The study of exotic hadrons in lattice QCD*

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Abstract

Recent studies of exotic hadrons from lattice QCD simulations are briefly reviewed. Hybrid $\bar{b}gb$ molecules in which the heavy $\bar{b}b$ pair is bound together by the excited gluon field g are investigated using the Born-Oppenheimer expansion and quenched numerical simulations. The consistency of results from the two approaches reveals a simple and compelling physical picture for heavy hybrid states. Results for the mass of the light-quark exotic meson from recent simulations are listed. These results agree with each other, but are significantly higher than the current experimental candidate.

1 Introduction

Hadronic states bound together by an *excited* gluon field, such as glueballs, hybrid mesons, and hybrid baryons, are a potentially rich source of information concerning the confining properties of QCD. Interest in such states has been recently sparked by observations of resonances with exotic 1^{-+} quantum numbers at Brookhaven[1]. In fact, the proposed Hall D at Jefferson Lab will be dedicated to the search for hybrid mesons and one of the goals of CLEO-c will be to identify glueballs and exotics. Although our understanding of these states remains deplorable, recent lattice simulations have shed some light on their nature. In this talk, I summarize our current knowledge about heavy- and light-quark hybrid mesons from lattice QCD simulations.

2 Heavy-quark hybrid mesons

One expects that a heavy-quark meson can be treated similar to a diatomic molecule: the slow valence heavy quarks correspond to the nuclei and the fast gluon and light sea quark

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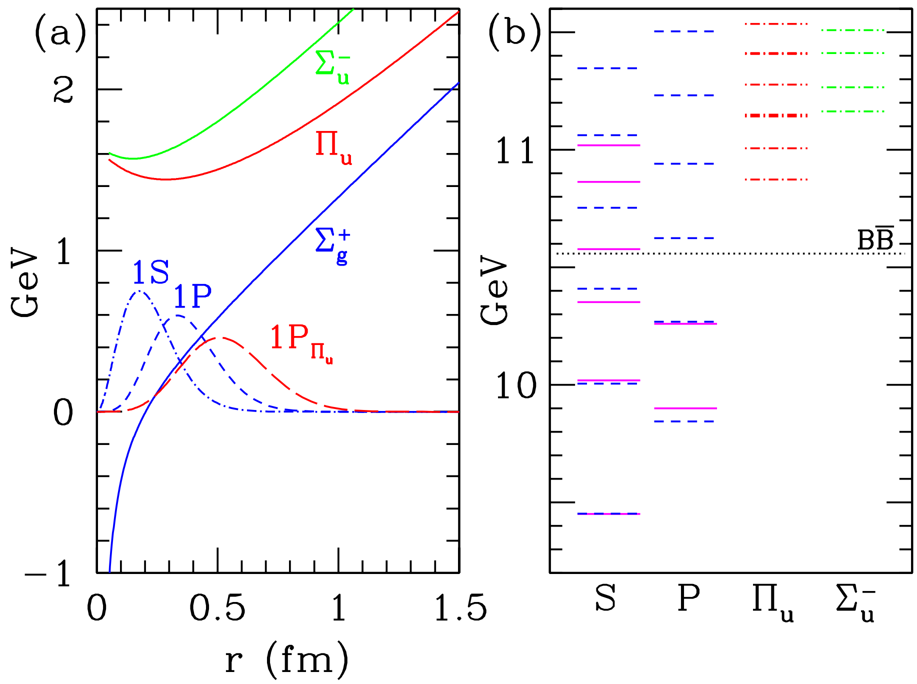

Figure 1: (a) Static potentials and radial probability densities against quark-antiquark separation r for $r_0^{-1} = 450$ MeV. (b) Spin-averaged $\bar{b}b$ spectrum in the LBO approximation (light quarks neglected). Solid lines indicate experimental measurements. Short dashed lines indicate the S and P state masses obtained using the Σ_g^+ potential with $M_b = 4.58$ GeV. Dashed-dotted lines indicate the hybrid quarkonium states obtained from the Π_u ($L = 1, 2, 3$) and Σ_u^- ($L = 0, 1, 2$) potentials. These results are from Ref. [5].

fields correspond to the electrons[2]. First, the quark Q and antiquark $\overline{Q}$ are treated as static color sources and the energy levels of the fast degrees of freedom are determined as a function of the $Q\overline{Q}$ separation r, each such energy level defining an adiabatic surface or potential. The motion of the slow heavy quarks is then described in the leading Born-Oppenheimer (LBO) approximation by the Schrödinger equation using each of these potentials. Conventional quarkonia are based on the lowest-lying potential; hybrid quarkonium states emerge from the excited potentials.

The spectrum of the fast gluon field in the presence of a static quark-antiquark pair has been determined in lattice studies[3, 4]. The three lowest-lying levels are shown in Fig. 1. Due to computational limitations, sea quark effects have been neglected in these calculations; their expected impact on the hybrid meson spectrum will be discussed below. The levels in Fig. 1 are labeled by the magnitude Λ of the projection of the total angular momentum $\mathbf{J}_g$ of the gluon field onto the molecular axis, and by $\eta = \pm 1$, the symmetry under the charge conjugation combined with spatial inversion about the midpoint between the Q and $\overline{Q}$. States with $\Lambda = 0, 1, 2, \ldots$ are denoted by $\Sigma, \Pi, \Delta, \ldots$, respectively. States which are even (odd) under the above-mentioned CP operation are denoted by the subscripts g (u). An additional $\pm$ superscript for the Σ states refers to even or odd symmetry under a reflection in a plane containing the molecular axis. The potentials are calculated in terms of the hadronic

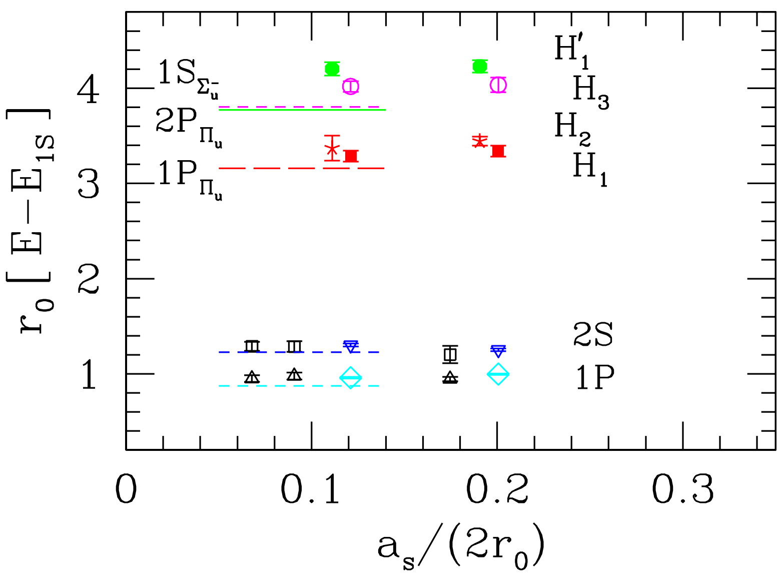

Figure 2: Simulation results from Ref. [5] for the heavy quarkonium level splittings (in terms of r_0 and with respect to the $1S$ state) against the lattice spacing a_s. Results from Ref. [6] using an NRQCD action with higher-order corrections are shown as open boxes and $\triangle$. The horizontal lines show the LBO predictions. Agreement of these splittings within 10% validates the Born-Oppenheimer approximation.

scale parameter r_0; in Fig. 1, $r_0^{-1} = 450$ MeV has been assumed.

The LBO spectrum[5] of conventional $\bar{b}b$ and hybrid $\bar{b}gb$ states are shown in Fig. 1. Below the $\overline{B}B$ threshold, the LBO results are in very good agreement with the spin-averaged experimental measurements of bottomonium states. Above the threshold, agreement with experiment is lost, suggesting significant corrections either from mixing and other higher-order effects or (more likely) from light sea quark effects. Note from the radial probability densities shown in Fig. 1 that the size of the hybrid state is large in comparison with the conventional $1S$ and $1P$ states.

The validity of such a simple physical picture relies on the smallness of higher-order spin, relativistic, and retardation effects and mixings between states based on different adiabatic surfaces. The importance of retardation and leading-order mixings between states based on different adiabatic potentials can be tested by comparing the LBO level splittings with those determined from meson simulations using a leading-order non-relativistic (NRQCD) heavy-quark action. Such a test was carried out in Ref. [5]. The NRQCD action included only a covariant temporal derivative and the leading kinetic energy operator (with two other operators to remove lattice spacing errors). The only difference between the leading Born-Oppenheimer Hamiltonian and the lowest-order NRQCD Hamiltonian was the $\boldsymbol{p}\cdot\boldsymbol{A}$ coupling between the quark color charge in motion and the gluon field. The level splittings (in terms of r_0 and with respect to the $1S$ state) of the conventional $2S$ and $1P$ states and four hybrid states were compared (see Fig. 2) and found to agree within 10%, strongly supporting the validity of the leading Born-Oppenheimer picture.

The question of whether or not quark spin interactions spoil the validity of the Born-

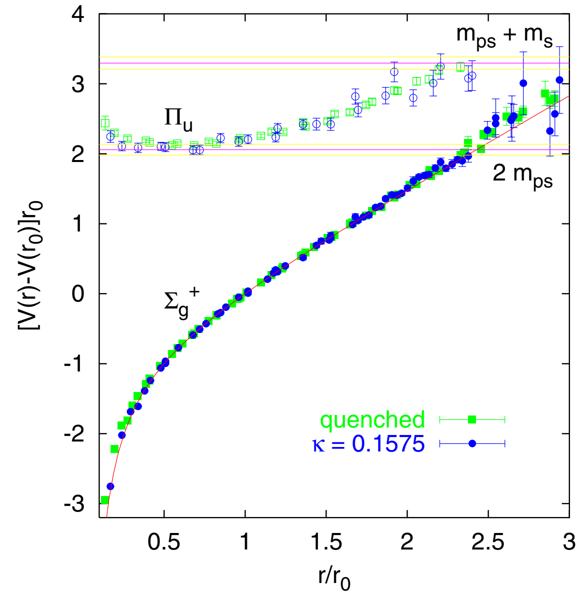

Figure 3: Ground Σ_g^+ and first-excited Π_u static quark potentials without sea quarks (squares, quenched) and with two flavors of sea quarks, slightly lighter than the strange quark (circles, $\kappa = 0.1575$). Results are given in terms of the scale $r_0 \approx 0.5$ fm, and the lattice spacing is $a \approx 0.08$ fm. Note that m_S and m_{PS} are the masses of a scalar and pseudoscalar meson, respectively, consisting of a light quark and a static antiquark. These results are from Ref. [9].

Oppenheimer picture for heavy-quark hybrids has been addressed in Ref. [7]. Simulations of several hybrid mesons using an NRQCD action including the spin interaction $c_B\boldsymbol{\sigma}\cdot\boldsymbol{B}/2M_b$ and neglecting light sea quark effects were carried out; the introduction of the heavy-quark spin was shown to lead to significant level shifts (of order 100 MeV or so) but the authors of Ref. [7] argue that these splittings do *not* signal a breakdown of the Born-Oppenheimer picture. First, they claim that no significant mixing of their non-exotic 0^{-+}, 1^{--}, and 2^{-+} hybrid meson operators with conventional states was observed; unfortunately, this claim is not convincing since a correlation matrix analysis was not used. Secondly, the authors argue that calculations using the bag model support their suggestion. These facts are not conclusive evidence that heavy-quark spin effects do not spoil the Born-Oppenheimer picture, but they are highly suggestive. Further evidence to support the Born-Oppenheimer picture has recently emerged in Ref. [8]. The NRQCD simulations carried out in this work examined the mixing of the Υ with a hybrid and found a very small probability admixture of hybrid in the Υ given by $0.0035(1)c_B^2$ where $c_B^2 \sim 1.5 - 3$ is expected.

The dense spectrum of hybrid states shown in Fig. 1 neglects the effects of light sea quark-antiquark pairs. In order to include these effects in the LBO, the adiabatic potentials must be determined fully incorporating the light quark loops. Such computations using lattice simulations are very challenging, but good progress is being made. For separations below 1 fm, the Σ_g^+ and Π_u potentials change very little[9] upon inclusion of the sea quarks (see Fig. 3), suggesting that a few of the lowest-lying hybrid states may exist as well-defined

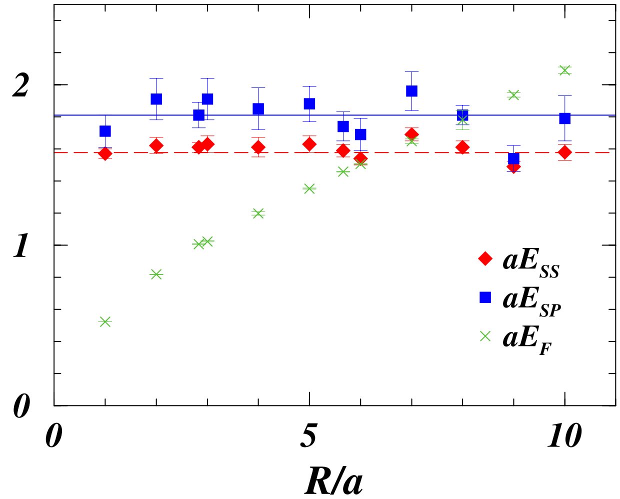

Figure 4: Evidence for "string breaking" at quark-antiquark separations $R \approx 1$ fm. E_{SS} is the energy of two S-wave static-light mesons (the light quark bound in an S-wave to the fixed static antiquark), E_{SP} is the energy of an S-wave and a P-wave static-light meson, and E_F is the energy of a static quark-antiquark pair connected by a gluonic flux tube. The distance of separation R refers to the distance between the static quark-antiquark pair. All quantities are measured in terms of the lattice spacing $a \approx 0.16$ fm. Two flavors of light sea quarks are present with masses such that $m_\pi/m_\rho \approx 0.36$. The dashed and solid lines give the asymptotic values $2am_S$ and $a(m_P + m_S)$, where m_S and m_P are the masses of individual S-wave and P-wave static-light mesons, respectively. Mixing between the flux tube and meson-meson channels was found to be very weak. Results are from Ref. [10].

resonances. However, for $Q\overline{Q}$ separations greater than 1 fm, the adiabatic surfaces change dramatically, as shown in Fig. 4 from Ref. [10]. Instead of increasing indefinitely, the static potential abruptly levels off at a separation of 1 fm when the static quark-antiquark pair, joined by flux tube, undergoes fission into two separate $\overline{Q}q$ color singlets, where q is a light quark. Clearly, such potentials cannot support the plethora of conventional and hybrid states shown in Fig. 1; the formation of bound states and resonances substantially extending over 1 fm seems unlikely. Whether or not the light sea quark-antiquark pairs spoil the Born-Oppenheimer picture is currently unknown. Future unquenched simulations should help to answer this question, but it is not unreasonable to speculate that the simple physical picture provided by the Born-Oppenheimer expansion for both the low-lying conventional and hybrid heavy-quark mesons will survive the introduction of the light sea quark effects. Note that the discrepancies of the spin-averaged LBO predictions with experiment above the $B\overline{B}$ threshold seen in Fig. 1 most likely arise from the neglect of light sea quark-antiquark pairs.

Table 1: Recent results for the light quark and charmonium 1^{-+} hybrid meson masses. Method abbreviations: W = Wilson fermion action; SW = improved clover fermion action; NR = nonrelativistic heavy quark action. N_f is the number of dynamical light quark flavors used.

Light quark 1^{-+}				Charmonium 1^{-+} $-$ $1S$		
Ref. & Method		N_f	M (GeV)	Ref. & Method		ΔM (GeV)
UKQCD 97[11]	SW	0	1.87(20)	MILC 97[12]	W	1.34(8)(20)
MILC 97[12]	W	0	1.97(9)(30)	MILC 99[13]	SW	1.22(15)
MILC 99[13]	SW	0	2.11(10)	CP-PACS 99[15]	NR	1.323(13)
LaSch 99[14]	W	2	1.9(2)	JKM 99[5]	LBO	1.19

3 Light-quark hybrid mesons

A summary of recent light-quark and charmonium 1^{-+} hybrid mass calculations is presented in Table 1. With the exception of Ref. [14], all results neglect light sea quark loops. The introduction of two flavors of dynamical quarks in Ref. [14] yielded little change to the hybrid mass, but this finding should not be considered definitive due to uncontrolled systematics (unphysically large quark masses, inadequate treatment of resonance properties in finite volume, *etc.*). All estimates of the light quark hybrid mass are near 2.0 GeV, well above the experimental candidates found in the range 1.4-1.6 GeV. Perhaps sea quark effects will resolve this discrepancy, or perhaps the observed states are *not* hybrids. Some authors have suggested that they may be four quark $\bar{q}\bar{q}qq$ states. Clearly, there is still much to be learned about these exotic QCD resonances.

4 Conclusion and Outlook

Our current understanding of hadronic states containing excited glue is poor, but recent lattice simulations have shed some light on their properties. The validity of a Born-Oppenheimer treatment for heavy-quark mesons, both conventional and hybrid, has been verified at leading order in the absence of light sea quark effects, and quark spin interactions do not seem to spoil this. Progress in including the light sea quarks is also being made, and it seems likely that a handful of heavy-quark hybrid states might survive their inclusion. Of course, much more work is needed. Future lattice simulations should provide insight into hybrid meson production and decay mechanisms and the spectrum and nature of hybrid baryons; virtually nothing is known about either of these topics. Glueballs, hybrid mesons, and hybrid baryons, remain a potentially rich source of information (and perhaps surprises) about the confining properties of QCD.

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