Recent highlights with baryons from lattice QCD

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12th Int. Workshop on the Physics of Excited Nucleons
Bonn, Campus Poppelsdorf

June 12, 2019

Overview

- recent highlights involving baryons in lattice QCD
  - proton mass decomposition
  - nucleon spin decomposition
  - percent level determination of nucleon axial coupling
  - proton and neutron electromagnetic form factors
  - parton distribution function
  - scattering amplitudes
  - baryon-baryon interactions with HAL QCD method
  - H-dibaryon warm up
- key progress
  - achieving much better precision with disconnected diagrams
  - ability to include multi-hadron operators
  - more and more studies being done at physical point

Temporal correlations from path integrals

- stationary-state energies from $N \times N$ Hermitian correlation matrix

$$C_{ij}(t) = \langle 0|\, O_i(t+t_0)\, \overline{O}_j(t_0)\ |0\rangle$$

- judiciously designed operators $\overline{O}_j$ create states of interest

$$O_j(t) = O_j[\overline{\psi}(t), \psi(t), U(t)]$$

- correlators from path integrals over quark $\psi, \overline{\psi}$ and gluon U fields

$$C_{ij}(t) = \frac{\int \mathcal{D}(\overline{\psi}, \psi, U)\ \ O_i(t+t_0)\, \overline{O}_j(t_0)\ \ \exp\left(-S[\overline{\psi}, \psi, U]\right)}{\int \mathcal{D}(\overline{\psi}, \psi, U)\ \exp\left(-S[\overline{\psi}, \psi, U]\right)}$$

- involves the action in imaginary time

$$S[\overline{\psi}, \psi, U] = \overline{\psi}\, K[U]\, \psi + S_G[U]$$

- $K[U]$ is fermion Dirac matrix
- $S_G[U]$ is gluon action

Integrating the quark fields

- integrals over Grassmann-valued quark fields done exactly
- meson-to-meson example:

$$\int \mathcal{D}(\overline{\psi},\psi)\ \psi_a\psi_b\ \overline{\psi}_c\overline{\psi}_d\ \exp\left(-\overline{\psi}K\psi\right)$$
$$= \left(K^{-1}_{ad}K^{-1}_{bc} - K^{-1}_{ac}K^{-1}_{bd}\right)\det K.$$

- baryon-to-baryon example:

$$\int \mathcal{D}(\overline{\psi},\psi)\ \psi_{a_1}\psi_{a_2}\psi_{a_3}\ \overline{\psi}_{b_1}\overline{\psi}_{b_2}\overline{\psi}_{b_3}\ \exp\left(-\overline{\psi}K\psi\right)$$
$$= \Big(-K^{-1}_{a_1b_1}K^{-1}_{a_2b_2}K^{-1}_{a_3b_3} + K^{-1}_{a_1b_1}K^{-1}_{a_2b_3}K^{-1}_{a_3b_2} + K^{-1}_{a_1b_2}K^{-1}_{a_2b_1}K^{-1}_{a_3b_3}$$
$$- K^{-1}_{a_1b_2}K^{-1}_{a_2b_3}K^{-1}_{a_3b_1} - K^{-1}_{a_1b_3}K^{-1}_{a_2b_1}K^{-1}_{a_3b_2} + K^{-1}_{a_1b_3}K^{-1}_{a_2b_2}K^{-1}_{a_3b_1}\Big)\ \det K$$

Monte Carlo integration

- correlators have form

$$C_{ij}(t) = \frac{\int \mathcal{D}U \ \det K[U] \ K^{-1}[U] \cdots K^{-1}[U] \ \ \exp\left(-S_G[U]\right)}{\int \mathcal{D}U \ \det K[U] \ \exp\left(-S_G[U]\right)}$$

- resort to Monte Carlo method to integrate over gluon fields
- use Markov chain to generate sequence of gauge-field configurations

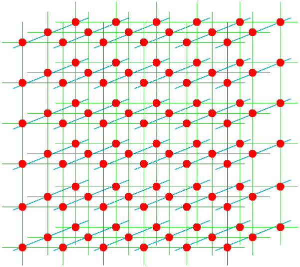
$$U_1, U_2, \ldots, U_N$$

- most computationally demanding parts:
  - including $\det K$ in updating
  - evaluating K^{-1} in numerator

Lattice QCD

- Monte Carlo method using computers requires formulating integral on space-time lattice (usually hypercubic)
- quarks reside on sites, gluons reside on links between sites
- integrate over gluon fields on each link

- Metropolis method with global updating proposal
  - RHMC: solve Hamilton equations with Gaussian momenta
- $\det K$ estimates with integral over pseudo-fermion fields
- systematic errors
  - discretization
  - finite volume
  - unphysical quark masses



Building blocks for single-hadron operators

- building blocks: covariantly-displaced LapH-smeared quark fields
- stout links $\widetilde{U}_j(x)$
- Laplacian-Heaviside (LapH) smeared quark fields

$$\widetilde{\psi}_{a\alpha}(x) = \mathcal{S}_{ab}(x,y)\,\psi_{b\alpha}(y), \qquad \mathcal{S} = \Theta\left(\sigma_s^2 + \widetilde{\Delta}\right)$$

- 3d gauge-covariant Laplacian $\widetilde{\Delta}$ in terms of $\widetilde{U}$
- displaced quark fields:

$$q^A_{a\alpha j} = D^{(j)}\widetilde{\psi}^{(A)}_{a\alpha}, \qquad \overline{q}^A_{a\alpha j} = \overline{\widetilde{\psi}}^{(A)}_{a\alpha}\gamma_4\, D^{(j)\dagger}$$

- displacement $D^{(j)}$ is product of smeared links:

$$D^{(j)}(x,x') = \widetilde{U}_{j_1}(x)\,\widetilde{U}_{j_2}(x+d_2)\,\widetilde{U}_{j_3}(x+d_3)\dots\widetilde{U}_{j_p}(x+d_p)\delta_{x',\,x+d_{p+1}}$$

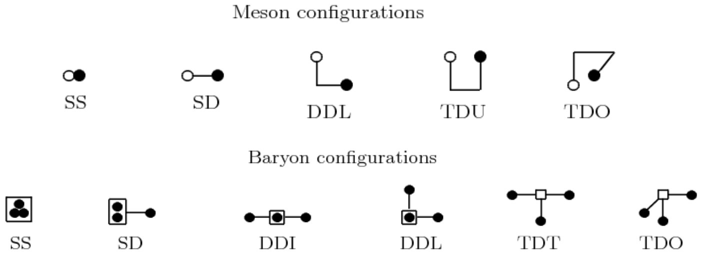
- to good approximation, LapH smearing operator is

$$\mathcal{S} = V_s V_s^\dagger$$

  - columns of matrix V_s are eigenvectors of $\widetilde{\Delta}$

Extended operators for single hadrons

- quark displacements build up orbital, radial structure



$$\overline{\Phi}^{AB}_{\alpha\beta}(\boldsymbol{p},t) = \sum_{\boldsymbol{x}} e^{i\boldsymbol{p}\cdot(\mathbf{x}+\frac{1}{2}(\boldsymbol{d}_\alpha+\boldsymbol{d}_\beta))}\delta_{ab}\ \overline{q}^B_{b\beta}(\boldsymbol{x},t)\ q^A_{a\alpha}(\boldsymbol{x},t)$$

$$\overline{\Phi}^{ABC}_{\alpha\beta\gamma}(\boldsymbol{p},t) = \sum_{\boldsymbol{x}} e^{i\boldsymbol{p}\cdot\mathbf{x}}\varepsilon_{abc}\ \overline{q}^C_{c\gamma}(\boldsymbol{x},t)\ \overline{q}^B_{b\beta}(\boldsymbol{x},t)\ \overline{q}^A_{a\alpha}(\boldsymbol{x},t)$$

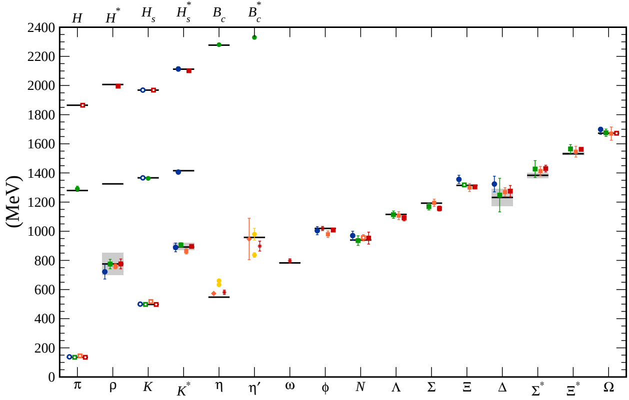
- group-theory projections onto irreps of lattice symmetry group

$$\overline{M}_l(t) = c^{(l)*}_{\alpha\beta}\ \overline{\Phi}^{AB}_{\alpha\beta}(t) \qquad \overline{B}_l(t) = c^{(l)*}_{\alpha\beta\gamma}\ \overline{\Phi}^{ABC}_{\alpha\beta\gamma}(t)$$

- definite momentum $\boldsymbol{p}$, irreps of little group of $\boldsymbol{p}$

Stable hadron mass success

- low-lying mass spectrum successfully determined
- level of precision: isospin breaking now relevant

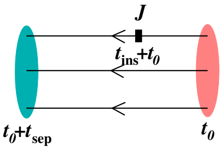


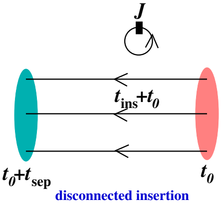
[A. Kronfeld, Ann. Rev. N.P. Sci 62, 265 (2012)]

- challenge: scattering amplitudes and resonances

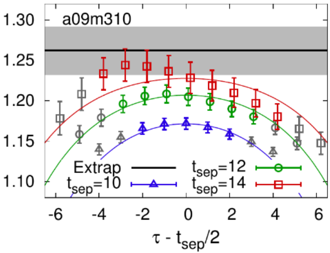
Matrix elements from lattice QCD

- standard method for matrix element calculations requires 3-point functions





- excited-state contamination removed by taking t_{sep}, t_{ins}, and $t_{sep} - t_{ins}$ large
- in practice, difficult to achieve due to signal-to-noise
- current requires renormalization for comparison to $\overline{\text{MS}}$
  - nonperturbative/perturbative methods



Proton mass decomposition

- recent determination of mass decomposition of proton
  [Y.Yang, J.Liang, Y.Bi, Y.Chen, T.Draper, K.F.Liu, Z.Liu, PRL121, 212001 (2018)]
- rest mass M of proton given by [Ji PRL74, 1071 (1995)]
  $$M = -\langle T_{44}\rangle = \langle H_m\rangle + \langle H_E\rangle(\mu) + \langle H_g\rangle(\mu) + \tfrac{1}{4}\langle H_a\rangle,$$
- $\langle T_{\mu\nu}\rangle$ expectation value of energy momentum tensor in hadron
- quark condensate $H_m = \sum_{u,d,s\cdots} \int d^3x\, m\, \overline{\psi}\psi$
- quark energy $H_E = \sum_{u,d,s\ldots} \int d^3x\, \overline{\psi}(\vec{D}\cdot\vec{\gamma})\psi$
- glue field energy $H_g = \int d^3x\, \frac{1}{2}(B^2 - E^2)$
- anomaly term $H_a = \sum_{u,d,s\cdots} \int d^3x\, \gamma_m m\, \overline{\psi}\psi - \int d^3x\, \frac{\beta(g)}{g}(E^2 + B^2)$
- $\langle H_m\rangle$, $\langle H_a\rangle$, $\langle H_E + H_g\rangle$ scale and scheme independent
- obtain from renormalized quark and gluon momentum fractions
  $\langle H_g\rangle = \frac{3}{4}M\langle x\rangle_g$ and $\langle H_E\rangle = \frac{3}{4}M\langle x\rangle_q - \frac{3}{4}\langle H_m\rangle$
- anomaly term from $\langle H_a\rangle = M - \langle H_m\rangle$

Proton mass decomposition (con't)

- determined mass M from two-point correlator
- used previous determination of $\langle H_m \rangle$ (2016)
- momentum fractions from

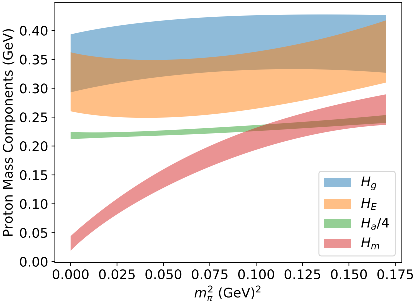
$$\begin{aligned} \langle x \rangle_{q,g} &\equiv -\frac{\langle N | \frac{4}{3} \overline{T}^{q,g}_{44} | N \rangle}{M \langle N | N \rangle}, \\ \overline{T}^{q}_{44} &= \int d^3x \overline{\psi}(x) \frac{1}{2} (\gamma_4 \overleftrightarrow{D}_4 - \frac{1}{4} \sum_{i=0,1,2,3} \gamma_i \overleftrightarrow{D}_i) \psi(x), \\ \overline{T}^{g}_{44} &= \int d^3x \frac{1}{2} (E(x)^2 - B(x)^2). \end{aligned}$$

- renormalization

$$\begin{aligned} \langle x \rangle^R_{u,d,s} &= Z^{\overline{\rm MS}}_{QQ}(\mu) \langle x \rangle_{u,d,s} + \delta Z^{\overline{\rm MS}}_{QQ}(\mu) \sum_{q=u,d,s} \langle x \rangle_q + Z^{\overline{\rm MS}}_{QG}(\mu) \langle x \rangle_g \\ \langle x \rangle^R_g &= Z^{\overline{\rm MS}}_{GQ}(\mu) \sum_{q=u,d,s} \langle x \rangle_q + Z^{\overline{\rm MS}}_{GG} \langle x \rangle_g, \end{aligned}$$

Proton mass decomposition (con't)

- obtained results on 4 ensembles ($N_f = 2+1$ DWF action, overlap valence)
- disconnected insertions: cluster-decomposition error reduction, all time slices looped over
- extrapolate with global fit including finite volume, spacing corrections, chiral behavior



- quark energy $32(4)(4)\%$
- glue energy $36(5)(4)\%$
- quark condensate $9(2)(1)\%$
- trace anomaly $23(1)(1)\%$
- with $N_f = 2+1$

Nucleon spin decomposition

- spin decomposition of nucleon
  [C.Alexandrou, M.Constantinou, K.Hadjiyiannakou, K.Jansen, C.Kallidonis, G.Koutsou, A.V.Avilés-Casco, C.Wiese, PRL 119, 142002 (2017)]
- from Ji sum rule [Ji, PRL78, 610 (1997)]

$$J_N = \sum_{q=u,d,s,c\cdots} \left(\tfrac{1}{2}\Delta\Sigma_q + L_q \right) + J_g$$

- obtain from nucleon matrix elements $(Q = p' - p, \quad P = \frac{1}{2}(p' + p))$

$$\langle N(p,s')|\bar{q}\gamma_\mu\gamma_5 q|N(p,s)\rangle = \bar{u}_N(p,s')\left[g_A^q\gamma^\mu\gamma_5\right]u_N(p,s),$$

$$\langle N(p',s')|\bar{q}\gamma^{\{\mu}\overleftrightarrow{D}^{\nu\}}q|N(p,s)\rangle = \bar{u}_N(p',s')\Lambda^q_{\mu\nu}(Q^2)u_N(p,s),$$

$$\Lambda^{\mu\nu}_{q(g)}(Q^2) = A_{20}^{q(g)}(Q^2)\gamma^{\{\mu}P^{\nu\}} + B_{20}^{q(g)}(Q^2)\frac{\sigma^{\{\mu\alpha}q_\alpha P^{\nu\}}}{2m} + C_{20}^{q(g)}(Q^2)\frac{1}{m}Q^{\{\mu}Q^{\nu\}},$$

Nucleon spin decomposition (con't)

- quark(gluon) total angular momentum and quark momentum fraction and spin from

$$\begin{aligned} J_{q(g)} &= \tfrac{1}{2}[A_{20}^{q(g)}(0) + B_{20}^{q(g)}(0)] \\ \langle x \rangle_q &= A_{20}^{q}(0), \qquad \Delta\Sigma_q = g_A^q \end{aligned}$$

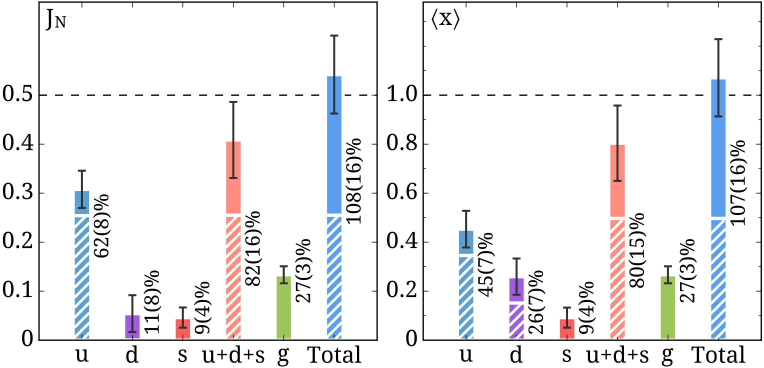
- gluon momentum fraction from $\mathcal{O}^g_{\mu\nu}{=}2\mathrm{Tr}[G_{\mu\sigma}G_{\nu\sigma}]$ with $\overline{\mathcal{O}}^g{\equiv}\mathcal{O}^g_{44} - \frac{1}{3}\mathcal{O}^g_{jj}$

$$\langle N(p,s')|\overline{\mathcal{O}}^g|N(p,s)\rangle{=}\left(-4E_N^2 - \frac{2}{3}\vec{p}^2\right)\langle x \rangle_g,$$

- one ensemble at physical point $48^3 \times 96$ twisted mass clover-improved $a = 0.0939(3)$ fm from nucleon mass
- u, d disconnected diagrams by exact deflation + one-end-trick
- s disconnected diagrams by truncated solver method
- renormalization factors determined nonperturbatively

Nucleon spin decomposition (con't)

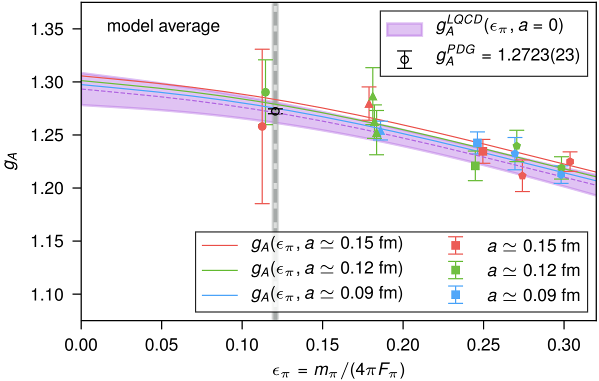
- nucleon spin (left) and momentum (right) decompositions
- striped segments → valence; solid → sea quark and gluon



Nucleon axial coupling

- recent percent level determination of g_A

[C.Chang et al., Nature 558, 91 (2018); arXiv:1805.12130]



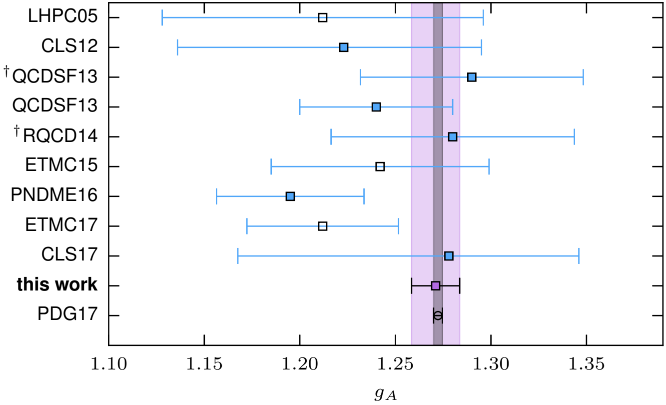
- use of Feynman-Hellman method

$$g_A = 1.2711(103)^s(39)^\chi(15)^a(19)^V(04)^I(55)^M$$

- errors: statistical,chiral,spacing,volume,isospin,model selection

Nucleon axial coupling (con't)

- comparison to other determinations

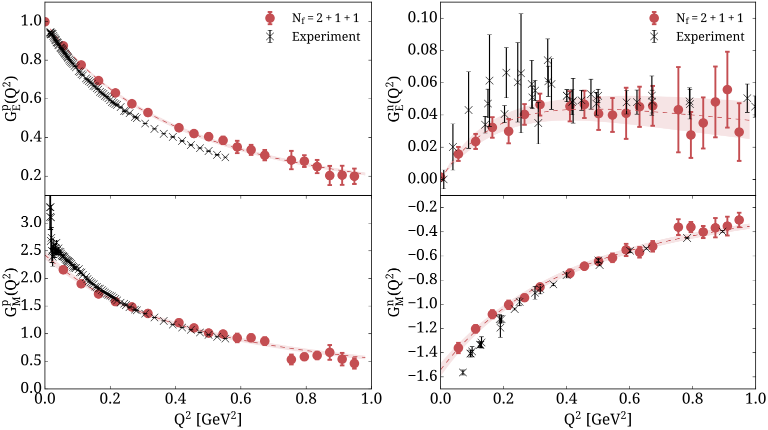


Proton/neutron electromagnetic form factors

- recent study of proton and neutron electromagnetic form factors [C.Alexandrou, S.Bacchio, M.Constantinou, J.Finkenrath, K.Hadjiyiannakou, K.Jansen, G.Koutsou, A.V.Aviles Casco, arXiv:1812.10311]
- one ensemble $N_f = 2+1+1$ twisted mass with $m_\pi = 130$ MeV
- two ensembles $N_f = 2$ twisted mass with $m_\pi = 130$ MeV and two volumes $Lm_\pi \sim 3$ and $Lm_\pi \sim 4$
- unprecedented precision of disconnected diagram contributions
  - hierarchical probing
  - low mode deflation
  - large numbers of smeared point sources to reduce gauge noise
- disconnected diagrams have nonnegligible effects
- thorough investigation of excited-state contamination
- further study of finite-volume effects at low Q^2 needed

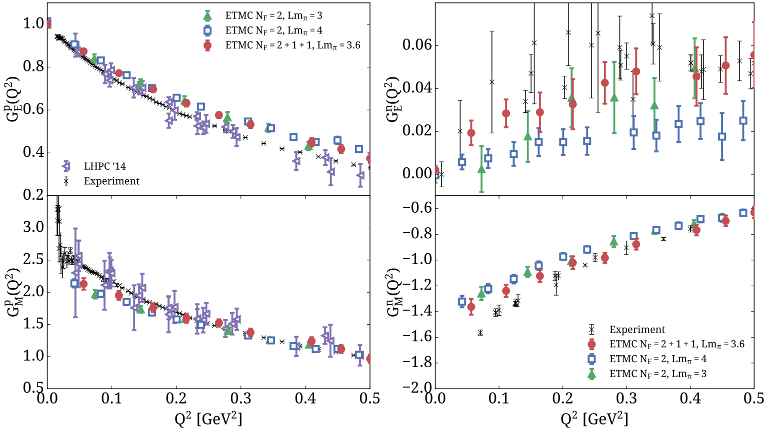
Proton/neutron electromagnetic form factors (con't)

- comparison of $N_f = 2 + 1 + 1$ results to experiment



Proton/neutron electromagnetic form factors (con't)

- comparing $N_f = 2 + 1 + 1$ and $N_f = 2$ (hollow symbols ignore disconnected)

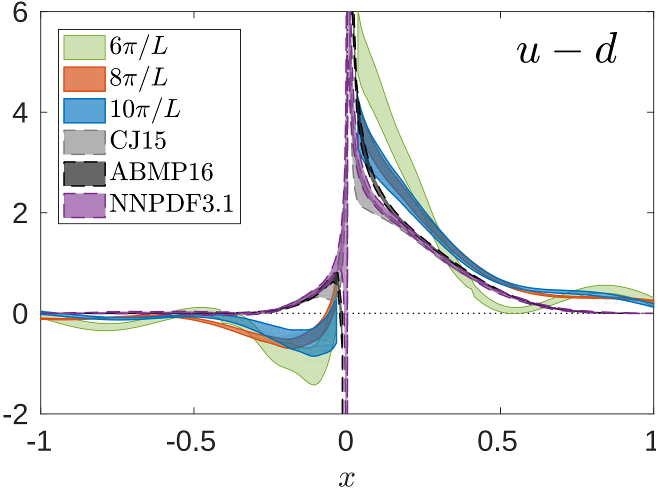


Light-cone parton distribution function

- first determination of unpolarized helicity parton distribution function at the physical point with nonperturbative renormalization and large momenta treated [C.Alexandrou, K.Cichy, M.Constantinou, K.Jansen, A.Scapellato, F.Steffens, PRL 121, 112001 (2018)]
- extracting PDFs from their moments impractical
- used method proposed by Ji [X.Ji, PRL110, 262002 (2013)] with subsequent refinements
  - compute spatial correlations between boosted nucleon states
  - Fourier transforms produce quasi-PDFs
  - take infinite-momentum limit via a refined matching procedure
  - target mass corrections
  - renormalization scheme for Wilson line operators
- one $48^3 \times 96$ twisted mass $N_f = 2$ ensemble $a = 0.0938(3)(2)$ fm and $m_\pi L = 2.98(1)$ at physical point

Light-cone parton distribution function (con't)

- unpolarized PDFs for three momenta compared to some phenomenological curves



Light-cone parton distribution function (con't)

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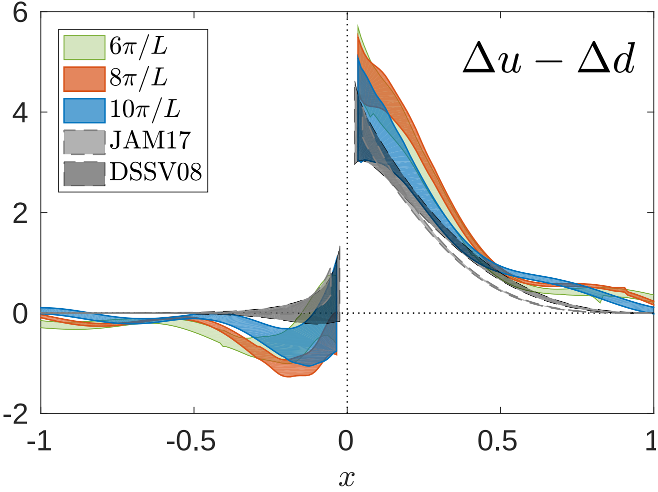


Excited states from correlation matrices

- in finite volume, energies are discrete (neglect wrap-around)

$$C_{ij}(t) = \sum_n Z_i^{(n)} Z_j^{(n)*} \; e^{-E_n t}, \qquad Z_j^{(n)} = \langle 0 | \; O_j \; | n \rangle$$

- not practical to do fits using above form
- define new correlation matrix $\widetilde{C}(t)$ using a single rotation

$$\widetilde{C}(t) = U^\dagger \; C(\tau_0)^{-1/2} \; C(t) \; C(\tau_0)^{-1/2} \; U$$

- columns of U are eigenvectors of $C(\tau_0)^{-1/2} \, C(\tau_D) \, C(\tau_0)^{-1/2}$
- choose τ_0 and τ_D large enough so $\widetilde{C}(t)$ diagonal for $t > \tau_D$
- effective energies

$$\widetilde{m}_\alpha^{\rm eff}(t) = \frac{1}{\Delta t} \ln \left(\frac{\widetilde{C}_{\alpha\alpha}(t)}{\widetilde{C}_{\alpha\alpha}(t+\Delta t)} \right)$$

  tend to N lowest-lying stationary state energies in a channel

- 2-exponential fits to $\widetilde{C}_{\alpha\alpha}(t)$ yield energies E_α and overlaps $Z_j^{(n)}$

Two-hadron operators

- our approach: superposition of products of single-hadron operators of definite momenta

$$c^{I_{3a}I_{3b}}_{\boldsymbol{p}_a\lambda_a;\ \boldsymbol{p}_b\lambda_b}\ B^{I_aI_{3a}S_a}_{\boldsymbol{p}_a\Lambda_a\lambda_a i_a}\ B^{I_bI_{3b}S_b}_{\boldsymbol{p}_b\Lambda_b\lambda_b i_b}$$

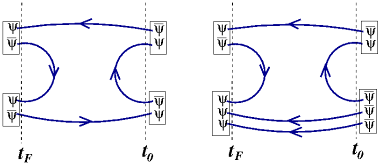
- fixed total momentum $\boldsymbol{p} = \boldsymbol{p}_a + \boldsymbol{p}_b$, fixed $\Lambda_a, i_a, \Lambda_b, i_b$
- group-theory projections onto little group of $\boldsymbol{p}$ and isospin irreps
- crucial to know and fix all phases of single-hadron operators for all momenta
  - each class, choose reference direction $\boldsymbol{p}_{\rm ref}$
  - each $\boldsymbol{p}$, select one reference rotation $R^{\boldsymbol{p}}_{\rm ref}$ that transforms $\boldsymbol{p}_{\rm ref}$ into $\boldsymbol{p}$
- efficient creating large numbers of two-hadron operators
- generalizes to three, four, . . . hadron operators

Quark propagation

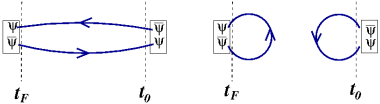
- quark propagator is inverse K^{-1} of Dirac matrix
  - rows/columns involve lattice site, spin, color
  - very large $N_{\rm tot} \times N_{\rm tot}$ matrix for each flavor
$$N_{\rm tot} = N_{\rm site} N_{\rm spin} N_{\rm color}$$
  - for $64^3 \times 128$ lattice, $N_{\rm tot} \sim 403$ million
- not feasible to compute (or store) all elements of K^{-1}
- solve linear systems $Kx = y$ for source vectors y
- translation invariance can drastically reduce number of source vectors y needed
- multi-hadron operators and isoscalar mesons require large number of source vectors y

Quark line diagrams

- temporal correlations involving our two-hadron operators need
  - slice-to-slice quark lines (from all spatial sites on a time slice to all spatial sites on another time slice)
  - sink-to-sink quark lines



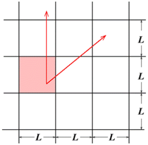
- isoscalar mesons also require sink-to-sink quark lines



- solution: the stochastic LapH method! [CM et al., PRD83, 114505 (2011)]

Quantum numbers in toroidal box

- periodic boundary conditions in cubic box
  - not all directions equivalent $\Rightarrow$ using J^{PC} is wrong!!



- label stationary states of QCD in a periodic box using irreps of cubic space group even in continuum limit
  - zero momentum states: little group O_h

$$A_{1a}, A_{2ga}, E_a, T_{1a}, T_{2a}, \quad G_{1a}, G_{2a}, H_a, \qquad a = g, u$$

  - on-axis momenta: little group C_{4v}

$$A_1, A_2, B_1, B_2, E, \quad G_1, G_2$$

  - planar-diagonal momenta: little group C_{2v}

$$A_1, A_2, B_1, B_2, \quad G_1, G_2$$

  - cubic-diagonal momenta: little group C_{3v}

$$A_1, A_2, E, \quad F_1, F_2, G$$

- include G parity in some meson sectors (superscript $+$ or $-$)

Spin content of cubic box irreps

- numbers of occurrences of Λ irreps in J subduced

J	A_1	A_2	E	T_1	T_2
0	1	0	0	0	0
1	0	0	0	1	0
2	0	0	1	0	1
3	0	1	0	1	1
4	1	0	1	1	1
5	0	0	1	2	1
6	1	1	1	1	2
7	0	1	1	2	2

J	G_1	G_2	H	J	G_1	G_2	H
$\frac{1}{2}$	1	0	0	$\frac{9}{2}$	1	0	2
$\frac{3}{2}$	0	0	1	$\frac{11}{2}$	1	1	2
$\frac{5}{2}$	0	1	1	$\frac{13}{2}$	1	2	2
$\frac{7}{2}$	1	1	1	$\frac{15}{2}$	1	1	3

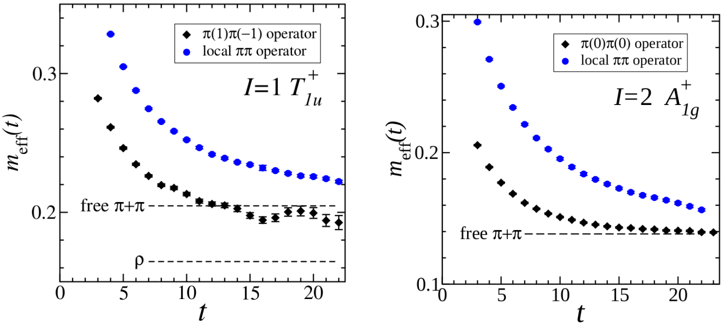
Common hadrons

- irreps of commonly-known hadrons at rest

Hadron	Irrep	Hadron	Irrep	Hadron	Irrep
π	A_{1u}^-	K	A_{1u}	η, η'	A_{1u}^+
ρ	T_{1u}^+	ω, ϕ	T_{1u}^-	K^*	T_{1u}
a_0	A_{1g}^+	f_0	A_{1g}^+	h_1	T_{1g}^-
b_1	T_{1g}^+	K_1	T_{1g}	π_1	T_{1u}^-
N, Σ	G_{1g}	Λ, Ξ	G_{1g}	Δ, Ω	H_g

Local multi-hadron operators

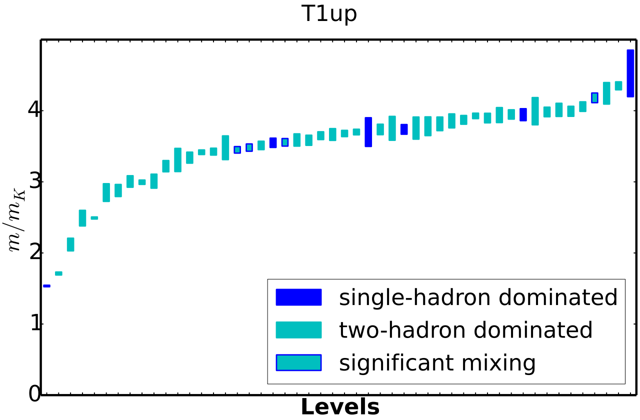
- comparison of $\pi(\boldsymbol{k})\pi(-\boldsymbol{k})$ and localized $\sum_{\boldsymbol{x}} \pi(\boldsymbol{x})\pi(\boldsymbol{x})$ operators



- much more contamination from higher states with local multi-hadron operators

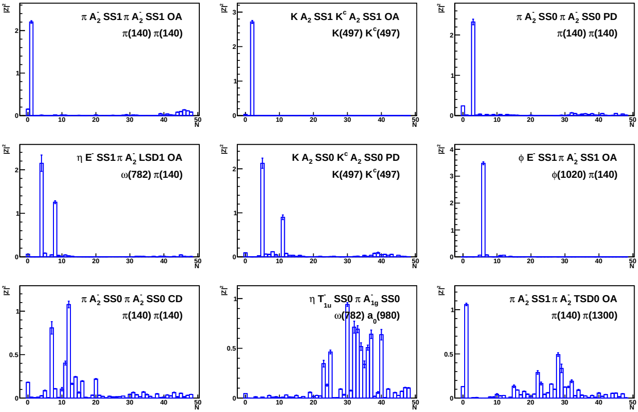
The challenge of excited states

- stationary state energies $I = 1,\ S = 0,\ T_{1u}^{+}$ channel on $(32^3 \times 256)$ anisotropic lattice $m_\pi \sim 240$ MeV



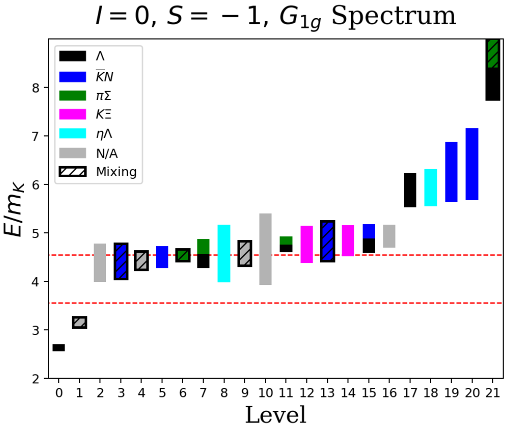
Level identification

- level identification inferred from $|Z|^2$ overlaps with probe operators
- overlaps for various operators



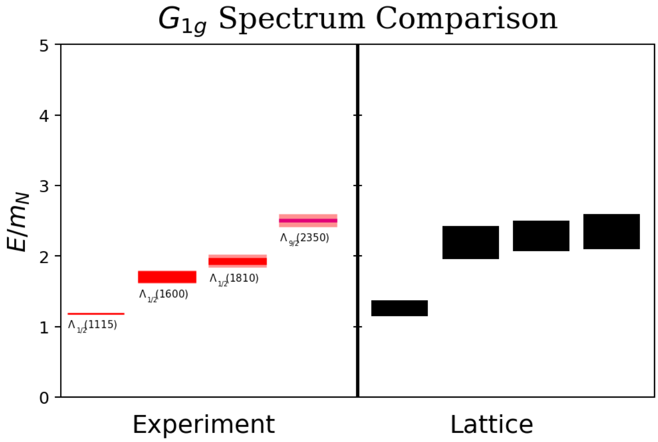
Staircase of energy levels

- stationary state energies $I = 0,\ S = -1,\ G_{1g}^{+}$ channel on $(32^3 \times 256)$ anisotropic lattice
- challenge: dashed horizontal lines show 3 and 4 particle thresholds



Comparison with experiment

- right: G_{1g} energies of $\overline{q}q$-dominant states as ratios over m_N for $(32^3|240)$ ensemble (resonance precursor states)
- left: experiment



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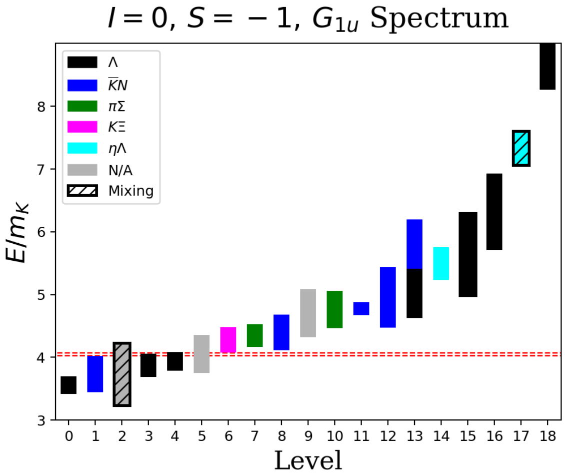

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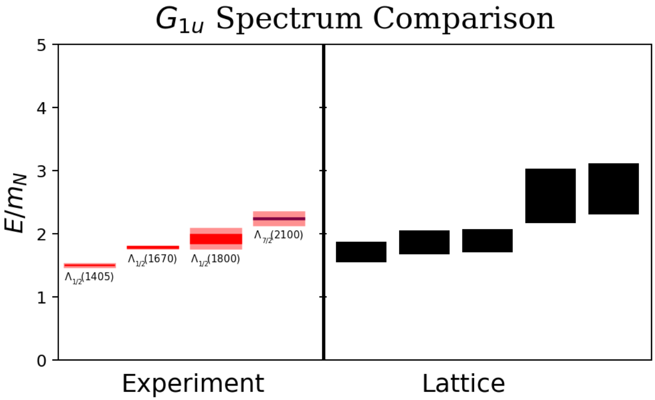
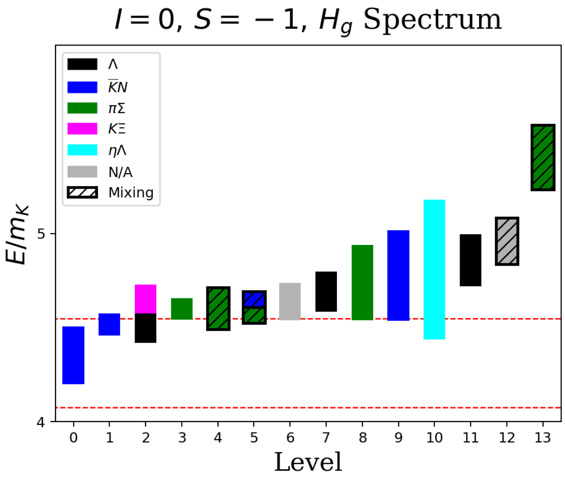


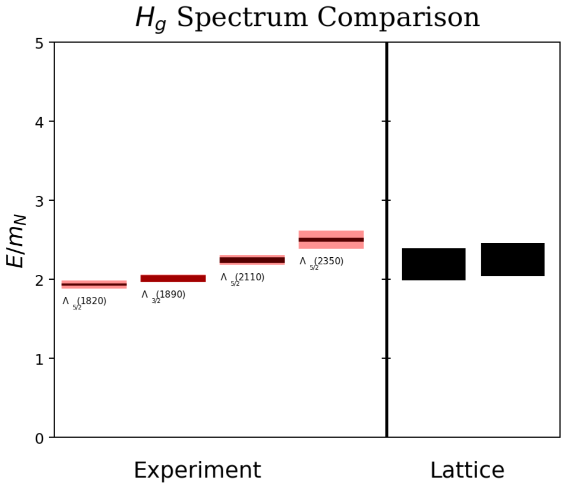
Staircase of energy levels

- stationary state energies $I = 0,\ S = -1,\ H_g^+$ channel on $(32^3 \times 256)$ anisotropic lattice



Comparison with experiment

- right: H_g energies of $\overline{q}q$-dominant states as ratios over m_N for $(32^3|240)$ ensemble (resonance precursor states)
- left: experiment



Staircase of energy levels

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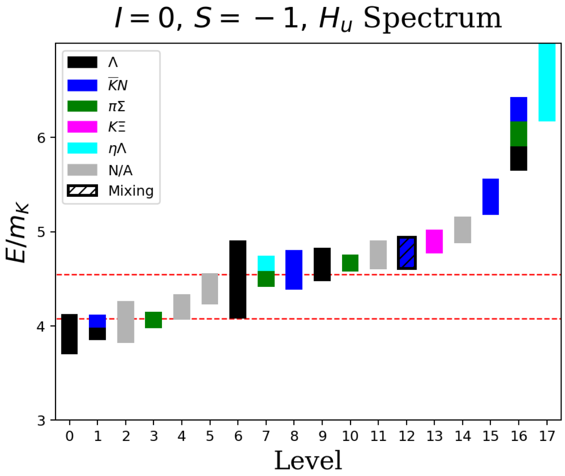

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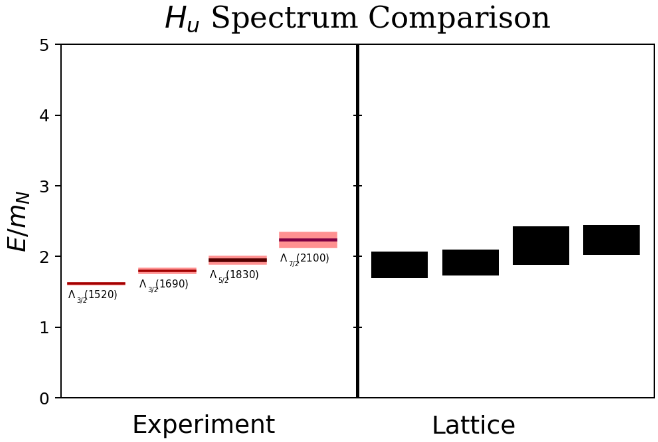


Scattering amplitudes from lattice QCD

- finite-volume energies E related to infinite-volume S matrix
  [M. Lüscher, NPB354, 531 (1991)]
- introduce K-matrix (Wigner 1946)

$$S = (1 + iK)(1 - iK)^{-1} = (1 - iK)^{-1}(1 + iK)$$

- $JLSa$ basis: total ang mom J, orbital L, spin S, species channel a
- introduce

$$K^{-1}_{L'S'a';\, LSa}(E) = q_{\mathrm{cm},a'}^{-L'-\frac{1}{2}}\ \widetilde{K}^{-1}_{L'S'a';\, LSa}(E_{\mathrm{cm}})\ q_{\mathrm{cm},a}^{-L-\frac{1}{2}}$$

- below 3-particle thresholds, quantization condition is

$$\det(1 - B^{(\boldsymbol{P})}\widetilde{K}) = \det(1 - \widetilde{K}B^{(\boldsymbol{P})}) = 0$$

- or

$$\det(\widetilde{K}^{-1} - B^{(\boldsymbol{P})}) = 0$$

- Hermitian “box matrix” $B^{(\boldsymbol{P})}$ encodes effects of cubic finite-volume

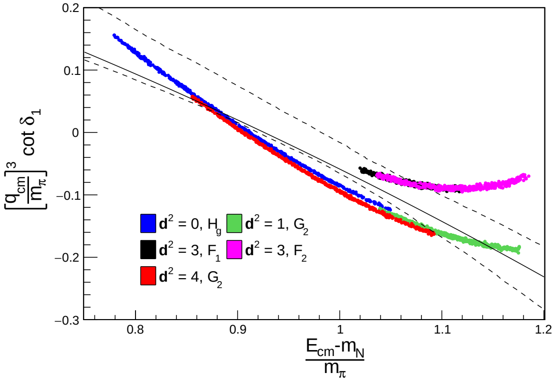
Scattering amplitudes from lattice QCD (con't)

- quantization condition relates single energy E to entire K-matrix
- cannot solve for K-matrix (except single channel, single wave)
- approximate K-matrix with functions depending on handful of fit parameters
- obtain estimates of fit parameters using many energies
- quantization condition involves infinite-dimensional determinant
  - make practical by (a) transforming to a block-diagonal basis and (b) truncating in orbital angular momentum
- meson-meson scattering becoming mature
- only a few meson-baryon scattering attempts
- baryon-baryon scattering currently gestating

Decay of Δ

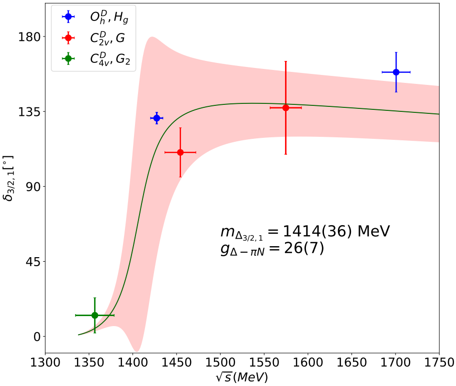
- recent study of $\Delta(1232) \rightarrow N\pi$ amplitude

  [C.W.Andersen, J.Bulava, B.Hörz, CM, PRD **97**, 014506 (2018)]
- included $L = 1$ wave only (for now)
- large $48^3 \times 128$ isotropic lattice, $m_\pi \approx 280$ MeV, $a \sim 0.076$ fm
- Breit-Wigner fit gives $m_\Delta/m_\pi = 4.738(47)$ and $g_{\Delta N\pi} = 19.0(4.7)$ in agreement with experiment ~ 16.9



Another recent Δ study

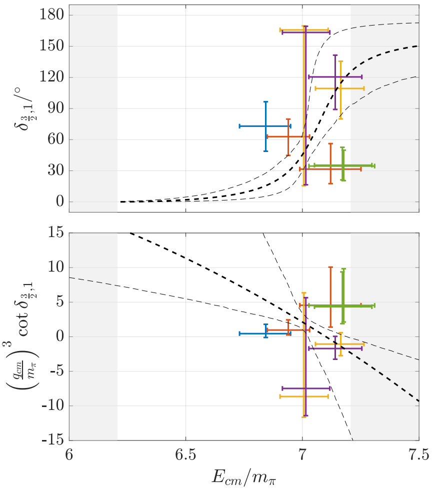
- Preliminary results $L = 2.8\ \text{fm},\ a = 0.116\ \text{fm}, m_\pi = 260\ \text{MeV}$
  [S.Paul, G.Silvi, C.Alexandrou, G.Koutsou, S.Krieg, L.Leskovec, S.Meinel, J.Negele, M.Petschlies, A.Pochinsky, G.Rendon, S.Syritsyn, Lattice 2018]



- no slice-to-slice propagators
- three total momenta
- ground and excited states
- single partial wave

Our Δ study in progress

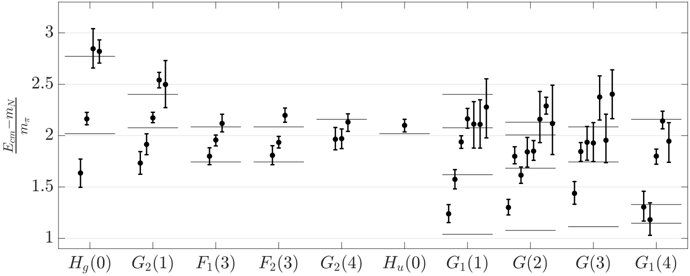
- Preliminary results $L = 4.2$ fm, $a = 0.065$ fm, $m_\pi = 200$ MeV
  [C.Andersen, B.Hörz, J.Bulava, CM, in prep.]



- five total momenta
- ground and excited states
- preliminary statistics: expect 6 times smaller errors
- light pion mass → small elastic region

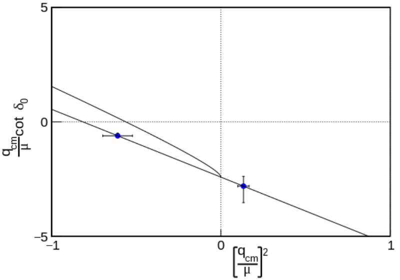
Our Δ study in progress (con't)

- fits include irreps which mix S and P waves
- relies on automated determination of B-matrix elements
  [CM et al., NPB924, 477 (2017)]
- finite-volume spectrum:



$\Lambda(1405) \to \Sigma\pi$ study in progress

- Preliminary results $L = 3.2$ fm, $a = 0.065$ fm, $m_\pi = 280$ MeV
  [B.Hörz, C.Andersen, J.Bulava, M.Hansen, D.Möhler, CM, H.Wittig, in prep.]



- $G_{1u}(0)$ below inelastic threshold only
- fit form

$$\frac{q}{\mu}\cot\delta_0 = \frac{1}{a_0\mu} + \frac{\mu r}{2}\left(\frac{q}{\mu}\right)^2$$

- best fit:

$$\frac{m_R}{\mu} = 6.143(77), \quad \frac{1}{a_0\mu} = -2.41(57), \quad \frac{\mu r}{2} = -2.9(1.1),$$
$$m_R = 1399(24)\ \text{MeV}$$

Time-like pion form factor (warm up for Δ)

- recent determination of time-like pion form factor
  [C.Andersen, J.Bulava, B.Hörz, CM, NPB939, 145 (2019)]
- extracted using

$$|F_\pi(E_{\rm cm})|^2 = g_\Lambda(\gamma)\left(q_{\rm cm}\frac{\partial\delta_1}{\partial q_{\rm cm}} + u\frac{\partial\phi_1^{(\boldsymbol{d},\Lambda)}}{\partial u}\right)\frac{3\pi E_{\rm cm}^2}{2q_{\rm cm}^5 L^3}\,|\langle 0|V^{(\boldsymbol{d},\Lambda)}|\boldsymbol{d}\Lambda n\rangle|^2$$

  where

$$\gamma = \frac{E}{E_{\rm cm}}, \qquad u = \frac{Lq_{\rm cm}}{2\pi}, \qquad g_\Lambda(\gamma) = \begin{cases} \gamma^{-1}, & \Lambda = A_1^+ \\ \gamma, & \text{otherwise} \end{cases}$$

  and δ_1 is the physical phase shift, and
  $B_{11}^{(\boldsymbol{d},\Lambda)} = (q_{\rm cm}/m_\pi)^3 \cot\phi_1^{(\boldsymbol{d},\Lambda)}$ gives the pseudophase $\phi_1^{(\boldsymbol{d},\Lambda)}$
- we compute the matrix element

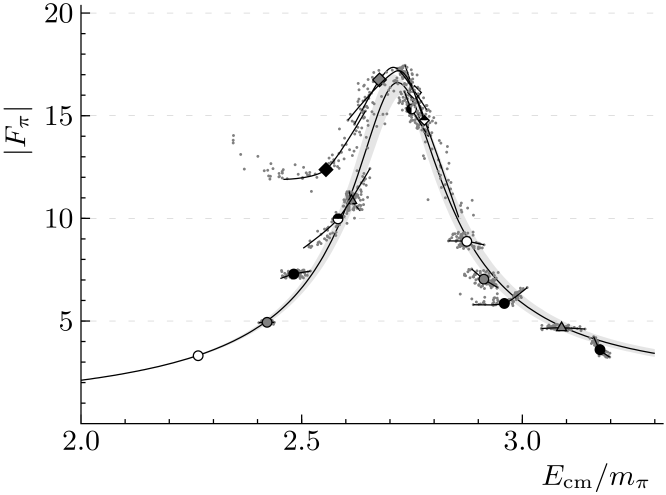
$$V^{(\boldsymbol{d},\Lambda)} = \sum_\mu b_\mu^{(\boldsymbol{d},\Lambda)} V_{R,\mu}, \qquad \sum_\mu b_\mu^{(\boldsymbol{d},\Lambda)*} b_\mu^{(\boldsymbol{d},\Lambda)} = 1,$$

$$V_{R,\mu} = Z_V(1 + ab_V m_1 + a\bar{b}_V {\rm Tr}M_q)\, V_{I,\mu}, \qquad V_{I,\mu} = V_\mu + ac_V\widetilde{\partial}_\nu T_{\mu\nu},$$

$$V_\mu^a = \tfrac{1}{2}\overline{\psi}\gamma_\mu\tau^a\psi, \qquad \widetilde{\partial}_\nu T_{\mu\nu}^a = \tfrac{1}{2}i\widetilde{\partial}_\nu\overline{\psi}\sigma_{\mu\nu}\tau^a\psi$$

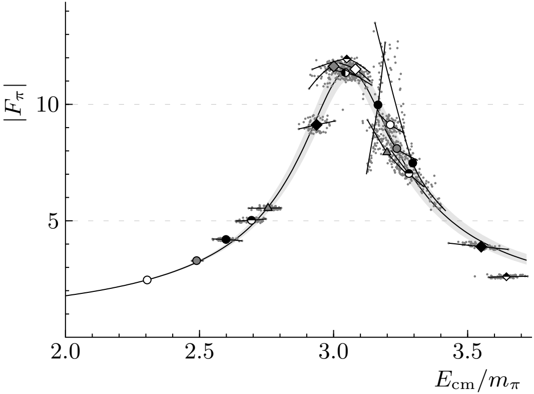
Time-like pion form factor results

- results for CLS N200 ensemble $48^3 \times 128$ with $a = 0.064$ fm and $m_\pi = 280$ MeV (curve is fit with thrice-subtracted dispersion)



Time-like pion form factor results

- results for CLS J303 ensemble $64^3 \times 192$ with $a = 0.050$ fm and $m_\pi = 260$ MeV (curve is fit with thrice-subtracted dispersion)

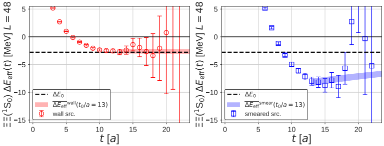


- similar method is now being used for Δ transition form factor needed by Deep Underground Neutrino Experiment

Baryon-baryon interactions in HAL QCD method

- HAL QCD collaboration has extensively studied NN interactions
- their method extracts observables from non-local kernels associated with tempo-spatial correlation functions
- controversy: disagreements with direct method
- recent study shows discrepany is from misidentification of energies in direct method

  [T.Iritani, S.Aoki, T.Doi, T.Hatsuda, Y.Ikeda, T.Inoue, N.Ishii, H.Nemura, K.Sasaki, JHEP03, 007 (2019)]
- used the $\Xi\Xi(^1S_0)$ temporal correlation functions



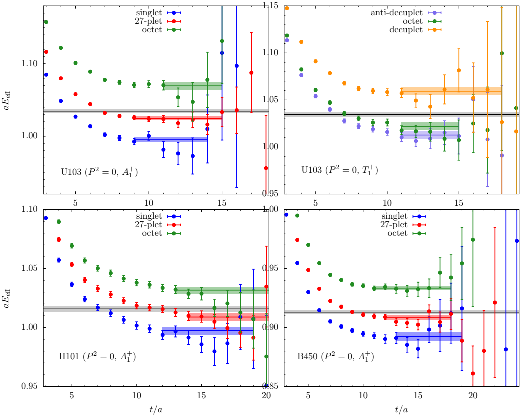
- accelerate progress in baryon-baryon scattering with this resolution

Recent H-dibaryon study

- recent report on ongoing study of the H-dibaryon [A.Hanlon, A.Francis, J.Green, P.Junnarkar, H.Wittig, arXiv:1810.13282]
- obtained results at the $SU(3)$ flavor symmetric point
- used baryon-baryon operators since previous study showed hexaquark operators would not saturate signal
- found several finite-volume energies below $\Lambda\Lambda$ threshold
- scattering amplitude analysis needed to determine if bound/resonance
- warm up exercise (small lattices, pion much too heavy)
- future work on larger lattices and lighter pions will involve stochastic LapH method

Recent H-dibaryon study (con't)

- effective masses for spin-0 and spin-1 operators of different flavor irreps using 3 ensembles
- horizontal black lines show two-octet baryon threshold



Conclusion

- recent highlights involving baryons in lattice QCD
  - proton mass decomposition
  - nucleon spin decomposition
  - percent level determination of nucleon axial coupling
  - proton and neutron electromagnetic form factors
  - parton distribution function
  - scattering amplitudes
  - baryon-baryon interactions with HAL QCD method
  - H-dibaryon warm up
- key progress
  - achieving much better precision with disconnected diagrams
  - ability to include multi-hadron operators
  - more and more studies being done at physical point
- excited-baryon resonances in near future