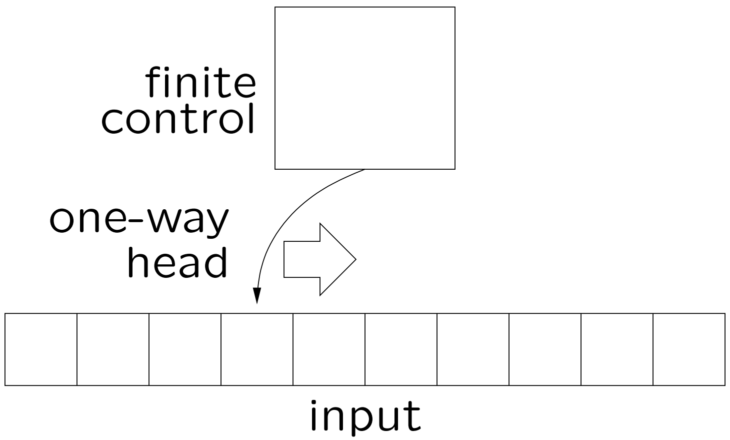
Deterministic moles cannot solve liveness

Christos Kapoutsis

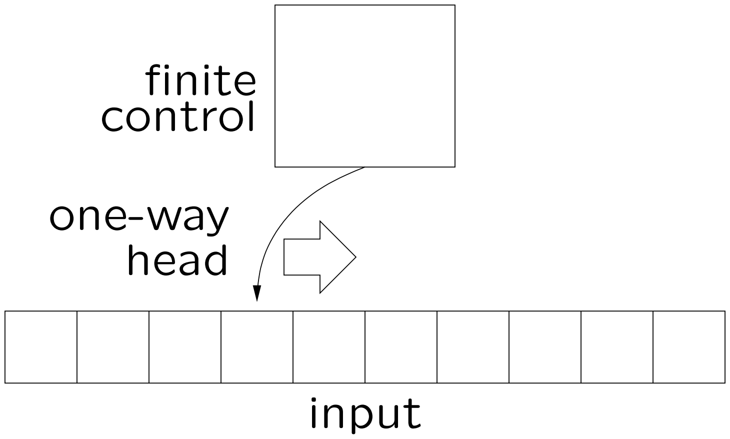
workshop on the
Descriptional Complexity of Formal Systems
Como, Italy, July 2005

1DFA

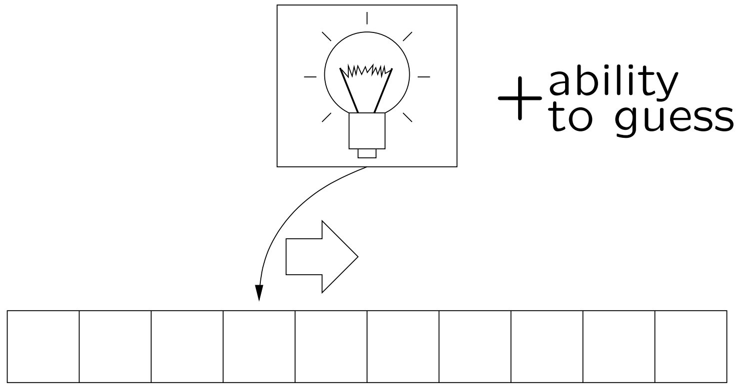


1DFAs versus 1NFAs

1DFA



1NFA



1DFAs versus 1NFAs

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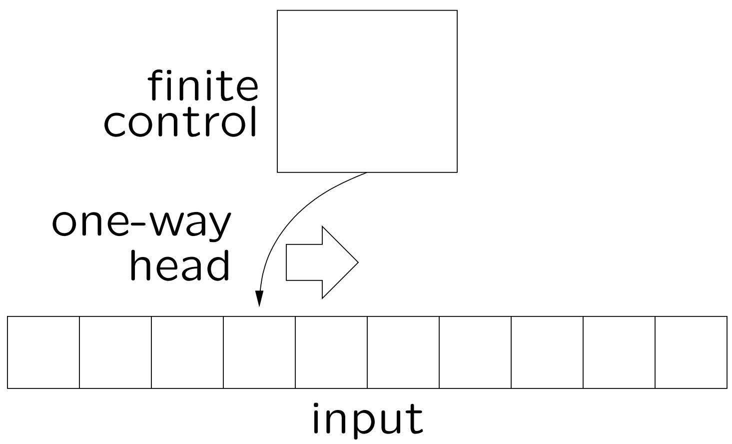

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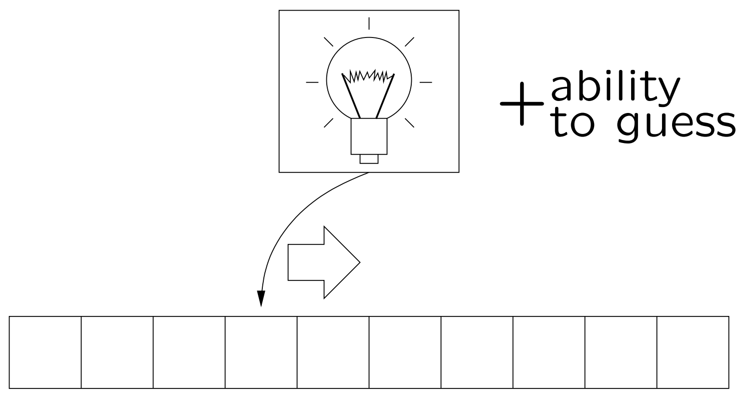
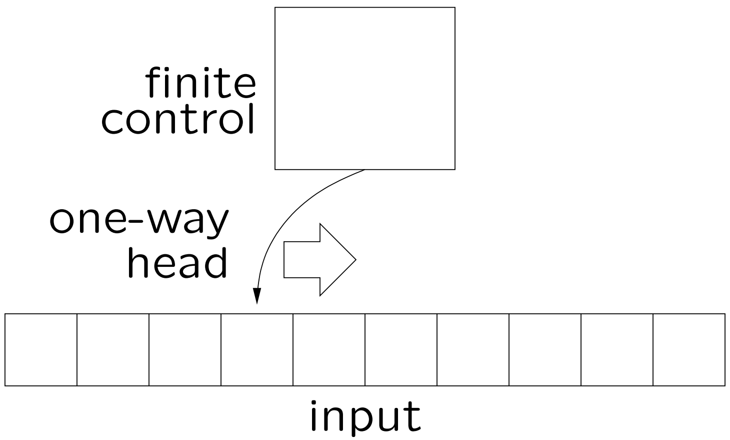


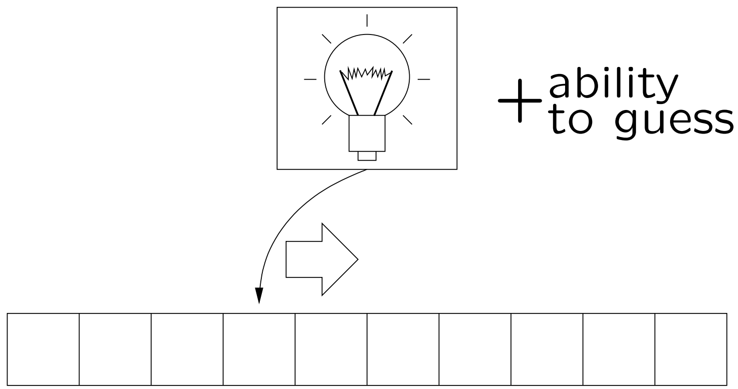
a 1DFA with $\leq 2^n - 1$ states $\xleftarrow{\text{can be converted to}}$ every 1NFA with n states

1DFAs versus 1NFAs

1DFA



1NFA



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and sometimes all these $2^n - 1$ states are necessary

1DFAs versus 1NFAs

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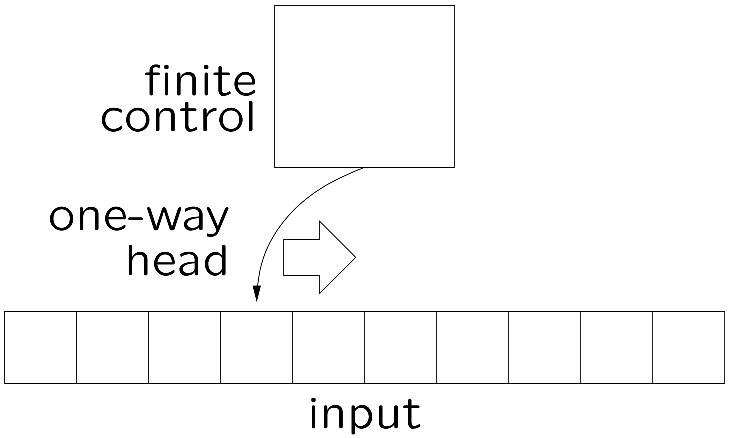
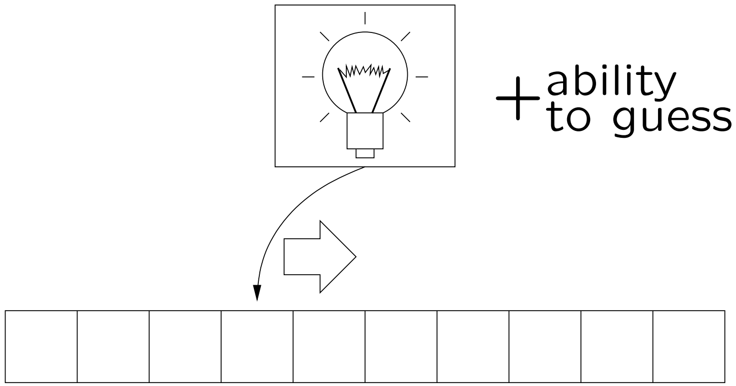


1NFA



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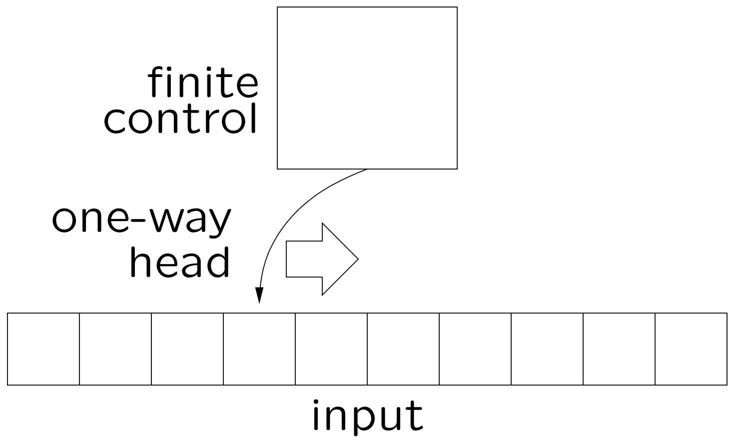
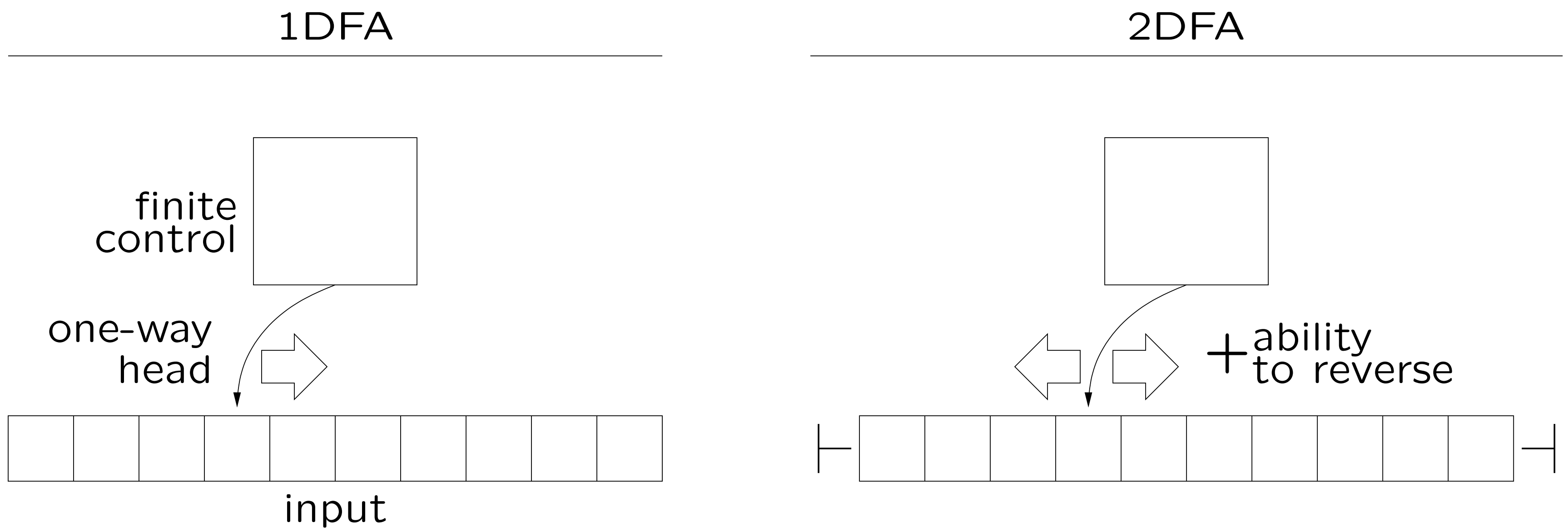
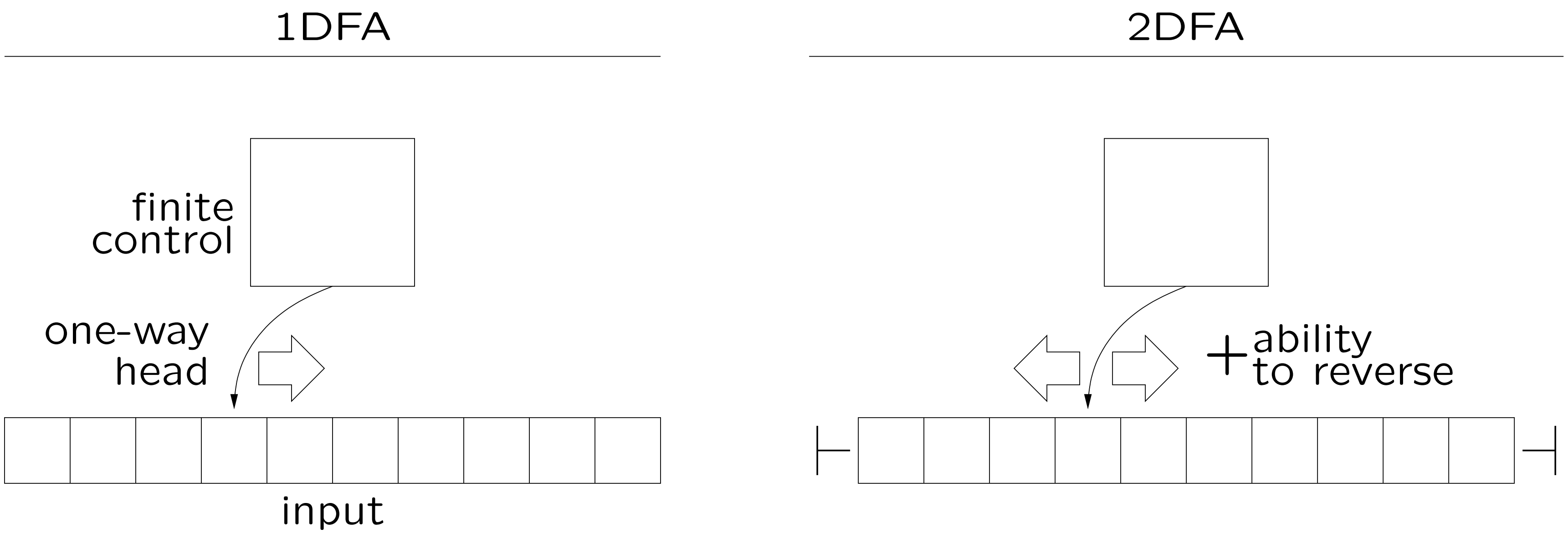
"*the trade-off is exactly* $2^n - 1$"

1DFA



1DFAs versus 2DFAs





a 1DFA with $\leq$ $\boxed{\ ?\ }$ states $\xleftarrow{\text{can be converted to}}$ every 2DFA with n states

and sometimes all these $\boxed{\ ?\ }$ states are necessary

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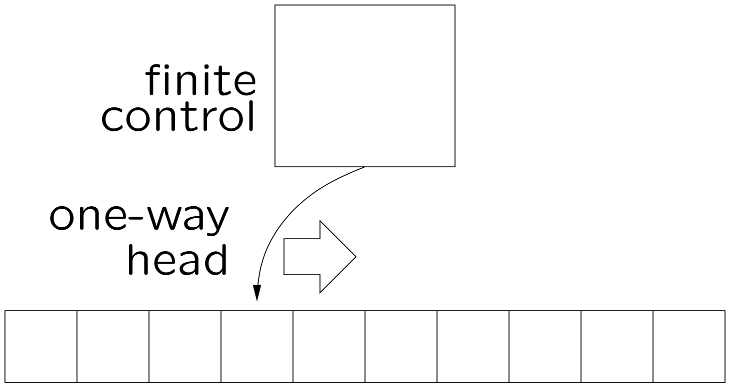
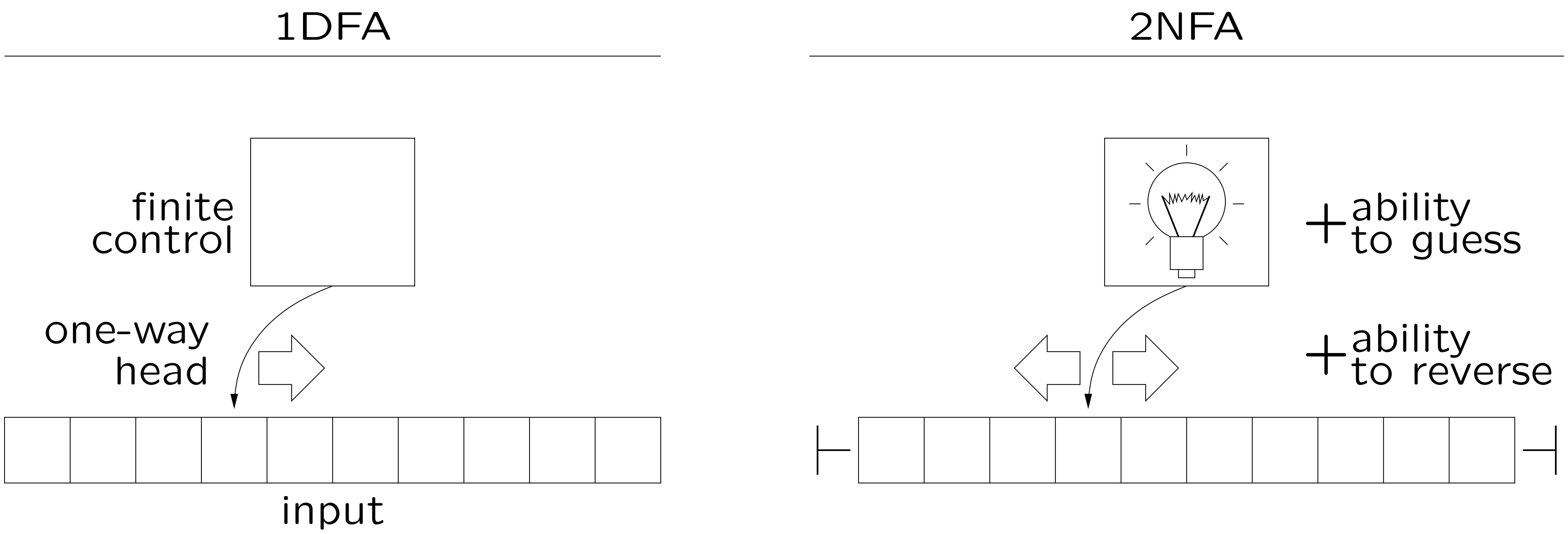
1DFA



1DFAs versus 2NFAs



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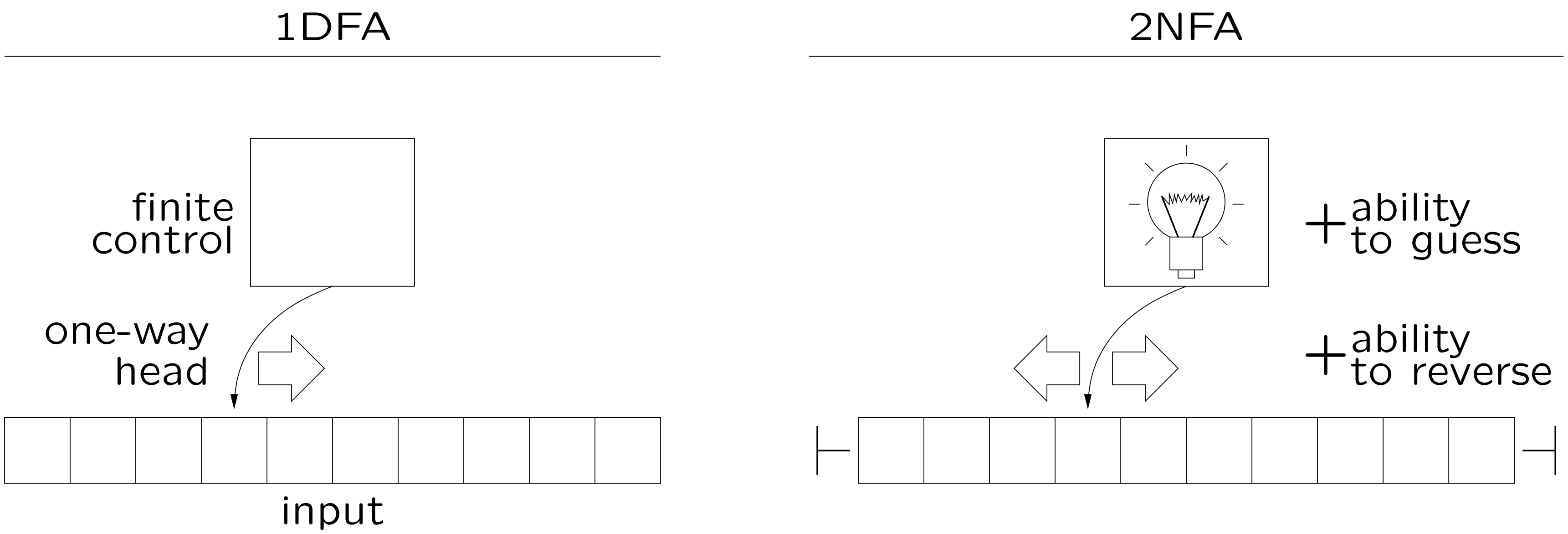

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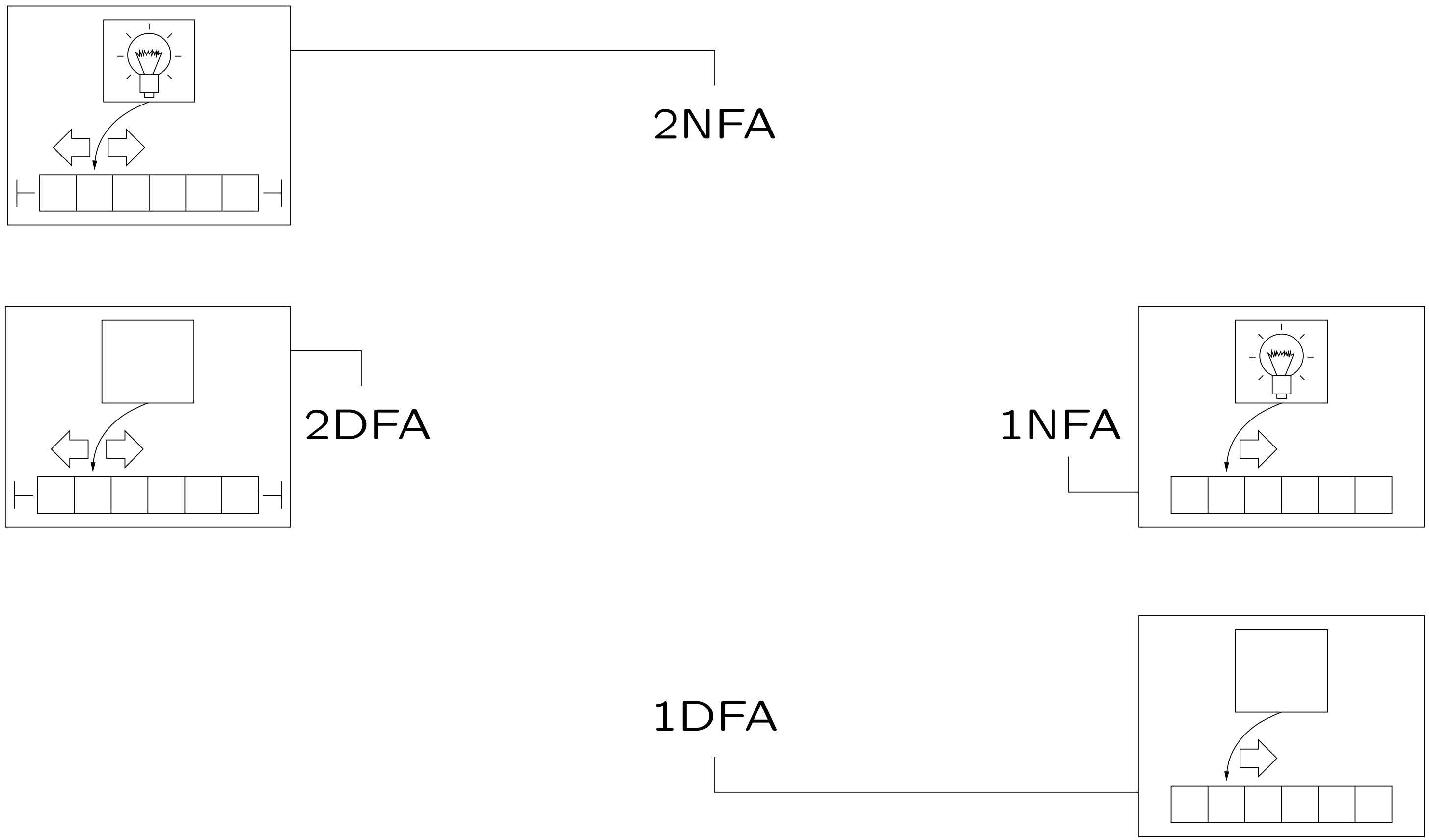
the big picture



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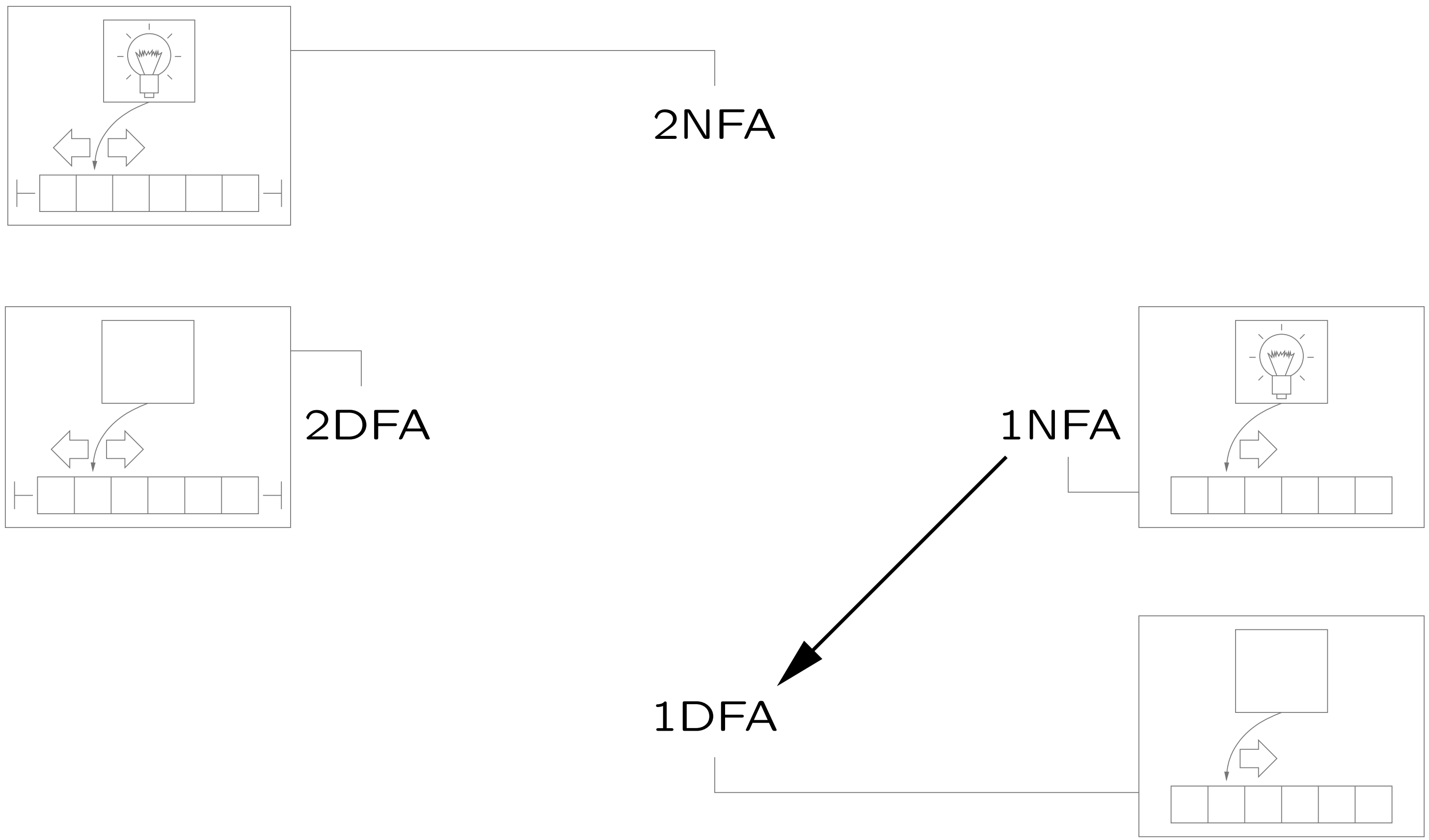
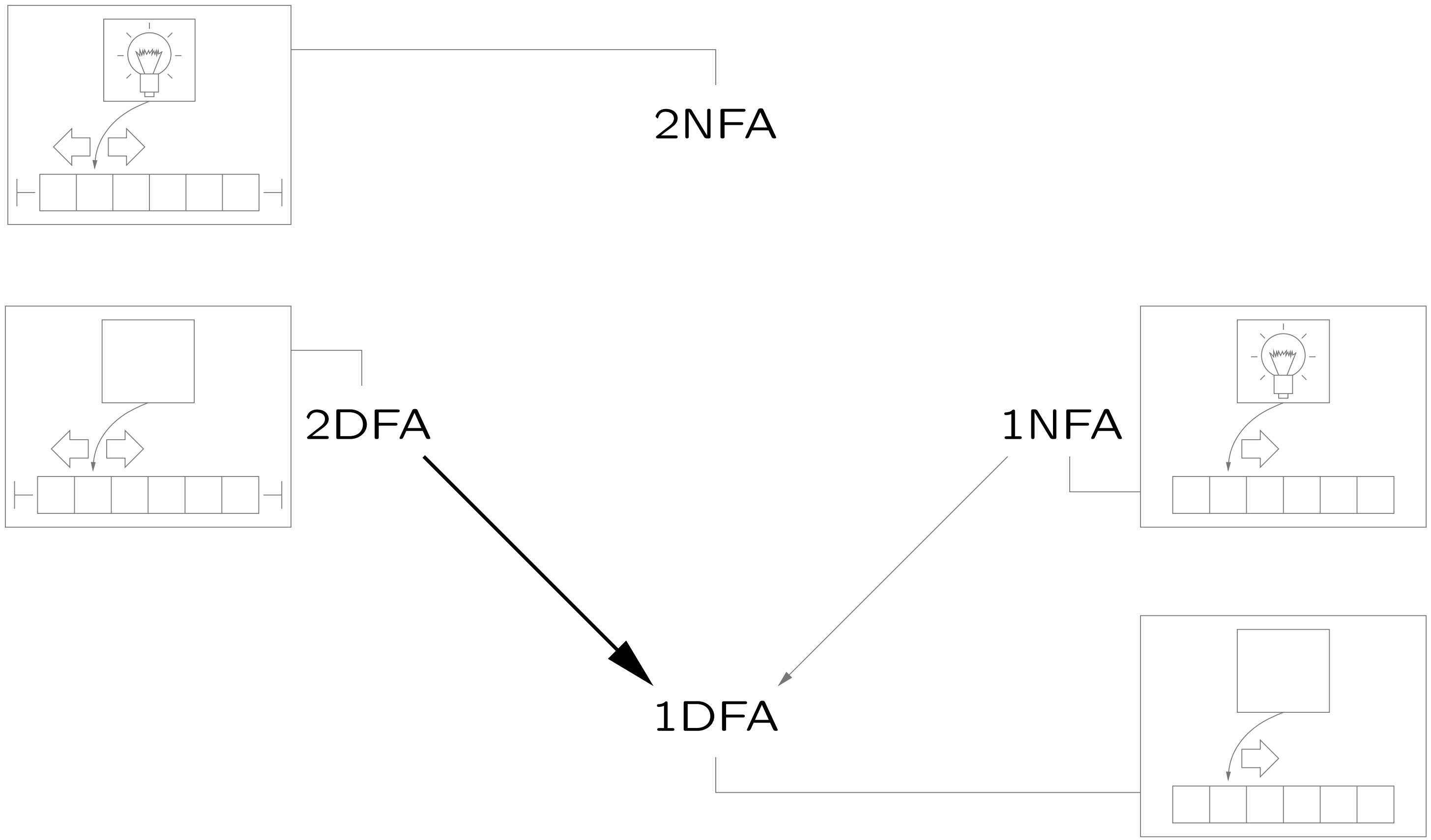


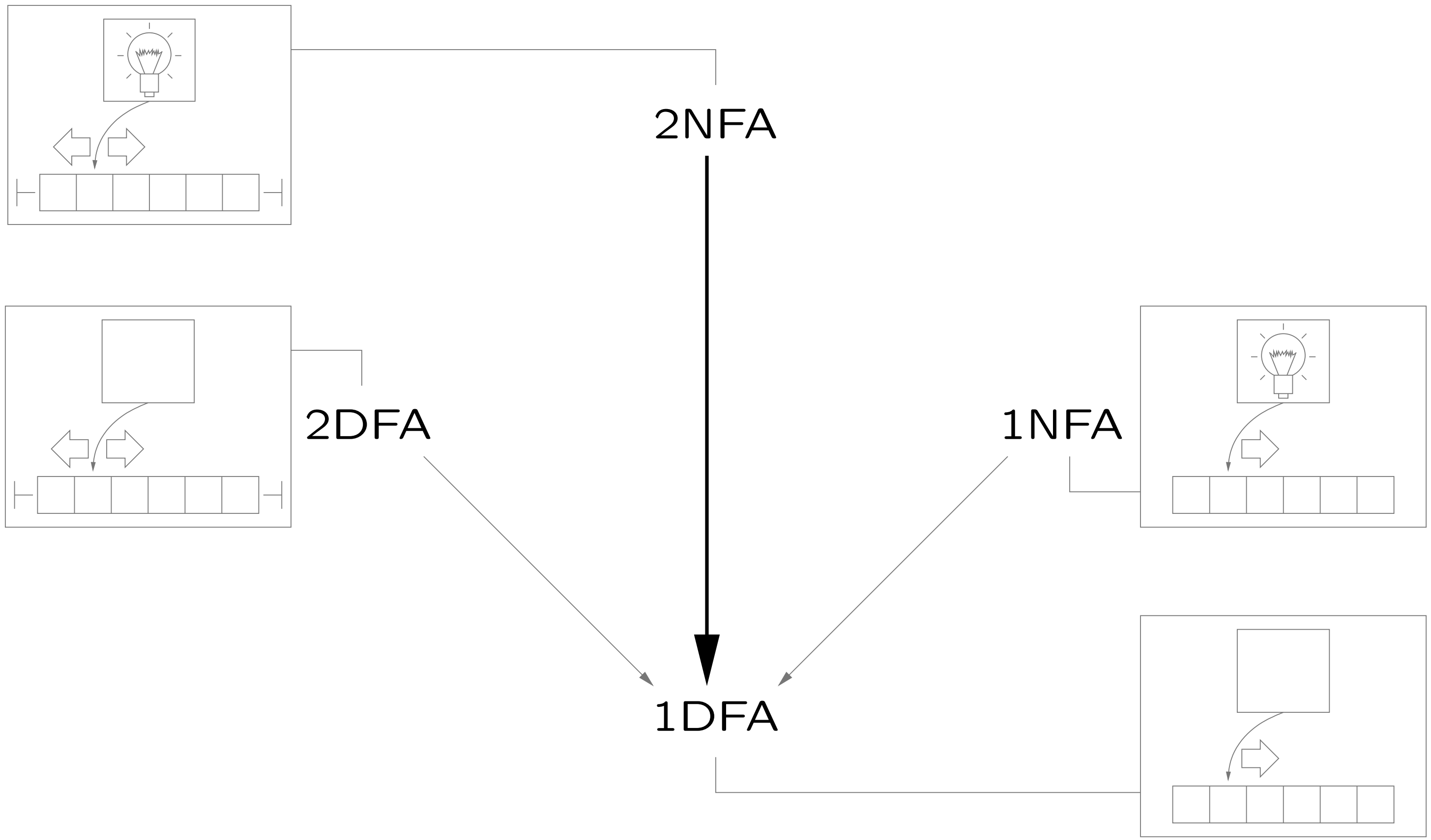
$2^n - 1$

the big picture

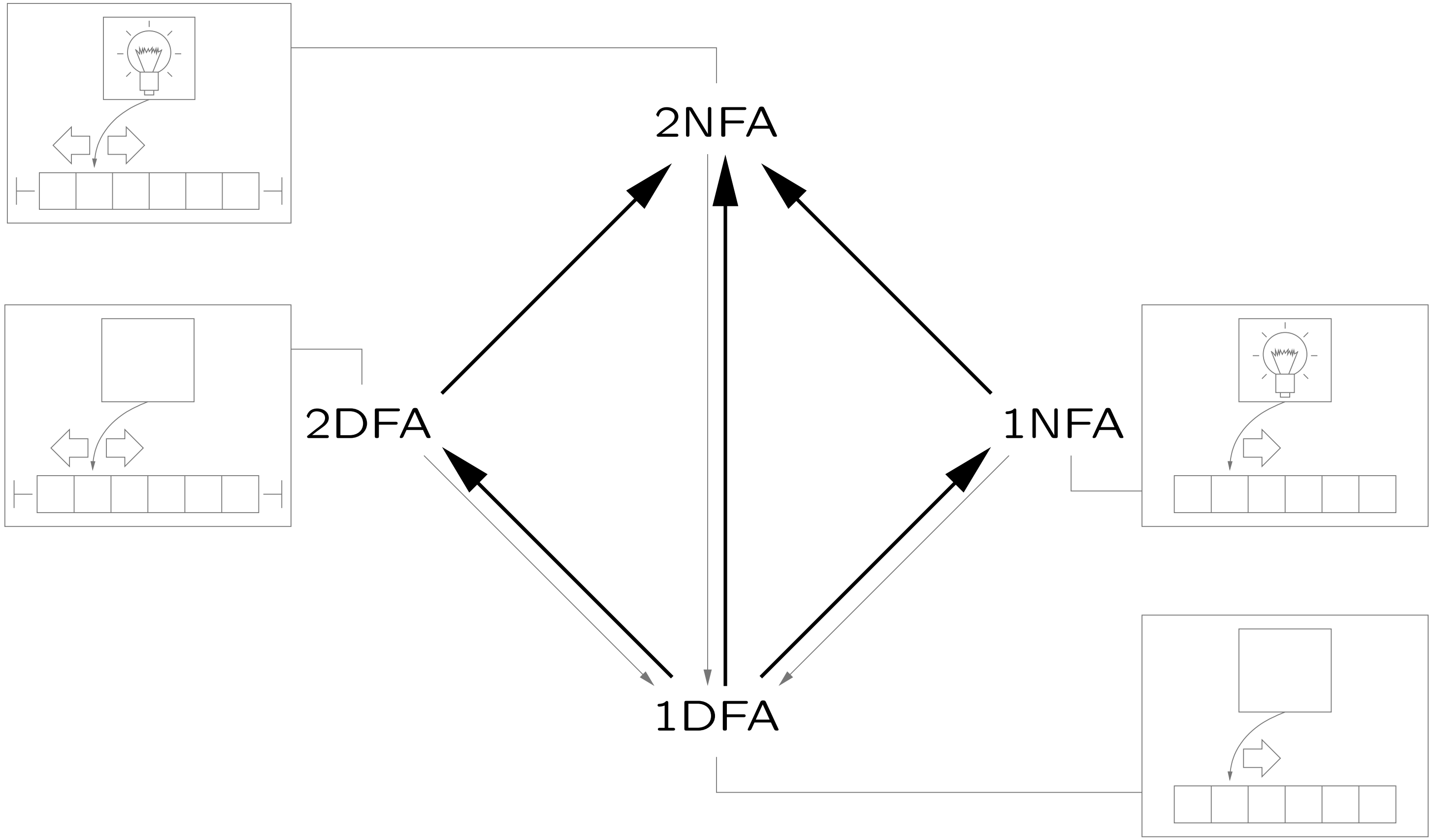


$2^n - 1$, $n(n^n - (n-1)^n)$

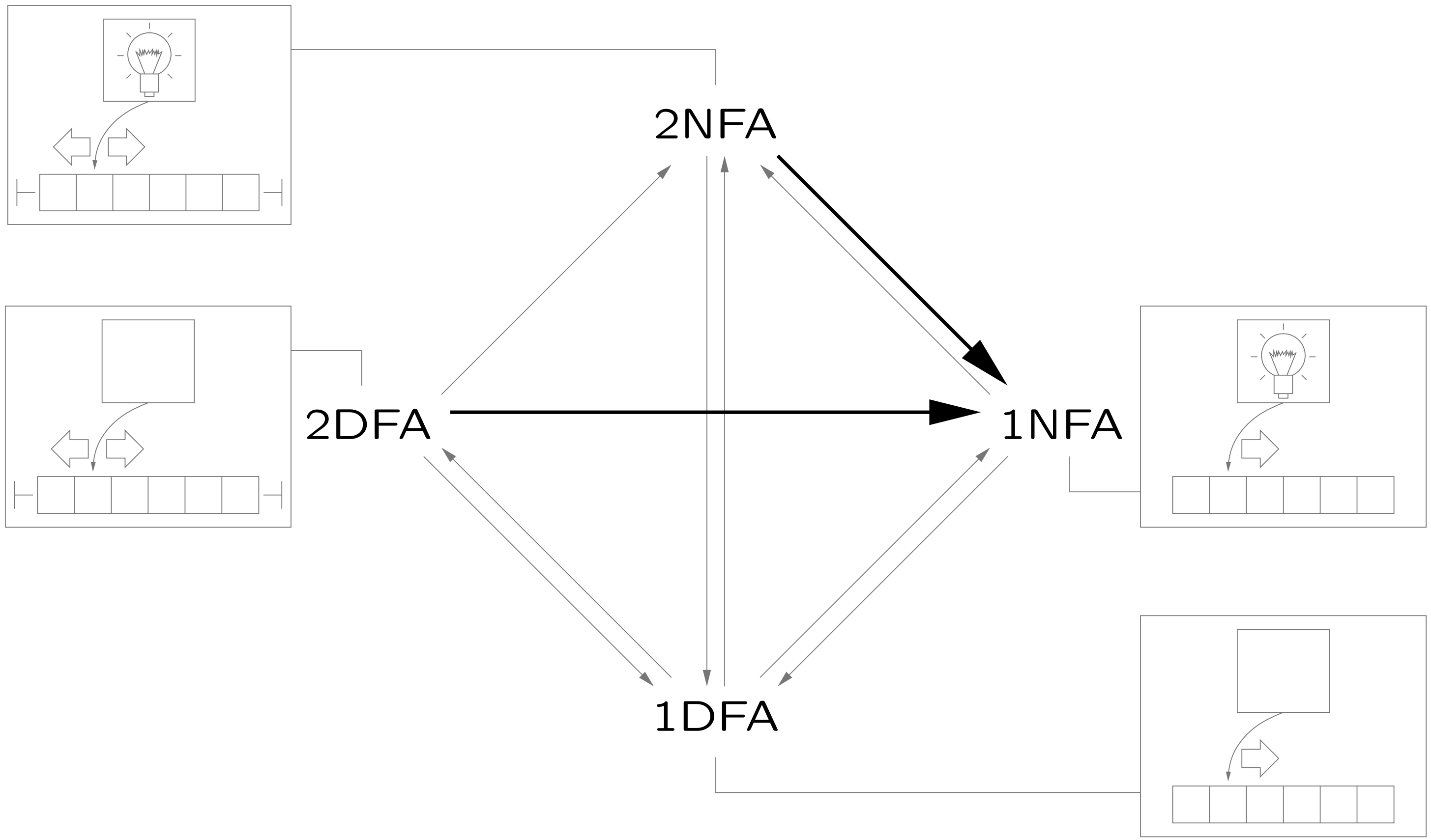
the big picture



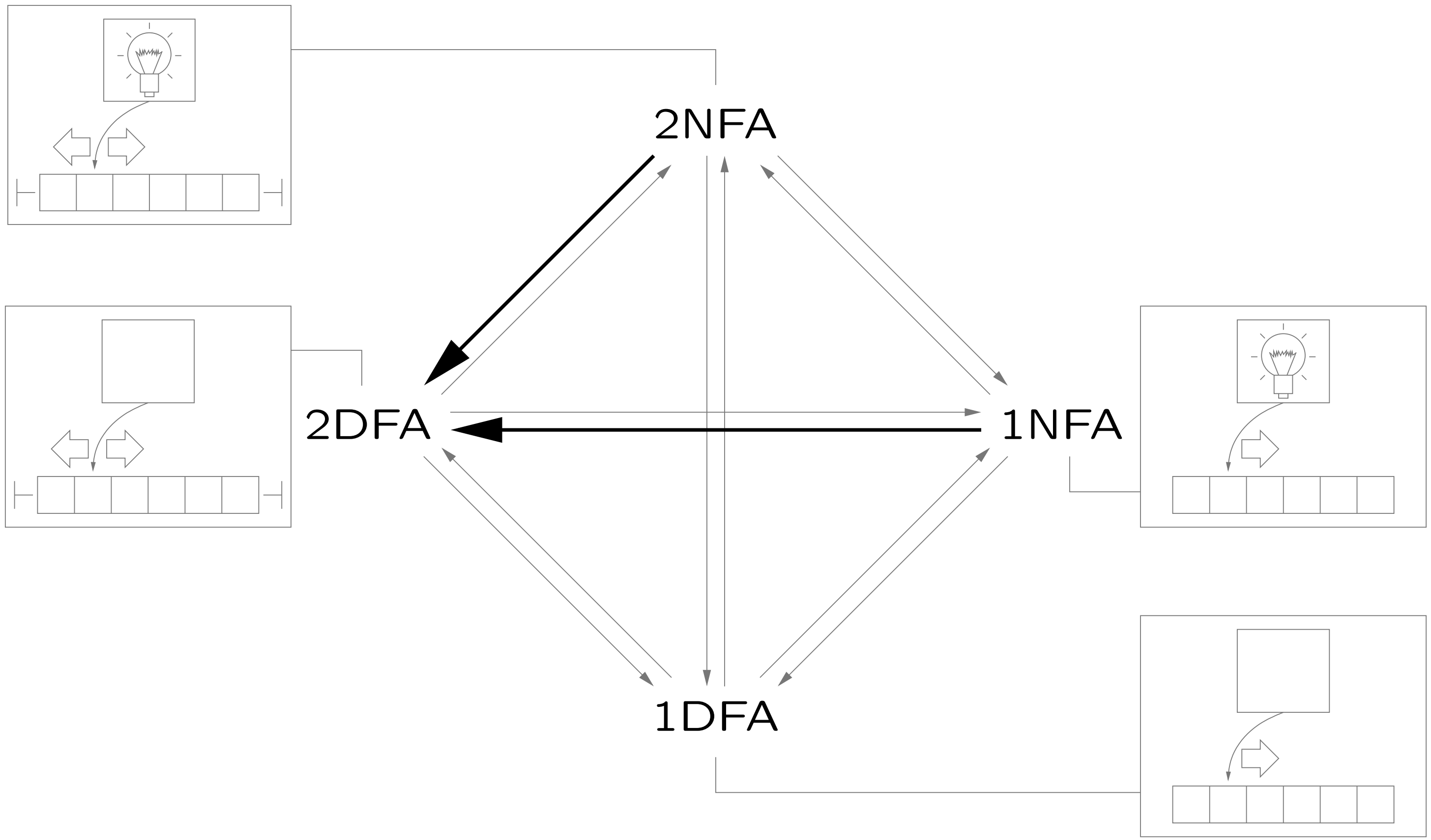
$2^n - 1,\ n(n^n - (n-1)^n),\ \sum_{i<n}\sum_{j<n}\binom{n}{i}\binom{n}{j}(2^i - 1)^j$



$2^n - 1$, $n(n^n - (n-1)^n)$, $\sum_{i<n} \sum_{j<n} \binom{n}{i}\binom{n}{j}(2^i - 1)^j$, n

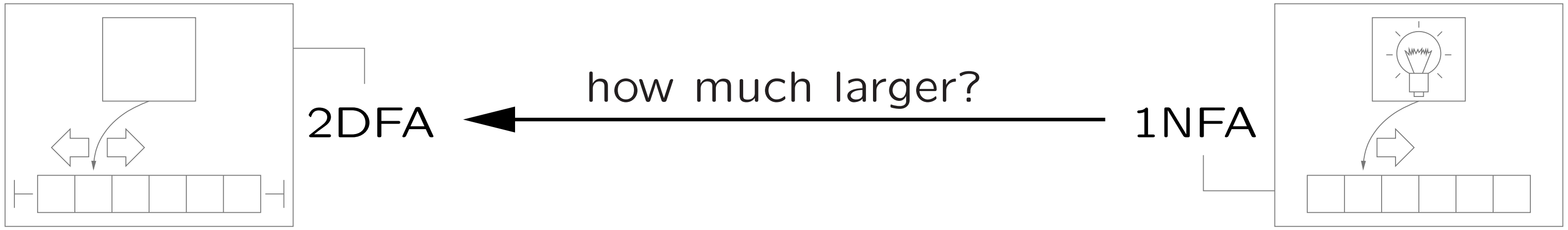


$2^n - 1$, $n(n^n - (n-1)^n)$, $\sum_{i<n} \sum_{j<n} \binom{n}{i}\binom{n}{j}(2^i - 1)^j$, n, $\binom{2n}{n+1}$

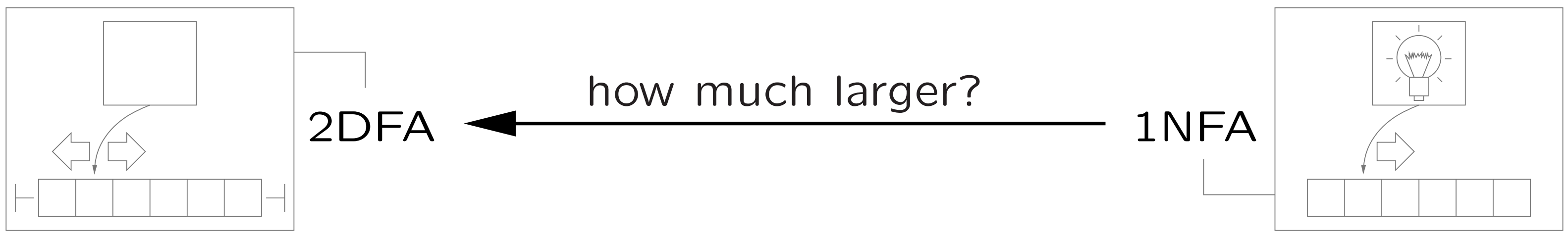


$2^n - 1$, $n(n^n - (n-1)^n)$, $\sum_{i<n}\sum_{j<n}\binom{n}{i}\binom{n}{j}(2^i-1)^j$, n, $\binom{2n}{n+1}$, ?, ?

the problem

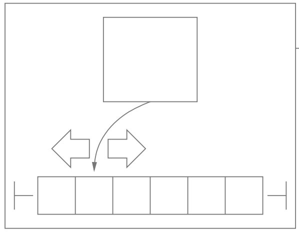


the problem

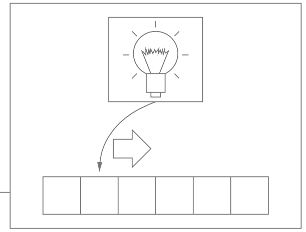


polynomially ? exponentially

the problem



2DFA ← how much larger? — 1NFA



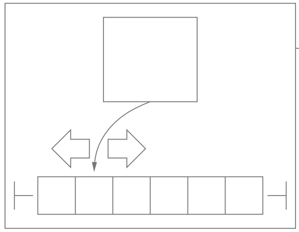
polynomially ? exponentially

determ Turing machine

DTM ← how much slower? — NTM

nondeterm Turing machine

the problem



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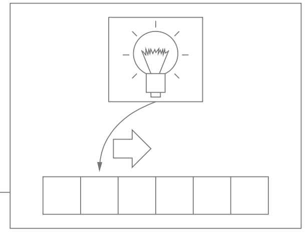

polynomially ? exponentially

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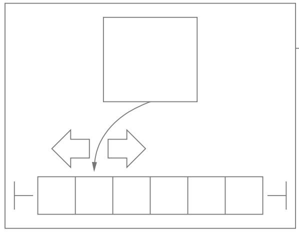

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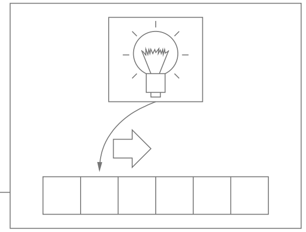

polynomially ? exponentially

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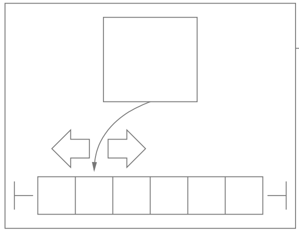
nondeterm Turing machine

polynomially ? exponentially

P=NP P≠NP



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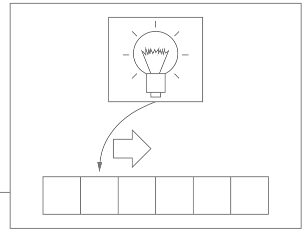


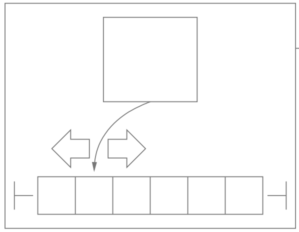
polynomially ? exponentially

determ Turing machine — DTM ← how much slower? ← NTM — nondeterm Turing machine

SATISFIABILITY

$$\overset{n}{\overleftrightarrow{(x_1 \vee \overline{x_3}) \wedge (\overline{x_2} \vee x_1) \wedge (x_3 \vee x_1 \vee x_2)}}$$

is there a satisfying assignment?



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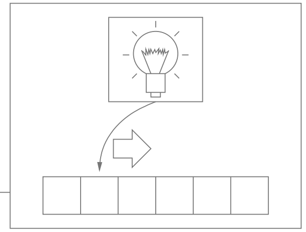
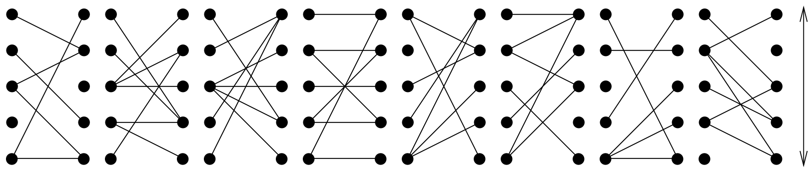


LIVENESS



n

is there a live path?

---

determ Turing machine

DTM ← how much slower? — NTM

nondeterm Turing machine

SATISFIABILITY

$$\overset{n}{\longleftrightarrow}$$

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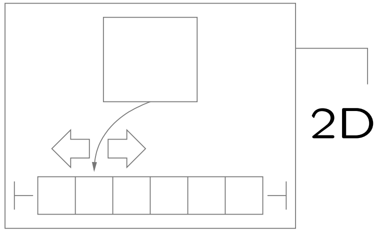

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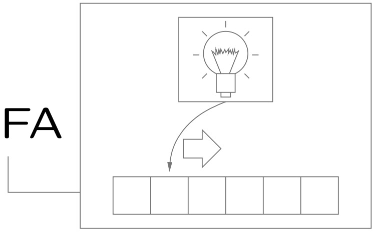
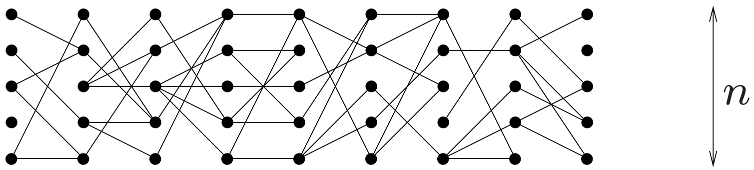


LIVENESS



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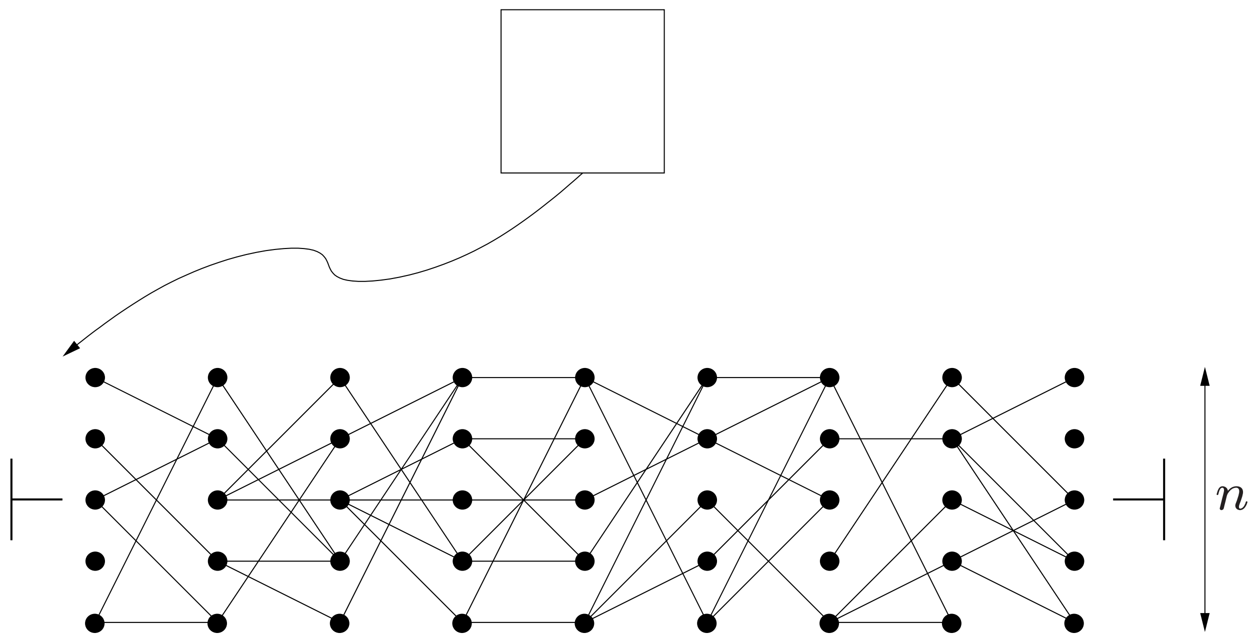
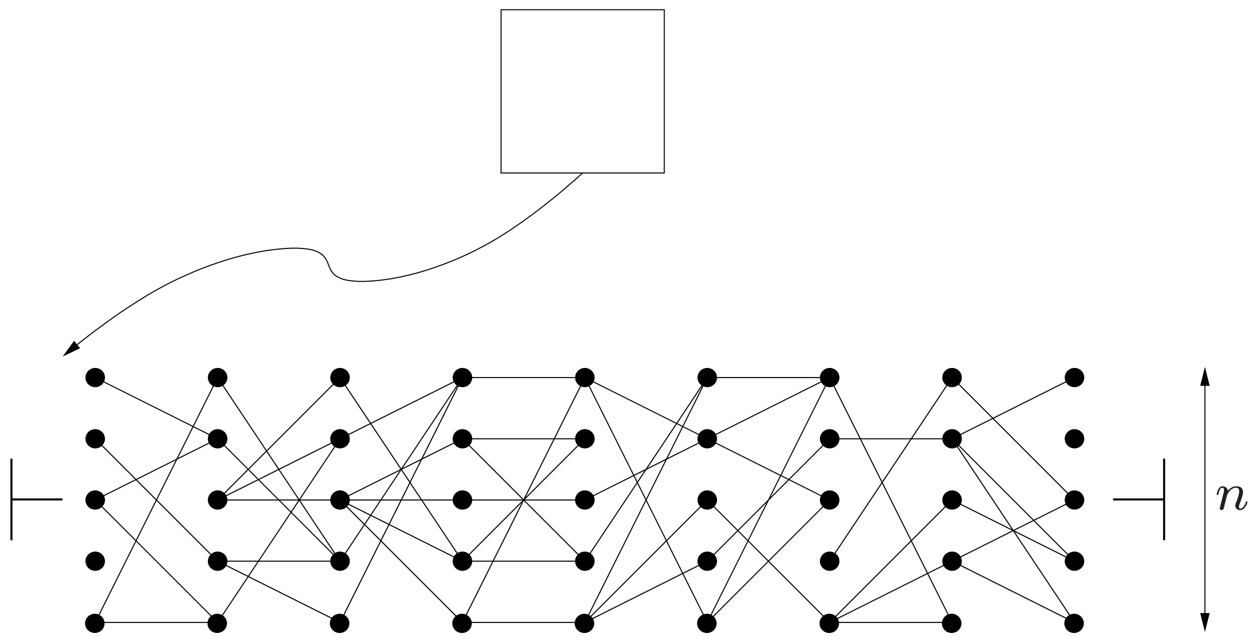
2DFA against LIVENESS

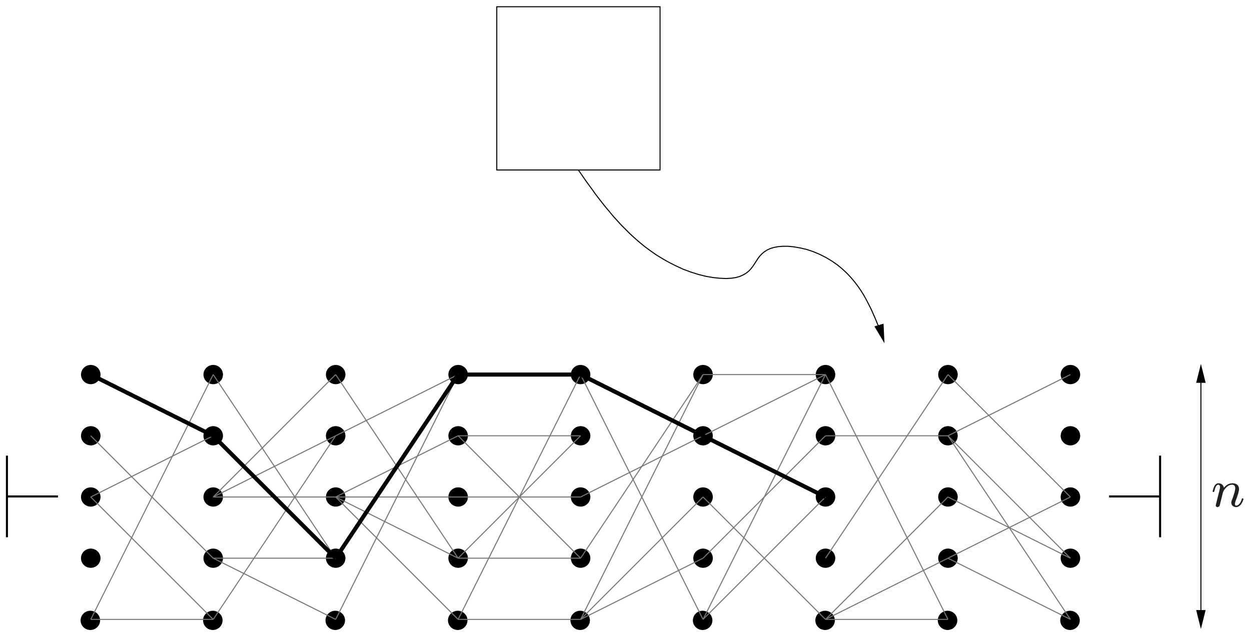


2DFA against LIVENESS: graph exploration



IDEA: depth first search!

2DFA against LIVENESS: graph exploration



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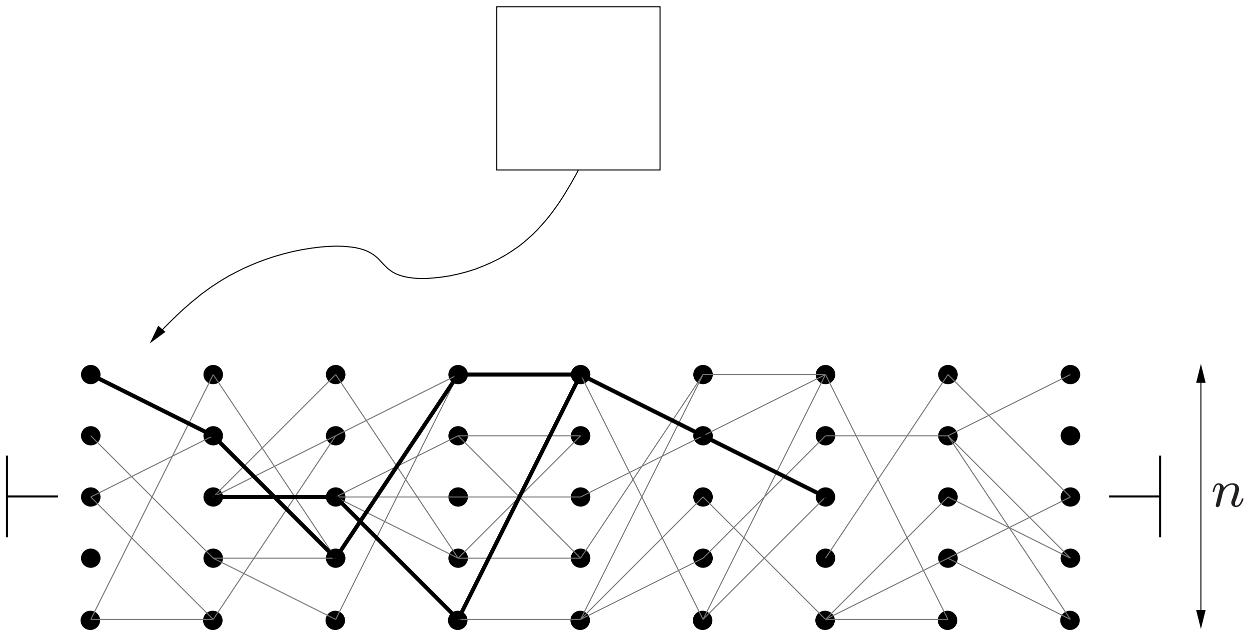

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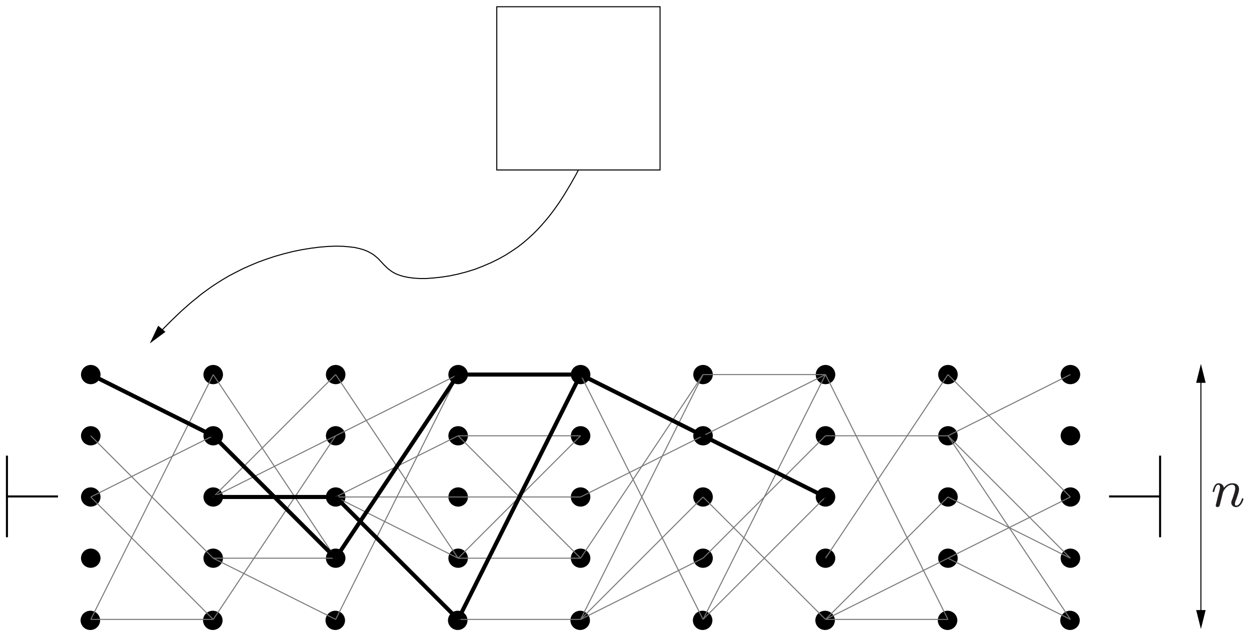
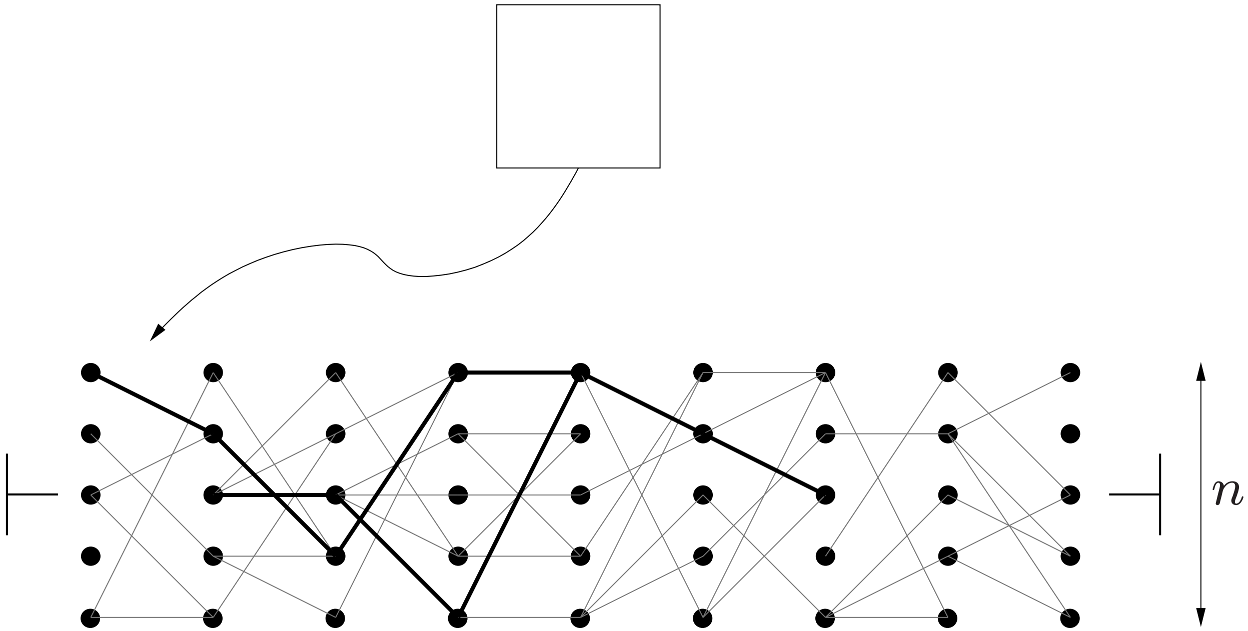


IDEA: depth first search!
PROBLEM: we get lost

2DFA against LIVENESS: graph exploration



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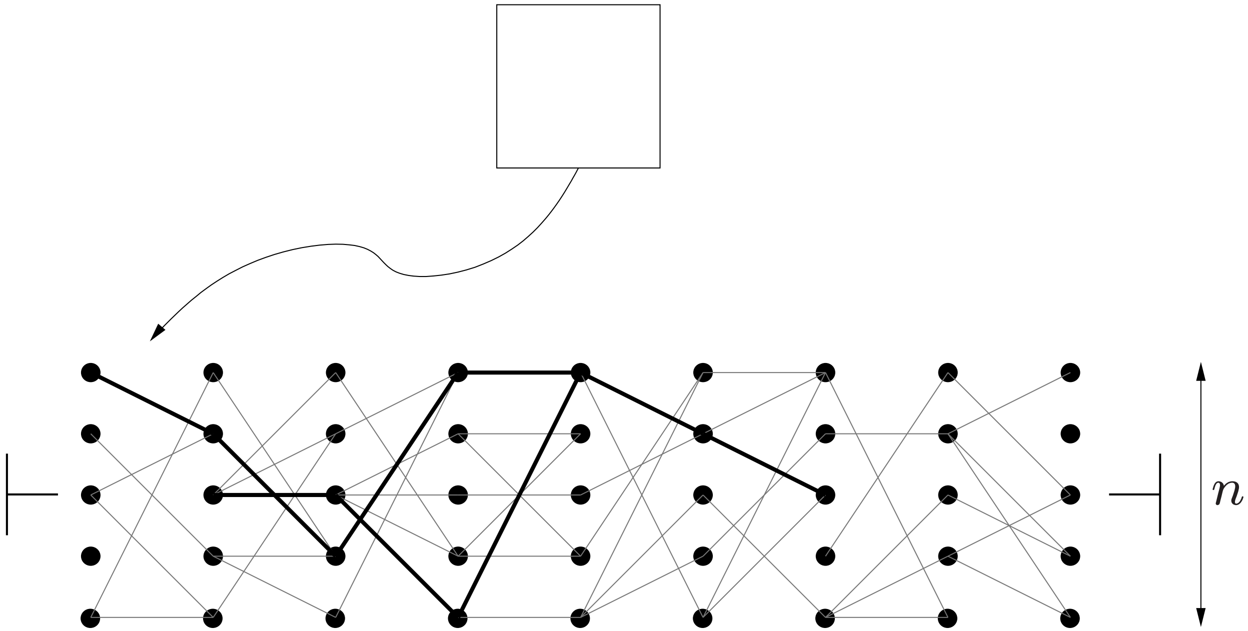
THEOREM: no graph exploration can work

2DFA against LIVENESS: graph exploration



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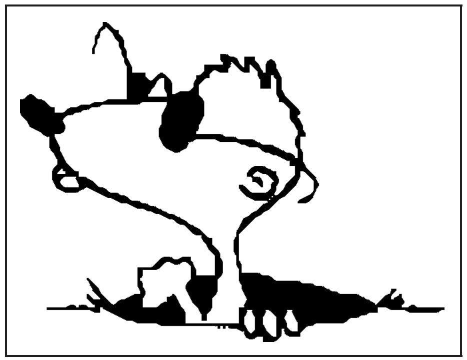
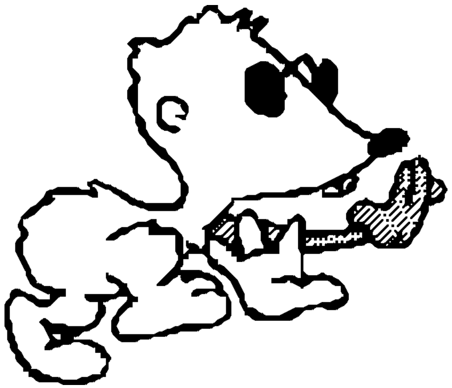
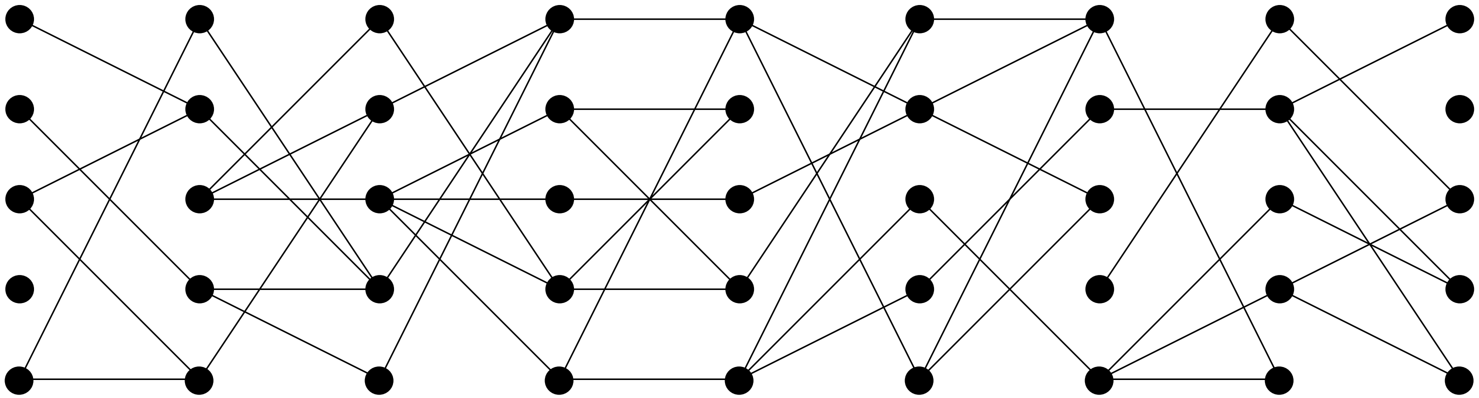


THEOREM: no graph exploration can work

MOLE against LIVENESS





PROOF PLAN: Construct a string of graphs w such that:

the mole fails on w

$\iff$ w is live and the mole rejects $\vee$ w is dead and the mole accepts

$\Longleftarrow$ w is dead and contains a live node that the mole never visits

PROOF PLAN: Construct a string of graphs w such that:

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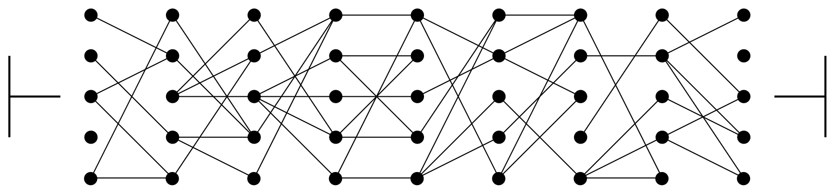
w
(dead)



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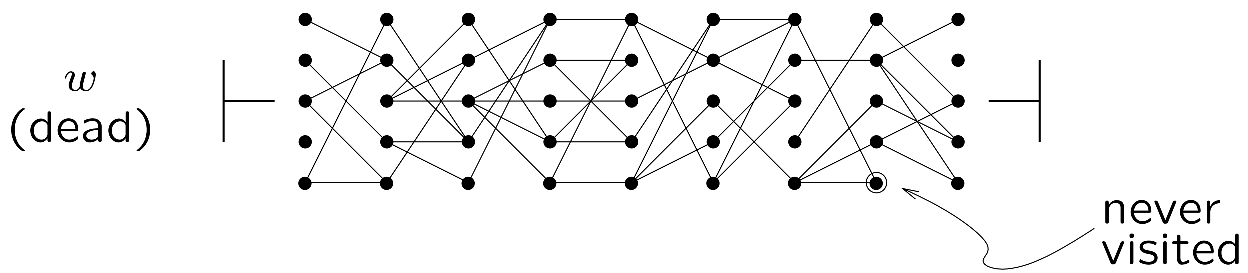

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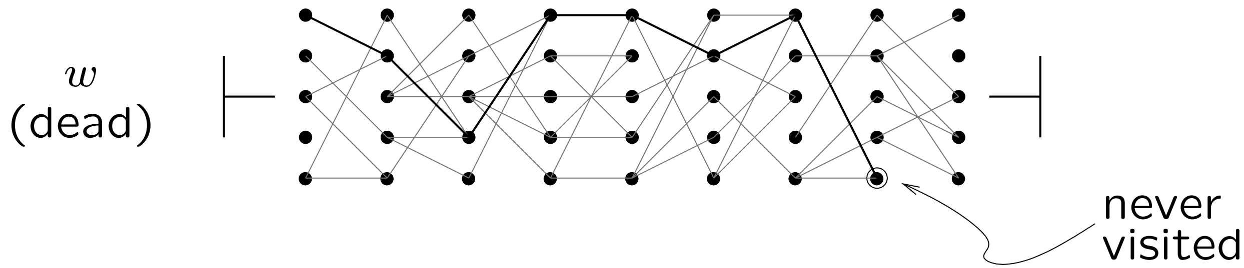

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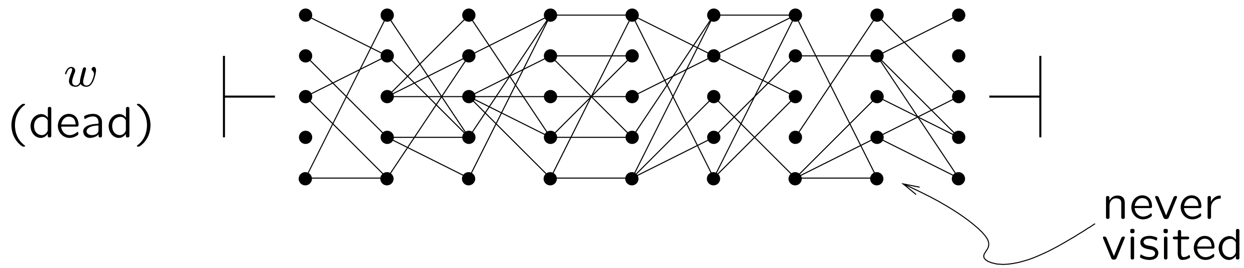

MOLE against LIVENESS

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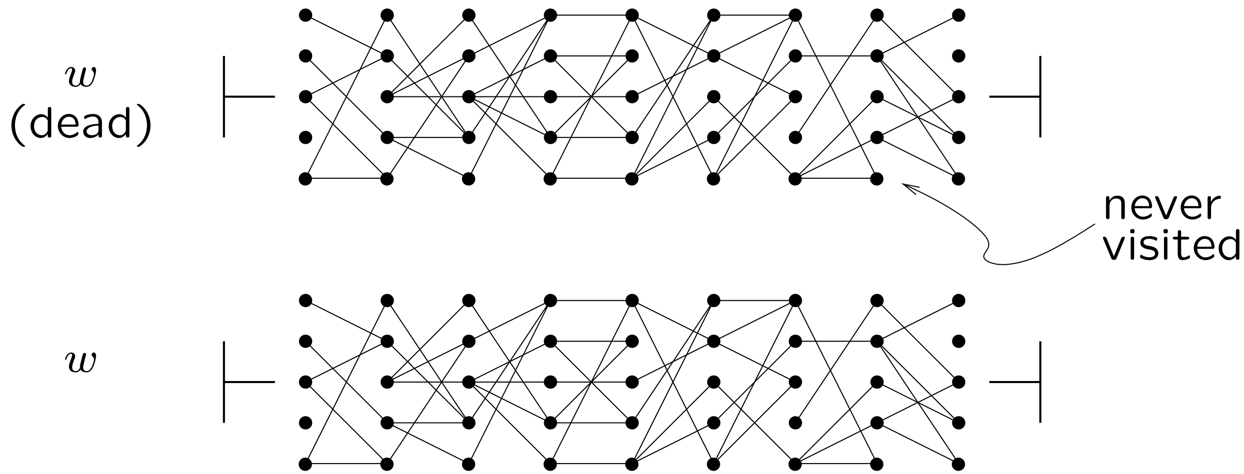

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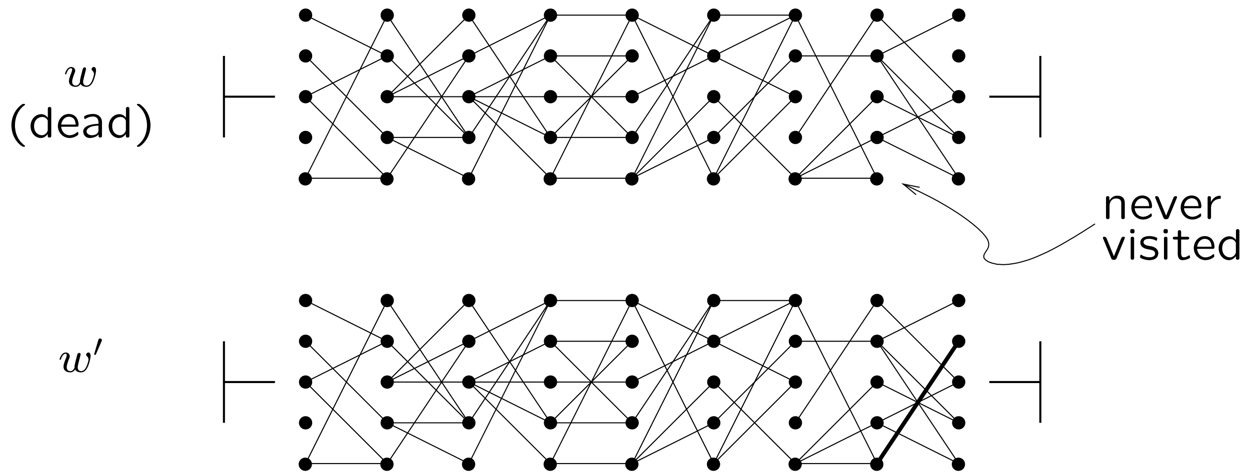

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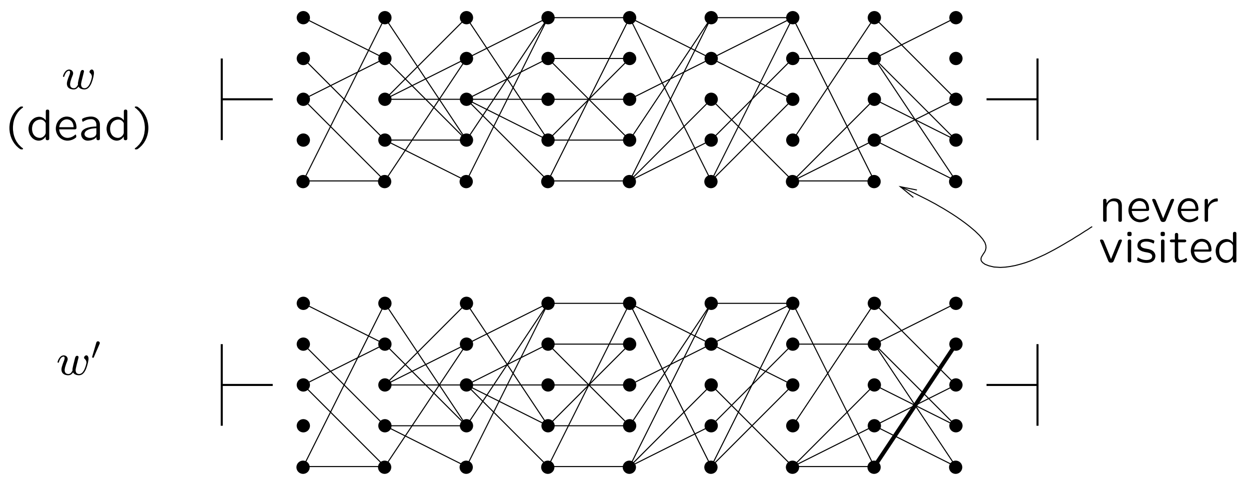

same computation!

MOLE against LIVENESS

PROOF PLAN: Construct a string of graphs w such that:

the mole fails on w

$\iff$ w is live and the mole rejects $\vee$ w is dead and the mole accepts

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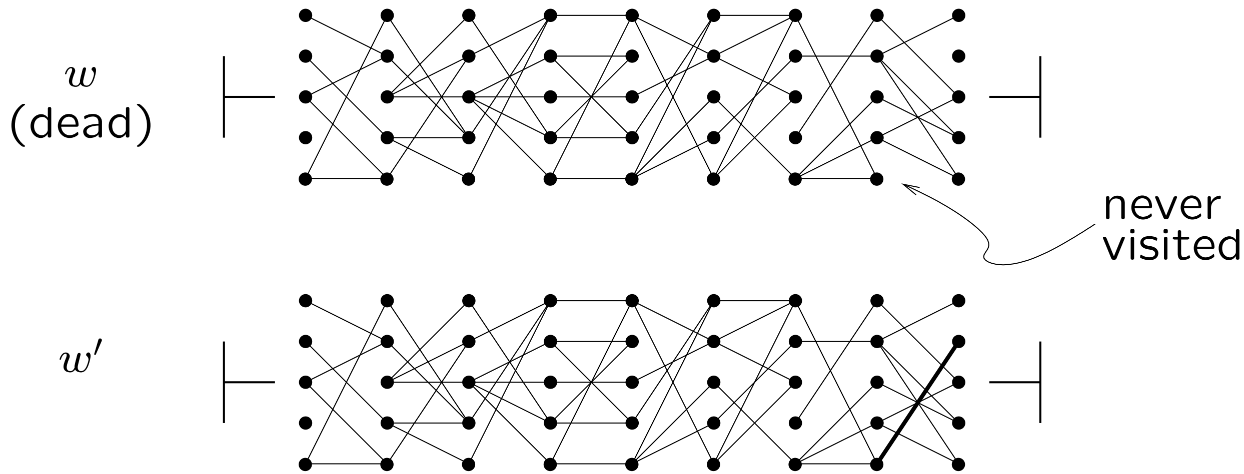

same computation $\Rightarrow$ same decision

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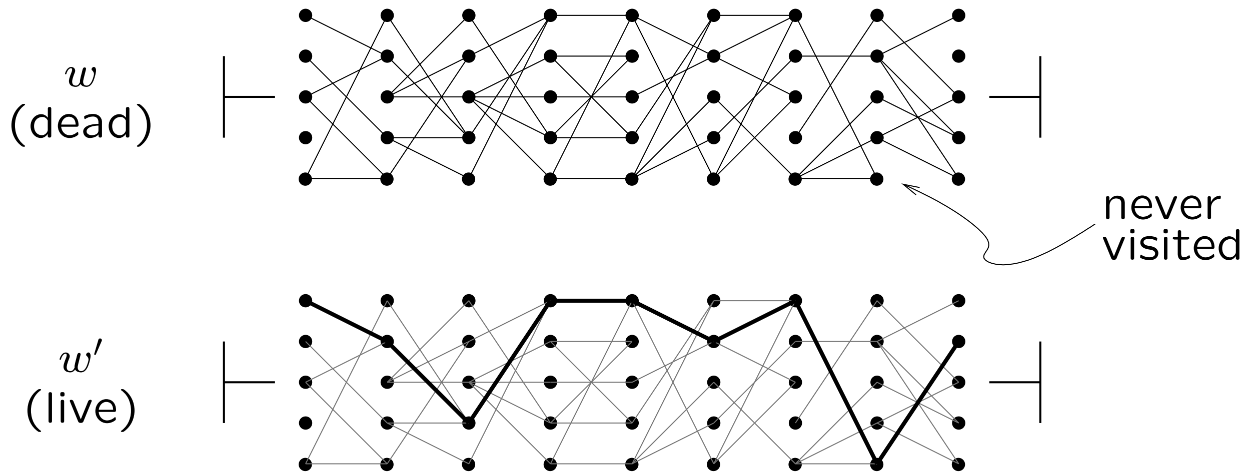


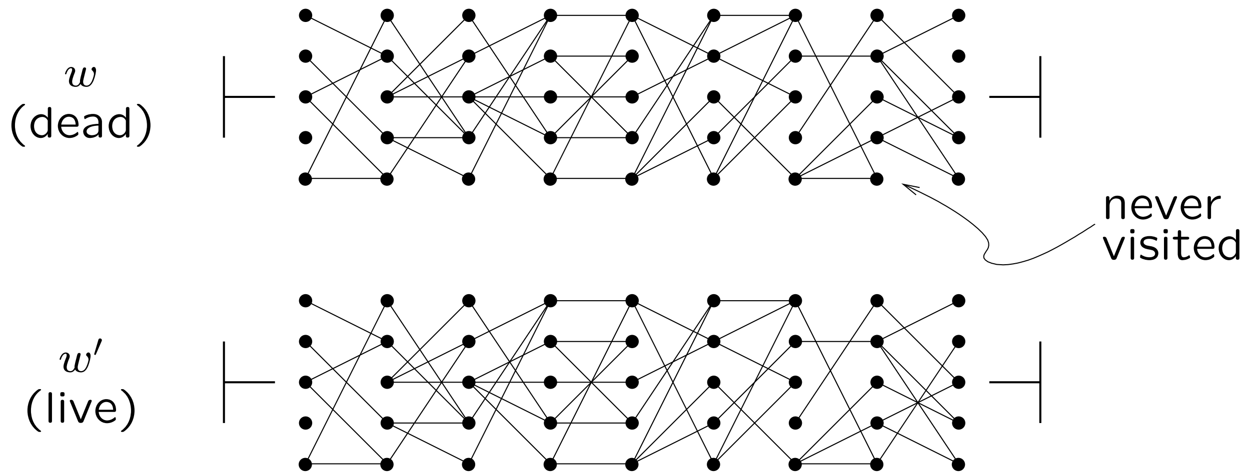
same computation $\Rightarrow$ same decision

PROOF PLAN: Construct a string of graphs w such that:

the mole fails on w

$\iff$ w is live and the mole rejects $\vee$ w is dead and the mole accepts

$\Longleftarrow$ w is dead and contains a live node that the mole never visits



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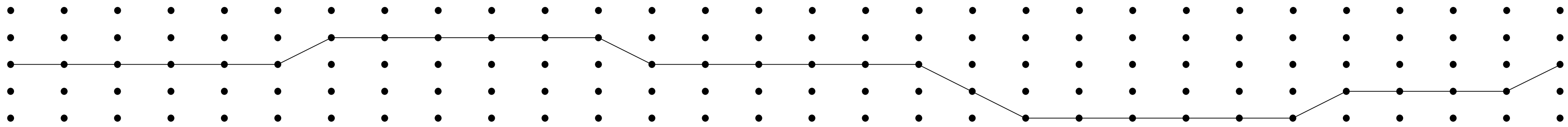
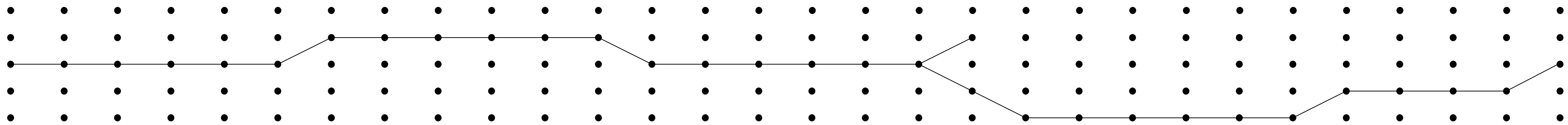
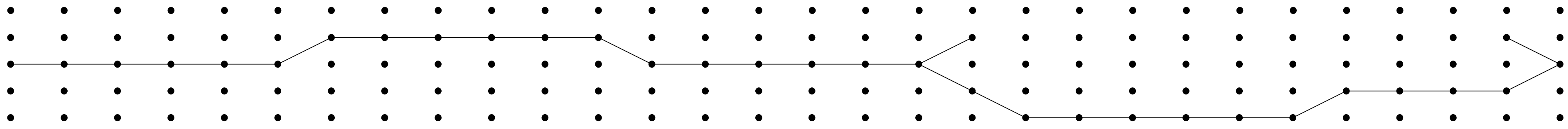
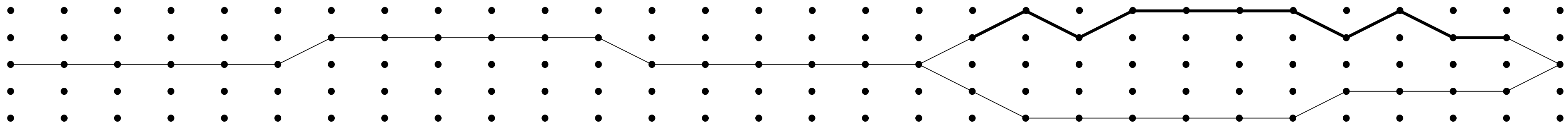
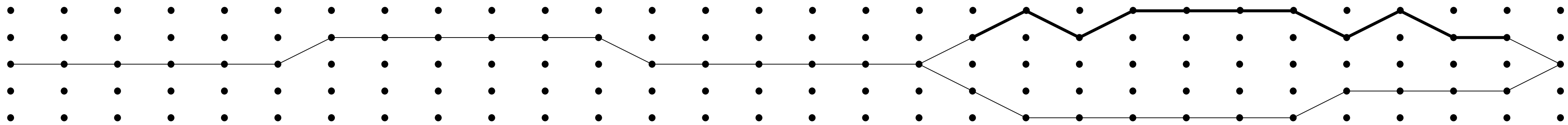


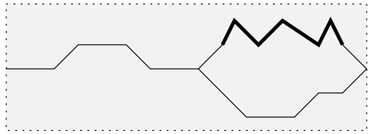


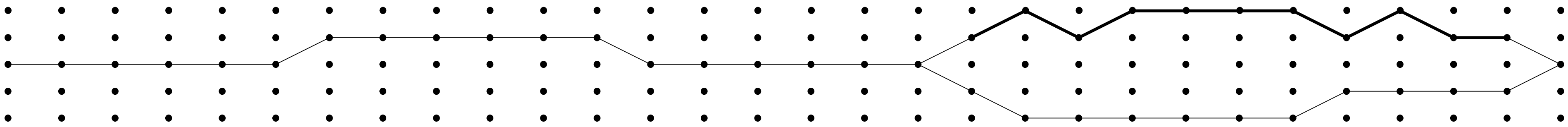


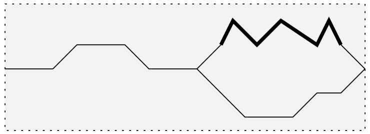
the main argument

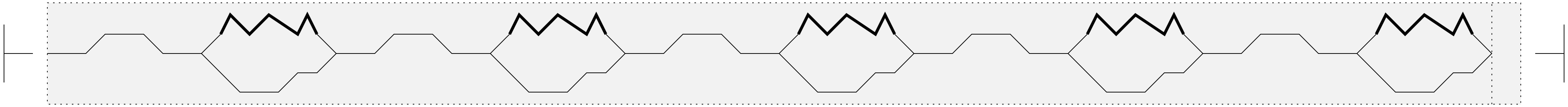


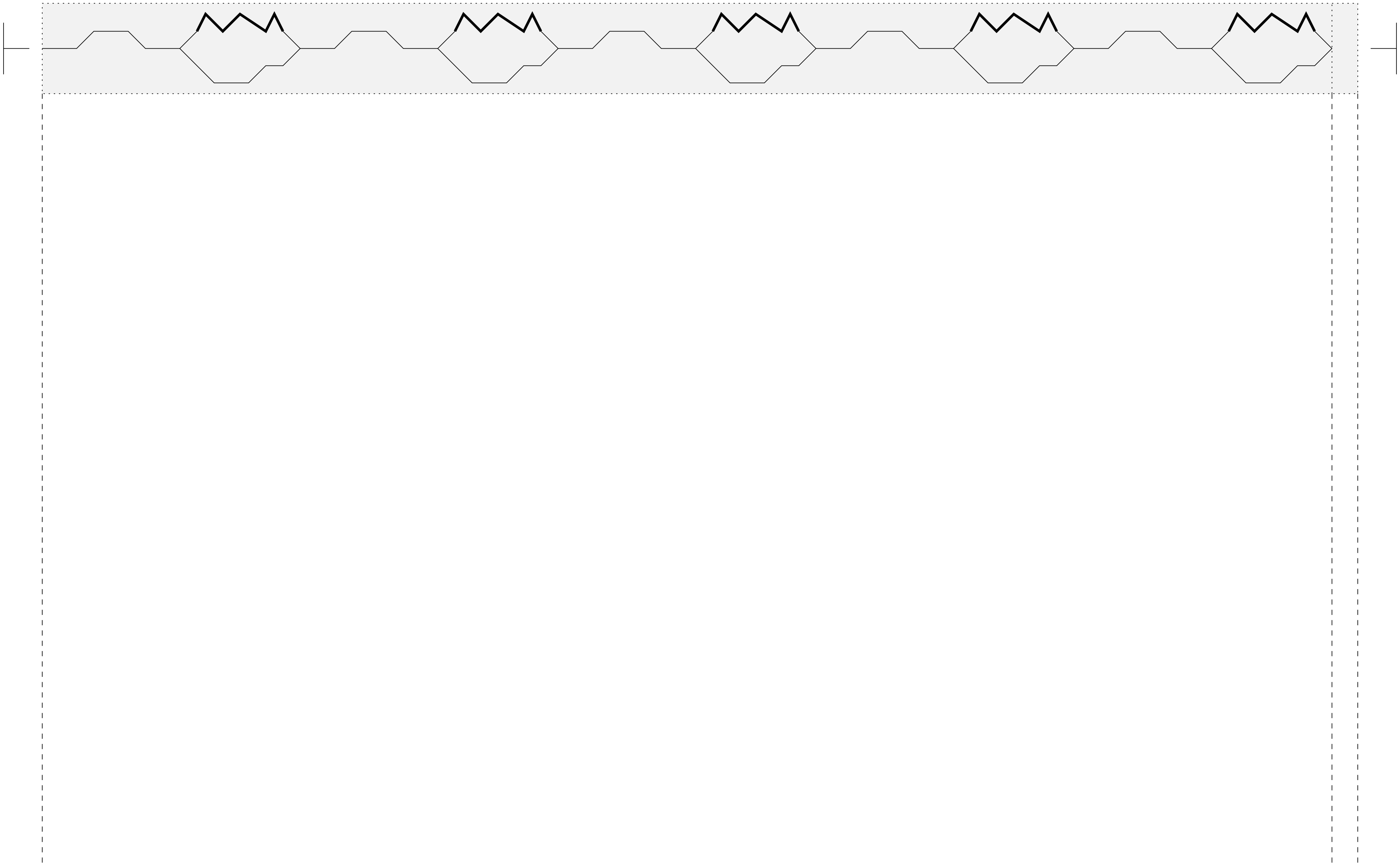


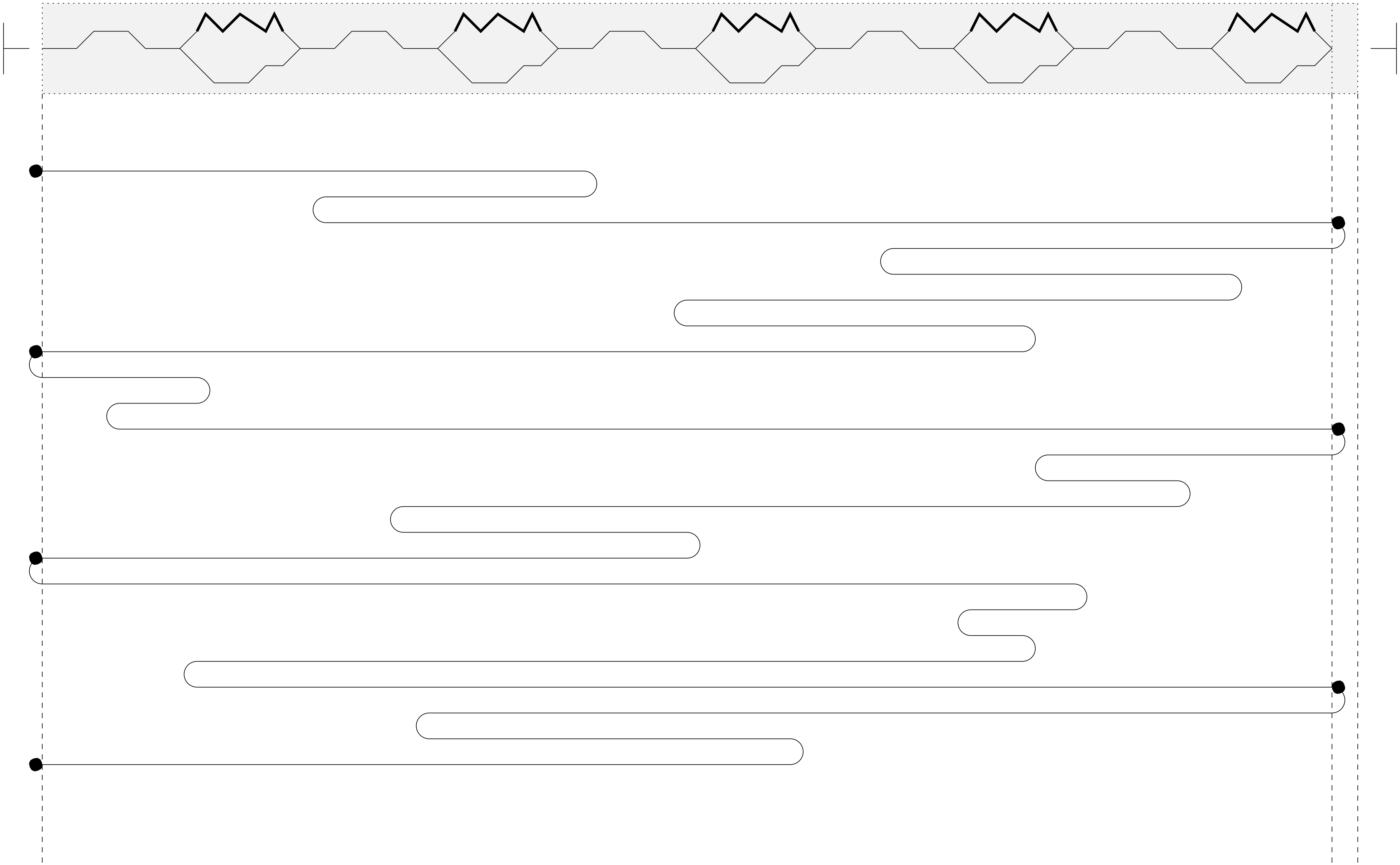












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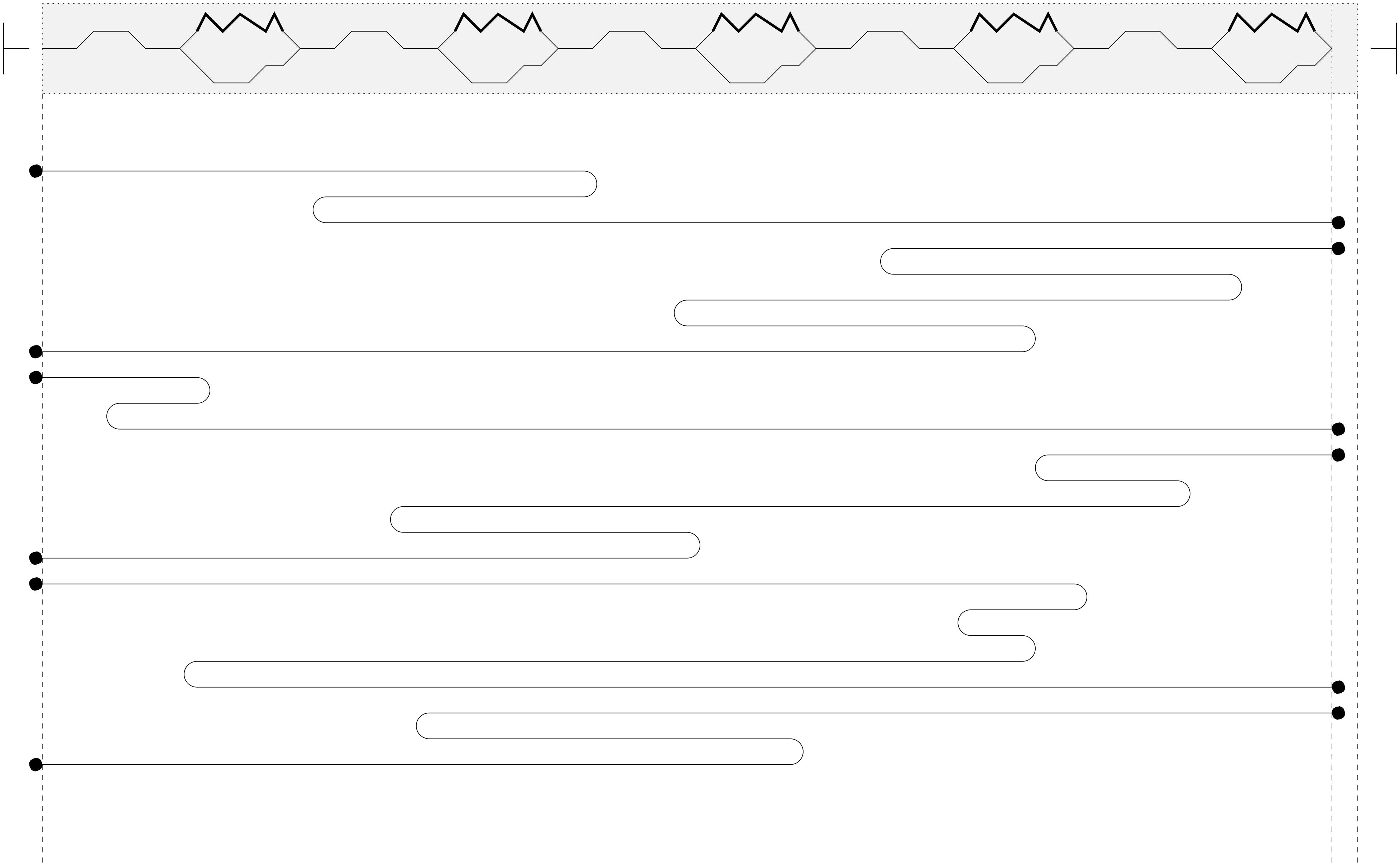

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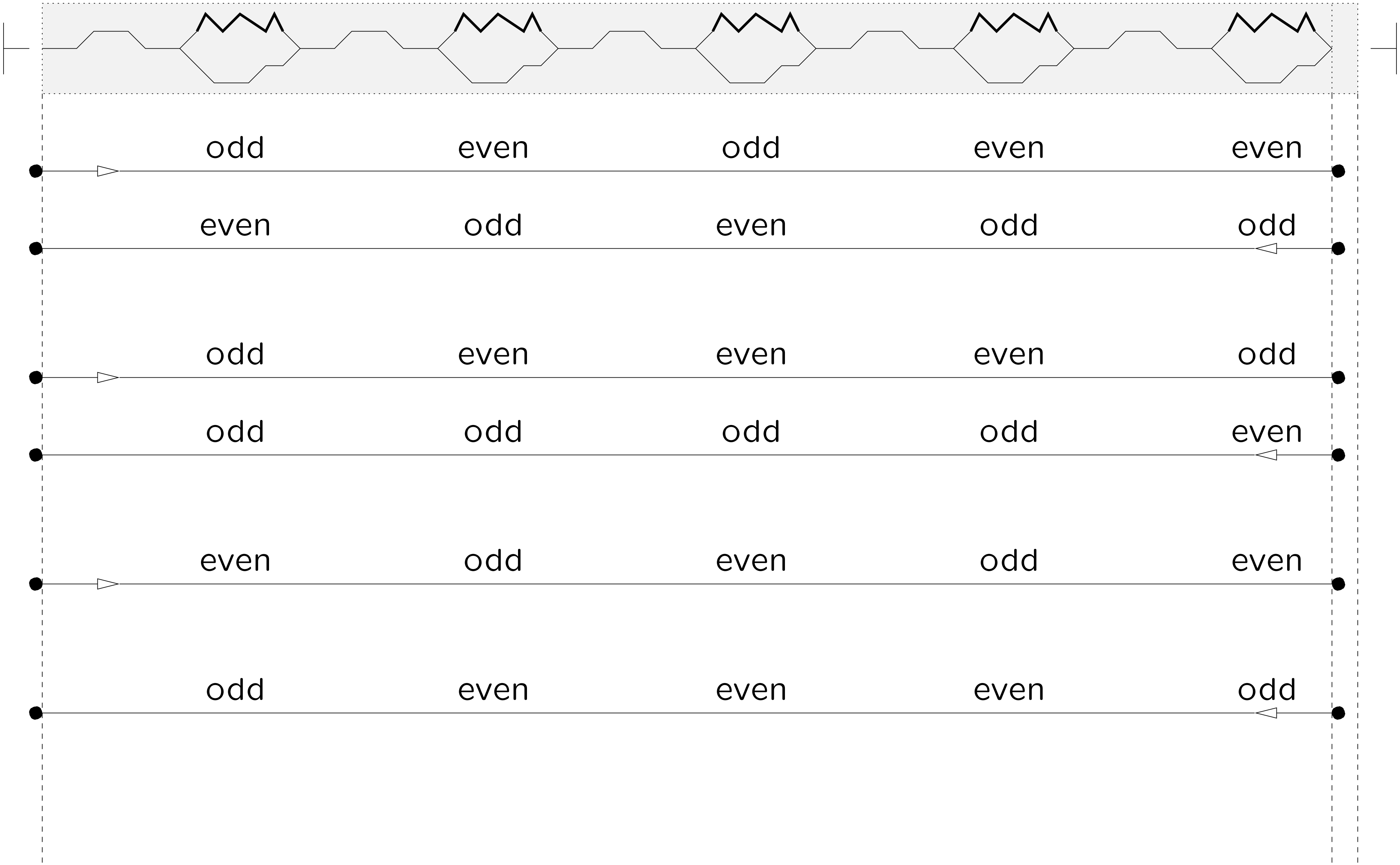

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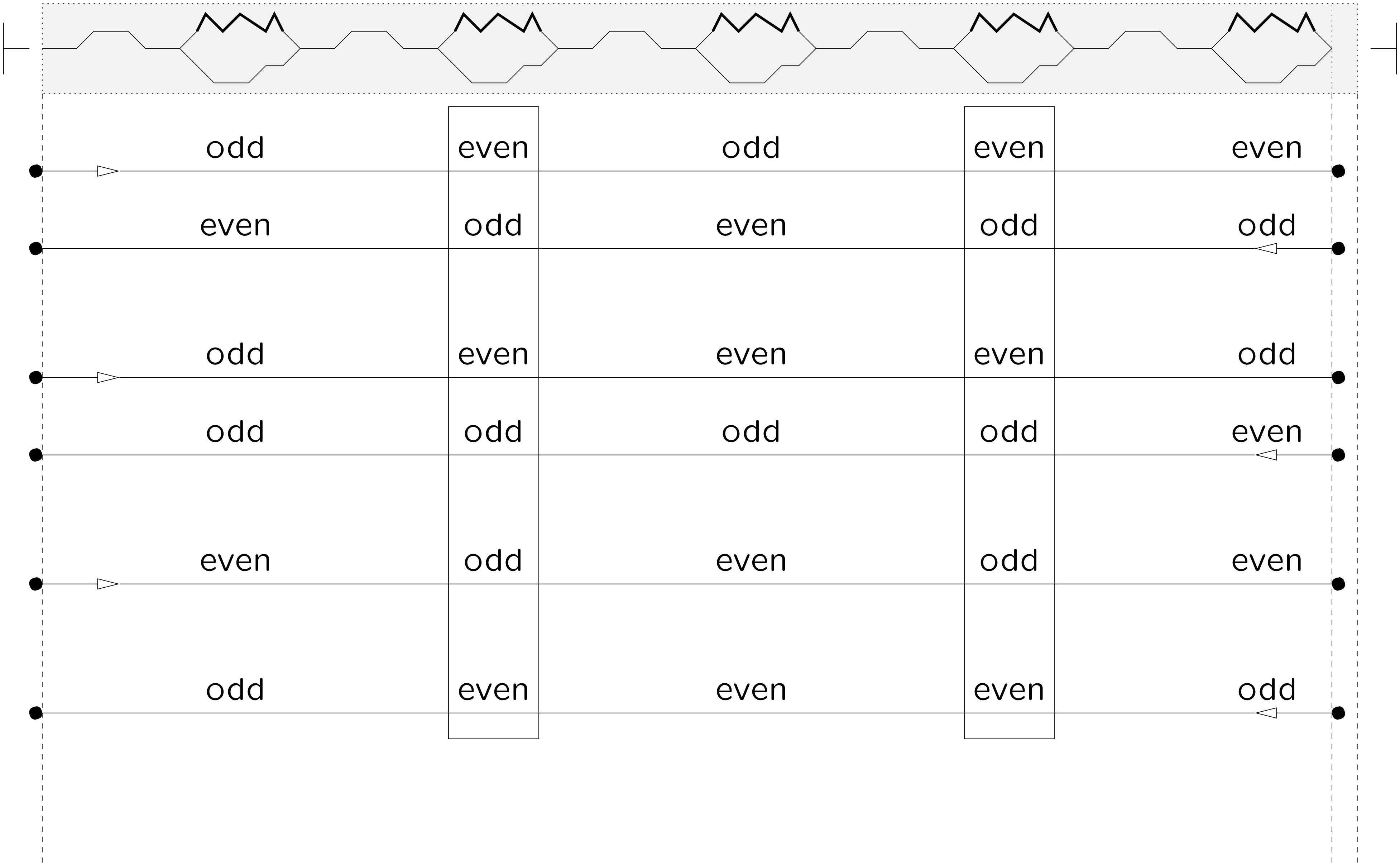

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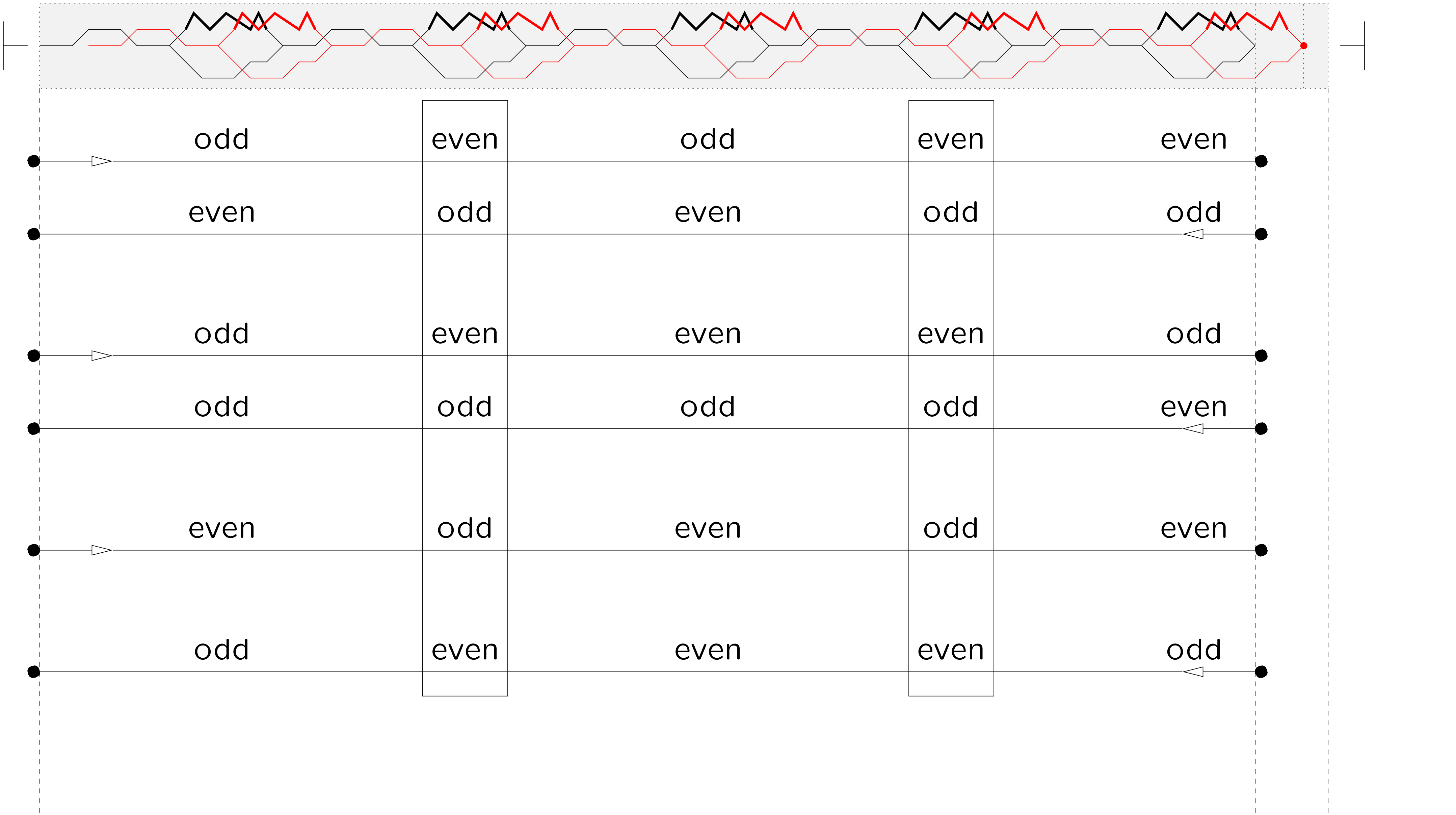

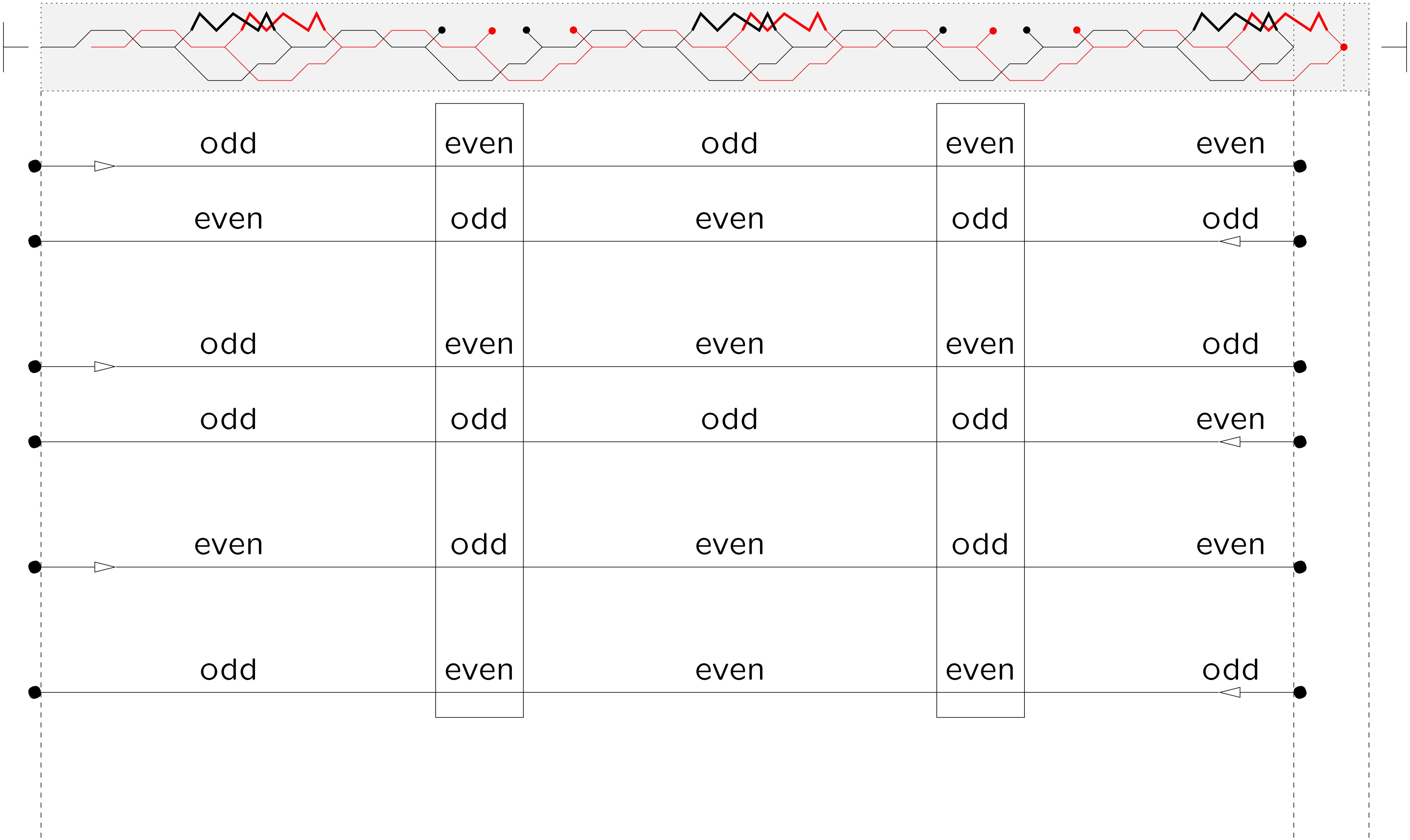
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the main argument

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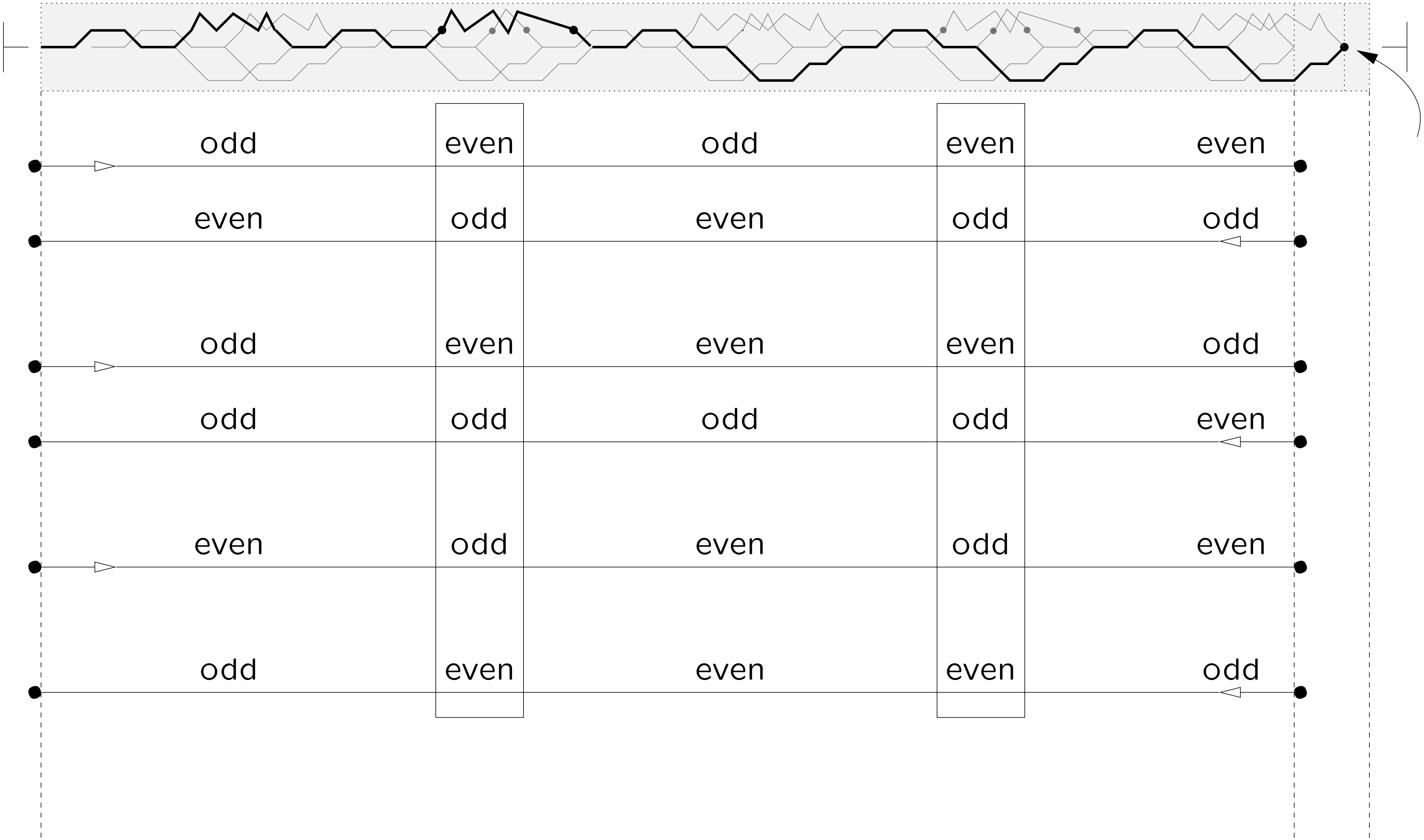

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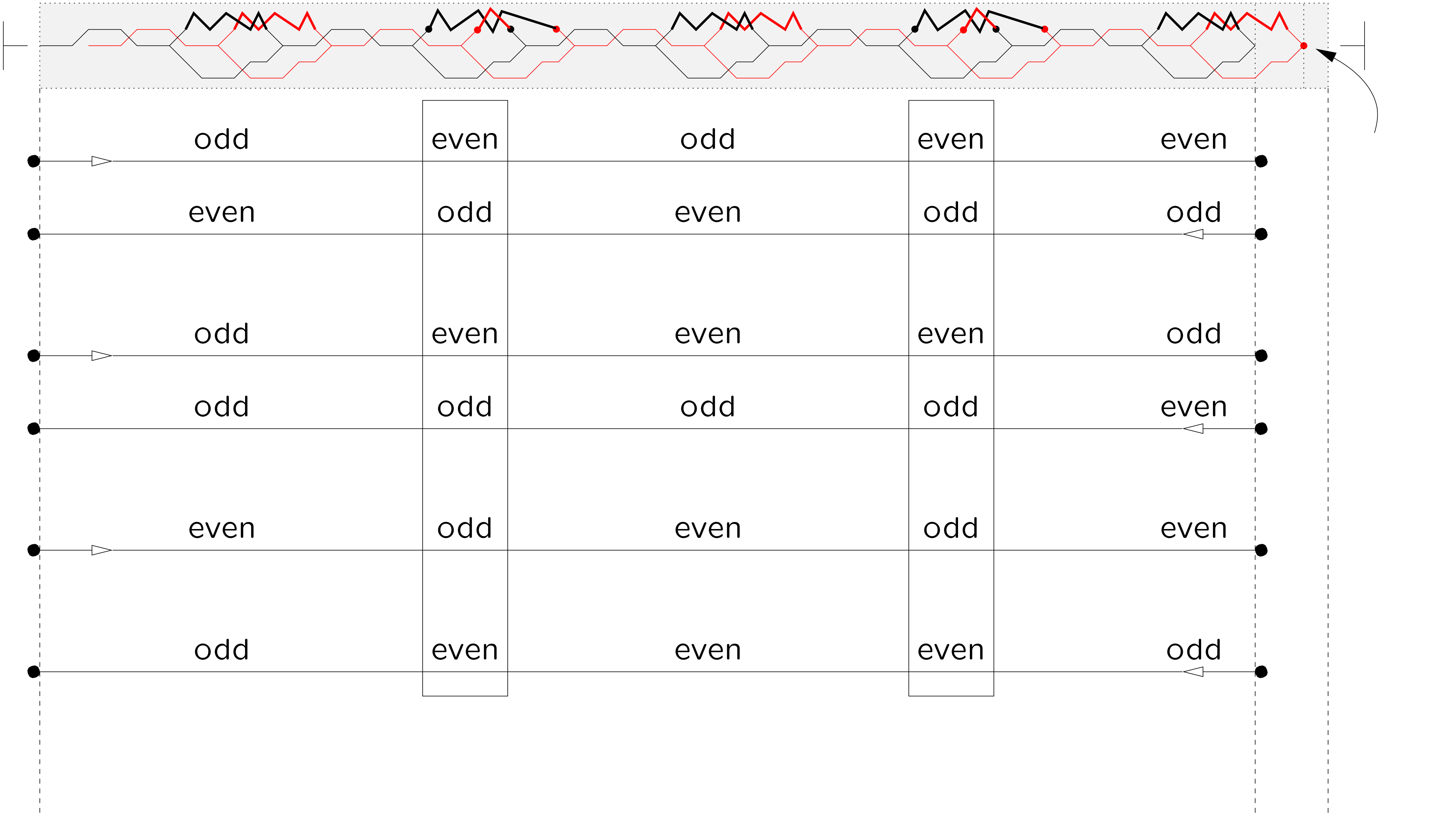

PROBLEM:	can small 2DFAs simulate small 1NFAs?
LIVENESS:	a complete problem for this conversion
MOLES:	a natural class of automata against liveness
GOAL:	show that **small** 2D moles cannot solve liveness
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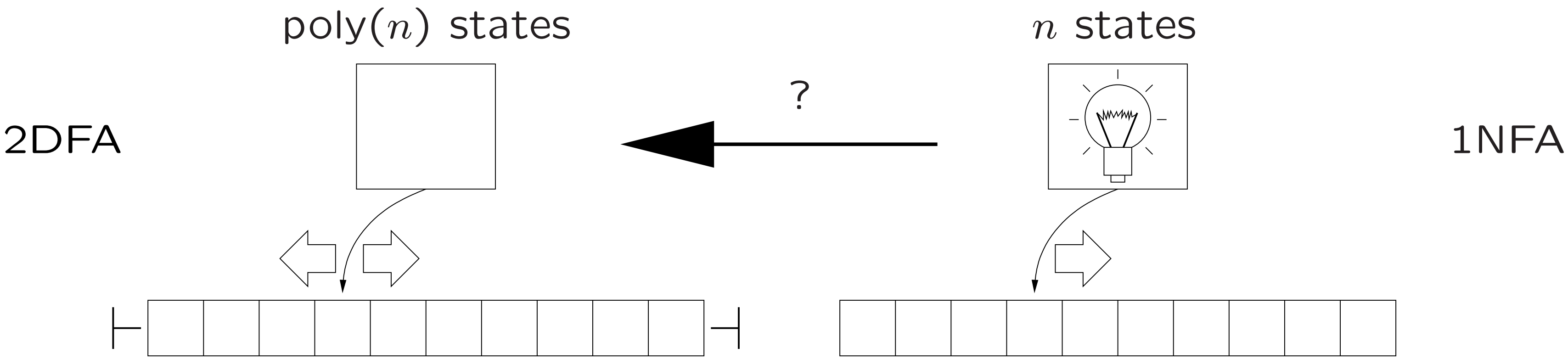
class too restricted:
1. computability answer to a complexity question
2. we have definitely missed the real reasons. . .

why care about this problem

REASON #1:

REASON #2:



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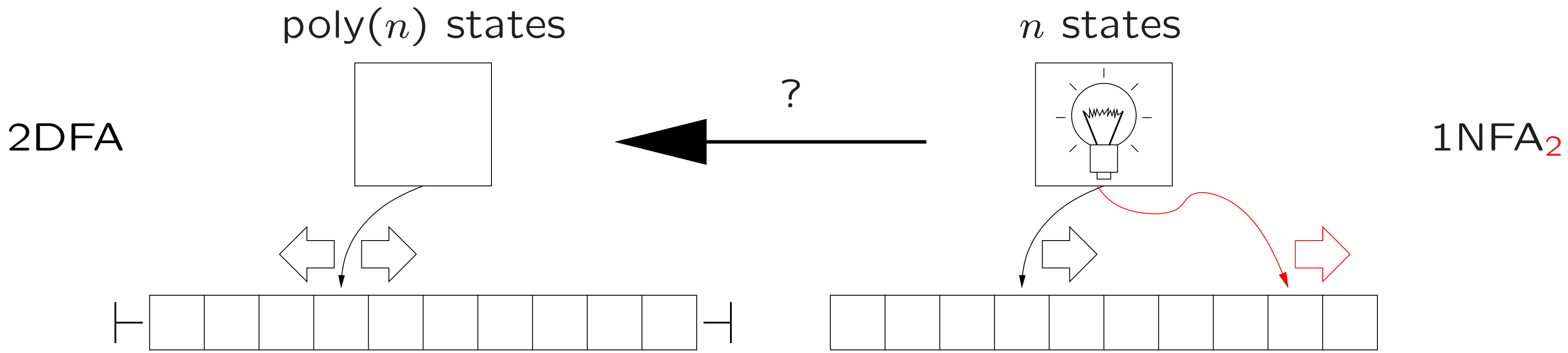

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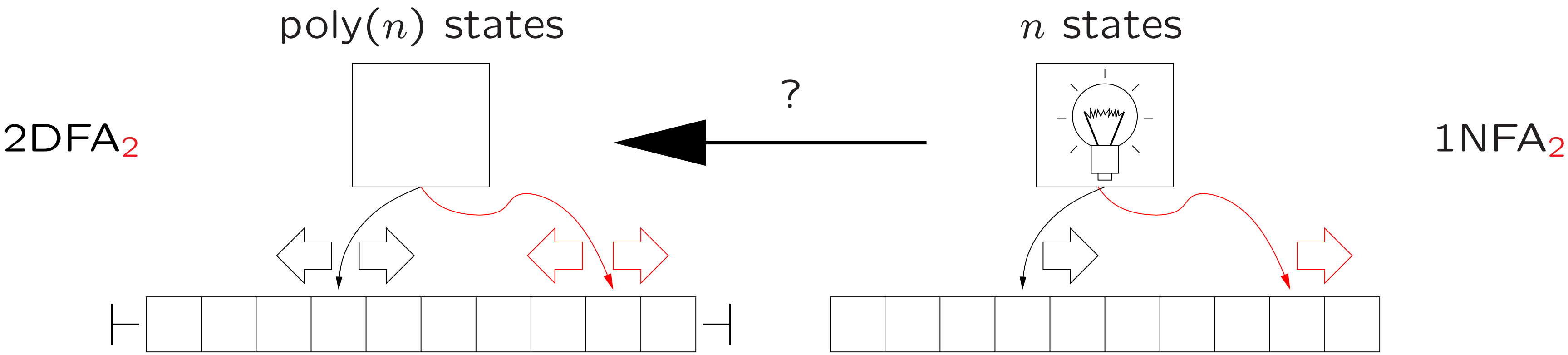

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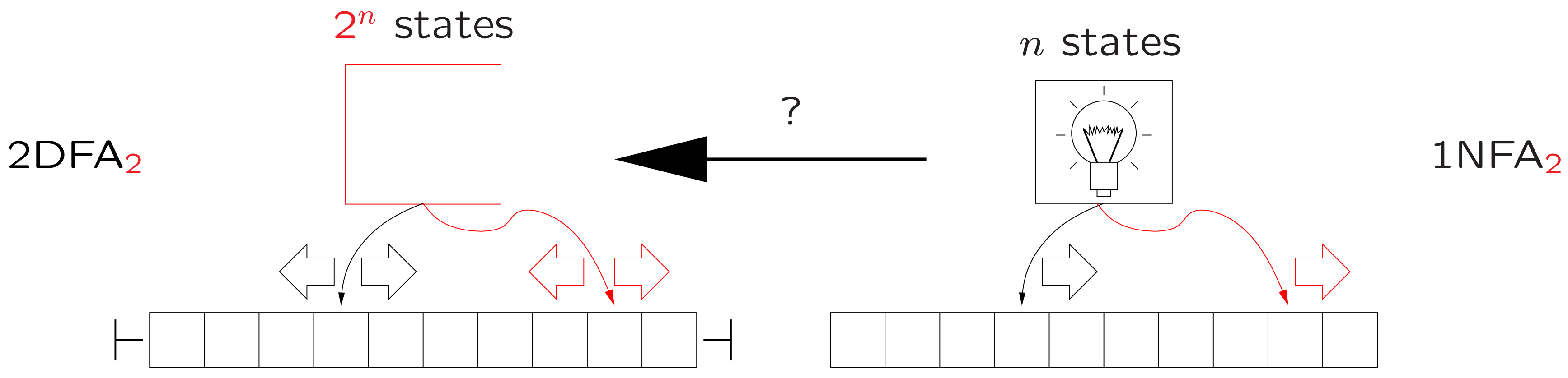

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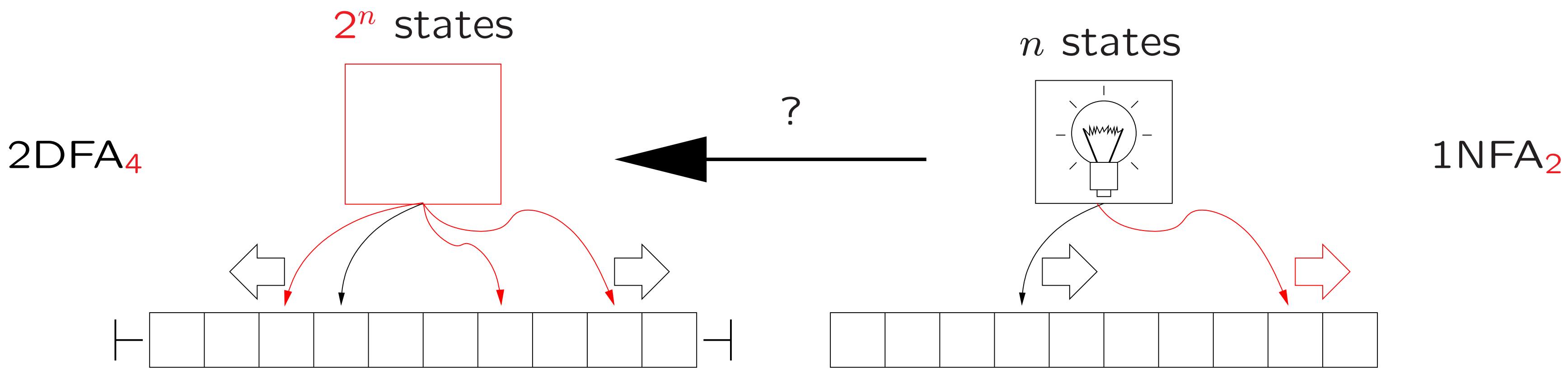

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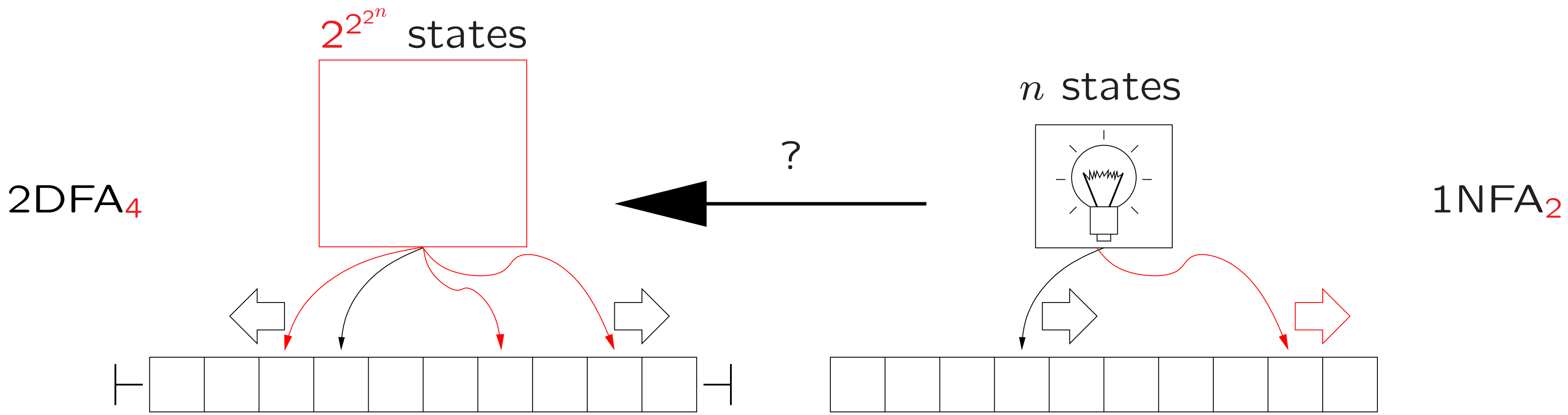

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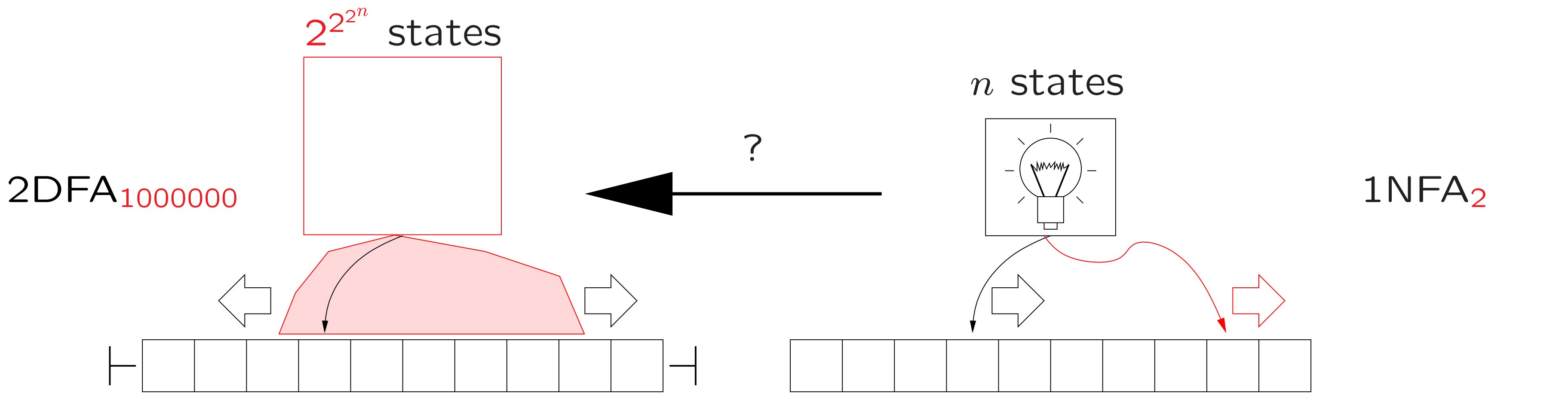

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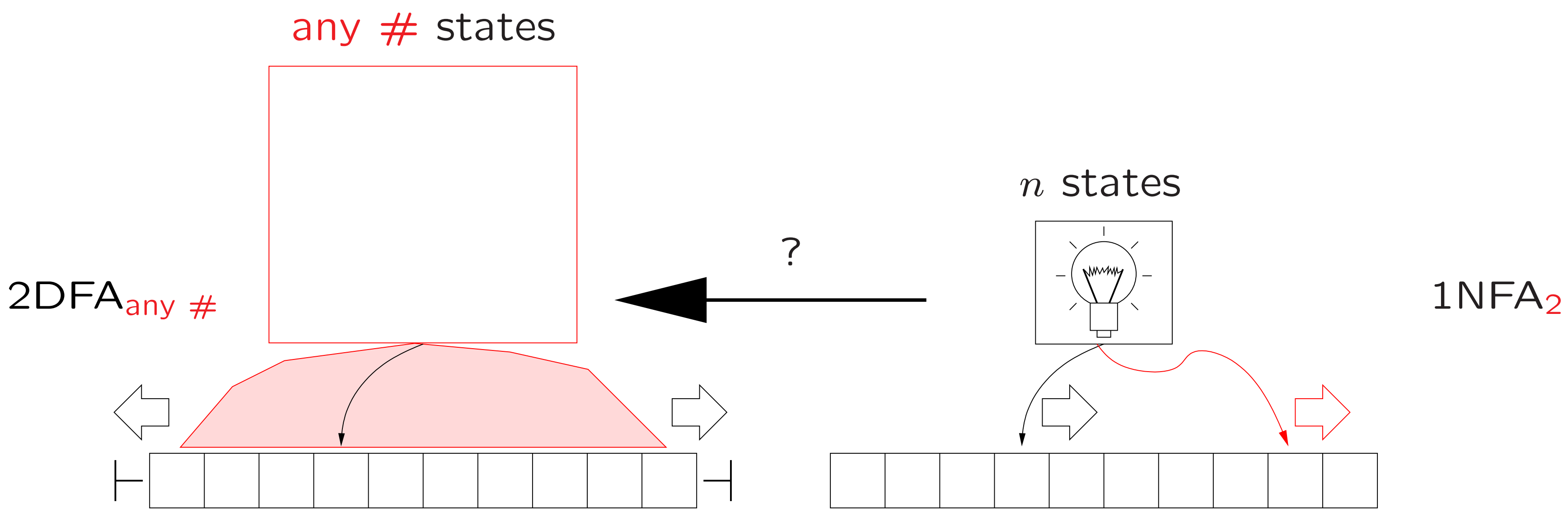

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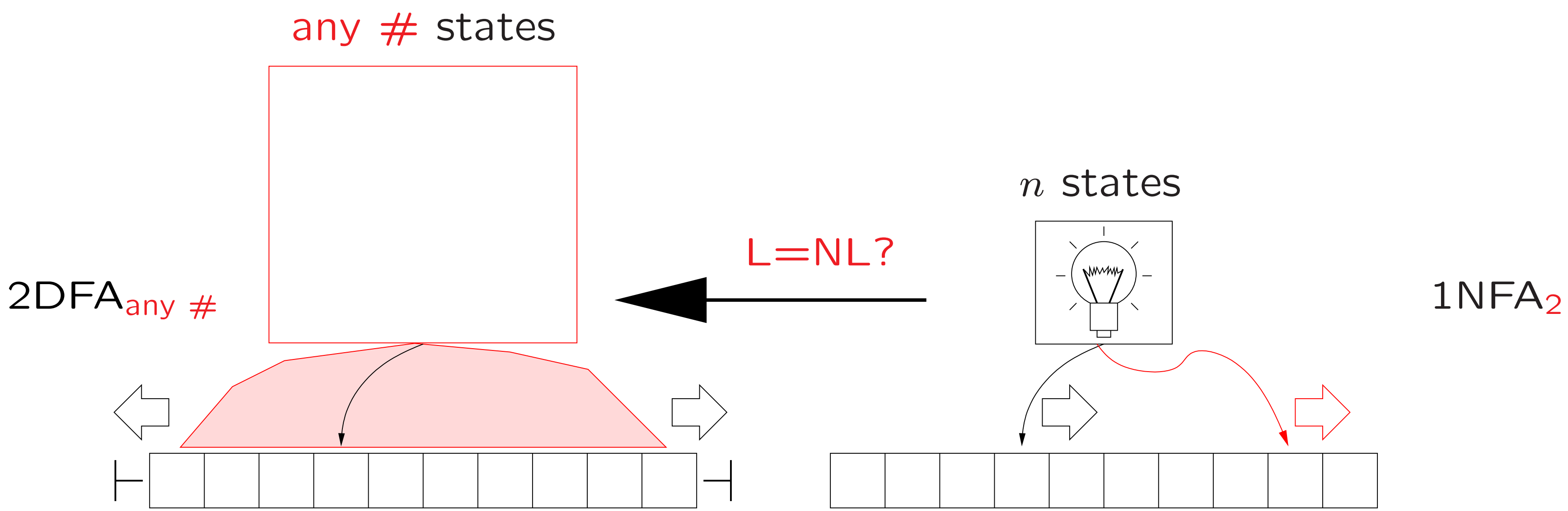
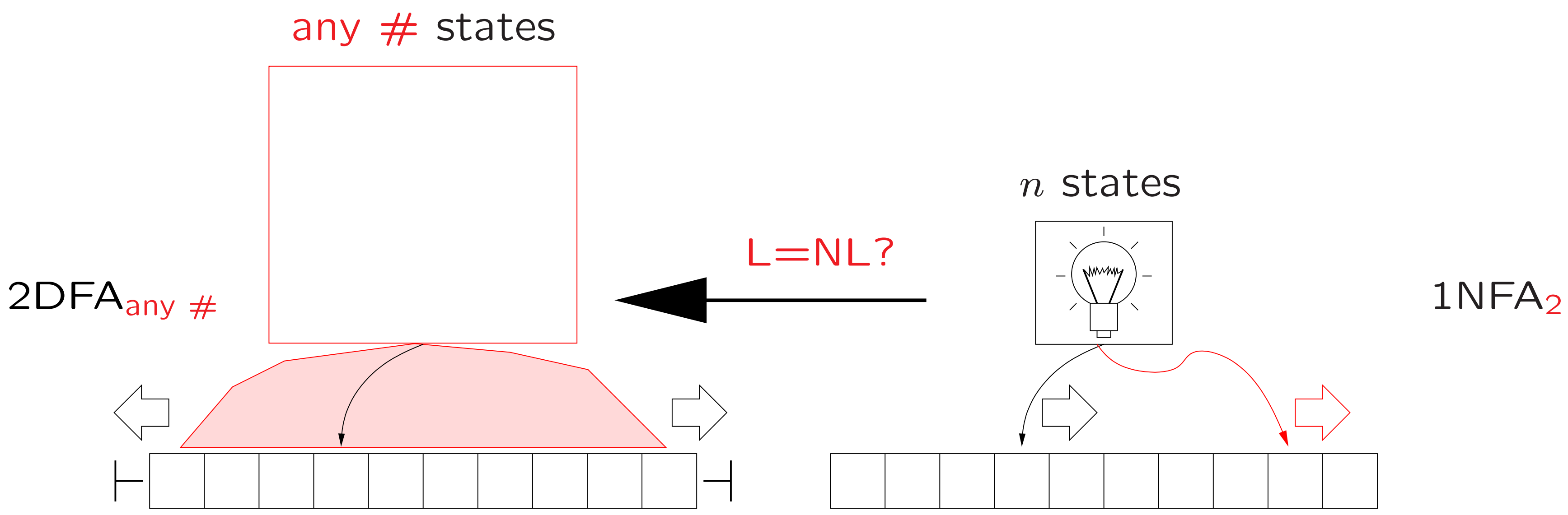


REASON #1:

REASON #2: two-way determinism vs. one-way nondeterminism



REASON #1: it is such a nice problem!

REASON #2: two-way determinism vs. one-way nondeterminism