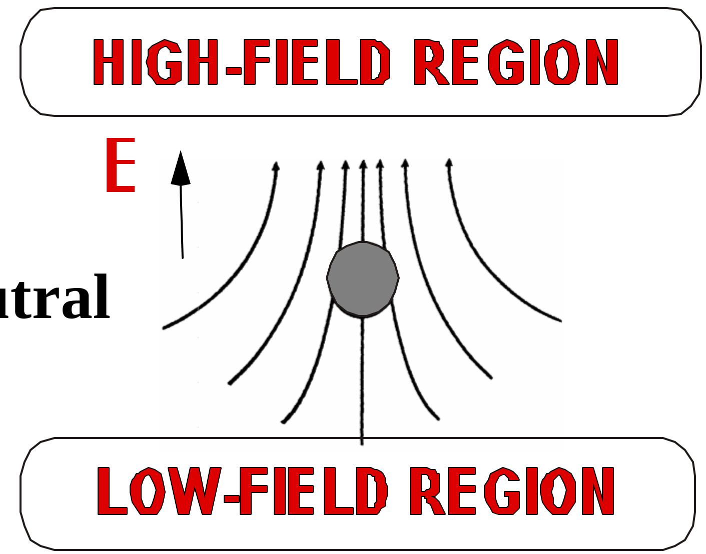
# Dielectrophoresis




**The force acting on a particle subject to a gradient electric field is**

$$\mathbf{F}_e = Q\mathbf{E} + (\mathbf{P}\cdot\nabla)\mathbf{E}$$



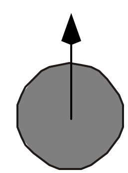

**Electrophoresis is the motion of a charged particle in a DC field**

**Dielectrophoresis is the motion of a neutral particle in gradient DC and AC fields**

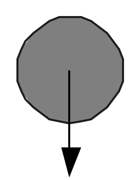
**The time average dielectrophoretic force in an AC field**

$$\langle \mathbf{F}_d \rangle = 2\pi\varepsilon_0\varepsilon_f a^3 \mathrm{Re}(\beta(\omega))\nabla \mathbf{E}^2_{rms}$$

$$\beta = \frac{\varepsilon_p^*(\omega) - \varepsilon_f^*(\omega)}{\varepsilon_p^*(\omega) + 2\varepsilon_f^*(\omega)}$$



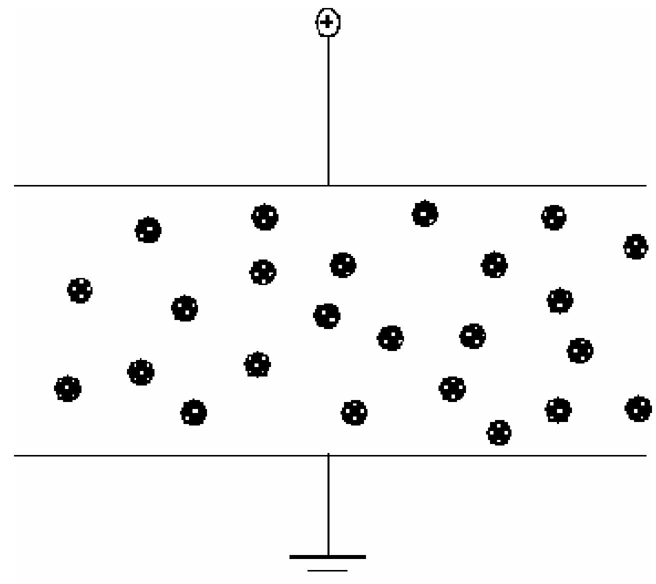
**Positive dielectrophoresis** $\mathrm{Re}(\beta) > 0$



**Negative dielectrophoresis** $\mathrm{Re}(\beta) < 0$

# Field-induced Phase Transition

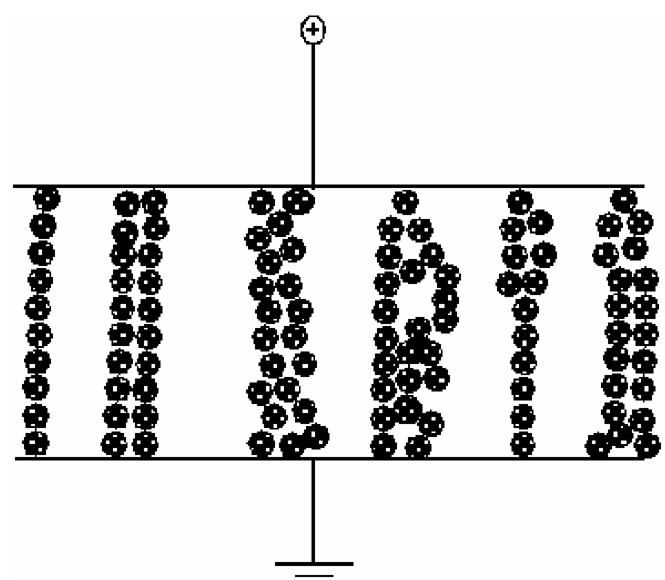
**A homogeneous random arrangement of particles**



$$\mathbf{E = 0}$$

**Electrorheological fluids**

**A variety of ordered aggregation patterns**



$$W_{el} \geq k_B T$$

# Measuring the Particle Polarization

**Dussaud, Khusid, Acrivos, J Appl Phys, 88, 2000**

- **Dielectric spectroscopy for measuring particle polarization for E~ 1 V/mm**

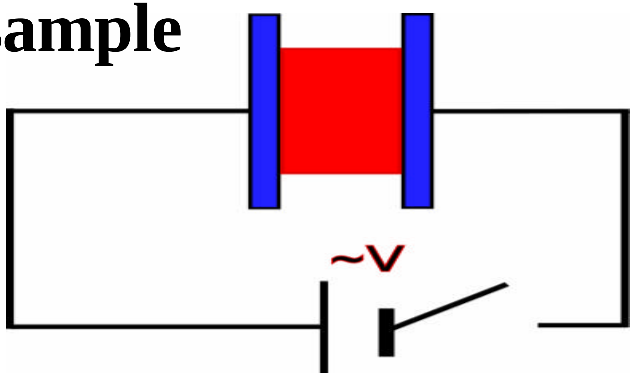
**The Maxwell-Wagner model**

$$\frac{\varepsilon_s^*(\omega, c) - \varepsilon_f^*}{\varepsilon_s^*(\omega, c) + 2\varepsilon_f^*} = c\beta(\omega)$$

- **Validated the equation for dielectrophoretic force for E~ 1 kV/mm**

## Dielectric Spectroscopy

**sample**




**E~1 V/mm**

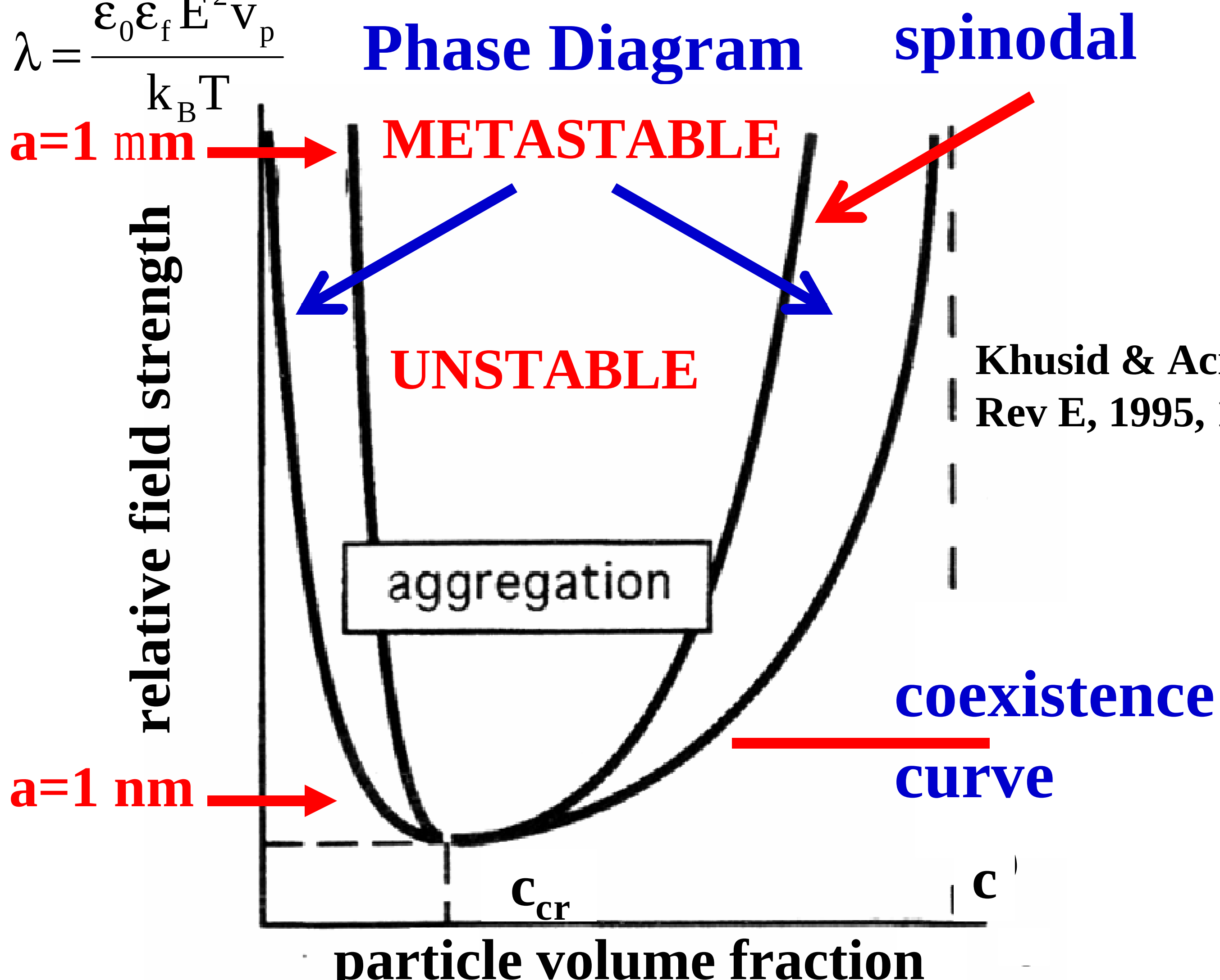
**DS measures the relation between time-varying voltage and current through a sample**

**NJIT W.M. Keck Laboratory**

# Field-induced Phase Separation

## Microscopic theory for field-induced phase transitions

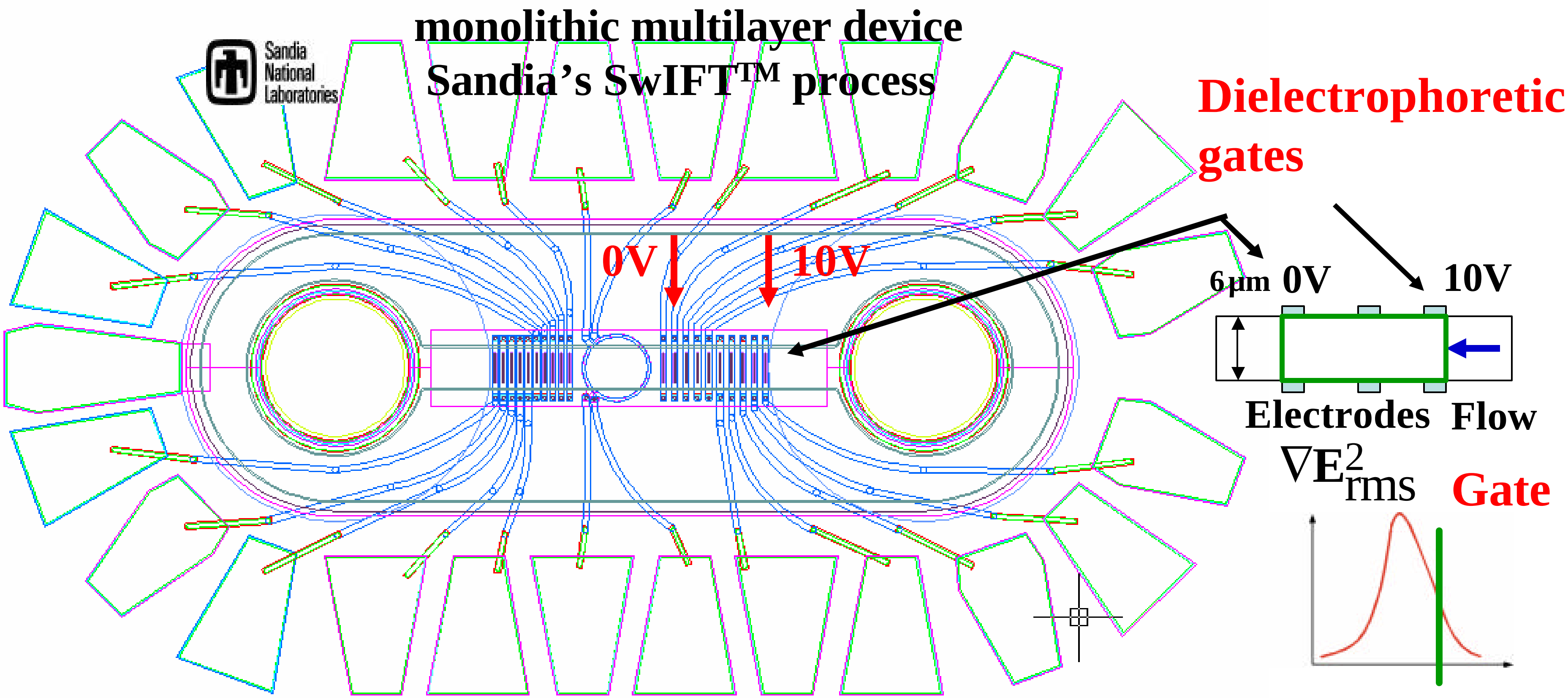
$$\lambda = \frac{\varepsilon_0 \varepsilon_f E^2 v_p}{k_B T}$$




Khusid & Acrivos, Phys Rev E, 1995, 1996, 1999

# Dielectrophoretic Particle Concentrator

$40\,\mu m\,(W) \times 6\,\mu m\,(H) \times 570\,\mu m\,(L)$ **10 Vptp, 15-30 MHz**




*Source:* **Bennett, Khusid, Galambos, James, Okandan, TRANSDUCERS'03, Boston, MA**

# Experimental Results

**1µm** **polystyrene spherical beads in DI water**, **0.1% (v/v)**

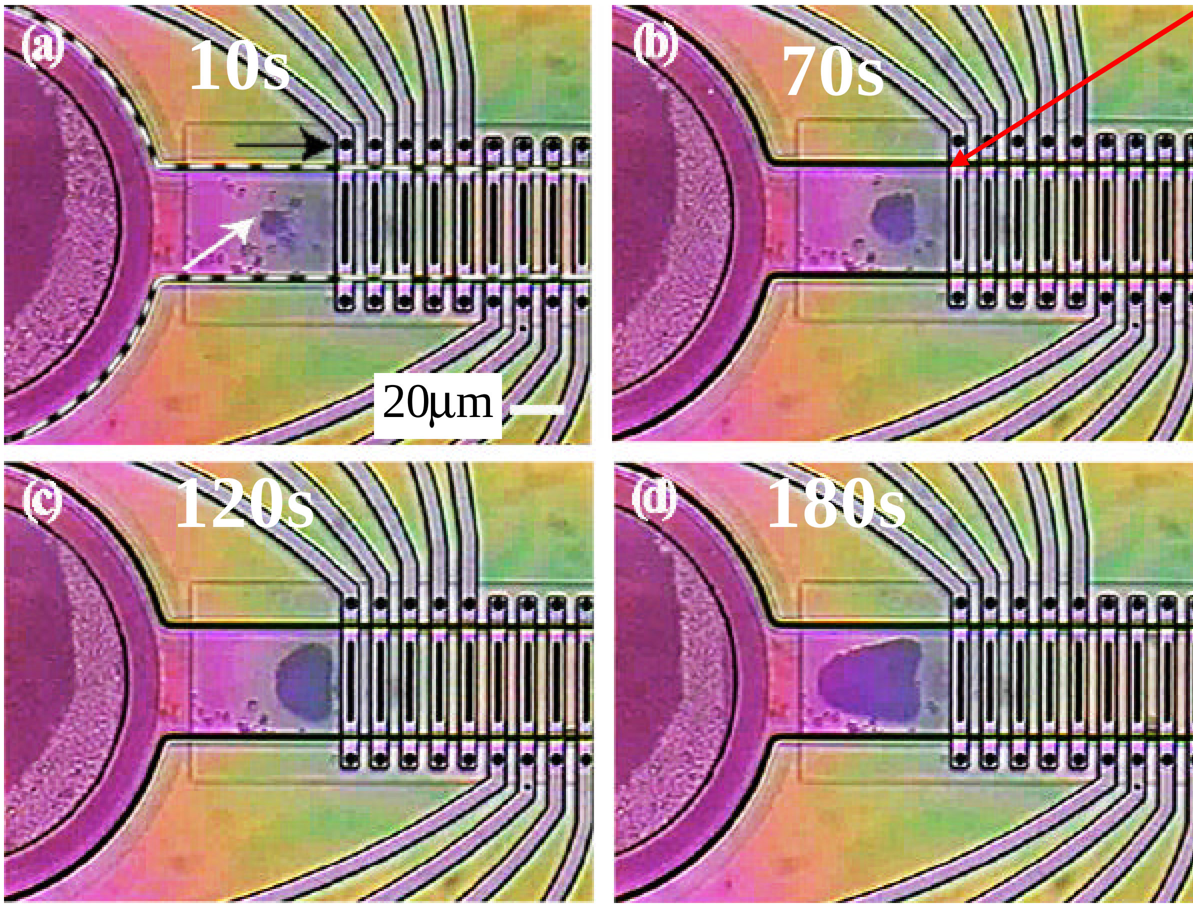
**Particle polarization** $ß = -0.45 - 0.27i$

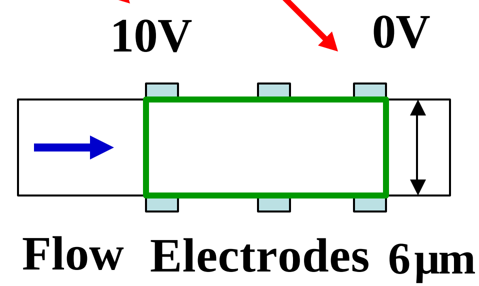
**Flow rate 0.24 pL/s to 9.6 pL/s; Re~$10^{-5}$- $10^{-3}$**

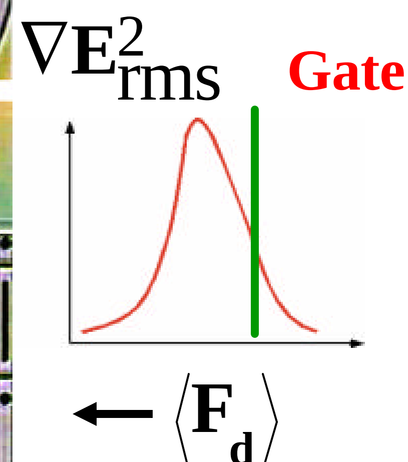
**Flow →** **$10V_{ptp}$ , 30 MHz**

**Dielectrophoretic gates**

**Phase transition**



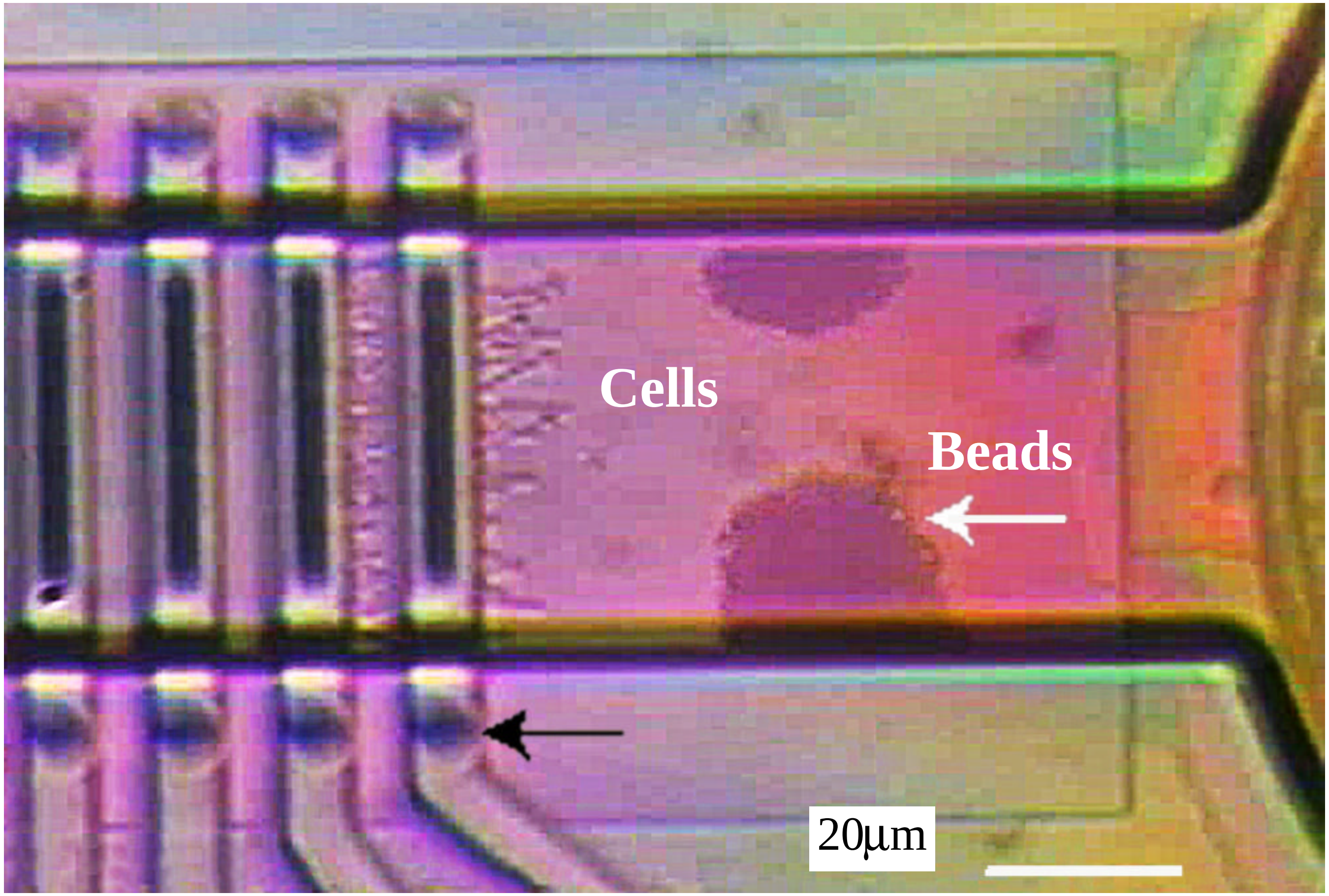




*Source:* Bennett, Khusid, Galambos, James, Okandan, Jacqmin, Acrivos, Appl Phys Lett, 2003

# Flowing Heterogeneous Mixture

## Beads and bacterial cells (heat-killed staphylococcus aureus)

**10 $V_{ptp}$, 15 MHz** **Flow rate 0.24 pL/s to 9.6 pL/s**




**Flow**

*Source:* **Bennett, Khusid, Galambos, James, Okandan, Jacqmin, Acrivos, Appl Phys Lett, 2003**

# Electro-hydrodynamic Model

**Chemical potential** (***Phys Rev E*, 1995-9**)

$$\mu_p = \frac{k_B T}{v_p}\frac{df_0}{dc} - \varepsilon_0 \left(\frac{\partial \varepsilon_s'}{\partial c}\right)_{\omega t_c} \left\langle \frac{\mathbf{E}^2}{2} \right\rangle$$

**Entropic factor** $f_0 = c(\ln c - 1) + c\int_0^c [(Z-1)/c]dc$

**Quasi-steady electrodynamic equations** $\langle \mathbf{E}^2 \rangle = \frac{1}{2}\left|\mathbf{E}_\omega^*(\mathbf{r})\right|^2$

$\nabla \mathbf{D}_\omega^*(\mathbf{r}) = 0$ $\nabla \times \mathbf{E}_\omega^*(\mathbf{r}) = 0$ **Electric displacement** $\mathbf{D}_\omega^* = \varepsilon_0 \varepsilon_s^*(\omega, c)\mathbf{E}_\omega^*$

**Suspension flow**

$$\rho_s\left(\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v}\nabla\mathbf{v}\right) = -\nabla p + \nabla \mathbf{s}^{vis} - c\nabla\mu_p + c(\rho_p - \rho_f)\mathbf{g} \qquad \nabla \mathbf{v} = 0$$

**Shear stress** $s_{ij}^{vis} = \eta_s\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i}\right)$ **Viscosity** $\eta_s = \eta_f\left(1 + \frac{1.5c}{1 - c/c_m}\right)^2$

**Particle balance** $\frac{\partial c}{\partial t} + \nabla(c\mathbf{v} + \mathbf{j}_p) = 0$ $\mathbf{j}_p = \frac{c(1-c)^2 v_p}{6\pi a \eta_s}\left[-\nabla\mu_p + (\rho_p - \rho_f)\mathbf{g}\right]$

**The particle polarization can be measured at low fields** → $\mathrm{Re}(\beta)$

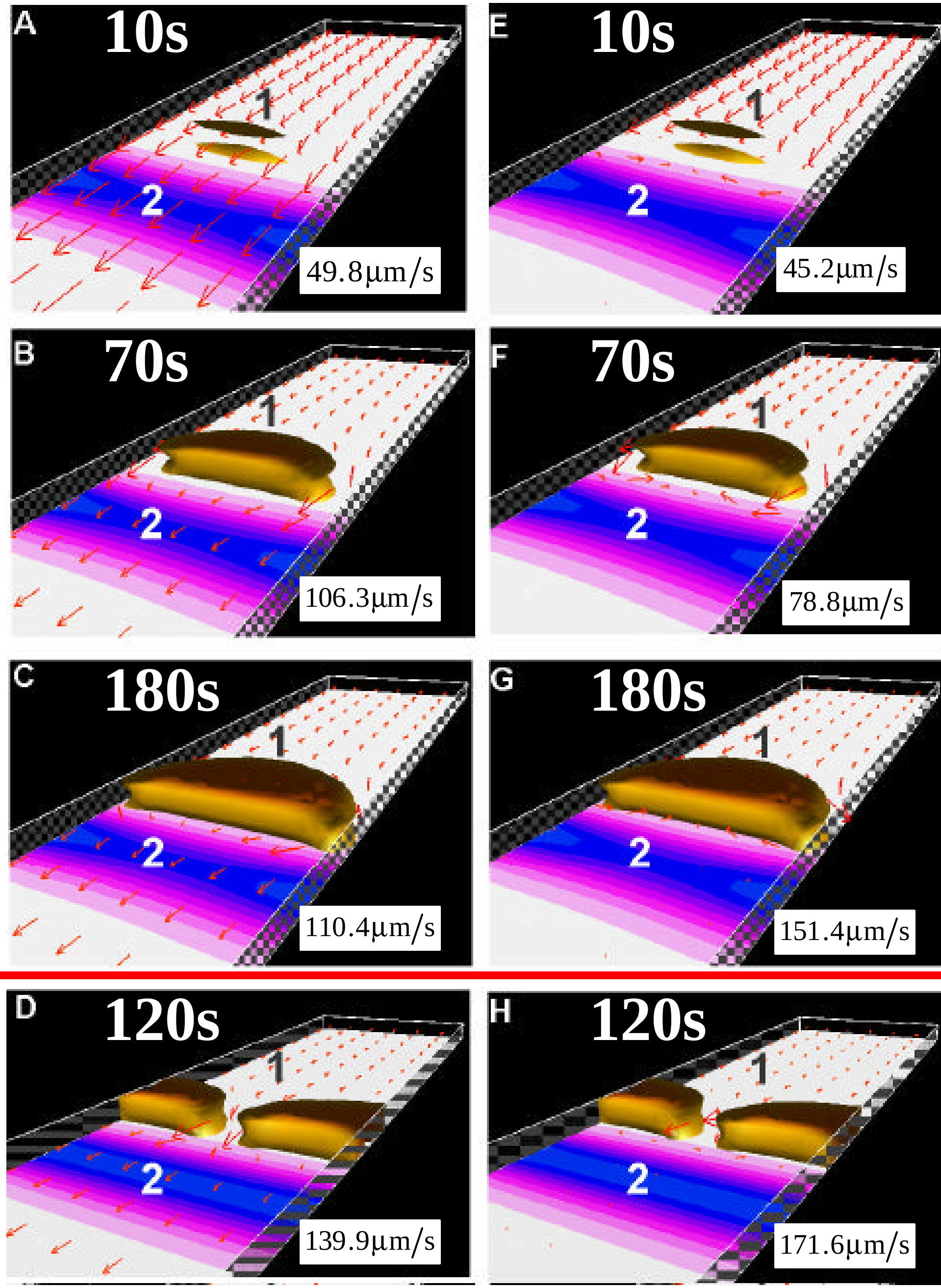
# Modeling

Flow velocity | Particle velocity

8.73%

45.4%

56.4%




← Single bolus

0.1%(v/v)-suspension

Flow rate 8.64 pL/s

Voltage $10V_{ptp}$

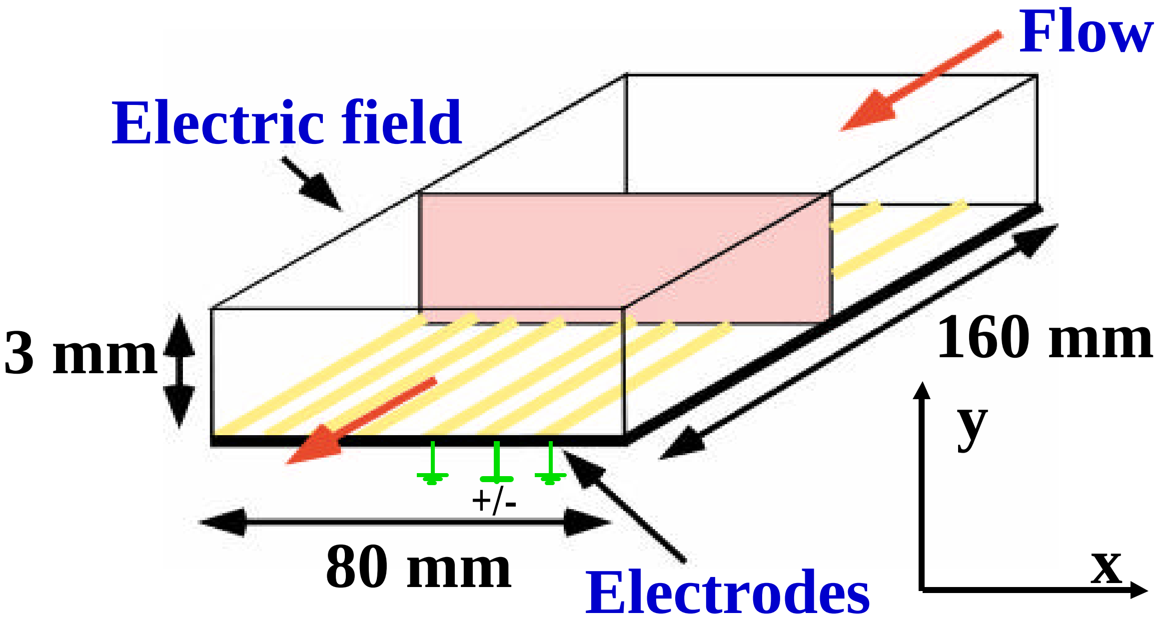
Average flow velocity 36 μm/s

1, concentration

2, field strength

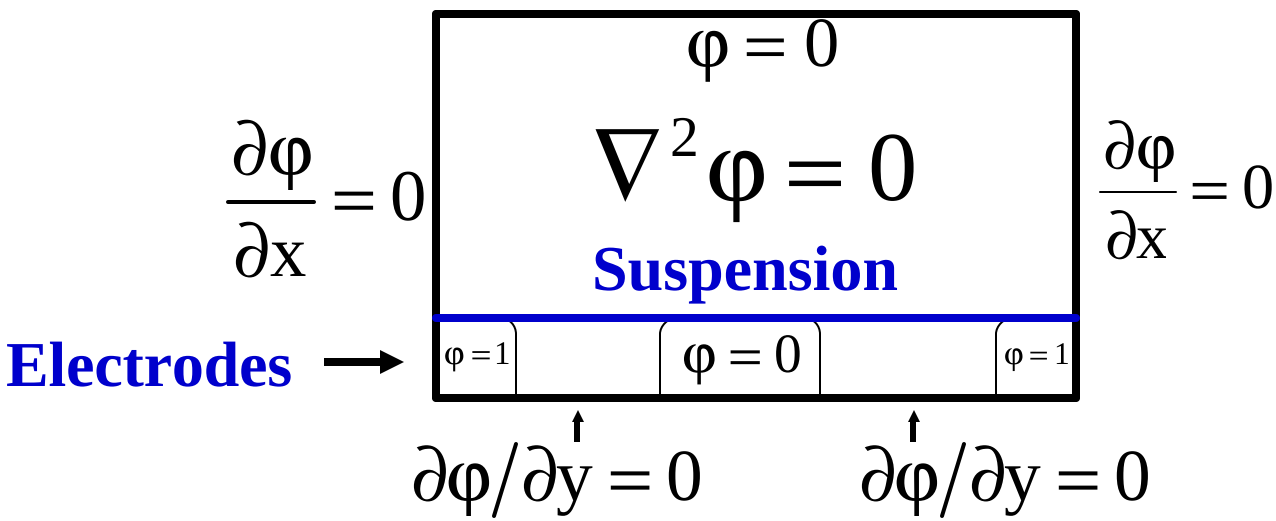
← Two boluses

# Electric Field Configuration



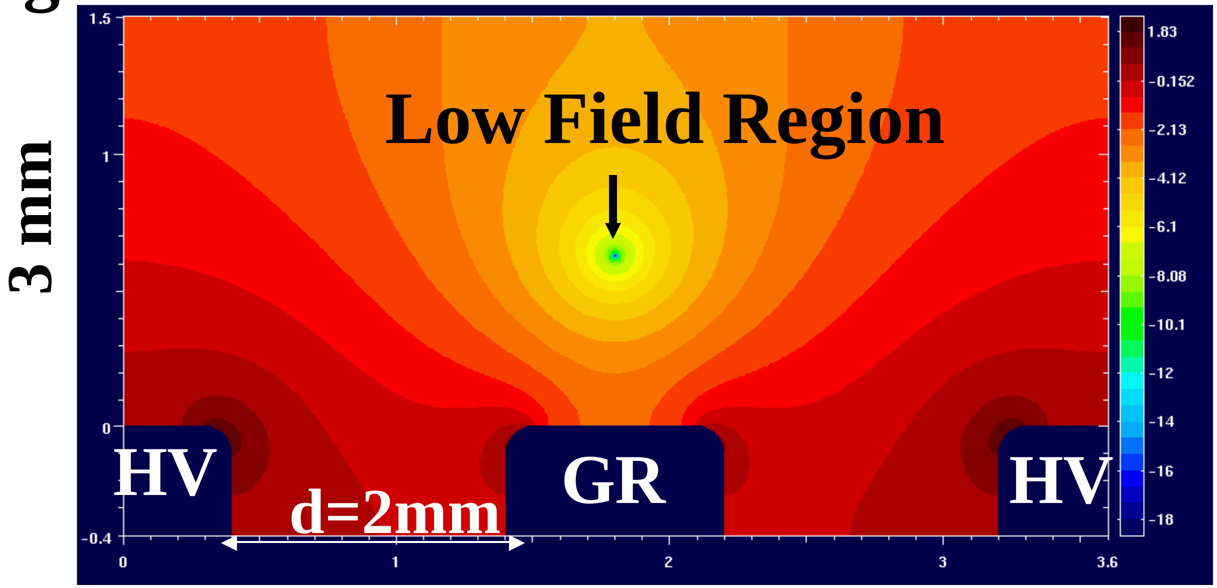

Dussaud, Khusid & Acrivos,
J Appl Phys., 88, 2000

## Computational cell



# Electric-field Strength

Log $E^2$ **Channel cross-section** $(V_{rms}/d)^2$




## Neutrally buoyant suspension

**Polyalphaolefin spheres (0.92 g/cm$^3$ , 90 mm)**
**in corn oil (0.92 g/cm$^3$, 0.06 Pa·s, eps=2.2)**

**Particle polarization in low field ~ 1V/mm**

**The Maxwell-Wagner model**

$$\frac{\varepsilon_s^*(\omega,\varphi)-\varepsilon_f^*}{\varepsilon_s^*(\omega,\varphi)+2\varepsilon_f^*}=\varphi\beta(\omega)$$

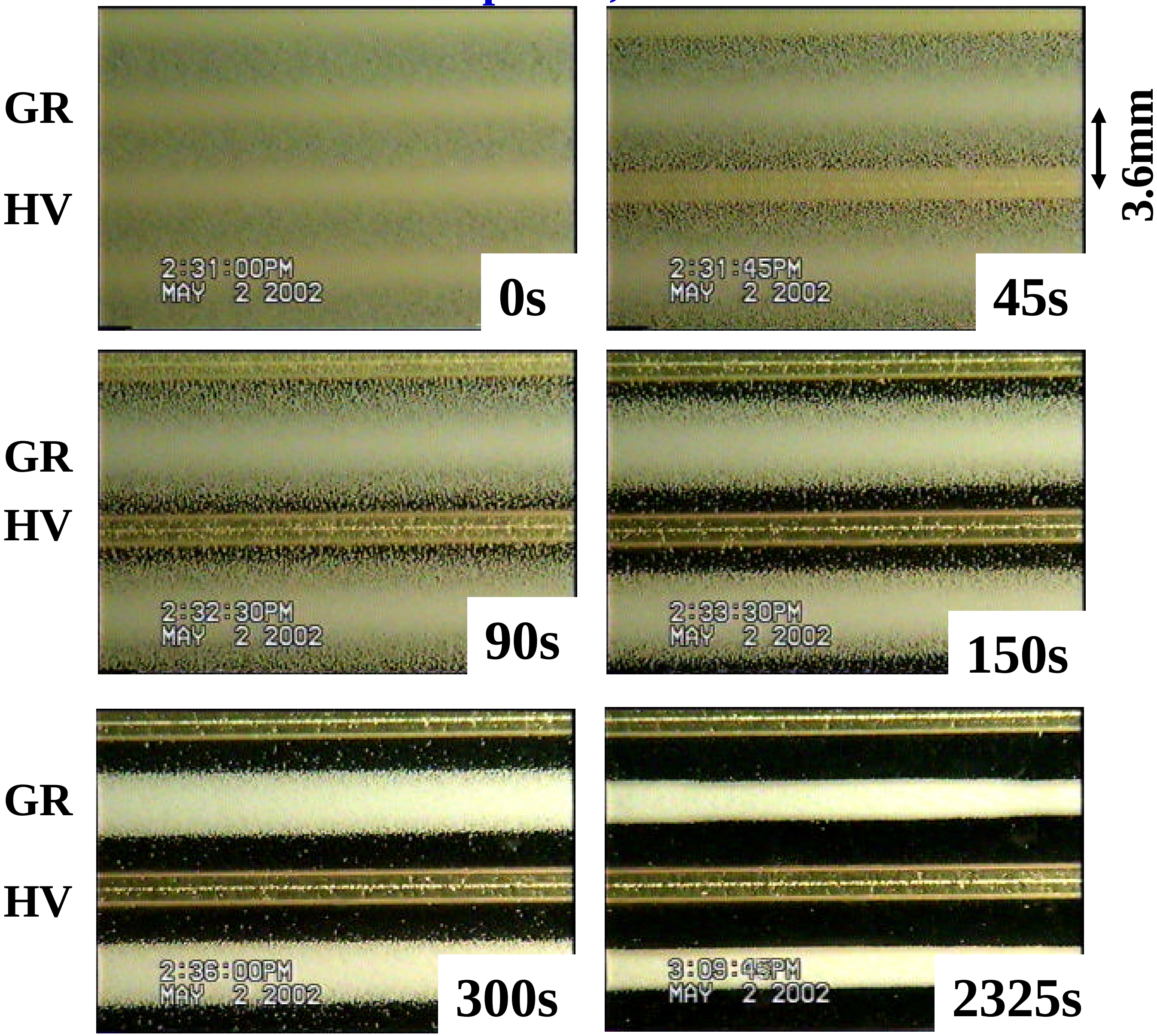
$$\mathrm{Re}(\beta)=-0.15$$

**for 100-1000 Hz**

*Source:* **Kumar, Qiu, Khusid, Jacqmin, Acrivos, Phys. Rev. E, 2004**

# Field-induced Segregation

Top view, 10%




**Neutrally buoyant polyalphaolefin spheres in corn oil**

$\mathrm{Re}(\beta) = -0.15$

**for 100-1000 Hz**

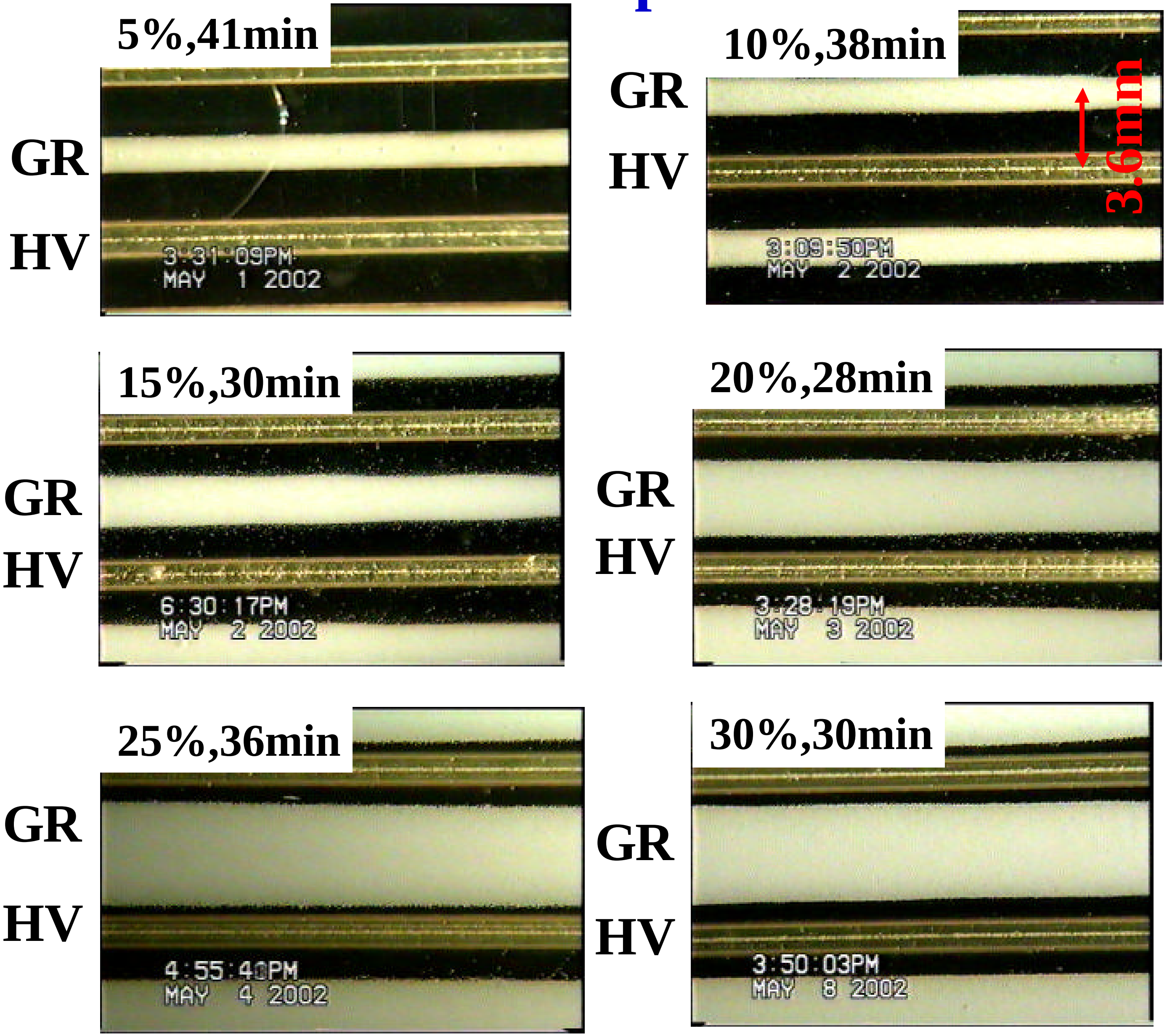
**5kv, 100Hz, without flow**

$V_{rms}/d$ **=2.5 kV/mm**

***Source:*** **Kumar, Qiu, Khusid, Jacqmin, Acrivos, Phys. Rev. E, 2004**

# Front Formation

## Top view

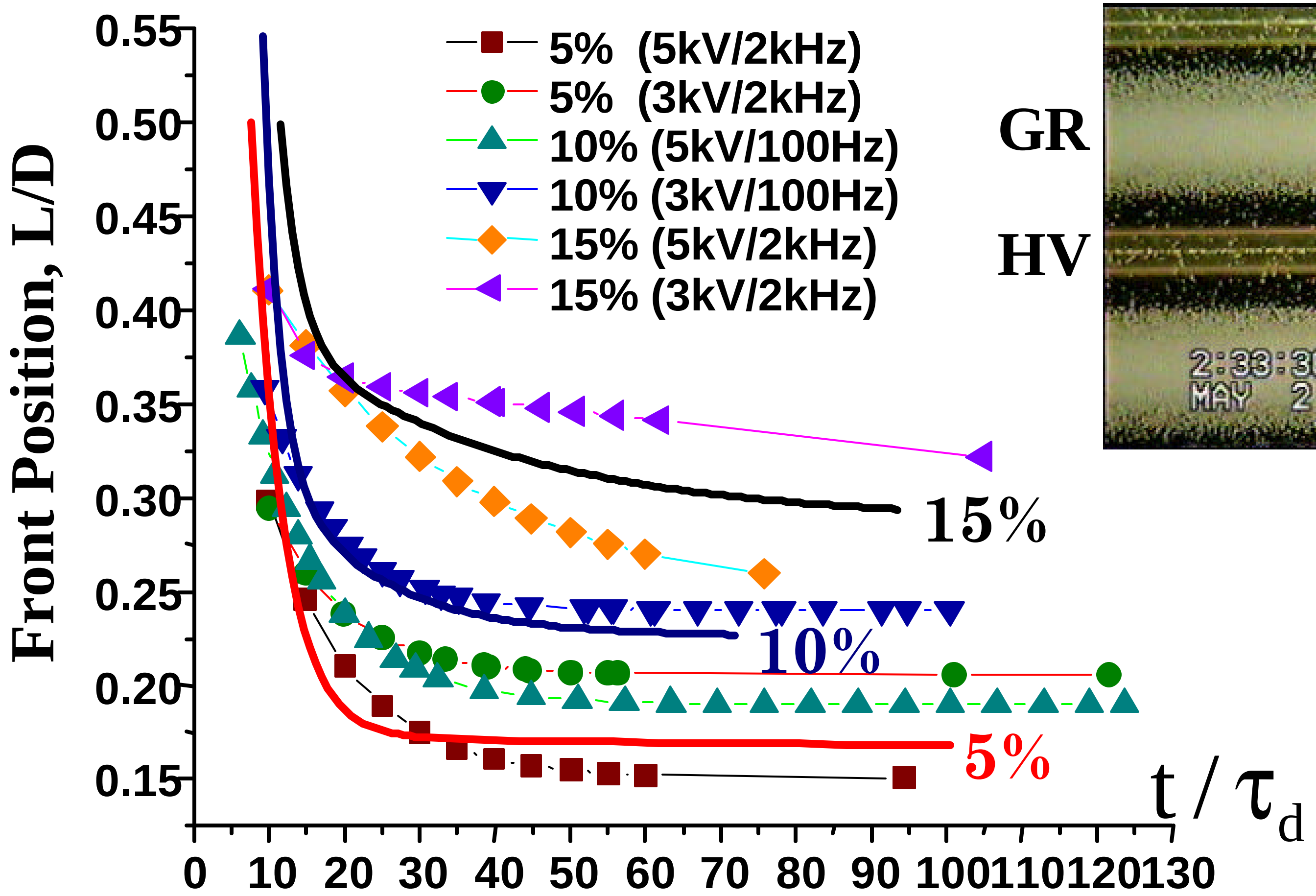


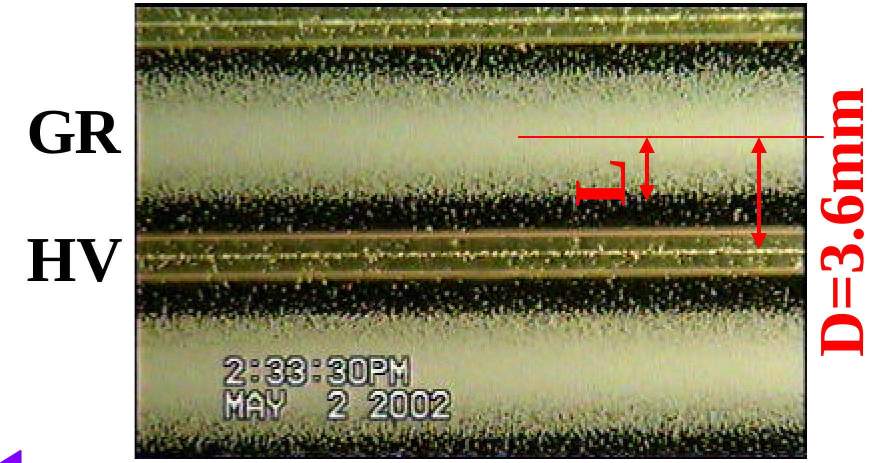

**5kV, 100Hz, without flow**

$$\frac{V_{rms}}{d} = 2.5\,\frac{\text{kV}}{\text{mm}}$$

***Source:*** **Kumar, Qiu, Khusid, Jacqmin, Acrivos, Phys. Rev. E, 2004**

# Comparison with Experiments



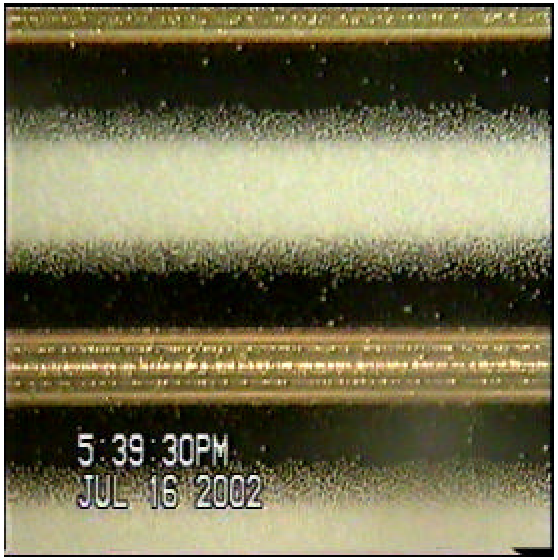

**Dielectrophoretic time**

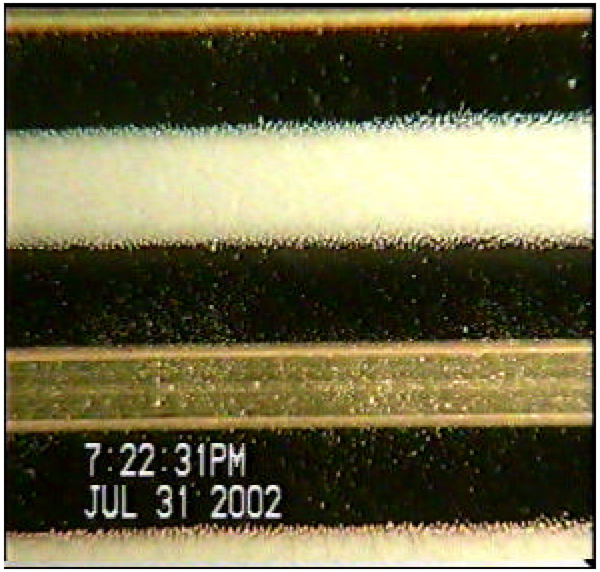
$$\tau_d = \frac{3d^4\eta_f}{a^2\varepsilon_0\varepsilon_f\left|\mathrm{Re}\left(\beta(\omega)\right)\right|V_{rms}^2}$$

***Source:*** **Kumar, Qiu, Khusid, Jacqmin, Acrivos, Phys. Rev. E, 2004**

# Field Strength and Frequency Effects

## Top view, 10% suspension

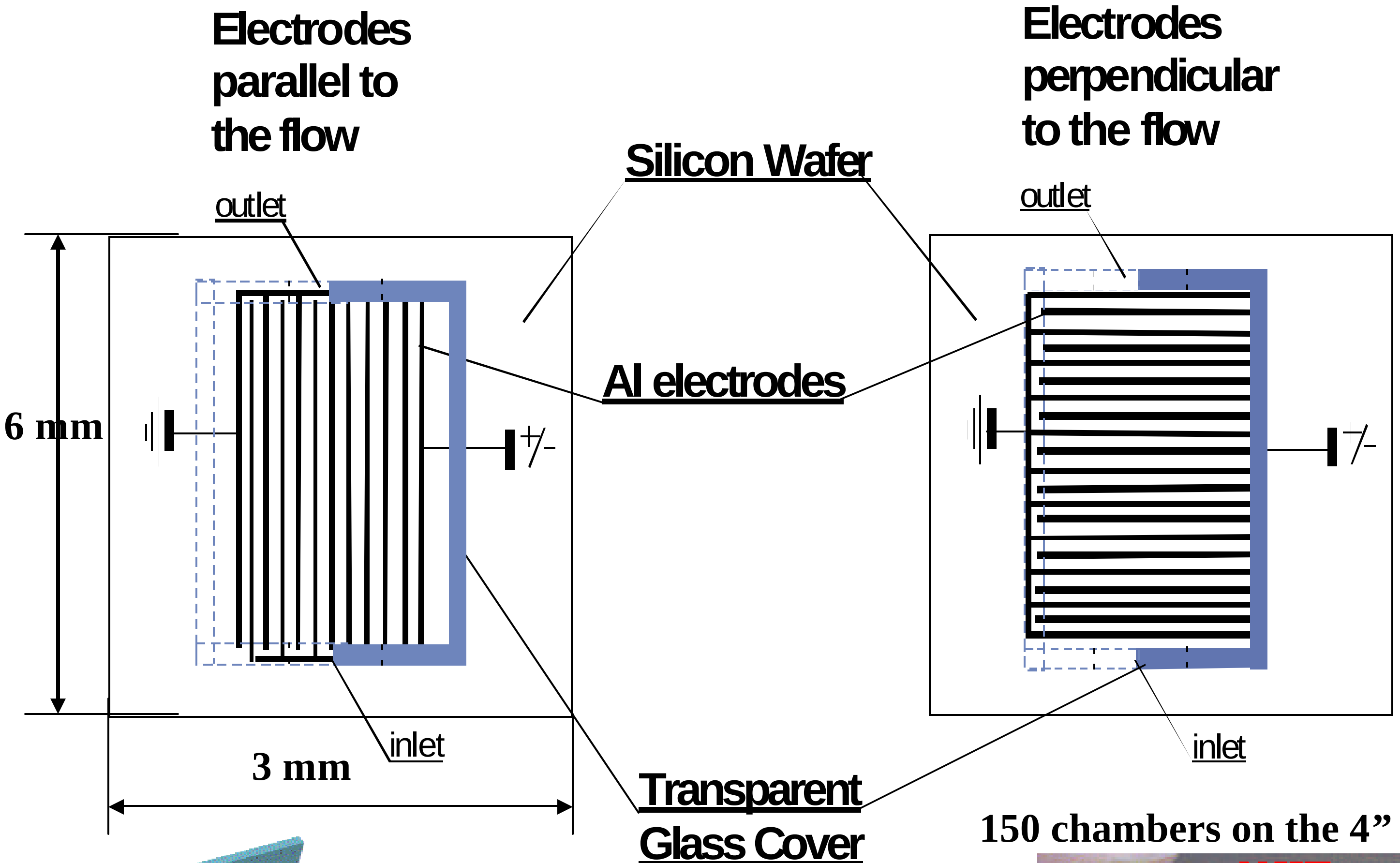




| 5kV, 100Hz | 3kV, 100Hz | 3kV, 2000Hz |
|---|---|---|
| t=20.5min | t=54.5min | t=52.5min |
| $t/\tau_d = 63.4$ | $t/\tau_d = 60.8$ | $t/\tau_d = 58.6$ |

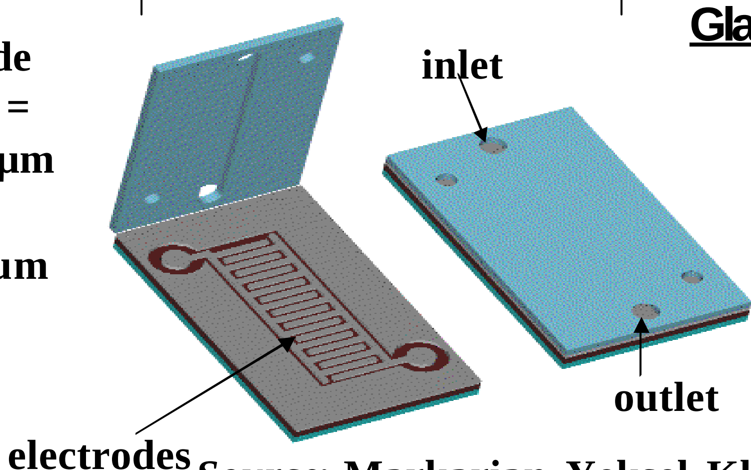
*Source:* **Kumar, Qiu, Khusid, Jacqmin, Acrivos, Phys. Rev. E, 2004**

# Multi-Channel Apparatus

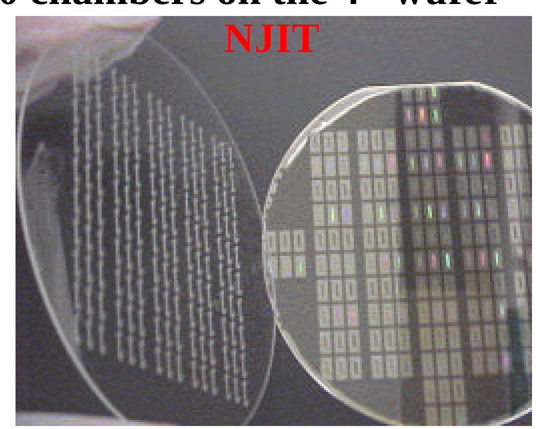



Electrode spacing = 2, 5, 10 μm

H = 30 μm




150 chambers on the 4” wafer




*Source*: Markarian, Yeksel, Khusid, Farmer, Acrivos, Appl Phys Lett, 82, 2003

# Model for Dilute Suspensions

**Qiu, Markarian, Khusid, Acrivos, J Appl Phys, 92, 2002**
**Dussaud, Khusid, Acrivos, J Appl Phys, 88, 2000**

- The balance of drag, dielectrophoretic, and gravitational forces

$$6\pi\eta_f a(\mathbf{u}-\mathbf{v}_f) = 2\pi\varepsilon_0\varepsilon_f a^3 \operatorname{Re}(\beta)\nabla E_{rms}^2 + \frac{4}{3}\pi(\rho_p-\rho_f)a^3 g\mathbf{e}$$

- The field-induced particle displacement

$$\frac{d\mathbf{r}}{dt} = \mathbf{u} \qquad \mathbf{r}|_{t=0} = \mathbf{r}_0$$

- The asymptotic expression for the spinodal

$$\varphi\lambda\Psi_\omega \approx 1 \qquad \Psi_\omega \sim 3\operatorname{Re}(\beta)^2 \qquad \lambda = \frac{\varepsilon_0\varepsilon_f E^2 v_p}{k_B T}$$

**The particle polarization can be measured at low fields** → $\operatorname{Re}(\beta)$

# Experimental Results

**Dioctyl Terephthalate:** $?_f$=0.98 g/cm$^3$ , $h_f$ =76 cp

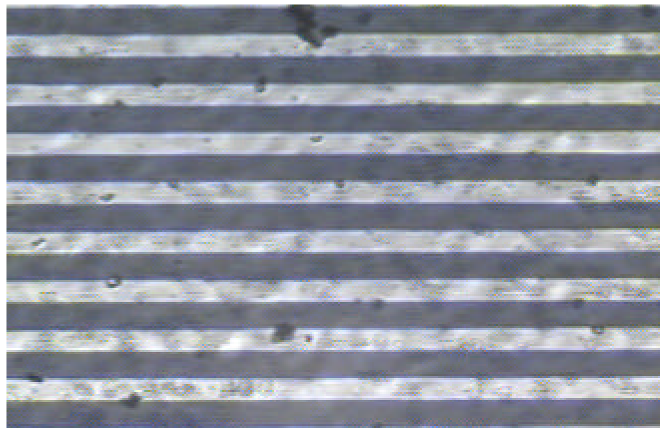
**$Al_2O_3$:** $?_p$=3.8 g/cm$^3$ a = 0.5-2 mm Re(b) = 0.35 for 1-10 kHz

**AC Field: 20V, 1kHz** 0.1% (v/v); Q = 0.05 µl/min; Re=$10^{-5}$

0 sec

X
Z

GR
HV
GR
HV
GR
HV
GR
HV

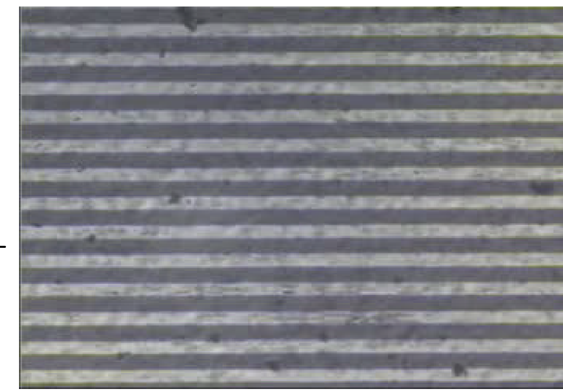


160 µm

240 µm

**Flow**

Movie, 0-900 s



**Measurement of Gray Level (GL)**

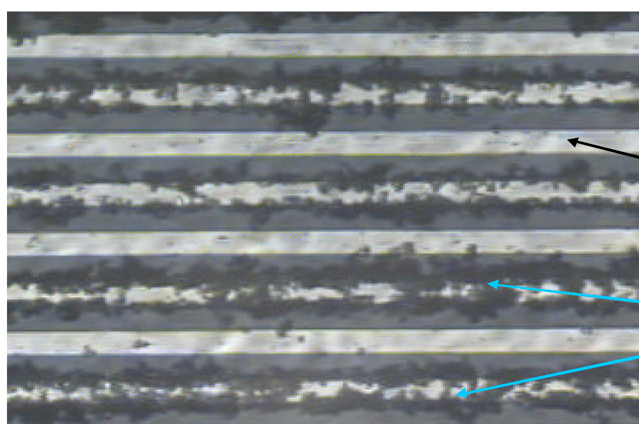
GL=0

$GL_0$ = 180

GL=255

900 sec

GR
HV
GR
HV
GR
HV
GR
HV



10 µm

electrodes → light

GL = $GL_0$

particles → dark

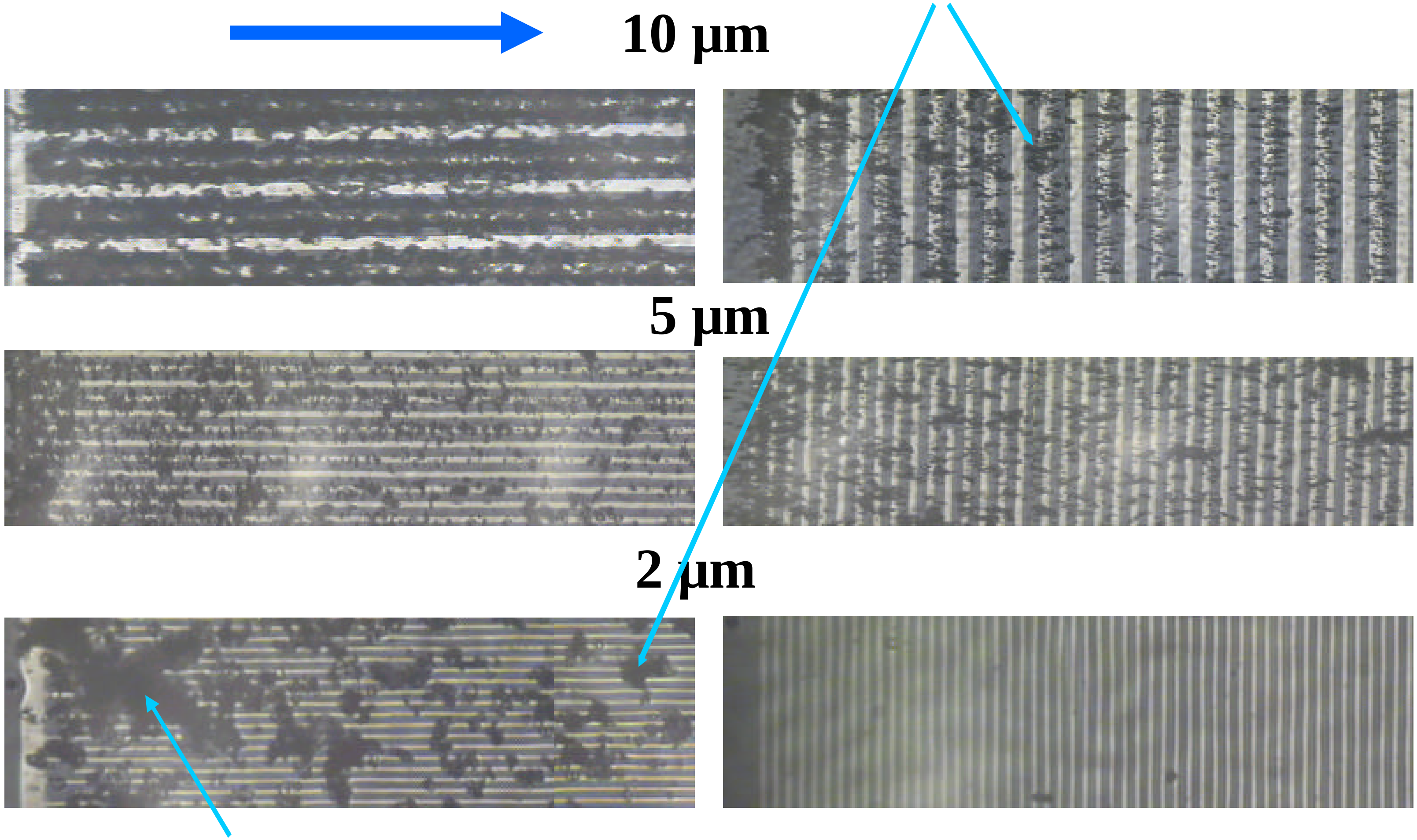
GL < $GL_0$

# Particle Positioning

t=900s

**Flow**

**Captured particles → black**




**Interparticle dipolar interactions prevail**

**Particle radius is 0.5-2.3 μm**

***Source*: Markarian, Yeksel, Khusid, Farmer, Acrivos, Appl Phys Lett, 82, 2003**

# Quantitative Comparison

$$\alpha \int_{z}^{L} (GL_0 - GL)\, dz$$ **(*empty triangles*) vs.** $$P(z)$$ **(*filled circles*)**

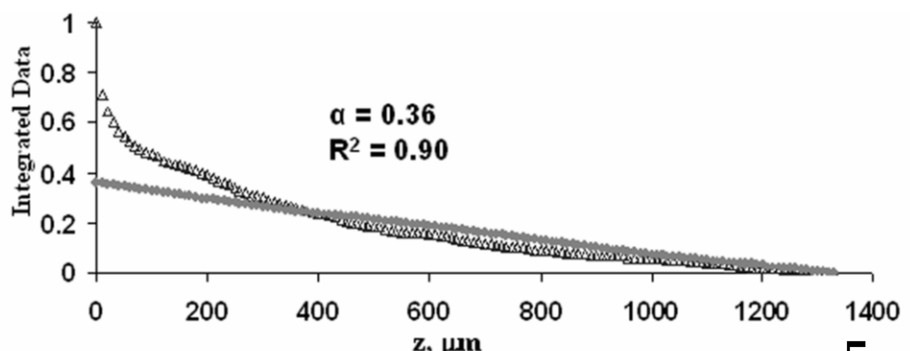
**P is the fraction of particles that had traveled beyond a certain length of the channel**

**$\alpha$, fitting parameter**

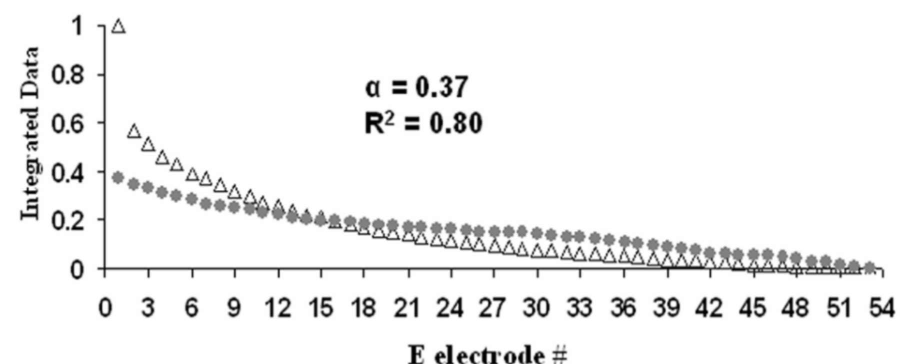
**$R^2$, the correlation coefficient**

## 10 µm

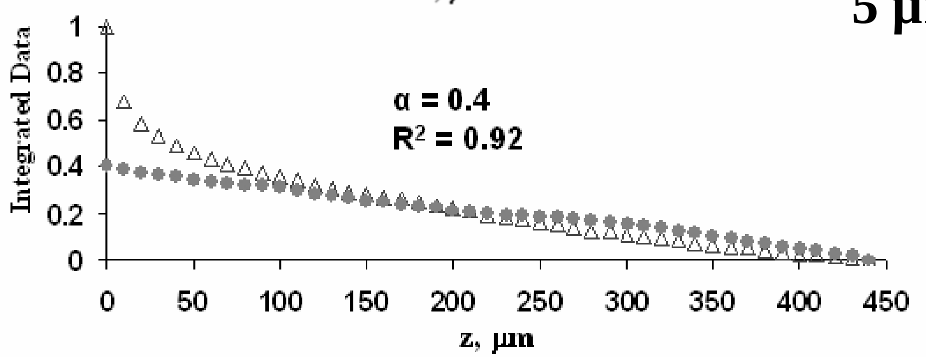
### Parallel Electrodes

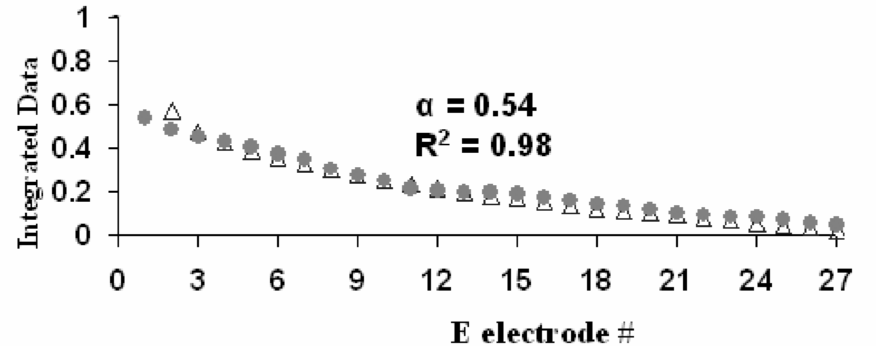



### Perpendicular Electrodes




## 5 µm




***Source:*** **Markarian, Yeksel, Khusid, Farmer, Acrivos, J Appl Phys, 2003**