Excited States from the Stochastic LapH Method

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Outline

- goals
  - comprehensive survey of spectrum of QCD stationary states in finite volume
  - hadron scattering phase shifts, decay widths, matrix elements
  - focus: large 32^3 lattices, $m_\pi \sim 240$ MeV, all 2-hadron operators
- extracting excited-state energies
- single-hadron and multi-hadron operators
- preliminary results in ρ-channel: $I = 1,\ S = 0,\ T_{1u}^+$
  - used 56×56 matrix of correlators
  - 12 single-hadron operators in first pass, more later
  - "$\pi\pi$", "$\eta\pi$", "$\phi\pi$", "$K\overline{K}$" operators
- level identification
- preliminary results using 59×59 matrix of correlators in the bosonic $I = \frac{1}{2},\ S = 1,\ T_{1u}$
- the stochastic LapH method

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Excited states from correlation matrices

- excited-state energies from $N \times N$ Hermitian correlation matrix

$$C_{ij}(t) = \langle 0|\, O_i(t+t_0)\, \overline{O}_j(t_0)\; |0\rangle$$

- estimate $C_{ij}(t)$ with Monte Carlo method in lattice QCD
- in finite volume, energies are discrete

$$C_{ij}(t) = \sum_n Z_i^{(n)} Z_j^{(n)*}\; e^{-E_n t}, \qquad\qquad Z_j^{(n)} = \langle 0|\; O_j\; |n\rangle$$

(no wrap-around)

- for t large such that only lowest N energies contribute, can solve for $E_n,\ Z_j^{(n)}$ using $C(t)$ at two time separations
- N *principal correlators* $\lambda_\alpha(t, \tau_0)$ are eigenvalues of

$$C(\tau_0)^{-1/2}\; C(t)\; C(\tau_0)^{-1/2}$$

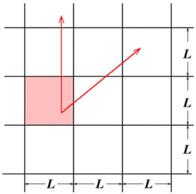
- large time separation: $\lim_{t\to\infty} \lambda_\alpha(t, \tau_0) = e^{-(t-\tau_0)E_\alpha}$
  to extract N lowest-lying stationary state energies in a channel

Excited states from correlation matrices (continued)

- simpler method: define new correlation matrix $\widetilde{C}(t)$ using a single rotation
$$\widetilde{C}(t) = U^\dagger\ C(\tau_0)^{-1/2}\ C(t)\ C(\tau_0)^{-1/2}\ U$$
- columns of U are eigenvectors of $C(\tau_0)^{-1/2}\, C(\tau_D)\, C(\tau_0)^{-1/2}$
- choose τ_0 and τ_D large enough such that $\widetilde{C}(t)$ remains diagonal for $t > \tau_D$
- produces results similar to principal correlator method
- avoids unpalatable eigenvector "pinning"
- effective masses
$$\widetilde{m}_\alpha^{\rm eff}(t) = \frac{1}{\Delta t} \ln\left(\frac{\widetilde{C}_{\alpha\alpha}(t)}{\widetilde{C}_{\alpha\alpha}(t+\Delta t)}\right)$$
tend to N lowest-lying stationary state energies in a channel
- exponential fits to $\widetilde{C}_{\alpha\alpha}(t)$ yield energies E_α and overlaps $Z_j^{(n)}$

Quantum numbers in toroidal box

- periodic boundary conditions in cubic box
  - not all directions equivalent $\Rightarrow$ using J^{PC} is wrong!!



- label stationary states of QCD in a periodic box using irreps of cubic space group even in continuum limit
  - zero momentum states: little group O_h

$$A_{1a}, A_{2ga}, E_a, T_{1a}, T_{2a}, \quad G_{1a}, G_{2a}, H_a, \qquad a = g, u$$

  - on-axis momenta: little group C_{4v}

$$A_1, A_2, B_1, B_2, E, \quad G_1, G_2$$

  - planar-diagonal momenta: little group C_{2v}

$$A_1, A_2, B_1, B_2, \quad G_1, G_2$$

  - cubic-diagonal momenta: little group C_{3v}

$$A_1, A_2, E, \quad F_1, F_2, G$$

- include G parity in some meson sectors (superscript $+$ or $-$)

Building blocks for single-hadron operators

- building blocks: covariantly-displaced LapH-smeared quark fields
- stout links $\widetilde{U}_j(x)$
- Laplacian-Heaviside (LapH) smeared quark fields

$$\widetilde{\psi}_{a\alpha}(x) = \mathcal{S}_{ab}(x,y)\ \psi_{b\alpha}(y), \qquad \mathcal{S} = \Theta\left(\sigma_s^2 + \widetilde{\Delta}\right)$$

- 3d gauge-covariant Laplacian $\widetilde{\Delta}$ in terms of $\widetilde{U}$
- displaced quark fields:

$$q^A_{a\alpha j} = D^{(j)} \widetilde{\psi}^{(A)}_{a\alpha}, \qquad \overline{q}^A_{a\alpha j} = \widetilde{\overline{\psi}}^{(A)}_{a\alpha}\, \gamma_4\, D^{(j)\dagger}$$

- displacement $D^{(j)}$ is product of smeared links:

$$D^{(j)}(x,x') = \widetilde{U}_{j_1}(x)\ \widetilde{U}_{j_2}(x+d_2)\ \widetilde{U}_{j_3}(x+d_3) \dots \widetilde{U}_{j_p}(x+d_p)\delta_{x',\ x+d_{p+1}}$$

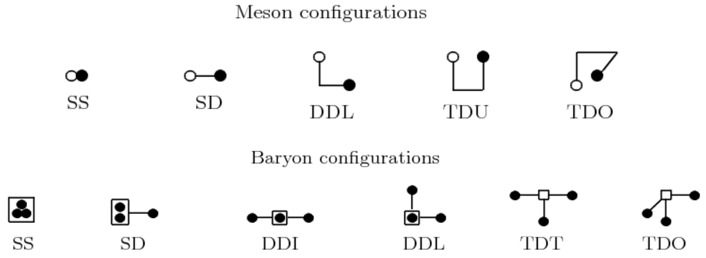
- to good approximation, LapH smearing operator is

$$\mathcal{S} = V_s V_s^\dagger$$

  - columns of matrix V_s are eigenvectors of $\widetilde{\Delta}$

Extended operators for single hadrons

- quark displacements build up orbital, radial structure



$$
\begin{aligned}
\overline{\Phi}^{AB}_{\alpha\beta}(t) &= \sum_{\boldsymbol{x}} e^{i\boldsymbol{p}\cdot(\mathbf{x}+\frac{1}{2}(\boldsymbol{d}_\alpha+\boldsymbol{d}_\beta))}\delta_{ab}\;\overline{q}^{B}_{b\beta}(\boldsymbol{x},t)\;q^{A}_{a\alpha}(\boldsymbol{x},t)\\
\overline{\Phi}^{ABC}_{\alpha\beta\gamma}(\boldsymbol{p},t) &= \sum_{\boldsymbol{x}} e^{i\boldsymbol{p}\cdot\mathbf{x}}\varepsilon_{abc}\;\overline{q}^{C}_{c\gamma}(\boldsymbol{x},t)\;\overline{q}^{B}_{b\beta}(\boldsymbol{x},t)\;\overline{q}^{A}_{a\alpha}(\boldsymbol{x},t)
\end{aligned}
$$

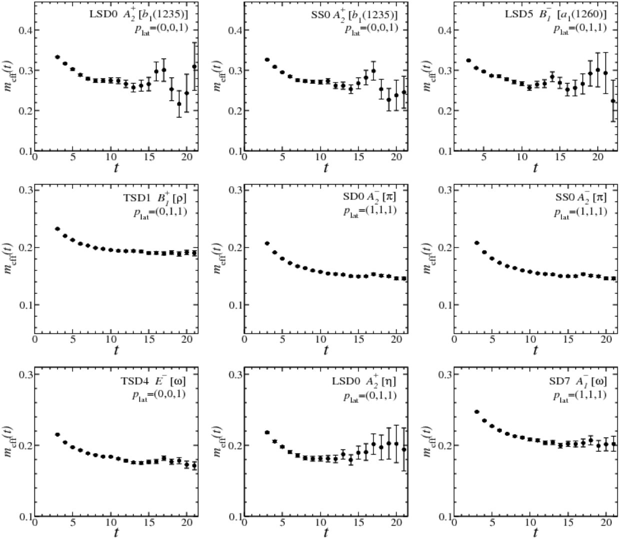
- group-theory projections onto irreps of lattice symmetry group

$$
\overline{M}_l(t) = c^{(l)*}_{\alpha\beta}\;\overline{\Phi}^{AB}_{\alpha\beta}(t) \qquad\qquad \overline{B}_l(t) = c^{(l)*}_{\alpha\beta\gamma}\;\overline{\Phi}^{ABC}_{\alpha\beta\gamma}(t)
$$

- definite momentum $\boldsymbol{p}$, irreps of little group of $\boldsymbol{p}$

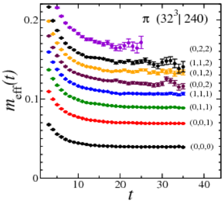
Testing single-hadron operators

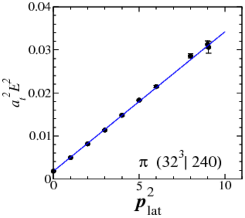
- meson effective masses on $(24^3|390)$ ensemble

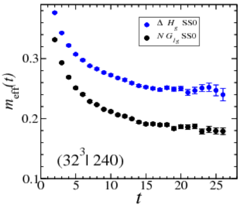


Testing single-hadron operators (con't)

- (left and center) pion energies on $(32^3|240)$ ensemble
- (right) nucleon and Δ baryons







Two-hadron operators

- our approach: superposition of products of single-hadron operators of definite momenta

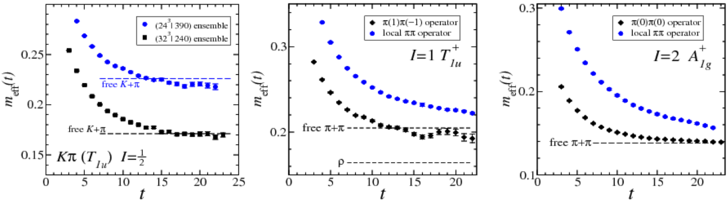
$$c^{I_{3a}I_{3b}}_{\boldsymbol{p}_a\lambda_a;\,\boldsymbol{p}_b\lambda_b}\ B^{I_aI_{3a}S_a}_{\boldsymbol{p}_a\Lambda_a\lambda_a i_a}\ B^{I_bI_{3b}S_b}_{\boldsymbol{p}_b\Lambda_b\lambda_b i_b}$$

- fixed total momentum $\boldsymbol{p}=\boldsymbol{p}_a+\boldsymbol{p}_b$, fixed $\Lambda_a, i_a, \Lambda_b, i_b$
- group-theory projections onto little group of $\boldsymbol{p}$ and isospin irreps
- restrict attention to certain classes of momentum directions
  - on axis $\pm\widehat{\boldsymbol{x}},\ \pm\widehat{\boldsymbol{y}},\ \pm\widehat{\boldsymbol{z}}$
  - planar diagonal $\pm\widehat{\boldsymbol{x}}\pm\widehat{\boldsymbol{y}},\ \pm\widehat{\boldsymbol{x}}\pm\widehat{\boldsymbol{z}},\ \pm\widehat{\boldsymbol{y}}\pm\widehat{\boldsymbol{z}}$
  - cubic diagonal $\pm\widehat{\boldsymbol{x}}\pm\widehat{\boldsymbol{y}}\pm\widehat{\boldsymbol{z}}$
- crucial to know and fix all phases of single-hadron operators for all momenta
  - each class, choose reference direction $\boldsymbol{p}_{\rm ref}$
  - each $\boldsymbol{p}$, select one reference rotation $R^{\boldsymbol{p}}_{\rm ref}$ that transforms $\boldsymbol{p}_{\rm ref}$ into $\boldsymbol{p}$
- efficient creating large numbers of two-hadron operators
- generalizes to three, four, . . . hadron operators

Testing our two-meson operators

- (left) $K\pi$ operator in T_{1u} $I = \frac{1}{2}$ channels
- (center and right) comparison with localized $\pi\pi$ operators

$$(\pi\pi)^{A_{1g}^+}(t) = \sum_{\boldsymbol{x}} \pi^+(\boldsymbol{x},t)\ \pi^+(\boldsymbol{x},t),$$
$$(\pi\pi)^{T_{1u}^+}(t) = \sum_{\boldsymbol{x},k=1,2,3}\left\{\pi^+(\boldsymbol{x},t)\ \Delta_k\pi^0(\boldsymbol{x},t) - \pi^0(\boldsymbol{x},t)\ \Delta_k\pi^+(\boldsymbol{x},t)\right\}$$



- less contamination from higher states in our $\pi\pi$ operators

Ensembles and run parameters

- plan to use three Monte Carlo ensembles
  - $(32^3|240)$: 412 configs $32^3 \times 256$, $m_\pi \approx 240$ MeV, $m_\pi L \sim 4.4$
  - $(24^3|240)$: 584 configs $24^3 \times 128$, $m_\pi \approx 240$ MeV, $m_\pi L \sim 3.3$
  - $(24^3|390)$: 551 configs $24^3 \times 128$, $m_\pi \approx 390$ MeV, $m_\pi L \sim 5.7$
- anisotropic improved gluon action, clover quarks (stout links)
- QCD coupling $\beta = 1.5$ such that $a_s \sim 0.12$ fm, $a_t \sim 0.035$ fm
- strange quark mass $m_s = -0.0743$ nearly physical (using kaon)
- work in $m_u = m_d$ limit so $SU(2)$ isospin exact
- generated using RHMC, configs separated by 20 trajectories

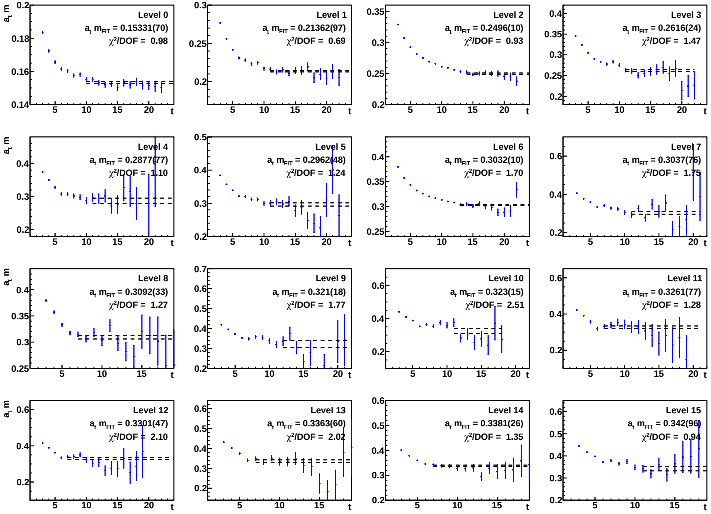
- stout-link smearing in operators $\xi = 0.10$ and $n_\xi = 10$
- LapH smearing cutoff $\sigma_s^2 = 0.33$ such that
  - $N_v = 112$ for 24^3 lattices
  - $N_v = 264$ for 32^3 lattices
- source times:
  - 4 widely-separated t_0 values on 24^3
  - 8 t_0 values used on 32^3 lattice

First results

- correlator software last_laph completed summer 2013
  - testing of all flavor channels for single and two-mesons completed
- first focus on the resonance-rich ρ-channel: $I = 1,\ S = 0,\ T_{1u}^{+}$
- experiment: $\rho(770), \quad \rho(1450), \quad \rho(1570), \quad \rho_3(1690), \quad \rho(1700)$
- first results: 56×56 matrix of correlators $(24^3|390)$ ensemble
  - 12 single-hadron (quark-antiquark) operators
  - 17 "$\pi\pi$" operators
  - 14 "$\eta\pi$" operators, 3 "$\phi\pi$" operators
  - 10 "$K\overline{K}$" operators
- inclusion of all possible 2-meson operators
- 3-meson operators currently neglected
- still finalizing analysis code

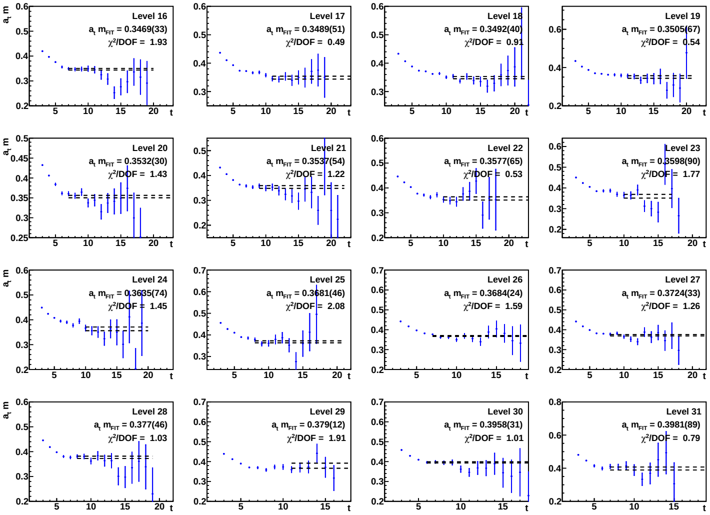
$I = 1,\ S = 0,\ T_{1u}^{+}$ channel

- effective masses $\widetilde{m}^{\text{eff}}(t)$ for levels 0 to 15
- dashed lines show energies from single exponential fits



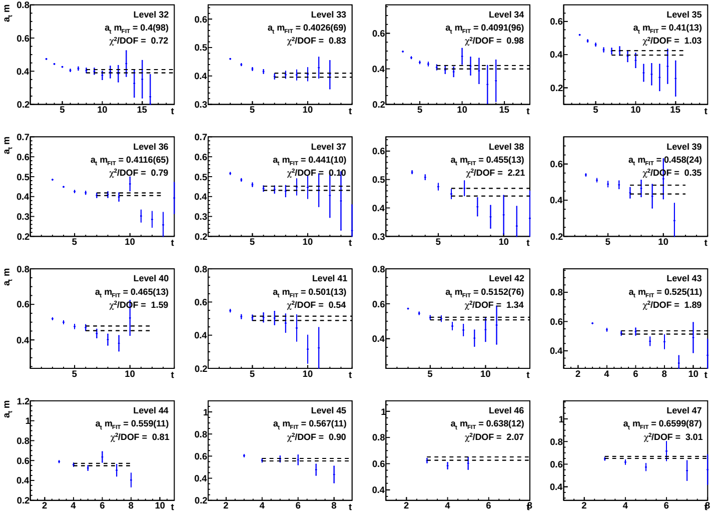
$I = 1,\ S = 0,\ T_{1u}^{+}$ energy extraction, continued

- effective masses $\widetilde{m}^{\mathrm{eff}}(t)$ for levels 16 to 31
- dashed lines show energies from single exponential fits



$I = 1,\ S = 0,\ T_{1u}^{+}$ energy extraction, continued

- effective masses $\widetilde{m}^{\rm eff}(t)$ for levels 32 to 47
- dashed lines show energies from single exponential fits



Level identification

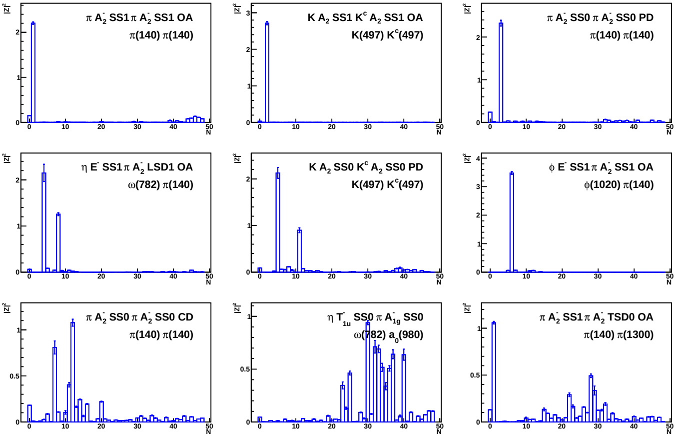
- level identification inferred from Z overlaps with probe operators
- analogous to experiment: infer resonances from scattering cross sections
- keep in mind:
  - probe operators $\overline{O}_j$ act on vacuum, create a "probe state" $|\Phi_j\rangle$, Z's are overlaps of probe state with each eigenstate

$$|\Phi_j\rangle \equiv \overline{O}_i|0\rangle, \qquad Z_j^{(n)} = \langle\Phi_j|n\rangle$$

  - have limited control of "probe states" produced by probe operators
    - ideal to be ρ, single $\pi\pi$, and so on
    - use of small$-a$ expansions to characterize probe operators
    - use of smeared quark, gluon fields
    - field renormalizations
  - mixing is prevalent
  - identify by dominant probe state(s) whenever possible

Level identification

- overlaps for various operators



Small$-a$ expansion of probes

- illustrate problem with real scalar field $\varphi(x)$
- consider three operators Φ_j for $j = 1, 2, 3$ defined by

$$\Phi_j(x) = \frac{1}{2a}\Big(\varphi(x+\widehat{j}) - \varphi(x-\widehat{j})\Big).$$

- this is forward-backward finite difference approx to derivative
- carry T_1 irrep of the octahedral point group O
- classical small-a expansion:

$$\Phi_j(x) \approx \partial_j\varphi(x) + \frac{1}{6}a^2\partial_j^3\varphi(x) + O(a^4).$$

- first term $\partial_j\varphi(x)$ is spin $J = 1$
- second term contains both $J = 1$ and $J = 3$
- radiative corrections modify relative weights (calculate in lattice perturbation theory, but difficult)

Small$-a$ expansion of probes

- link variables in terms of continuum gluon field

$$U_\mu(x) = \mathcal{P}\exp\left\{ ig\int_x^{x+\hat{\mu}} d\eta\cdot A(\eta)\right\},$$

- classical small$-a$ expansion of displaced quark field:

$$U_j(x)U_k(x+\widehat{j})\psi_\alpha(x+\widehat{j}+\widehat{k}) = \exp(a\mathcal{D}_j)\ \exp(a\mathcal{D}_k)\ \psi_\alpha(x).$$

- where $\mathcal{D}_j = \partial_j + igA_j$ is covariant derivative
- must take smearing of fields into account
- radiative corrections of expansion coefficients (hopefully small due to smearing)
- work in progress for our probe operators

J^{PG} of continuum probe operators

- isovector meson continuum probe operators

$$M_{\mu j_1 j_2 \cdots} = \chi^d \Gamma_\mu \mathcal{D}_{j_1} \mathcal{D}_{j_2} \cdots \psi^u, \qquad \chi = \overline{\psi}\gamma_4$$

- where $\Gamma_0 = 1$ and $\Gamma_k = \gamma_k$ (analogous table inserting $\gamma_4,\ \gamma_5,\ \gamma_4\gamma_5$)

J^{PG}	O_h^G irrep	Basis operator
0^{++}	A_{1g}^+	M_0
1^{-+}	T_{1u}^+	M_1
1^{--}	T_{1u}^-	M_{01}
0^{+-}	A_{1g}^-	$M_{11} + M_{22} + M_{33}$
1^{+-}	T_{1g}^-	$M_{23} - M_{32}$
2^{+-}	E_g^- T_{2g}^-	$M_{11} - M_{22}$ $M_{23} + M_{32}$
0^{++}	A_{1g}^+	$M_{011} + M_{022} + M_{033}$
1^{+-}	T_{1g}^-	$M_{023} - M_{032}$
2^{++}	E_g^+ T_{2g}^+	$M_{011} - M_{022}$ $M_{023} + M_{032}$

J^{PG} of continuum probe operators (continued)

- isovector meson continuum probe operators

$$M_{\mu j_1 j_2 \cdots} = \chi^d \Gamma_\mu \mathcal{D}_{j_1} \mathcal{D}_{j_2} \cdots \psi^u, \qquad \chi = \overline{\psi}\gamma_4$$

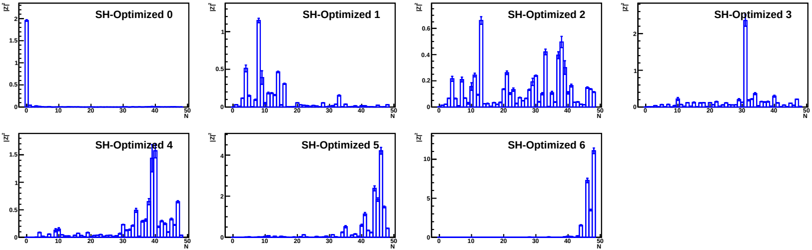
- where $\Gamma_0 = 1$ and $\Gamma_k = \gamma_k$ (analogous table inserting $\gamma_4,\ \gamma_5,\ \gamma_4\gamma_5$)

J^{PG}	O_h^G irrep	Basis operator
0^{--}	A_{1u}^-	$M_{123} + M_{231} + M_{312} - M_{321} - M_{213} - M_{132}$
1^{-+}	T_{1u}^+	$M_{111} + M_{122} + M_{133}$
1^{-+}	T_{1u}^+	$2M_{111} + M_{221} + M_{331} + M_{212} + M_{313}$
1^{--}	T_{1u}^-	$M_{221} + M_{331} - M_{212} - M_{313}$
2^{--}	E_u^-	$M_{123} + M_{213} - M_{231} - M_{132}$
	T_{2u}^-	$M_{221} - M_{331} + M_{313} - M_{212}$
2^{-+}	E_u^+	$M_{123} + M_{213} - 2M_{321} - 2M_{312} + M_{231} + M_{132}$
	T_{2u}^+	$M_{221} - M_{331} - 2M_{122} + 2M_{133} - M_{313} + M_{212}$
3^{-+}	A_{2u}^+	$M_{123} + M_{231} + M_{312} + M_{213} + M_{321} + M_{132}$
	T_{1u}^+	$2M_{111} - M_{221} - M_{331} - M_{212} - M_{313} - M_{122} - M_{133}$
	T_{2u}^+	$M_{331} - M_{212} + M_{313} - M_{122} + M_{133} - M_{221}$

Identifying resonances

- resonances: finite-volume "precursor states"
- probes: *optimized* single-hadron operators
  - analyze matrix of just single-hadron operators $O_i^{[SH]}$ (12×12)
  - perform single-rotation as before to build probe operators
    $O_m'^{[SH]} = \sum_i v_i'^{(m)*} O_i^{[SH]}$
- obtain Z' factors of these probe operators

$$Z_m'^{(n)} = \langle 0 | \; O_m'^{[SH]} \; | n \rangle$$

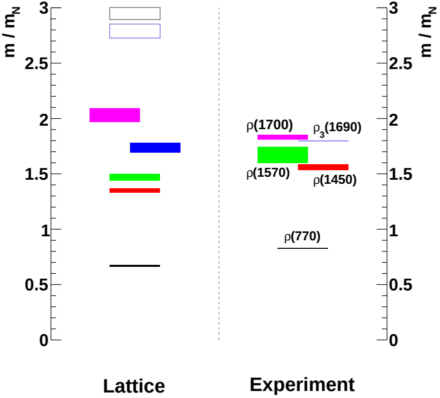


List of tentative level identifications

Level	Dominant Probe	Level	Dominant Probe
0	$\rho(770)$	16	$\omega(782)\pi(140)$ (PD)
1	$\pi(140)\pi(140)$ (OA)	17	$\phi(1020)\pi(140)$ (PD)
2	$K(497)K^c(497)$ (OA)	18	$\phi(1020)\pi(140)$ (PD)
3	$\pi(140)\pi(140)$ (PD)	20	$K(497)K^c(497)$ (CD)
4	$\omega(782)\pi(140$ (OA)	21	$K(497)K^c(497)$ (CD)
5	$K(497)K^c(497)$ (PD)	22	$\rho(770)\rho(770)$ (OA)
6	$\phi(1020)\pi(140)$ (OA)	30	$\eta(547)\rho(770)$ (PD)
7	$\pi(140)\pi(140)$ (CD) (12)	31	$\rho(1690)$
8	$\rho(1450)$	36	$h_1(1170)\pi(140)$ (OA)
9	$\eta(547)\rho(770)$ (OA)	40	$\rho(1700)$
11	$\pi(140)a_1(1260)$ (AR)	41	$\eta(547)b_1(1235)$ (AR)
12	$\pi(140)\pi(140)$ (CD) (7)	46	$\rho(?)$
13	$\rho(1570)$	48	$\rho(?)$
14	$K^*(892)K^c(497)$ (OA)		

Summary and comparison with experiment

- left: energies of $\overline{q}q$-dominant states as ratios over m_N for $(24^3|390)$ ensemble (resonance precursor states)
- right: experiment



Issues

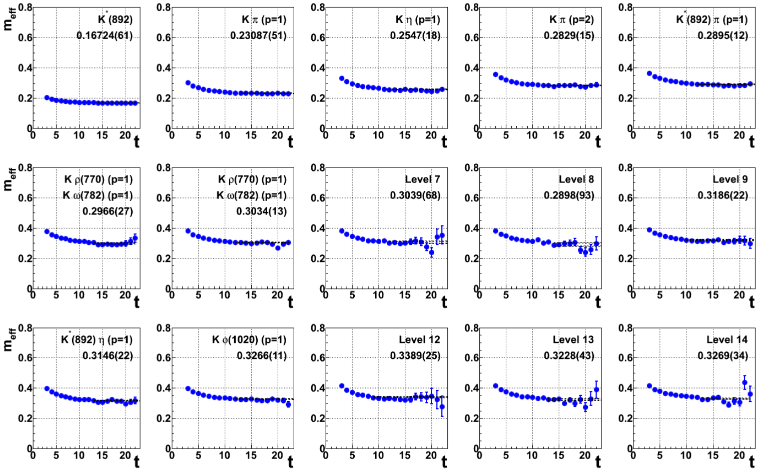
- address presence of 3 and 4 meson states
- in other channels, address scalar particles in spectrum
  - scalar probe states need vacuum subtractions
  - hopefully can neglect due to OZI suppression
- infinite-volume resonance parameters from finite-volume energies
  - Luscher method too cumbersome, restrictive in applicability
  - need for new hadron effective field theory techniques

Bosonic $I = \frac{1}{2},\ S = 1,\ T_{1u}$ channel

- also have results for the kaon channel: $I = \frac{1}{2},\ S = 1,\ T_{1u}$
- experiment: $K^*(892)$, $K^*(1410)$, $K^*(1680)$, $K_3^*(1780)$
- first results: 59×59 matrix of correlators $(24^3|390)$ ensemble
  - 10 single-hadron (quark-antiquark) operators
  - 25 "$K\pi$" operators
  - 12 "$K\eta$" operators, 12 "$K\phi$" operators

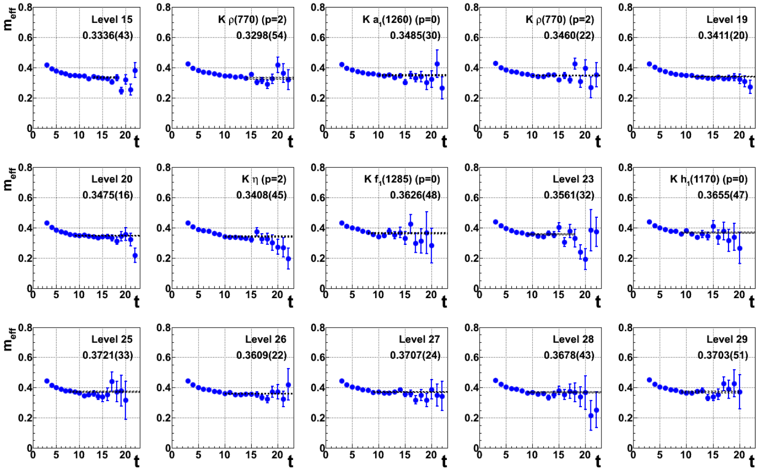
Bosonic $I = \frac{1}{2},\ S = 1,\ T_{1u}$ channel

- effective masses $\widetilde{m}^{\text{eff}}(t)$ for levels 0 to 14
- dashed lines show energies from single exponential fits



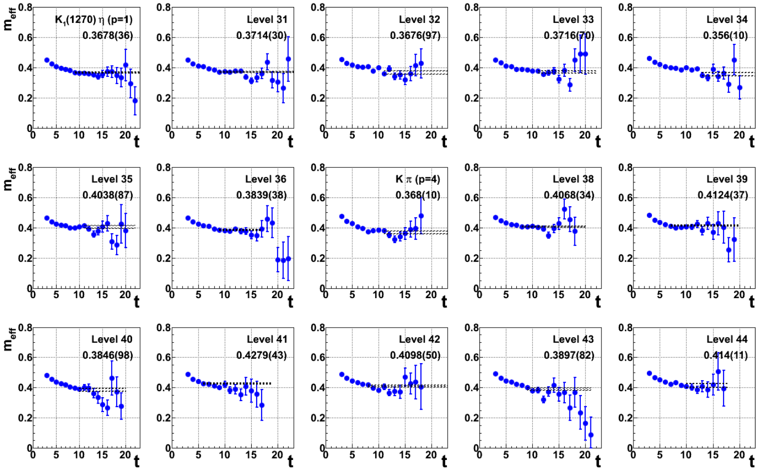
Bosonic $I = \frac{1}{2},\ S = 1,\ T_{1u}$ channel

- effective masses $\widetilde{m}^{\text{eff}}(t)$ for levels 15 to 29
- dashed lines show energies from single exponential fits



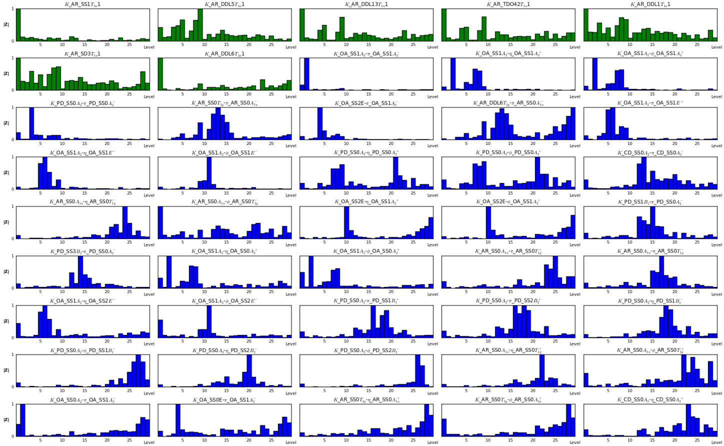
Bosonic $I = \frac{1}{2},\ S = 1,\ T_{1u}$ channel

- effective masses $\widetilde{m}^{\mathrm{eff}}(t)$ for levels 30 to 44
- dashed lines show energies from single exponential fits



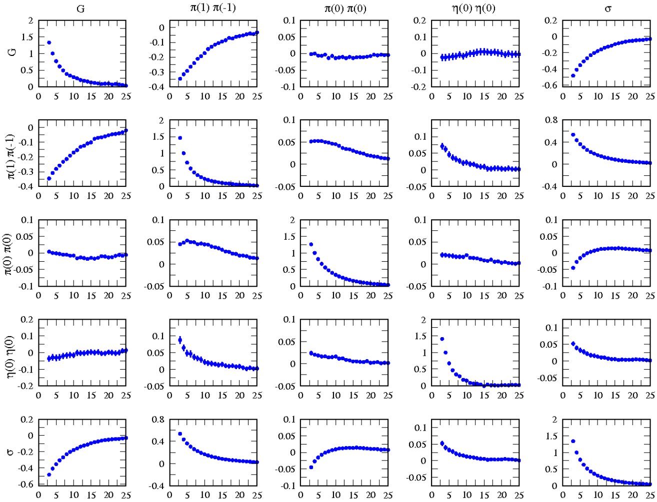
Bosonic $I = \frac{1}{2},\ S = 1,\ T_{1u}$ channel

- *preliminary* estimates of Z overlaps for various operators:



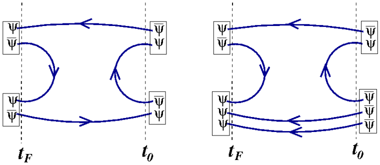
The scalar isoscalar sector

- 5×5 correlator matrix mixing glueball G, two $\pi\pi$, an $\eta\eta$, and a $\overline{q}q$ operator for $(24^3|390)$ ensemble

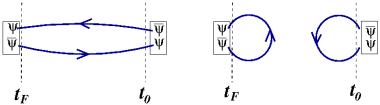


Quark line diagrams

- temporal correlations involving our two-hadron operators need
  - slice-to-slice quark lines (from all spatial sites on a time slice to all spatial sites on another time slice)
  - sink-to-sink quark lines



- isoscalar mesons also require sink-to-sink quark lines



- solution: the stochastic LapH method!

Stochastic estimation of quark propagators

- do not need exact inverse of Dirac matrix $K[U]$
- use noise vectors η satisfying $E(\eta_i) = 0$ and $E(\eta_i \eta_j^*) = \delta_{ij}$
- Z_4 noise is used $\{1, i, -1, -i\}$
- solve $K[U]X^{(r)} = \eta^{(r)}$ for each of N_R noise vectors $\eta^{(r)}$, then obtain a Monte Carlo estimate of all elements of K^{-1}

$$K_{ij}^{-1} \approx \frac{1}{N_R} \sum_{r=1}^{N_R} X_i^{(r)} \eta_j^{(r)*}$$

- variance reduction using noise dilution
- dilution introduces projectors

$$P^{(a)} P^{(b)} = \delta^{ab} P^{(a)}, \qquad \sum_a P^{(a)} = 1, \qquad P^{(a)\dagger} = P^{(a)}$$

- define

$$\eta^{[a]} = P^{(a)} \eta, \qquad X^{[a]} = K^{-1} \eta^{[a]}$$

to obtain Monte Carlo estimate with drastically reduced variance

$$K_{ij}^{-1} \approx \frac{1}{N_R} \sum_{r=1}^{N_R} \sum_a X_i^{(r)[a]} \eta_j^{(r)[a]*}$$

Stochastic LapH method

- introduce Z_N noise in the LapH subspace

$$\rho_{\alpha k}(t), \qquad t = \text{time},\ \alpha = \text{spin},\ k = \text{eigenvector number}$$

- four dilution schemes:

$$\begin{array}{lll}
P_{ij}^{(a)} = \delta_{ij} & a = 0 & \text{(none)} \\
P_{ij}^{(a)} = \delta_{ij}\delta_{ai} & a = 0,1,\ldots,N-1 & \text{(full)} \\
P_{ij}^{(a)} = \delta_{ij}\delta_{a,Ki/N} & a = 0,1,\ldots,K-1 & \text{(interlace-}K\text{)} \\
P_{ij}^{(a)} = \delta_{ij}\delta_{a,i \bmod k} & a = 0,1,\ldots,K-1 & \text{(block-}K\text{)}
\end{array}$$

- apply dilutions to
  - time indices (full for fixed src, interlace-16 for relative src)
  - spin indices (full)
  - LapH eigenvector indices (interlace-8 mesons, interlace-4 baryons)

Quark line estimates in stochastic LapH

- each of our quark lines is the product of matrices

$$\mathcal{Q} = D^{(j)}\mathcal{S}K^{-1}\gamma_4\mathcal{S}D^{(k)\dagger}$$

- displaced-smeared-diluted quark source and quark sink vectors:

$$\begin{aligned} \varrho^{[b]}(\rho) &= D^{(j)}V_sP^{(b)}\rho \\ \varphi^{[b]}(\rho) &= D^{(j)}\mathcal{S}K^{-1}\gamma_4\, V_sP^{(b)}\rho \end{aligned}$$

- estimate in stochastic LapH by (A, B flavor, u, v compound: space, time, color, spin, displacement type)

$$\mathcal{Q}_{uv}^{(AB)} \approx \frac{1}{N_R}\delta_{AB}\sum_{r=1}^{N_R}\sum_b \varphi_u^{[b]}(\rho^r)\ \varrho_v^{[b]}(\rho^r)^*$$

- occasionally use γ_5-Hermiticity to switch source and sink

$$\mathcal{Q}_{uv}^{(AB)} \approx \frac{1}{N_R}\delta_{AB}\sum_{r=1}^{N_R}\sum_b \overline{\varrho}_u^{[b]}(\rho^r)\ \overline{\varphi}_v^{[b]}(\rho^r)^*$$

defining $\overline{\varrho}(\rho) = -\gamma_5\gamma_4\varrho(\rho)$ and $\overline{\varphi}(\rho) = \gamma_5\gamma_4\varphi(\rho)$

Source-sink factorization in stochastic LapH

- baryon correlator has form

$$C_{l\bar{l}} = c^{(l)}_{ijk}\, c^{(\bar{l})*}_{\bar{i}\bar{j}\bar{k}}\, \mathcal{Q}^{A}_{i\bar{i}}\, \mathcal{Q}^{B}_{j\bar{j}}\, \mathcal{Q}^{C}_{k\bar{k}}$$

- stochastic estimate with dilution

$$\begin{aligned} C_{l\bar{l}} \quad &\approx \quad \frac{1}{N_R}\sum_{r}\sum_{d_A d_B d_C} c^{(l)}_{ijk}\, c^{(\bar{l})*}_{\bar{i}\bar{j}\bar{k}} \left(\varphi^{(Ar)[d_A]}_{i}\, \varrho^{(Ar)[d_A]*}_{\bar{i}}\right) \\ &\times \quad \left(\varphi^{(Br)[d_B]}_{j}\, \varrho^{(Br)[d_B]*}_{\bar{j}}\right)\left(\varphi^{(Cr)[d_C]}_{k}\, \varrho^{(Cr)[d_C]*}_{\bar{k}}\right) \end{aligned}$$

- define baryon source and sink

$$\begin{aligned} \mathcal{B}^{(r)[d_A d_B d_C]}_{l}(\varphi^{A}, \varphi^{B}, \varphi^{C}) \quad &= \quad c^{(l)}_{ijk}\; \varphi^{(Ar)[d_A]}_{i}\, \varphi^{(Br)[d_B]}_{j}\, \varphi^{(Cr)[d_C]}_{k} \\ \mathcal{B}^{(r)[d_A d_B d_C]}_{l}(\varrho^{A}, \varrho^{B}, \varrho^{C}) \quad &= \quad c^{(l)}_{ijk}\; \varrho^{(Ar)[d_A]}_{i}\, \varrho^{(Br)[d_B]}_{j}\, \varrho^{(Cr)[d_C]}_{k} \end{aligned}$$

- correlator is dot product of source vector with sink vector

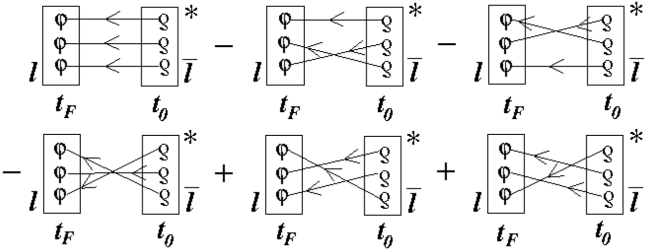
$$C_{l\bar{l}} \approx \frac{1}{N_R}\sum_{r}\sum_{d_A d_B d_C} \mathcal{B}^{(r)[d_A d_B d_C]}_{l}(\varphi^{A}, \varphi^{B}, \varphi^{C})\, \mathcal{B}^{(r)[d_A d_B d_C]}_{\bar{l}}(\varrho^{A}, \varrho^{B}, \varrho^{C})^{*}$$

Correlators and quark line diagrams

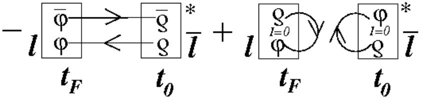
- baryon correlator

$$C_{l\bar{l}} \approx \frac{1}{N_R}\sum_{r}\sum_{d_A d_B d_C} \mathcal{B}_l^{(r)[d_A d_B d_C]}(\varphi^A,\varphi^B,\varphi^C)\,\mathcal{B}_{\bar{l}}^{(r)[d_A d_B d_C]}(\varrho^A,\varrho^B,\varrho^C)^*$$

- express diagrammatically

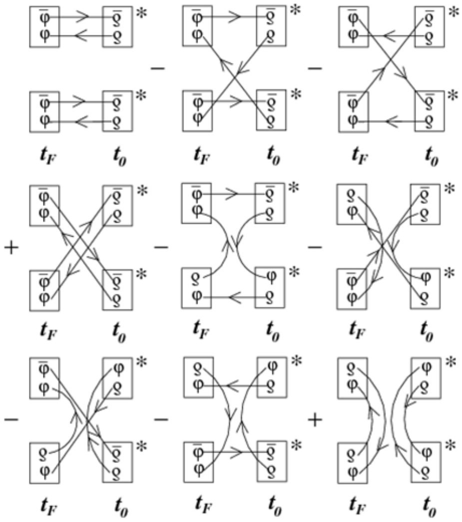


- meson correlator



More complicated correlators

- two-meson to two-meson correlators (non isoscalar mesons)



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Conclusion and future work

- goal: comprehensive survey of energy spectrum of QCD stationary states in a finite volume
- stochastic LapH method works very well
  - allows evaluation of all needed quark-line diagrams
  - source-sink factorization facilitates large number of operators
  - last_laph software completed for evaluating correlators
- showed first results in ρ-channel: $I = 1,\ S = 0,\ T_{1u}^{+}$ using 56×56 matrix of correlators
- preliminary results using 59×59 matrix of correlators in the bosonic $I = \frac{1}{2},\ S = 1,\ T_{1u}$
- large number of channels to study over the next year!
- first peek: results on $(32^3|240)$ ensemble look even better so far!!
- study various scattering phase shifts also planned
- infinite-volume resonance parameters from finite-volume energies $\longrightarrow$ need new effective field theory techniques