Efficient Financial Crises*

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ABSTRACT

We develop a theory of systemic financial crises. We obtain conditions under which a single bank optimally chooses a fragile capital structure that is subject bank runs. When depositors are unable to commit to long-term lending arrangements, they optimally finance the bank using short-term debt. With multiple banks, lack of depositors commitment leads depositors to invest via short-term debt in a financial system in which all banks make loans with correlated, volatile returns. The optimal financial system features occasional, costly systemic crises in which all banks are subject to ex post inefficient liquidations. In this sense, financial crises are efficient.

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1. Introduction

Financial crises are a pervasive feature of modern economies: Reinhart and Rogoff (2008) document the occurrence of 435 crises featuring the closure, merging, or takeover by the public sector of one or more financial institutions typically following bank runs in 70 developed and developing economies since 1800. Given that these financial crises are frequent and associated with severe economic recessions, it is important to understand the reasons which lead banks and those who finance banks to create a fragile financial system with the potential for crises. Such an understanding is necessary in order to evaluate policy recommendations aimed at preventing future crises or mitigating their consequences. The fact that banks and other financial firms typically rely heavily on short-term debt to finance their assets naturally exposes them to runs or other panic-like phenomena as in Diamond and Dybvig (1983) for example and suggests their capital structure is *fragile*.

In this paper, we analyze the incentives of both a single bank and a financial system with multiple banks to adopt a fragile capital structure and create the potential for systemic banking crises. We develop conditions under which a single bank chooses to finance its loans with primarily short-term claims which resemble short-term debt and are subject to bank runs. The bank's reliance on short-term debt leads to occasional terminations of the bank's activity which are inefficient and, in this sense, resemble bank runs.

We then use the model to analyze the efficiency of *systemic crises* where multiple banks are subject to runs at the same time by extending the model to feature multiple banks. We show that a financial system in which bank returns are correlated and sometimes all banks fail together is more efficient than a system in which bank returns are independent and at most a single bank fails at any point in time. Equilibrium outcomes with a fragile financial system feature occasional, costly crises. These outcomes are efficient in the sense that any alternative arrangement leads to lower welfare. We conclude that in the absence of other spillover effects, systemic banking crises are efficient.

The main contribution of this paper is to demonstrate that systemic banking crises may play a socially desirable role in spite of the costs associated with these crises. To arrive at this result, we first examine when the optimal capital structure for a single bank is fragile in the sense that there are some histories in which the bank is inefficiently liquidated. A critical finding is that when depositors cannot commit to long-term lending arrangements and they experience privately known shocks to their discount factors, short-term debt is essential for attaining the optimal, fragile capital structure. An important feature of these optimal lending arrangements is that equilibrium outcomes exhibit events which resemble costly bank runs along the equilibrium path.

We then analyze optimal lending arrangements when there are multiple banks. We show that if banks undertake projects with independent returns, short-term debt may not longer suffice to attain commitment outcomes. Our primary contribution is to demonstrate that commitment outcomes are attainable when bankers undertake projects with correlated returns and finance their investments with short-term debt. Thus, a financial system in which banks take correlated risks and occasionally, many banks suffer from bank runs simultaneously is superior in terms of ex ante depositors' welfare than one in which banks undertake projects or loans with independent returns. In this sense, the possibility and realization of systemic banking crises plays a socially desirable role in allowing depositors to commit to contracts which they could not otherwise do.

To illustrate these results, we begin by analyzing the optimal capital structure of a single bank. Motivated in part by the idea that short-term debt may serve as a useful disciplining device for bank managers, as in Calomiris and Kahn (1991) and Diamond and Rajan (2001), we develop a model in which the capital structure of banks is optimally designed to solve incentive problems. Specifically, we develop a model in which depositors to a bank must design compensation contracts for a bank manager, or *banker*, to provide incentives to exert effort in a dynamic environment. The banker is protected by limited liability. In the model, the banker's unobservable effort affects the distribution of future loan returns. In this sense, one may interpret the banker's effort as the amount of time and energy the banker expends monitoring potential borrowers. High effort implies that the loans the banker makes are likely to repay and yield high returns, and low effort implies that the banker's loans are likely to default and yield low returns (see Hölmstrom (1979) for an example of this kind of incentive problem). One way the depositors may provide the banker with incentives to exert high effort is for them to commit to dismiss the banker and terminate the bank if loan returns are low. Such dismissal, which we call *liquidation* henceforth, typically, is costly not just for the banker but for the depositors as well.

Consider the tradeoffs involved with liquidating the bank after poor loan returns. In any history in which the depositors continue the project, the banker receives strictly positive expected value net of effort costs, or a rent, due to the combination of moral hazard and limited liability. When the depositors liquidate the bank, the banker does not receive the rents involved with continuing the bank. By liquidating the banker after poor loan returns, depositors align the banker's ex ante incentives with their own, and, therefore, can save on how much they must compensate the banker after high returns are realized while still providing appropriate incentives for effort. The benefit to depositors from such a liquidation strategy is cost savings in terms of providing the banker with incentives to exert effort. These costs savings must be compared to the direct costs to depositors of liquidation, which is forgone profits earned from continuing the bank. When the likelihood of poor loan returns is low if the banker exerts high effort and when the moral hazard problem of the banker is severe, we show it is efficient to liquidate the bank following poor loan returns in spite of the costs of such liquidation to the depositors.

We then explain how the optimal provision of the banker's incentives can be used to explain why banks and financial institutions rely heavily on short-term debt.[1] Because liquidation is costly for both depositors and the banker, depositors may be tempted to try to re-negotiate the contract ex post to avoid liquidation. We formalize this temptation by analyzing the same environment under an assumption that depositors cannot commit to the entirety of their long-term contract and show that this additional friction leads depositors to prefer to lend to the banker via short-term debt.

One consequence of assuming that depositors lack full commitment is a contract which calls for liquidation after poor loan returns may no longer be feasible. The reason is that after low returns are realized, both the depositors and the banker stand to gain by renegotiating their contract and allowing the banker to continue. If the banker and the depositors expect such renegotiation, then the banker rationally chooses a low level of effort

[1] The main result here is that contracts are short-term in nature. That contracts resemble short-term debt, per se, relies on the structure of the economy. A more detailed discussion on when optimal contracts resemble short-term debt in moral hazard problems can be found in Innes (1990).

(ex ante). Thus, if bankers and depositors cannot commit to carrying out their contracts, outcomes are worse on average than with commitment. In this sense, lack of commitment creates a *time inconsistency problem* (see Kydland and Prescott (1977) for an example of this problem).

Our analysis of optimal financing for a single bank demonstrates that the use of short-term debt introduces a coordination problem among depositors that can help resolve the time inconsistency problem. In particular, if an agreement to renegotiate the contract requires all or a substantial fraction of depositors to agree to a renegotiation and each depositor privately knows her own discount factor, we show that the time inconsistency problem can be resolved. The basic idea is that an agreement to renegotiate creates incentives on the part of each depositor to threaten to disagree unless that depositor is paid a large fraction of the bank's future loan returns. Such incentives make it difficult for depositors to renegotiate the terms of the contract and help ensure that the original contract is implemented even when it is undesirable from the perspective of the collective interests of the depositors. Since the banker anticipates the likelihood of such disagreement, the banker expects the original contract to be implemented and rationally chooses to exert high effort to reduce the likelihood of poor outcomes.

Coordination problems of the kind studied here are well known in the literature on the problem of providing public goods which serve common purposes such as military defense or pollution control (see Rob (1989) or Mailath and Postlewaite (1990) for examples). This literature has emphasized that requiring all or most citizens to agree to an appropriate level of defense or pollution control is difficult and has emphasized that government action might be desirable in such circumstances. The theoretical result, that coordination problems can be used to resolve time inconsistency problems demonstrates how such coordination problems can actually serve a desirable social role.

Short-term debt and the bank runs which this financing structure creates play a desirable social role by introducing a coordination problem among depositors that allow them to, in effect, commit to dismiss the banker after poor loan returns. When the depositors lend to the bank with short-term debt contracts, low performance of the banker's loans trigger actions that look like they could not be part of an ex ante efficient contract – e.g., bank runs.

Each depositor refuses to roll-over their debt even though it is in the collective interest of the depositors to do so.

The usefulness of short-term debt in this model relies on the lack of outside depositors with deep pockets who are willing to invest in the bank when some of the original depositors are unwilling to do so. This raises a concern about the usefulness of short-term debt if there are multiple banks or other depositors with funds they are willing to invest. In other words, although it may be optimal for a single bank to be fragile, it is not clear why it is optimal or even feasible for a financial system consisting of multiple banks to be fragile. We address this concern by analyzing a version of our model with multiple banks. We show that the same motive that leads a single bank to use short-term debt will lead multiple banks to optimally undertake investments with correlated returns that are riskier than those they would choose with independent returns.

Consider first possible outcomes when bank loan returns are independent. One can show that under full commitment, the optimal liquidation strategy is similar to that which arises in the single bank outcome. Specifically, if a bank realizes high returns, independent of the returns earned by other banks, the bank should be continued, and if a bank realizes low returns, that bank should be liquidated.

When bankers' loans are independent and if all bankers exert effort, when a single bank realizes low loan returns, it is likely that other banks have realized high loan returns. This means that even if the depositors in the bank with low loan returns would like to liquidate the bank as called for in the optimal commitment strategy, the depositors in banks with high loan returns may be willing to lend to their own bank and the bank which earned low returns. They may be willing to do so because they have resources beyond what they need to finance their own bank and because even the bank which realized low returns has a profitable loan opportunity. If depositors are unable to commit to prevent dilution of deposits, then it is likely if a single bank realizes low returns that all banks will be continued. Again, if the banker and the depositors expect such dilution to occur, then the banker rationally chooses a low level of effort (ex ante). Thus, if bankers and depositors cannot commit to prevent rollover from outside depositors, outcomes on average are worse than with commitment.

Consider next outcomes when bank loan returns are perfectly correlated across banks.

In this case, if one bank realizes low returns, then all banks realize low returns. If low returns are sufficiently small, then all of the depositors will find it difficult to agree to finance even a single bank. We construct such a correlated return economy so that under full commitment depositors are indifferent between a financial sector in which bank returns are independent and have a low degree of risk in loan returns and one with correlated returns and a high degree of risk in loan returns. When depositors and bankers lack full commitment, they strictly prefer a financial sector with correlated and highly volatile returns. In this sense, facing limited commitment, the efficient financial sector features banks financed with short-term debt that undertake correlated, volatile investments. Along the outcome path, with strictly positive probability, all banks earn low returns and are liquidated.

An efficient financial sector in our model is fragile and susceptible to systemic rises. In the model, such fragility and susceptibility serves an important social purpose by providing bankers of financial institutions the incentives needed to achieve high loan returns. Equilibrium outcomes in the model are efficient in the sense that no planner confronted with the same informational structures as other agents could achieve a better outcome. In this sense, government interventions can only be harmful. Of course, there may be other spillover costs associated with bank runs or bank liquidations in which case equilibrium outcomes in this model are likely to be inefficient. An extended version of the model here could be useful for analyzing the best way to mitigate the probability of financial crises and to address them appropriately when they do occur.

It is important here to discuss the closely related work of Calomiris and Kahn (1991) and Diamond and Rajan (2001) on the usefulness of short-term debt as a disciplining device. In both of these papers, the optimal financing structure between a group of depositors and a single bank is chosen to resolve a moral hazard problem on the part of the bank or banker. In both of these papers, liquidation, or the threat of liquidation is a feature of optimal contracts. In these papers, however, there are *zero* costs associated with liquidation. Specifically, in Calomiris and Kahn (1991), liquidation is ex post inefficient in the sense that when the optimal contract calls for liquidation of the bank, there is no alternative arrangement in which the bank is continued and the welfare of the depositors is improved. In Diamond and Rajan (2001), bank runs only occur off the equilibrium path, and so only the threat of bank runs are needed. Given the finding that liquidations in these papers are not costly, it is not surprising that their policy recommendations are to maintain the fragility (or susceptibility to bank runs) of the banking sector. Admati and Hellwig (2013) have criticized this literature on the usefulness of short-term debt for precisely this reason.

In our model, liquidations occur with positive probability, have the feature that ex post, welfare of the depositors could be improved by continuing the bank, and are a feature of optimal lending arrangements between depositors and the banker. As a result, even when there are potentially large ex post costs associated with the liquidation of a single bank, it is still the case that policymakers should not intervene to prevent such liquidations. Moreover, this paper develops new insights into why it is efficient for an entire financial sector to be fragile and susceptible to systemic crises. We leave a more complete discussion of the related literature to Section 4C.

The remainder of the paper is organized as follows. Section 2 contains a benchmark moral hazard problem between a single banker and and a large number of depositors. This sections develops conditions under which a fragile capital structure is optimal and demonstrates the essentiality of short-term debt. Section 3 extends the benchmark model to one with multiple banks. This section demonstrates why a fragile financial system is optimal. Section 4 discusses policy implications and the related literature. Section 5 concludes.

2. The One Bank Economy

We begin with a version of our economy with only a single bank and a large group of small depositors. We develop conditions under which, in this economy, optimal contracts under full commitment feature events along the outcome path that resemble bank runs. We then introduce a limited commitment constraint and show that when depositors lack full commitment, optimal contracts feature early payouts to depositors and bank runs. We argue in a simple decentralization of this benchmark economy that optimal contracts resemble short-term debt with the possibility of bank runs. This contract dominates contracts which resemble long-term debt or equity contracts. We discuss an extension which suggests reasons why this feature of optimal contracts is likely to apply to financial firms such as banks and not to non-financial firms.

Consider a three period environment with $N+1$ agents. Let the periods be indexed by $t = 0, 1, 2$. We call the $N+1$st agent a *banker* and the remaining agents *depositors*. The banker has the ability to make loans but requires resources from the depositors to do so. In addition, the distribution of outcomes from any loans made by the banker depend on the screening effort expended by the banker. The banker may make a loan of fixed size which requires I units of resources in period 0 and effort of the banker which we denote by $e_0 \in \{\pi_l, \pi_h\}$. The loans yields a gross payout, or *return*, of $I + y_1$ with $y_1 \in \{0, y_h\}$ in period 1 where $y_1 = y_h$ with probability e_0 and $y_1 = 0$ with prob. $1 - e_0$. To conserve on notation, we index the banker's effort choice by the probability of a high returns following that effort choice.

If the loan made by the banker is continued from period 1 to 2, and the banker expends additional screening effort e_1, then the loan yields gross returns of $I + \rho y_1 + z_2$, $z_2 \in \{0, z_h\}$ with $z_h > 0$, $\rho > 0$, and $z_2 = z_h$ with probability e_1. Since $\rho > 0$, the returns from a loan in period 2 which yielded high returns in period 1 are larger than those obtained from a loan which yielded low returns in period 1. Thus, there is persistence in loan returns which is independent of the banker's previous effort level. If the loan is not continued, we will say it has been *liquidated*.

Only the initial banker who makes a loan in period 0 can provide continuing monitoring effort from period 1 to 2. In other words, in the absence of the banker, the loan if continued yields zero net returns from period 1 to 2.[2] In this sense, each banker is linked to the loans he or she makes, so we may also say the bank is liquidated in the event that it does not continue its loan.

The banker must in turn raise resources to finance its loans from depositors. Each depositor i is endowed with $k_0^i = I/N$ in period 0, may choose how much to lend to the banker, and may store the remainder at a one-for-one rate. Since the banker requires I resources, the banker necessarily requires participation of each depositor in order to undertake

[2] The assumption that the loan yields zero net returns if the original banker no longer participates in continuing the loan can be relaxed. A necessary assumption for the results that follow is simply that if an alternative banker is used to continue the bank's loan from period 1 to period 2, then this banker is less efficient than the original banker. In other words, we require that replacing the banker in the middle of the bank's loans is costly.

a loan.[3]

The banker is risk neutral over streams of consumption c_t and has disutility of effort e_t given by $q(e_t)$. The banker's inter-temporal preferences over consumption and effort is given by

$$c_0 - q(e_0) + c_1 - q(e_1) + \beta c_2,$$

where β is the banker's discount factor from period 1 to period 2 and $q(e_t)$ is the cost of effort in period t. For simplicity, we normalize $q(\pi_l) = 0$ and denote $q(\pi_h) = \bar{q}$.

Each depositor also has linear preferences over streams of consumption according to

$$U(c_0, c_1, c_2) = c_0 + c_1 + v_i c_2,$$

where v_i is a preference shock realized at the beginning of period 1. The preference shock of each agent, v_i, is drawn independently across depositors from the cumulative distribution function $G_i(v_i)$ which has support $[\underline{v}, \bar{v}]$. Let G denote the joint distribution over v. The depositors' preference shocks can be thought of as liquidity shocks to the depositors, causing them to have a stronger preference for period 1 consumption when they realize lower values of v_i as in Diamond and Dybvig (1983). Additionally, the preference shocks are privately known by each individual depositor with $\beta < \underline{v} < \bar{v} < 1$.

2A. The Case of Full Commitment

We analyze a benchmark economy under full commitment to contracts by the banker and the depositors. The time-line of outcomes in this environment is described in Figure 1.

We begin by describing direct mechanisms, or contracts which specify recommended effort levels, payments to the banker and depositors, and a period 1 continuation rule of whether to continue the banker's loan or not, all as functions of the relevant history. We focus on *loan contracts* in which the depositors lend their resources to the bank in period 0 and later ensure that such a loan contract yields superior expected returns relative to autarky from the ex ante perspective of each of the depositors.

[3]The assumption that exactly N depositors are required to finance the bank can be relaxed by appropriately modifying some of the later assumptions.

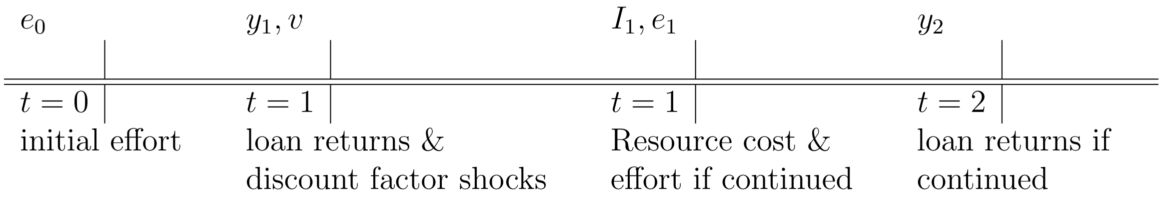

Figure 1: Timeline for the model described in Section 2.

Let p_t^i denote a payment to depositor i in period t. Let P^d denote the set of all payments to depositors so that

$$P^d = \left\{ \left(p_0^i, p_{1c}^i(y_1, v), p_{1n}^i(y_1, v), p_2^i(y_1, z_2, v)\right)_{i \in \{1, \ldots, N\}} \right\},$$

where p_{1c}^i denotes a payment to depositor i if the vector of reported discount factors is v and the realized return is y_1 and the bank is continued while p_{1n}^i is a similar payment which occurs if the bank is not continued or liquidated. Similarly, we let p_t^b denote the payment to the banker in period t and $P^b = \left\{p_1^b(y_1), p_2^b(y_1, z_2, v)\right\}$. A loan contract is then a collection of functions, $\left\{p^d, p^b, x(y_1, v)\right\}$ where $p^d \in P^d$, $p^b \in P^b$ and $x(y_1, v) \in [0, 1]$ where $x = 1$ represents a continuation decision and $x = 0$ represents a liquidation decision.

Next, we discuss the constraints a loan contract must satisfy. First, the contract must satisfy nonnegativity constraints so that in each period consumption of the banker and depositors are positive. These constraints require $p_{1c}^i, p_{1n}^i, p_2^i, c_t \geq 0$. The nonnegativity constraint for the banker is critical because it ensures that in order to provide the banker with incentives to exert high effort, the banker necessarily obtains expected payments in excess of her outside option which are sometimes called incentive rents. As we make clear below, these rents are critical for ensuring the optimal loan contract sometimes features liquidation. The nonnegativity constraints for the depositors simply provide content to the notion that each depositor is small so that a single depositor cannot finance the bank's loan out of negative consumption.

Second, the mechanism must be *resource feasible*. That is, we require period 1 payments to be less than period 1 resources in the event of continuation

$$p_1^b(y_1) + \sum_{i=1}^{N} p_{1c}^i(y_1, v) \leq y_1$$

and in the event of liquidation

$$p_1^b(y_1) + \sum_{i=1}^{N} p_{1n}^i(y_1, v) \leq I + y_1.$$

We write these two constraints compactly using the continuation rule $x(y_1, v)$ as

$$p_1^b(y_1) + \sum_{i=1}^{N} \left[x(y_1, v) p_{1c}^i(y_1, v) + (1 - x(y_1, v)) p_{1n}^i(y_1, v) \right] \leq I + y_1 - I x(y_1, v). \tag{1}$$

The contract must also be feasible in period 2 (if the bank is continued) so that

$$p_2^b(y_1, z_2, v) + \sum_{i=1}^{N} p_2^i(y_1, z_2, v) \leq I + \rho y_1 + z_2.$$

We write this constraint, after taking an expectation across period 2 loan returns, z_2, under the recommended effort level of the banker as

$$E_{e_1(y_1,v)} \sum_{i=1}^{N} p_2^i(y_1, z_2, v) \leq I + \rho y_1 + E_{e_1(y_1,v)} \left(z_2 - p_2^b(y_1, z_2, v) \right). \tag{2}$$

Third, the loan contract must satisfy two types of incentive compatibility conditions. The contract must induce the banker to choose the recommended level of effort in each period and the contract must induce truth-telling of the depositors. We say a contract is *incentive compatible for the banker* if for all y_1 and v,

$$\begin{aligned} &\beta e_1(y_1, v) p_2^b(y_1, z_h, v) + \beta(1 - e_1(y_1, v)) p_2^b(y_1, z_l, v) - q(e_1(y_1, v)) \\ &\geq \max_{e'} \beta e' p_2^b(y_1, z_h, v) + \beta(1 - e') p_2^b(y_1, z_l, v) - q(e'), \end{aligned} \tag{3}$$

and

$$E_{e_0}\left[p_1^b(y_1)+\int_v U_1(y_1,v)dG(v)\right]-q(e_0)\geq \max_{e'} E_{e'}\left[p_1^b(y_1)+\int_v U_1(y_1,v)dG(v)\right]-q(e'), \quad (4)$$

where $U_1(y_1, v) = x(y_1, v)\left[\beta E_{e_1(y_1,v)} p_2^b(y_1, z_2, v) - q(e_1(y_1, v))\right]$. Constraint (3) requires that the banker prefers the stream of payments from period 1 to period 2 expected under effort level $e_1(y_1, v)$ net of the disutility of effort to that which could be obtained under an alternative effort level e'. Constraint (4) is similar but provides the banker incentives to choose effort level e_0 in period 0 taking as given period 1 consumption and the continuation utility the banker will receive in each state of the world.

To define incentive compatibility for the depositors, it is useful to define the continuation utility of a depositor conditional on realizing a preference shock v_i and reporting preference shock $\hat{v}_i$, which we denote by $w_i(y_1, \hat{v}_i, v_i)$ and is given by

$$\begin{aligned} w_i(y_1,\hat{v}_i,v_i) &= \int_{v_{-i}} x(y_1,\hat{v}_i,v_{-i})\left(p_{1c}^i(y_1,\hat{v}_i,v_{-i})+v_i p_2^i(y_1,\hat{v}_i,v_{-i})\right)dG_{-i}(v_{-i}) \\ &\quad +\int_{v_{-i}}(1-x(y_1,\hat{v}_i,v_{-i})p_{1n}^i(y_1,\hat{v}_i,v_{-i})dG_{-i}(v_{-i}). \end{aligned}$$

In evaluating this continuation utility, the depositor assumes the other depositors will report truthfully in which case with probability $x(y_1, \hat{v}_i, v_{-i})$ the bank will be continued and the depositor will receive payment $p_{1c}^i(y_1, \hat{v}_i, v_{-i})$ in period 1 and, with a slight abuse of notation, a payment $p_2^i(y_1, \hat{v}_i, v_{-i})$ in period 2 (this payment represents the expected period 2 payment where the expectation is taken with respect to period 2 loan returns).

We say a contract is *incentive compatible for the depositors* if for each y_1 and v_i,

$$w_i(y_1,v_i,v_i)\geq \max_{\hat{v}_i\in[\underline{v},\bar{v}]} w_i(y_1,\hat{v}_i,v_i). \quad (5)$$

Lastly, because each depositor is free to store their endowment which earns a rate of return of 1 and since $\bar{v} < 1$, if the loan contract is superior to autarky, it must satisfy the *voluntary participation constraint*

$$E_{e_0}\left[\int_{v_i} w_i(y_1, v_i, v_i)dG_i(v_i)\right] \geq \frac{I}{N}. \tag{6}$$

As well, the loan contract must satisfy the participation constraint of the banker, however, we will show this constraint is necessarily slack.

We focus on contracts which maximize the ex ante expected welfare of the depositors as the resulting allocation will coincide with that which would occur in a decentralized environment when bankers must compete for depositors' resources. Ex ante welfare of the depositors is given by

$$E_{e_0}\left[\sum_i \int_{v_i} w_i(y_1, v_i, v_i)dG_i(v_i)\right]. \tag{7}$$

An *optimal loan contract* maximizes (7) among all loan contracts which satisfy the nonnegativity constraints, the resource feasibility constraints (1) and (2), the incentive constraints (3), (4), and (5), and the participation constraint (6).

We now characterize the optimal loan contract under the assumption that it is always optimal to induce high effort.[4] We begin by providing a sharp characterization of optimal payments to the banker for any set of payments to the depositors and continuation rule. As in most moral hazard models, the principal, who here can be thought of as representing the coalition of depositors, would like to use the payments to the banker to align the banker's incentives to exert high effort with her own. This implies making zero payments to the banker following a realization of low loan returns in either period 1 or period 2. Moreover, following low period 1 returns, the banker's incentive constraint must bind as must the banker's period 0 incentive constraint. We state this result as the following Lemma.

Lemma 1. Without loss of generality, we may restrict attention to contracts which feature zero payment to the banker following low returns in either period 1 or period 2 and (3) holds with equality when $y_1 = y_l$. Moreover, the banker's incentive constraint in period 0 (4) also holds with equality.

[4] The assumption that it is always optimal to induce effort of the banker is simply an assumption that loans in the absence of high screening effort of the banker yield sufficiently low returns in expectation.

Substituting for $p_2^b(y_1, y_l, v) = 0$ and $p_2^b(y_l, z_h, v) = \bar{q}/(\beta(\pi_h - \pi_l))$ into (3), we find that in the optimal contract, the period 2 payment to the banker is given by $p_2^b(y_1, y_h, v) = \bar{q}/\beta\,(\pi_h - \pi_l)$. Consequently, the banker's continuation utility in period 1 satisfies

$$U_1(y_1, v) = x(y_1, v)[\beta\pi_h p_2^b(y_1, x(y_1, v) - \bar{q}] \geq x(y_1, v)\frac{\pi_l \bar{q}}{\pi_h - \pi_l}.$$

The fact that the banker receives a strictly positive rent in the second period arises from the assumptions of risk-neutrality and non-negativity of the banker's consumption. Lemma 1 and $p_1^b(y_l) = 0$ allows us to simplify (4) as

$$p_1^b(y_h) = \frac{\bar{q}}{\pi_h - \pi_l} + \frac{\pi_l \bar{q}}{\pi_h - \pi_l}\int_v x(y_l, v)G(dv) - \int_v x(y_h, v)\left[\beta\pi_h p_2^b(y_h, z_h, v) - \bar{q}\right]G(dv). \quad (8)$$

An important feature of the banker's period 0 incentive constraint is that a commitment to continue the bank after high loan returns are realized in period 1 relaxes the banker's incentive constraint and allows the principal to reduce the payment $p_1^b(y_h)$ to the banker. A commitment to continue the bank after low returns are realized in period 1, however, actually tightens the incentive constraint. This tightening of the incentive constraint requires the principal to make a larger payment to the banker in period 1 after high returns are realized to preserve the banker's incentives to exert effort in period 0. We summarize this discussion in the following lemma.

Lemma 2. The optimal payment to the banker in period 1 following a realization of high returns is decreasing in the probability of continuation after high returns and increasing in the probability of continuation after low returns.

We use Lemma 2 to develop conditions under which the optimal contract under commitment features events that resemble bank runs. In particular, we ask whether the optimal contract calls for liquidation in period 1 after some loan return realizations when this liquidation is inefficient at least under full information of the depositors' discount factors. To answer this question, we develop sufficient conditions to ensure that liquidation is optimal for all reported discount factors following a realization of low returns in period 1.

Towards this end, we re-write the depositors' ex ante welfare by substituting the resource constraints and using Lemma 1 as

$$I + \pi_h \left[y_h - p_1^b(y_h) + \int_v x(y_h, v) \left[-I + \sum_i v_i p_2^i(y_h, v) \right] dG(v) \right]$$
$$+ (1 - \pi_h) \int_v x(y_l, v) \left[-I + \sum_i v_i p_2^i(y_l, v) \right] dG(v). \tag{9}$$

From Lemma 2, we know that $p_1^b(y_h)$ is increasing in $\int x(y_l, v)\, dG(v)$. Thus, in states where $-I + \sum_i v_i p_2^i(y_l, v)$ is positive, there is a tradeoff between providing the banker with incentives and future returns. When depositors realize high values of v_i, liquidation after low returns in period 1 are realized is costly (even considering the future rents the banker will obtain), but such liquidation relaxes the incentive constraint and reduces the payment to the banker after high returns are realized in period 1.

Consider the benefit from reducing $\int_v x(y_l, v) dG(v)$. Using equation (8), the benefit from relaxing the incentive constraint is $\pi_h \pi_l \bar{q} / (\pi_h - \pi_l)$. The cost of such a reduction, in ex ante terms, is bounded above by

$$(1 - \pi_h) \left[-I + \bar{v} \left(I + \pi_h z_h - \frac{\pi_h \bar{q}}{\beta(\pi_h - \pi_l)} \right) \right]$$

which is the cost in terms of forgone future returns (net of the banker's rent) of reducing the continuation probability if all depositors have the highest discount factor. As long as the benefit exceeds this maximum cost, or,

$$\frac{\pi_h \pi_l \bar{q}}{\pi_h - \pi_l} - (1 - \pi_h) \left[-I + \bar{v} \left(I + \pi_h z_h - \frac{\pi_h \bar{q}}{\beta(\pi_h - \pi_l)} \right) \right] > 0 \tag{10}$$

then the optimal contract will satisfy $x(y_{l,} v) = 0$ for all v.

Consider next the optimal continuation rule following a realization of high returns in period 1. From Lemma 2, the payment to the banker, $p_1^b(y_h)$ is decreasing in the continuation probability $x(y_h, v)$. This suggests that as long as bank lending opportunities are profitable following high first period returns, then continuing the bank improves surplus available to pay depositors and relaxes the banker's incentive constraint thereby increasing welfare. This suggestive logic neglects the fact that it is possible that by committing to liquidate the bank for some reports v, the principal may be able to better allocate period 2 resources to the most patient depositors. Nonetheless, as we prove in the appendix, as long as

$$\underline{v}\left[I+\rho y_h+\pi_h\left(z_h-\frac{\bar{q}}{\beta(\pi_h-\pi_l)}\right)\right]\geq I, \tag{11}$$

then depositor welfare is maximized when $x(y_h, v) = 1$ for all v. Note that (11) assumes that if all of the depositors are impatient, it is ex post efficient to continue the bank even when accounting for the cost of providing the banker with incentives to exert high effort.

We have developed conditions such that the continuation rule which maximizes depositor welfare has the property that $x(y_l, v) = 0$ and $x(y_h, v) = 1$ for all v. We summarize these results in Proposition 1.

PROPOSITION 1. *Suppose the banker is more impatient than all of the depositors, the expected incentive benefits of liquidation exceed the expected costs (inequality* (10) *is satisfied), and continuing the bank yields positive net present value following high first period returns (inequality* (11) *is satisfied). Then the optimal loan contract calls for continuation after a high return realization in period 1 and liquidation after a low return realization in period 2.*

Proposition 1 illustrates conditions under which liquidation of the bank in period 1 is ex ante optimal. A consequence of the proposition is that each depositor receives an equal share of their original investment, I/N in period 1 after low returns are realized and the bank is liquidated since, without the promise of future transfers, no other transfer scheme is incentive compatible for the depositors.

To argue that such interim period liquidations are inefficient, we appeal to a notion of ex post efficiency in which the depositors' discount factors are observable but the effort of the banker is unobservable. Formally, we say a continuation rule is *ex post efficient* if there is no other continuation rule and associated payments to the depositors which is incentive compatible for the banker, is feasible, and weakly increases the utility of each depositor and strictly increases at least one depositor's utility.

Consider the ex post efficiency of the optimal contract under commitment. It is straightforward to show that the continuation of the optimal contract following high period

1 returns is ex post efficient, so we focus on the continuation outcome following a low first period return realization. Along this outcome path, each depositor receives a transfer of I/N and the bank is liquidated. We now ask whether an alternative contract which calls for continuation of the bank can improve depositor welfare. Such an alternative contract, denoted with ˆ's, would necessarily satisfy the resource constraint

$$\sum_i \hat{p}_2^i(v) = I + \pi_h z_h - \frac{\pi_h \bar{q}}{\beta(\pi_h - \pi_l)},$$

and to improve welfare for each depositor would require $\hat{p}_2^i(v) \geq I/(v_i N)$. Such a scheme exists if

$$I + \pi_h z_h - \frac{\pi_h \bar{q}}{\beta(\pi_h - \pi_l)} > \frac{I}{N} \sum_i \frac{1}{v_i}. \tag{12}$$

Hence, the optimal contract is ex post efficient only when the inequality in (12) is violated. As long as there are realizations of the vector of discount factors v that violate the inequality in (12), for example if $\bar{v}\left[I + \pi_h z_h - \pi_h \bar{q}/\beta(\pi_h - \pi_l)\right] > I$, then the optimal contract features ex post inefficient liquidations. In order to make a more precise statement about ex post inefficiency of liquidations in limiting economies as the number of depositors becomes large, we make an additional assumption as in Proposition 2.

Proposition 2. *As the number of depositors tends to infinity, if the discount factor of depositors is such that*

$$IE\left[\frac{1}{v_i}\right] < I + \pi_h z_h - \frac{\pi_h \bar{q}}{\beta(\pi_h - \pi_l)},$$

then the probability that the optimal continuation rule following a low first period return realization is ex post efficient tends to zero.

Propositions 1 and 2 imply that the optimal ex ante contract necessarily features ex post inefficient outcomes. This feature of the optimal contract arises in many environments with informational frictions (see Phelan and Townsend (1991) and Yared (2010) among many others for examples). In this benchmark model, the ex post inefficiency takes the form of forgone surplus. Along the outcome path of the optimal contract, a large number of depositors refuse to roll-over their loans and liquidate the bank. Because both the banker and the depositors could be made better off (with arbitrarily high probability as $N \to \infty$) by continuing the bank following a low period 1 return realization and because this information is publicly known, this outcome would resemble a bank run to an outside policymaker. As a result, we view our benchmark economy when the average depositor is patient and liquidations are ex post inefficient as a complement to the model in Calomiris and Kahn (1991) when such liquidations are assumed to be ex post efficient.

As is clear, however, this version of the benchmark model cannot be immediately applied to discuss the optimal maturity structure of banks. In the benchmark model, depositors and the banker are able to write long-term state-contingent contracts and are able to commit to the future terms of the contract. We now turn to a modified version of our benchmark economy in which we introduce a form of limited commitment and derive implications for the optimal maturity of bank debt.

2B. The Case of Limited Commitment

In this section, we describe our notion of limited commitment and characterize optimal contracts. We show, perhaps surprisingly, that the limited commitment constraint does not bind in the sense that it does not strictly reduce the attainable level of ex ante depositors' welfare. The result holds only when the preference shocks of the depositors are private information. Although the limited commitment constraint does not bind, it does restrict set of contracts which can yield the highest level of depositors' welfare. In particular, we show that contracts which resemble short-term debt can attain the optimal contract while long-term debt or equity contracts do not.

We assume that the depositors and the banker are free to re-negotiate certain features of the contract after first period returns and the depositors discount factors have been realized. To allow for some, limited form of commitment, we divide period 1 and allow for payments to be made after returns are realized but before depositors realize their discount factors. We call these payments *early payments* and denote them by $p_1^i(y_1)$. The payments made after depositors realize their discount factors $p_{1c}^i(y_1, v)$ and $p_{1n}^i(y_1, v)$ we call *late payments*. An individual depositor's expected payoffs from a loan contract are now

$$E_{e_0}\left[p_1^i(y_1)+\int_v \left[x(v)\left(p_{1c}^i(v)+v_i p_2^i(y_1,v)\right)+(1-x(v))p_{1n}^i(v)\right]dG(v)\right].$$

We assume that the banker and depositors can commit to early payments as well as those to the banker in period 1, but may freely choose to alter the remaining components of the contract.

The only remaining restrictions on the new, re-negotiated contract are that it cannot deliver negative consumption to any agent in period 1 and no agent can be coerced into participating. The nonnegativity constraint limits any single depositor or a small block of depositors from fully financing the bank in period 1. The no coercion constraint serves to implicitly define the outside option of any individual depositor, which is simply $p_1^i(y_1)$. Importantly, we allow the depositors to choose new contracts which may be worse for individual depositors than the continuation value associated with the status quo. This distinction will be important in obtaining a unique optimal maturity of bank debt.

To be precise, we define a *continuation contract* as the following collection of functions,

$$\left\{\left(\hat{p}_{1c}^i(v),\hat{p}_{1n}^i(v),\hat{p}_2^i(v)\right)_{i\in\{1,\ldots,N\}},\hat{x}(v),\hat{e}_1(v),\hat{p}_2^b(z_2,v)\right\}.$$

Facing an arbitrary, status quo contract, a continuation contract, denoted with ˆ's must be incentive compatible for the depositors so that if

$$\hat{w}_i(\hat{v}_i,v_i)=\int_{v_{-i}}\left[x(\hat{v}_i,v_{-i})\left(\hat{p}_{1c}^i(\hat{v}_i,v_{-i})+v_i\hat{p}_2^i(\hat{v}_i,v_{-i})\right)+(1-x(\hat{v}_i,v_{-i}))\hat{p}_{1n}^i(\hat{v}_i,v_{-i})\right]dG_{-i}(v_{-i})$$

represents the continuation utility a depositor with discount factor v_i would receive from the alternative contract when reporting $\hat{v}_i$, then

$$\hat{w}^i(v_i,v_i)\geq\max_{\hat{v}_i\in[\underline{v},\bar{v}]}\hat{w}_i(\hat{v}_i,v_i).$$

Since depositors cannot be coerced to participate, we require $\hat{w}^i(v_i,v_i)\geq 0$.

The contract must also be incentive compatible for the banker so that

$$\begin{aligned}
&\hat{e}_1(v)\hat{p}_2^b(z_h, v) + (1 - \hat{e}_1(v))\hat{p}_2^b(z_l, v) - q(\hat{e}_1(v)) \\
&\quad \geq \max_{e'} e'\hat{p}_2^b(z_h, v) + (1 - e')\hat{p}_2^b(z_l, v) - q(e').
\end{aligned}$$

The banker's incentive constraint places weakly tighter bounds on the continuation utility of the banker than the status quo contracts incentive constraints so that the banker will also choose to participate in any re-negotiation.

The continuation contract must be feasible implying

$$\sum_{i=1}^{N} \hat{p}_2^i(v) \leq I + \rho y_1 + E_{\hat{e}_1(v)}\left[z_2 - \hat{p}_2^b(z_2, v)\right]$$

and

$$\begin{aligned}
\sum_{i=1}^{N} \hat{p}_{1c}^i(v) &\leq y_1 - \sum_{i=1}^{N} p_1^i(y_1) - p_1^b(y_1), \\
\sum_{i=1}^{N} \hat{p}_{1n}^i(v) &\leq I + y_1 - \sum_{i=1}^{N} p_1^i(y_1) - p_1^b(y_1).
\end{aligned}$$

Again, we combine these later constraints into a single, ex ante constraint as

$$\begin{aligned}
&\sum_{i=1}^{N} \int_v \left[\hat{x}(v)\hat{p}_{1c}^i(v) + (1 - \hat{x}(v))\hat{p}_{1n}^i(v)\right] dG(v) \\
&\quad \leq I + y_1 - \sum_{i=1}^{N} p_1^i(y_1) - p_1^b(y_1) - I \int_v \hat{x}(v) dG(v). \qquad (13)
\end{aligned}$$

Lastly, the continuation contract must satisfy the nonnegativity constraints for each depositor

$$p_1^i(y_1) + \hat{x}(v)\hat{p}_{1c}^i(v) + (1 - \hat{x}(v))\hat{p}_{1n}^i(v) \geq 0. \qquad (14)$$

We say a contract is *enforceable* if there is no continuation contract which satisfies depositor and banker incentive compatibility, satisfies nonnegative consumption of depositors and yields strictly greater utility to the depositors in ex ante terms than the status quo contract, or,

$$\sum_i \int_{v_i} \hat{w}^i(v_i, v_i) dG_i(v_i) > \sum_i \int_v \left[x(v)\left(p^i_{1c}(v) + v_i p^i_2(y_1, v)\right) + (1 - x(v))p^i_{1n}(v)\right] dG(v). \tag{15}$$

Observe that elements of the status quo contract appear only in the resource constraint in equation (13), the nonnegativity constraint in equation (14), and in the objective in equation (15). This notion of enforceability makes clear the distinction between period 1 payments made before depositors realize their preference shock (early payments) and period 1 payments made after depositors realize their type (late payments). Because we assume that early payments are made before new continuation contracts can be designed, these early payments affect the set of feasible continuation contracts. By allocating positive early payments in period 1, the limited liability constraints of the depositors and the resource constraints in any continuation contract become more stringent. If early payments are all equal to zero, then the limited liability constraints are weak – any contract, including a contract that calls for continuation, is feasible as long as it delivers at least 0 payments to each depositor.

Before analyzing optimal contracts in this environment, it is useful to consider outcomes with limited commitment when the depositors discount factor shocks are observable. Clearly, the optimal contract under commitment is not enforceable. The reason is that under the assumption of Proposition 2, any contract which calls for liquidation can be dominated ex post by one which calls for continuation. The fact that we analyze a situation in which liquidations are ex post inefficient creates the potential for a time inconsistency problem when the discount factors of the depositors are observable. We summarize this discussion in Lemma 3.

Lemma 3. If depositors' discount factors are observable and if $IE\left[\frac{1}{v_i}\right] < I + \pi_h z_h - \frac{\pi_h \bar{q}}{\beta(\pi_h - \pi_l)}$, then the optimal contract under commitment is not enforceable with limited commitment.

We now argue that the welfare obtained by the optimal contract under commitment is attainable with limited commitment so that in terms of welfare, it appears the limited commitment constraint does not bind. However, we also show that the timing of contracts is determinate and that to obtain the commitment outcome, the principal must use early payments. Formally, we will show that if $p_1^i(y_1) = (I + y_1 - p_1^b(y_1))/N$ for $y_1 = y_l$ and $y_1 = y_h$ then the optimal contract under commitment is enforceable without commitment. Moreover, if $p_1^i(y_l) = 0$ and the contract calls for liquidation when $y_1 = y_l$, then the contract is not enforceable. Early payments to depositors are a necessary feature of optimal contracts. To establish these results, we use the following assumptions.

ASSUMPTION 1. The support of the depositors' discount factors satisfy

$$\underline{v}\left[I + \rho y_l + \pi_h\left(z_h - \frac{\bar{q}}{\beta\left(\pi_h - \pi_l\right)}\right)\right] < I < \underline{v}\left[I + \rho y_h + \pi_h\left(z_h - \frac{\bar{q}}{\beta\left(\pi_h - \pi_l\right)}\right)\right].$$

Assumption 1 plays a key role in characterizing the nature of the coordination problem that arises in period 1 following low or high returns. The left-hand side inequality asserts that if all of the depositors have the lowest rate of time preference, then it would be ex post inefficient for them to continue the bank following a low period 1 return. Of course, as the number of agents becomes large, the probability of this outcome becomes arbitrarily small under the assumption of Proposition 2. Nonetheless, the fact that it is possible for all of the depositors to have a preference shock of $\underline{v}$ implies that each individual depositor must be provided with incentives to reveal their type truthfully, and these incentives do not become arbitrarily small as N tends to ∞.

The incentive problem in renegotiated contracts facing the depositors following high period 1 returns is different however, due primarily to the right inequality in Assumption 1. This inequality asserts that even if all of the depositors have the lowest rate of time preference, then it is ex post efficient to continue the bank after high period 1 outcomes. As a result, even with a probability of continuation equal to 1, there exist payments to depositors satisfying voluntary participation and incentive compatibility of the depositors. For example, a constant transfer scheme will satisfy these constraints. As a result, the depositors will always efficiently continue the bank after high period 1 returns.

We also impose the following regularity assumptions on the distribution of depositor discount factors.

Assumption 2. The distribution $G_i(v_i)$ is such that $(1 - G_i(v_i))/g_i(v_i)$ is decreasing in v_i, there exists $\kappa > 0$ such that $g_i^n(v_i) > \kappa$.

We are now ready to demonstrate how the depositors attain the commitment outcome under limited commitment. Given the nature of the time inconsistency, we focus on enforcing the continuation rule $x(y_l, v) = 0$ following low returns in period 1. The simplest contract that attains the commitment outcome sets early payments equal to a pro-rata share of bank returns, or $p_1^i(y_l) = I/N$.

Consider the renegotiation problem facing depositors. We ask whether the depositors can design renegotiation contracts which call for continuation. First, observe that by the nonnegativity constraint, any such renegotiation contract must satisfy $\hat{p}_{1c}^i = -I/N$ since the depositors must roll over at least I resources. Using this fact, we simplify the constraints of the renegotiation problem to

$$\hat{w}_i(\hat{v}_i, v_i) = \int_{v_{-i}} \left[\hat{x}(\hat{v}_i, v_{-i}) \left(\frac{-I}{N} + v_i \hat{p}_2^i(\hat{v}_i, v_{-i})\right)\right] dG_{-i}(v_{-i}) \tag{16}$$

$$\hat{w}_i(v_i, v_i) \geq \hat{w}_i(\hat{v}_i, v_i) \tag{17}$$

$$\hat{w}_i(v_i, v_i) \geq 0 \tag{18}$$

$$\sum_i \hat{p}_2^i(v) \leq I + E_{\pi_h} z_2 - \frac{\pi_h \bar{q}}{\beta(\pi_h - \pi_l)} \tag{19}$$

Notice, the banker's incentive constraint is nested in the resource constraint in period 2.

To further simplify these constraints, we appeal to results from Myerson (1981) and Myerson and Satterthwaite (1983) which allow us to characterize the global incentive constraints as local constraints and allow us to eliminate period 2 payments from the problem (albeit under a weaker, ex ante version of the period 2 resource constraint in inequality (19)).

Lemma 4. A contract satisfies depositor incentive compatibility if and only if the function $\rho_i(v_i)$ defined by

$$\rho_i(v_i) = \int_{v_{-i}} \hat{x}(v_{-i}, v_i) dG_{-i}(v_{-i})$$

is increasing in v_i for all i, v_i and

$$u^i(v_i) \equiv w^i(v_i, v_i) = v_i \left[\frac{u_i(\underline{v})}{\underline{v}} + \frac{I}{N} \int_{\underline{v}}^{v_i} \frac{1}{z^2} \rho_i(z) dz \right].$$

Moreover, the contract satisfies voluntary participation if and only if $u^i(\underline{v}) \geq 0$.

Combining Lemma 4 with the ex ante version of the period 2 resource constraint, we have the following result.

LEMMA 5. Suppose that $\hat{x}$ is such that ρ_i is increasing in v_i. There exist payments $\hat{p}_2^i$ such that $(\hat{p}_2^i, \hat{x})$ satisfy depositors' incentive compatibility, voluntary participation, and the period 2 resource constraint if and only if

$$\int_v \hat{x}(v) \left[I + E_{\pi_h} z_h - \frac{\pi_h \bar{q}}{\beta(\pi_h - \pi_l)} - \frac{I}{N} \sum_i \left[\frac{1 - G_i(v_i)}{v_i^2 g_i(v_i)} + \frac{1}{v_i} \right] \right] dG(v) \geq 0. \qquad (20)$$

Using this lemma and following a result in Mailath and Postlewaite (1990), it is straightforward to demonstrate that as as $N \to \infty$, the maximal probability of continuation for which there exist transfers so that the continuation rule and transfers satisfy the constraints of the renegotiation problem converges to zero following low period 1 outcomes. We state this result in the following proposition, which makes use of the conditions in Assumption 1 as well as the regularity conditions of Assumption 2.

PROPOSITION 3. *Following a low period 1 outcome, the maximum probability that the bank is continued, or* $\hat{x}(n)$ *satisfying*

$$\begin{aligned} \tilde{x}(N) \;\; = \;\; & sup \big\{ E\hat{x}(v) : \exists (\hat{p}_2^i) \textit{ such that } (\hat{x}, \hat{p}_2^i) \textit{ satisfies depositor IC, participation,} \\ & \textit{and the Resource Constraint in the N-agent economy } (16)\text{-}(19) \big\} \end{aligned}$$

converges to 0 as N goes to infinity. Furthermore, the probability that it is ex post efficient to continue the bank goes to 1.

To understand the result, first recall that the most impatient depositor is unwilling to participate if promised only a pro-rata share of future returns. Hence, contracts which call for continuation must necessarily subsidize impatient depositors in favor of patient depositors. Consider next the tradeoff a single patient depositor faces when deciding which discount rate to report, $\hat{v}_i$. By under-reporting her discount factor, a depositor can attain a higher share of period 2 returns if the bank is continued. Such under-reporting however is costly because by under-reporting her discount factor, the probability of continuation declines since ρ_i is increasing.

As in Mailath and Postlewaite (1990), as $N \to \infty$, the cost of under-reporting shrinks to zero since the likelihood that a single depositor is pivotal converges to zero, but the benefits of under-reporting do not. In other words, the probability the depositor's report leads to a break-down in renegotiation becomes arbitrarily small, but the costs of providing incentives for the depositors to report their types truthfully do not shrink to zero since depositors can always gain a strictly higher payment by under-reporting. As a result, in the limit the incentive costs are so large as to make the value of continuation, net of incentive costs, equivalent to that which would occur in a full information economy if all depositors had the lowest discount factor; at this rate, under Assumption 1 the depositors prefer not to continue the bank.

This result can be seen from (20). Consider maximizing the probability of continuation subject to (20). The maximal rule would set $x(v) = 1$ whenever

$$I + E_{\pi_h} z_h - \frac{\pi_h \bar{q}}{\beta(\pi_h - \pi_l)} - \frac{I}{N} \sum_i \left[\frac{1 - G_i(v_i)}{v_i^2 g_i(v_i)} + \frac{1}{v_i} \right] \geq 0. \tag{21}$$

As N becomes large, the term involving v_i's, which Myerson (1981) terms virtual utility, tends to $1/\underline{v}$. By Assumption 1, this inequality is never satisfied for N sufficiently large.

We have argued that a contract with $p_1^i(y_l) = I/N$ and a continuation rule $x(y_l, v) = 0$ for all v is enforceable. It remains to prove that the remaining features of the optimal contract with the limited commitment coincide with those under full commitment. Conditional on the continuation rule and the banker's consumption, the optimal transfer scheme with limited commitment must coincide with that under full commitment. Moreover, optimality of transfers under full commitment ensures that these transfers are enforceable with limited commitment (were they not enforceable, then there would exist superior transfers under full commitment). As a result, we have proved that the solutions to the commitment and limited commitment problem coincide (as $N \to \infty$). We summarize this discussion in the following proposition.

PROPOSITION 4. *As the number of depositors becomes large, if $p_1^i(y_l) = I/N$, then the depositors can attain their commitment value when contracts are subject to limited commitment.*

Proposition 4 illustrates that the limited enforcement of contracts, in this environment, does not lead to welfare losses. However, it also illustrates which types of contracts can enforce the optimal commitment outcome. These contracts resemble short-term debt because they require a payment (or demandable claim to a payment) roughly equal to the remaining resources of the bank. To further illustrate this result, in the Appendix, we prove that the optimal contract without commitment can be implemented in a decentralized economy in which a banker offers a short-term debt contract in each period. We state this result as Proposition 5

PROPOSITION 5. *Under Assumptions 1 and 2, if $y_h = \bar{q}(1-\pi_l)/(\pi_h - \pi_l)$, as the number of depositors becomes large, the optimal contract is implemented with short-term debt contracts. Short-term debt is non-contingent, is rolled over if period 1 returns are high and is not rolled over if period 1 loan returns are low. When debt is not rolled over, such liquidation is ex post inefficient.*

The benefit of short-term debt contracts in this environment is that such contracts make the continuation decision or the debt-rollover decision resemble a problem of providing public goods. Such contracts introduce ex post public goods problems which make renegotiation difficult; by doing so, such contracts limit the extent of renegotiation which is beneficial from the perspective of providing ex ante incentives to the banker.

The constraints also make plain why by using all long-term debt (i.e. $p_1^i(y_1) = 0$), even when depositors have the option to walk away from the contract cannot implement the optimal continuation rule. With this contract, an individual depositor's payoff from walking away is simply 0. Thus, a re-negotiation contract that delivers 0 late payments in period 1 and constant payments in period 2 will be incentive compatible, satisfy the limited liability constraints and the participation constraints. Since this contract in expectation yields the average valuation of continuing the bank which is assumed to be larger than the liquidation value, this contract strictly improves depositors' ex ante welfare. This proves that the optimal commitment contract is not enforceable with contracts in which early payments are 0. In this sense, contracts which resemble short term debt are both necessary and sufficient for attaining commitment outcomes.

As discussed earlier, the form of limited commitment constraint analyzed here does not require renegotiated contracts to yield Pareto improvements. This assumption is critical for obtaining a determinate maturity structure. The assumption is that early payments yield formal property rights to agents whereas promises to late payments such as equity claims do not. This result suggests an important feature of short-term versus long-term debt is not the timing of payments per se but the allocation of property rights over liquidation decisions.

Optimal Maturity of Financial vs. Non-Financial Firms.— A natural question is why we view the benchmark model as particularly useful model to think about banks as opposed to other non-financial firms. The simplest answer is simply one of scale. In order for the depositors to be able to commit to the optimal liquidation rule, sufficiently many depositors must be needed to agree to roll over the bank's debt. If instead a single depositor can finance the bank, then no contracts can commit the depositors to the optimal liquidation rule. Consequently, depositors are indifferent between short- and long-terms debt.

We arrive at a more subtle answer by contrasting our model with the ideas in Dang et al. (2014) and Monnet and Quintin (2013). Those authors argue that bank loans and investments are typically more opaque than those of non-financial firms and that investing in opaque assets is necessary for banks to engage in maturity transformation. One interpretation of the results in this paper is that if banks invest in opaque assets, then it is optimal for the to finance their investment with short-term debt. Depositors in the model have no ability to evaluate the quality of the loan made by the banker besides observing its gross returns. Zetlin-Jones (2012) analyzes outcomes when depositors receive additional information besides gross payouts and finds that when this information is sufficiently precise, depositors strictly prefer lending via long-term debt. Hence, this theory yields a partial converse to that of Dang et al. (2014) and Monnet and Quintin (2013). If a firm's investments are sufficiently opaque, then we predict that the firm should find it optimal to finance its investments with short-term debt. It is the opacity of bank loans which leads them to choose short-term debt.

3. The Multiple Bank Model

We use the results of the benchmark model to understand the efficiency properties of *systemic crises* in which multiple banks face difficulty rolling over their debt at the same time. In this section, w e argue that the usefulness of short term debt in allowing depositors to commit to provide a single banker with efficient incentives to exert effort is undermined in the context of a richer financial system with multiple banks. Our analysis of the multiple bank model reveals that in the presence of the same limited commitment frictions described above, obtaining commitment requires banks to undertake more risky loans which are correlated with the loans made by other banks. We show that the optimal financial system features systemic crises and that these crises play an important role in allowing depositors to provide efficient incentives to bankers.

We analyze a version of the benchmark model with multiple banks each of which initially borrows from its own set of depositors. After bank returns are observed in period 1, depositors may share resources and future returns across all banks in an attempt to improve ex post outcomes. We analyze this possibility and demonstrate that when the returns to bank projects are independent, such an outcome indeed occurs and limits the ability of depositors of all banks to provide their own banker with incentives to exert effort ex ante. We go on to show that if bankers undertake projects with correlated, riskier returns, depositors can credibly commit to liquidate banks after low project returns and thus provide better ex ante incentives to bankers. We conclude that in this framework, depositors and bankers prefer their bankers to make loans with risky, correlated returns. As a consequence, optimal outcomes feature systemic crises in which in some histories all banks earn low returns simultaneously and all banks face difficulty rolling over their debt.

For simplicity, we analyze a version of our benchmark model with two banks or bankers, each of which is initially paired with its own set of N depositors. We index the two banks as bank A and bank B. There are $2N + 2$ agents in this model. Preferences are identical to those in the benchmark model and each depositor is endowed with I/N. The loan technology across banks may be correlated, in which case banks' loan returns may depend on the joint effort of both bankers. We let the probability over joint bank returns be denoted by π which is a mapping from the effort level of each banker, $\left(e^A, e^B\right)$ to the returns from each bank $\left(y^A, y^B\right)$. We begin by analyzing the case where loans are uncorrelated and returns to one bank are independent of the effort choice of and returns to the other bank. In this sense, we study a *replica economy* of the single-bank model. Below we analyze a case where project returns are perfectly correlated.

3A. The Replica Economy

Consider first a replica economy of the benchmark, single bank model in which the bankers' loans are independent. In this replica economy, banker A's effort level has no effect on the distribution of returns obtained by banker B and similarly for banker B's effort on returns by banker A. Moreover, conditional on the effort choices, loan returns are independent across the two banks. Recall that the value of low returns to either bank in period 1 is given by $y_l = 0$ and the value of high returns in period 1 is given by y_h.

In terms of maximizing resources available to depositors (or minimizing the cost of providing both bankers with incentives to exert effort), the complete independence of the two banks suggests that optimal payments to the bankers and the optimal continuation rules are identical to the case of a single banker. That is, it is optimal to continue bank A if and only if bank A earns high returns in period 1 and similarly for bank B. Given that principal may transfer resources across the depositors, and the ability of the principal to allocate period 2 returns to the ex post most patient depositors depends on the amount of aggregate resources available in each period, this suggestive logic does not immediately yield the desired result. In the Appendix, we show that under the stronger condition,

$$\pi_h \frac{\pi_l \bar{q}}{\pi_h - \pi_l} - (1 - \pi_h)\left[\left(\bar{v} + \frac{1}{g_i\left(\underline{v}\right)}\right) Y_{2l} - I\right] > 0,$$

the optimal contract under commitment has the feature that bank A is continued if and only if bank A earns high returns. Consequently, the depositors would like to commit to liquidate a bank when that bank earns low period 1 returns regardless of the returns earned by the other bank.

We now show that the optimal contract under commitment in the replica economy suffers from two time inconsistency problems associated with the different possible realizations of returns across the two banks. One of these time inconsistencies follows when both banks realize low returns in period 1; the other corresponds to the case when one bank realizes high returns and the other realizes low returns. Each of these time inconsistencies suggests a preference of depositors and bankers for banks to make loans with different return structures.

For illustrative purposes, we restrict our attention to asking whether it is feasible for the depositors to implement the commitment continuation rule when there is limited commitment, as they were able in the single bank context. Since the renegotiation constraints are tightest when all of the returns are paid out in period one, we focus on determining whether choosing $p_1^i(y_l) = (I+y_l)/N$ for both banks can implement the commitment outcome of liquidate either bank if either bank yields low first period returns.

Suppose first that both banks realize low returns in period 1. Aggregate resources available to depositors following such a realization of returns is $2I$. Clearly, the depositors cannot continue both banks. The reason is that exactly $2N$ depositors, each of whom received a payment I/N are needed to finance both banks. However, the most impatient depositors would not value rolling over their debt in either bank, exactly as in the single bank model. As a consequence, there are no incentive-feasible renegotiation contracts which call for both banks to be continued.

However, under a condition on the asymptotic behavior of the *median* depositor, it is possible for the depositors to continue one bank. Specifically, we make the following assumption.

Assumption 3. Assume the distribution of depositors' discount factors satisfies

$$G_i\left(\frac{I}{I+\pi_h z_h - \pi_h \bar{q}/(\beta(\pi_h-\pi_l))}\right) < \frac{1}{2}.$$

Assumption 3 implies that the median depositor would strictly prefer to continue a single bank following low first period returns. After both banks realize low returns in period

1, among the $2N$ depositors, the N most patient depositors would be willing to continue at least one bank even if they receive only an equal share of the future returns (for N sufficiently large). Intuitively, since the median depositor values renewing its debt with one of the banks, for large enough N at least one half of the depositors are willing to continue one of the banks. Because the enforcement constraint effectively allows the depositors to coordinate their rollover decision and the depositors are indifferent between rolling over either of the banks, we assume that the depositors randomize equally between banks when they continue one of the two. Thus, when both banks earn low returns, each banker expects their bank to be continued with probability 1/2.

Lemma 6. With independent bank returns, if both banks realize low returns, then each bank is continued with probability 1/2. In expectation, the banker expects to receive a rent $(1/2)\pi_l\bar{q}/\left(\pi_h - \pi_l\right)$ in this history.

As a consequence of Lemma 6, the optimal commitment outcome is not consistent with outcomes under limited commitment. The reason is that the depositors cannot commit to liquidate the bank with probability 1 when both banks realize low returns. This tightens the period 0 effort constraint of the banker and implies that even with short-term debt, the depositors cannot obtain commitment outcomes.

Very naturally, one way to resolve this time inconsistency problem is to require the banks to invest in projects with riskier returns. In particular, the depositors would like the bankers to invest in projects with greater losses in the event of low returns to ensure that there are limited resources available to continue even a single bank. One way to do this would be for the bankers to choose loans which yield returns $y_l \leq -I/2$ so that when both banks earn low returns, there are only enough to resources in aggregate to rollover at most one bank. Indeed, if aggregate resources after this outcome are exactly equal to I, then the public goods problem studied with N depositors and a single bank applies equally to the problem of $2N$ depositors attempting to finance a single bank. Thus, by increasing the riskiness over returns, depositors may be able to commit to liquidate all banks when all banks earn low returns. We formalize this argument below.

Suppose next that without loss of generality only bank A realizes low returns. In this case, the optimal contract under commitment calls for bank A to be liquidated and bank B to be continued. We show that there exist payments to depositors which satisfy the renegotiation constraints, continue both projects, and necessarily improve ex ante welfare of the depositors. We conclude that with independent returns, depositors can never fully commit to liquidate a bank after it earns low period 1 returns.

First, consider the status quo value of the optimal contract under commitment. Let Y_1 denote the aggregate resources available to depositors in period 1 under this outcome along the optimal commitment contract. The value Y_1 is $Y_1 = 2I + y_h - p_1^{bB}(y_h)$ where $p_1^{bB}(y_h)$ is the payment to banker B following high returns to bank B in period 1. Suppose $Y_1 \geq 2I$ (we will formalize this below) so that there are sufficient resources to continue both banks. We develop conditions consistent with those above such that pro-rata contracts are incentive-feasible and strictly improve ex ante welfare relative to the optimal commitment contract.

If both banks are continued, the depositors must contribute $2I$ resources and they earn an aggregate return $Y_h + Y_l$ where

$$Y_i = I + \rho y_i + \pi_h z_h - \frac{\pi_h \bar{q}}{\beta(\pi_h - \pi_l)}.$$

A *pro-rata contract* is a contract for each depositor such that if the period 1 payment is p_1, the period 2 payment satisfies

$$p_2 = p_1 \frac{Y_h + Y_l}{2I}.$$

That is, pro-rata contracts yield the same aggregate rate of return to each depositor. Consider a pro-rata contract with a required period 1 payment I/N for each depositor. Such a contract is clearly incentive compatible, resource feasible and satisfies the participation constraints as long as

$$v_i \frac{I}{N} \frac{Y_h + Y_l}{2I} \geq \frac{I}{N}$$

or

$$v_i \left(\frac{1}{2} Y_h + \frac{1}{2} Y_l \right) \geq I. \tag{22}$$

The aggregate continuation utility associated with this status quo contract is

$$Y_1 - 2I + E[v_i]\left(Y_h + Y_l\right).$$

This contract implies the optimal contract under commitment is not enforceable as long as it yields strictly higher welfare to the aggregate of depositors than does the optimal commitment outcome.

Since the commitment outcome only continues bank B, it is sufficient to prove that the pro-rata contract delivers more utility than the continuation value associated with a contract which delivers all of the period 2 returns to the most patient depositor since this is an upper bound on the continuation utility associated with the optimal commitment contract. This upper bound is given by

$$Y_1 - I + \bar{v}Y_h.$$

If

$$E[v_i](Y_l) - I > (\bar{v} - E[v_i])Y_h \tag{23}$$

then it is straightforward to show that the pro-rata contract yields strictly higher welfare to the $2N$ depositors than that which can be obtained through the optimal commitment contract. We have proved Lemma 7.

LEMMA 7. Suppose (22) and (23) are satisfied. With independent bank returns, if in period 1 bank A realizes low returns and bank B realizes high returns the optimal commitment outcome is not enforceable. In this case, both banks would be continued ex post. And similarly if bank A realizes high returns and bank B realizes low returns.

Lemma 7 implies that the optimal commitment outcome is not consistent with outcomes under limited commitment even when only one bank realizes low returns. In this case, period 1 and period 2 resources can be shared across both banks to improve outcomes for all depositors. Of course, the bankers rationally expect their contracts to be re-negotiated and thus the depositors lack of commitment limits their ability to provide incentives to the bankers. A straightforward way to resolve this time inconsistency problem is to limit the possibility of such situations arising by requiring the bankers to take correlated risks. In this case, the likelihood of one banker earning high period 1 returns and the other earning low period 1 returns is reduced and so allows the depositors to provide better period 0 incentives to both bankers.

We now formalize the above results on the potential value of increasing the risk and correlation of returns by analyzing an alternative economy with these features.

3B. The Perfectly Correlated Economy with Greater Return Risk

We construct an example in which bankers invest in correlated loans which are riskier than those considered in the independent case, and we show that welfare in this economy is strictly higher than in the replica economy under limited commitment. The parameters of this economy are chosen to ensure that under full commitment, the depositors can obtain the same welfare as in the replica economy. These choices allows us to demonstrate that there is a desirable social role for increased risk and correlation in returns for banks. This desirable social role arises from limited commitment frictions which prevent depositors from providing incentives associated with commitment outcomes.

We alter the production technology to allow for perfectly correlated returns. Specifically, we assume that for any effort levels of the bankers, $\pi\left(y_h, y_l; e^A, e^B\right) = 0$. Moreover, we assume that $\pi\left(y_h, y_h; e^A, e^B\right) = \min\left\{e^A, e^B\right\}$ so that if either banker chooses low effort, the probability of high returns for both banks is low. This assumption ensures that depositors face the same difficulties in providing bankers with incentives to exert effort as in the independent case. While the assumption is stark, it ensures that under full commitment, a social planner has no advantage over the replica economy in terms of providing incentives to the bankers.

Bankers' loans are also riskier in the sense that there is more spread between y_h and y_l than in the independent case. We denote period 1 returns in this economy as $\hat{y}_h$ and $\hat{y}_l$ to distinguish these values from those in the single bank economy. In particular, we set $\hat{y}_l = -I/2$ so that if both banks realize low returns, aggregate resources available to depositors are exactly equal to I. We then choose $\hat{y}_h$ so that under full commitment, the depositors are indifferent between the independent banker and the correlated banker economies (in the Appendix, we prove that such a value $\hat{y}_h$ exists).

In this correlated return environment, it is not surprising that the optimal liquidation policy under full commitment resembles that of the benchmark model. When both banks earn low returns, both banks should be liquidated and when both banks earn high returns, both banks should be continued. With limited commitment, it is immediate that when both bankers realize low returns, neither project can be continued. Since each of the $2N$ depositors is needed to continue even just one of the banks, the set of $2N$ depositors face exactly the renegotiation problem which a single group of depositors faced in the single bank model. As such, depositors can clearly de facto commit to the commitment outcomes. Moreover, because the probability that returns in bank A are high when returns in bank B are low is zero, depositors can also (de facto) commit to liquidate both banks.[5] We have proved the following proposition.

Proposition 6. *If returns are perfectly correlated and sufficiently risky, and depositors lack commitment, then depositors strictly prefer a financial system which features perfectly correlated returns across banks. The optimal financial system features systemic crises with strictly positive probability in which all banks earn low returns and are liquidated simultaneously.*

The above proposition implies that optimal outcomes feature a financial system which is subject to systemic crises. In order for the depositors to commit to provide the optimal incentives to the bankers, they require bankers to undertake investments which are risky and correlated with each other. With probability $(1 - \pi_h)$, both banks realize low returns, and, although there are sufficient resources to finance at least one of the banks and such

[5]Notice that we are changing both the riskiness of returns and the correlation of returns at the same time. If we only change the riskiness over returns, the optimal commitment outcome is not enforceable. The reason is that when bank A earns low returns and bank B earns high returns, as long as in the replica economy, the high return in period 1 satisfies

$$y_h - \frac{\bar{q}(1-\pi_l)}{(\pi_h - \pi_l)(1-\pi_h)} \geq \frac{I}{2}$$

then even with increased risk, both banks can be continued. Since the risky return economy has $\hat{y}_l = -I/2$, clearly in order for the this economy to be equivalent under full commitment, we require $\hat{y}_h \geq y_h$. For any $\hat{y}_h \geq y_h$, this new assumption ensures that aggregate resources independent of the required payment to bank B are larger than $2I$ when bank A earns low returns and bank B earns high returns. Consequently, there are sufficient resources for the depositors to construct re-negotiation contracts which call for continuation of both banks, which implies as in the analysis of the replica economy proves that depositors cannot commit to liquidate bank A.

financing is ex post efficient, none of the banks receive financing. We interpret this result as illustrating the optimality of a fragile financial system.

4. Discussion

In this section, we discuss implications of the model for policy, under what conditions correlated risk-taking by bankers may be an equilibrium outcome of a decentralized economy, as well as a more detailed account of the related literature.

4A. Policy Implications

The policy implications of this model are stark. The model suggests policymakers should exercise caution in restricting the use by banks of short-term debt, or other short-term liabilities; in limiting the correlation or riskiness of returns across banks; and in intervening ex post when they observe many banks facing difficulty rolling over their debt. While these conclusions are stark, our model provides a formal framework for addressing several concerns of those who have questioned the usefulness of short-term debt as a disciplining device.

In particular, Admati and Hellwig (2013) have issued criticisms of the view that short-term debt may play a desirable social role in disciplining bank managers. Among their critiques, they argue that (i) the theory is silent about the costs to the bank and society of depositors suddenly withdrawing their funds and (ii) in the years before the 2007-2009 financial crisis in the U.S., creditors did not monitor banks.

In our model, the costs of systemic crises are forgone profits associated with the banking sector. Implicit are the assumptions that on average, continuing the financial system is a positive net present value investment. In this sense, there are potentially large costs associated with crises and they are realized with positive probability. How to quantify these costs is a different question. However, to the extent that there are external costs from crises (say a disruption in the payment system) which are not internalized by creditors and bankers in the model, there may be a role for policy. Incorporating such external effects could potentially yield interesting policy recommendations on limiting the use of short-term debt by banks.

Moreover, unlike in Calomiris and Kahn (1991) or Diamond and Rajan (2001), there is no monitoring by depositors along the optimal contract. They lend via short-term debt to the banks and when they costlessly observe that banks have earned low returns, they simply refuse to roll-over their debt claims. Our model then is consistent with the observation that banks were able to roll-over their short-term debt until adverse general information about the returns of the banking sector were realized at which point many banks faced difficulty rolling over their short-term debt.

Lastly, some have suggested that correlated risk taking on the part of banks is simply a response to expectations of bailouts, or other blunt policy instruments (see Acharya and Yorulmazer (2007) and Farhi and Tirole (2012) for examples). While it is not surprising that correlated risk taking may be a privately optimal response to a limited commitment problem on the part of a regulator, our model demonstrates in the absence of such a regulator, correlated risk taking by banks may play a desirable social role. In this sense, limiting the correlation of bank returns may be detrimental.

Bailouts.— An important assumption on the efficiency of short-term debt and correlated risk taking in this paper is that depositors have no expectations of bailouts by the government. Some have argued that following a systemic crisis, the incentives of a government to intervene may be stronger than those of private individuals (see Chari and Kehoe (2013) for an argument of this kind). Consequently, the threat by the government of bailouts might unwind the ability of depositors to provide optimal incentives. While there is no role for such bailouts in the model in this paper, it is straightforward to incorporate expectations of bailouts. We focus on the benchmark model with a single bank. Suppose the depositors and the banker expect the government to inject resources if low returns are realized by the bank. By injecting resources, the government effectively relaxes the participation constraints of the depositors in renegotiation and will allow them to construct contracts which call for continuation of the bank.

One would expect there to be several possible outcomes in the optimal lending arrangement between the depositors and the banker which depend on the parameters of the model economy and the bailout policy. If the bailout policy is sufficiently generous, it is possible that optimal contract between the depositors and the bankers resembles short-term debt but has the banker exert low screening effort. The optimal contract has this feature because all agents are better off by extracting the bailout resources from the government.

We conjecture that there are parameter constellations such that the depositors still find it optimal to induce high effort by the banker. Nonetheless, they prefer to use short-term debt as way to commit to extract resources from the government. With the bailout, for all return realizations, the depositors will continue the bank. If they use long-term debt, their private renegotiations will succeed and they will not obtain outside resources from the government. By using short-term debt, the depositors ensure that private re-negotiations fail to continue the bank in the absence of government bailout funding, and they force the government to intervene. In this scenario, however, depositor welfare remains lower (with the bailouts) than in the no-bailout economy. These conjectures are reminiscent of results in Kareken and Wallace (1978) where the promise of bailouts, or deposit insurance, leads to excessive risk-taking in the banking sector.

Securitization.— A natural extension of the ideas in our paper is an application to the securitization market. A key feature of securitized loans is that banks were able to package a large number of loans and sell them to a dispersed group of investors. During the financial crisis, policymakers pointed to the dispersed group of investors in home loans created by securitization as a deterrent in home loan renegotiation and recommended policies such as the mortgage modification program to ease such re-negotations. Our model suggests creating dispersed groups of investors with heterogenous and privately known discount factors is a feature which allows for greater commitment to force homes into default on the part of lenders. This commitment allows lenders to provide better incentives to homeowners to expend effort to be able to repay their loans. Expectations of such modification programs in the future, or limits to the ability of banks to securitize home loans may have the unintended consequence of limiting the extension of home loan credit in the future by making it more difficult for lenders to commit to push homeowners into default when they are unable to repay.

4B. Correlated Risk Taking as an Equilibrium Phenomena

Our analysis of the multibank economy focused on a study of optimal contracts. In particular, we compared outcomes of optimal contracts in economies where risks across banks were either independent or correlated. One way to richen our interpretation of the multibank model would be to increase the choice set of the principal in the optimal contracting framework to include the type of risks undertaken by bankers. Because the existing framework involved no incentive constraints regarding whether bankers undertook independent or correlated risks, implicit in this analysis is the assumption that the type of risks undertaken by bankers is observable and contractable. Any decentralization which yields the same outcomes as the optimal contract studied here, then, would assume such risk-taking choices by bankers are observable. To the extent such choices are observable and decentralized contracts can condition payments to bankers on these choices, it is natural to conjecture that one can construct a decentralization in which both (or all) bankers take correlated risks when the optimal contract has this feature.

One way to interpret the contractibility of bankers' decision to correlated with other bankers or not would be to assume there are bankers who specialize in lending to borrowers whose repayment decisions are correlated with aggregate outcomes and there are those who do not. Each banker has the option to offer deposit contracts and the depositors determine which bankers to invest in. When the optimal contract calls for correlated risk taking by bankers, those bankers who specialize in correlated risk can offer better loan terms to depositors and would therefore attract funding as opposed to those who specialize in independent returns.

An interpretation of these assumptions is as follows. There are bankers who specialize in home mortgage lending and those who specialize in corporate lending. It is observable whether a bank has undertaken home mortgage lending or corporate lending, however it is unobservable whether the bank expended effort to screen potential borrowers in either of these areas. To the extent that home mortgage loan repayments are more sensitive to fluctuations in aggregate house prices, home mortgage lenders returns are more correlated. This interpretation of the model suggests there is a benefit in terms of attaining commitment to provide optimal incentives to having banks invest in an aggregate housing sector.

4C. Related Literature

As discussed in the introduction, this paper is related to an extensive literature on bank runs and the role of demand deposits or short-term debt as in Cole and Kehoe (2000) and Diamond and Dybvig (1983). The theoretical results on the use of short-term debt as a commitment device are closest in nature to those found in Diamond and Rajan (2001), Diamond and Rajan (2000), Diamond (2004), Calomiris and Kahn (1991), and Bolton and Scharfstein (1990), and we view our results as a generalization of the ones in these papers. Specifically, in Diamond and Rajan (2001), bank runs do not occur along the equilibrium path; in Diamond and Rajan (2000), inefficient dismissal of the banker is not a feature of the optimal contract; and in Calomiris and Kahn (1991), when the optimal contract is, in their terminology, "short-term debt," dismissal of the banker is desirable from the perspective of the collective interest of the depositors. Bolton and Scharfstein (1990) consider an arbitrary ex post coordination game between two lenders. When terminating the firm is costly from the collective interests of the lenders, the lenders have strong incentives to coordinate and roll over their debt. In this paper, we allow the lenders to coordinate to develop arbitrary incentive-feasible contracts that induce the lenders as a whole to roll their debt over and demonstrate that even when the option to do so exists, short-term debt prevents them from doing so. Generalizing these results to the case where bank runs occur in equilibrium and are a feature of the optimal contract is a necessary first step in building a framework to analyze the effects of regulatory policy on the capital structure of banks.

The idea that bank runs may be a feature of optimal lending arrangements is related to results in Allen and Gale (1998) and Allen and Gale (2004). In these papers, when intermediaries are restricted to offer demand deposits, bank default or crises allow intermediaries to share risk and effectively offer fully state-contingent contracts. Here. we analyze general optimal contracts and show under what conditions particular frictions of moral hazard on the part of the bank manager and incomplete information regarding depositors' liquidity shocks give rise to crises as a feature of optimal lending arrangements. Of course, there is a long literature on understanding the potential for bank runs that occur along the equilibrium path when banks are restricted to using particular, incomplete contracts. Recent examples of such findings can be found in Goldstein and Pauzner (2005) and He and Xiong (2012).

The idea that a coordination problem can resolve a time inconsistency problem is related to the results in Laffont and Tirole (1988) and Netzer and Scheuer (2010). In their environments, a risk-neutral principal wants to provide both incentives for effort and insurance to a risk-averse principal. Under commitment, the principal provides incentives by delivering less than full insurance to the agent. In both of these papers, when the principal or markets lack commitment, the optimal contract introduces an adverse selection problem ex post, which limits the ability of the principal to provide full insurance after effort has been provided. This adverse selection problem allows the principal to commit to deliver less than full insurance and is the efficient way to provide ex ante incentives. In this paper, because the agent or banker is risk-neutral, a different type of ex post informational problem is necessary for the principal to commit to deliver the appropriate incentives.

Additionally, we provide new results regarding the optimality of short-term contracts in long term agency relationships. Fudenberg et al. (1990) develop conditions under which spot contracts implement optimal commitment outcomes in a long-run relationship. One key condition for their result is that the utility frontier describing payoffs of the principal and payoffs of the agent must be decreasing. In other words, after each history, continuation utilities for the principal and the agent lie on the set of efficient continuation allocations. The main result in this paper demonstrates that short-term contracts may implement long-run commitment outcomes even when long-run commitment outcomes feature histories where continuation outcomes are ex post inefficient. In this sense, our results differ from those found in Brunnermeier and Oehmke (2010), where a lack of commitment causes short-term contracts to deliver worse outcomes than long-run commitment outcomes.

5. Conclusion

We have argued that systemic banking crises may play a desirable social role. Fragility for a single bank serves a useful purpose in providing bank managers appropriate incentives to exert effort to yield a superior distribution of returns. When depositors and bankers have limited ability to commit to long-term, state contingent outcomes, short-term debt can allow depositors to replicate a commitment to liquidate banks when it is ex post inefficient to do so. This commitment is beneficial from an ex ante perspective because it allows depositors to obtain greater ex ante returns from the bank. With multiple banks, short-term debt may not be sufficient to allow depositors to commit to the optimal liquidation strategy. Instead, depositors find it optimal for their banks to engage in riskier, correlated investment strategies to limit their ability to finance banks in the midst of a crisis. Systemic banking crises then serve as a commitment device among depositors to the entire financial system to provide appropriate incentives to bankers to exert effort. One interpretation of this finding is that it illustrates the appropriateness of modeling the financial system as a representative, fragile bank.

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Appendix

A1. Proof of Lemma 1

Here, we prove the following features of optimal contracts under commitment:

1. $p_1^b(y_l) = 0$,
2. $p_2^b(y_l, z_l, v) = p_2^b(y_h, y_l, v) = 0$,
3. $p_2^b(y_l, z_h, v) = \bar{q}/(\beta(\pi_h - \pi_l))$,
4. If the banker's period 0 outside option is sufficiently low, then

$$p_1^b(y_h) + \int_v U_1(y_1, v)G(dv) = \frac{\bar{q}}{\pi_h - \pi_l} + \int_v U_1(y_l, v)G(dv).$$

We show these results in turn.

1. **Proof that $p_1^b(y_l) = 0$.** First, note that the period 0 effort constraint for the banker can be written as

$$\begin{aligned} &p_1^b(y_h) + \int_v \left[E_{\pi_h}\beta p_2^b(y_h, z_2, v) - \bar{q}\right] dG(v) \\ &\quad\geq \frac{\bar{q}}{\pi_h - \pi_l} + p_1^b(y_l) + \int_v \left[E_{\pi_h}\beta p_2^b(y_l, z_2, v) - \bar{q}\right] dG(v). \end{aligned} \tag{A1}$$

Suppose at an optimum, $p_1^b(y_l) > 0$. Consider an alternative allocation in which $\hat{p}_1^b(y_l) = 0$, $\hat{p}_{1c}^i(y_l, v) = p_{1c}^i(y_l, v) + \frac{1}{N}p_1^b(y_l)$, and $\hat{p}_{1n}^i(y_l, v) = p_{1n}^i(y_l, v) + \frac{1}{N}p_1^b(y_l)$. This alternative contract is feasible since

$$\sum_i \hat{p}_{1c}^i(y_l, v) = \sum_i p_{1c}^i(y_l, v) + p_1^b(y_l) \leq y_l$$

and

$$\sum_i \hat{p}_{1n}^i(y_l, v) = \sum_i p_{1n}^i(y_l, v) + p_1^b(y_l) \leq I + y_l$$

where the inequalities follow from the feasibility of the conjectured optimum. The alternative contract is incentive compatible since the utility a depositor receives from reporting her type as $\hat{v}_i$ when her true type is v_i under the alternative satisfies

$$\begin{aligned}
\hat{w}_i(y_l, \hat{v}_i, v_i) &= \int_{v_{-i}} x(y_l, \hat{v}_i, v_i) \left(\hat{p}^i_{1c}(y_l, \hat{v}_i, v_{-i}) + v_i p^i_2(y_l, \hat{v}_i, v_{-i})\right) dG_{-i}(v_{-i}) \\
&\quad + \int_{v_{-i}} x(y_l, \hat{v}_i, v_i)\hat{p}^i_{1n}(y_l, \hat{v}_i, v_{-i}) dG_{-i}(v_{-i}) \\
&= \int_{v_{-i}} x(y_l, \hat{v}_i, v_i) \left(p^i_{1c}(y_l, \hat{v}_i, v_{-i}) + v_i p^i_2(y_l, \hat{v}_i, v_{-i})\right) dG_{-i}(v_{-i}) \\
&\quad + \int_{v_{-i}} x(y_l, \hat{v}_i, v_i) p^i_{1n}(y_l, \hat{v}_i, v_{-i}) dG_{-i}(v_{-i}) + \frac{1}{N} p^b_1(y_l) \\
&= w_i(y_l, \hat{v}_i, v_{-i}) + \frac{1}{N} p^b_1(y_l).
\end{aligned}$$

Consequently, since $w_i(y_l, v_i, v_{-i}) \geq \max_{\hat{v}_i} w_i(y_l, \hat{v}_i, v_{-i})$, it must be that the same holds for the alternative contract. Moreover, welfare of the depositors under this alternative allocation is strictly higher by the amount $(1 - \pi_h)p^b_1(y_l)$. This contradicts optimality of the conjectured optimum.

2. **Proof that $p^b_2(y_l, z_l, v) = p^b_2(y_h, z_l, v) = 0$.** Exactly as above, the first term, $p^b_2(y_l, z_l, v)$ enters the right hand side of (A1) and so reducing this payment relaxes the incentive constraint (as long as this payment is positive on some measureable set of reported types v for which $x(y_l, v) > 0$). Hence, by setting $p^b_2(y_l, z_l, v) = 0$, which relaxes the period 1 effort constraint, the value $p^b_1(y_h)$ can be reduced and paid to depositors after high first period returns have been realized in an incentive compatible way. (For example, choose a contract with $\hat{p}^b_1(y_h) = p^b_1(y_h) - \varepsilon$ and for $j = c, n$, set $\hat{p}^i_{1j}(y_h, v) = p^i_{1j}(y_h, v) + \frac{\varepsilon}{N}$. Exactly as in the proof that $p^b_1(y_l) = 0$, this alternative contract is incentive feasible but strictly raises welfare of the depositors). Consequently, in any optimal contract, $p^b_2(y_l, z_l, v) = 0$.

   We may choose $p^b_2(y_h, z_l, v) = 0$ without loss of generality. To see this, take an arbitrary contract with $p^b_2(y_h, z_l, v) > 0$. Consider an alternative contract which has $\hat{p}^b_2(y_h, z_l, v) = 0$ and

$$\hat{p}^b_2(y_h, z_h, v) = p^b_2(y_h, z_h, v) + \frac{1 - \pi_h}{\pi_h} p^b_2(y_h, z_l, v).$$

   The remainder of the alternative contract is identical to the original. Then, the period 1 effort constraint (following high returns in the first period) is satisfied because

$$\begin{aligned}
\hat{p}^b_2(y_h, z_h, v) &= p^b_2(y_h, z_h, v) + \frac{1 - \pi_h}{\pi_h} p^b_2(y_h, z_l, v) \\
&\geq \frac{\bar{q}}{\beta(\pi_h - \pi_l)} + \frac{1 - \pi_h}{\pi_h} p^b_2(y_h, z_l, v) \\
&\geq \frac{\bar{q}}{\beta(\pi_h - \pi_l)}.
\end{aligned}$$

By construction, $\beta E_{\pi_h}\hat{p}_2^b(y_h, z_2, v) - \bar{q} = \beta E_{\pi_h} p_2^b(y_h, z_2, v) - \bar{q}$ and thus the period 1 effort is constraint is satisfied. Moreover, the payments to the depositors are unchanged.

3. **Proof that** $p_2^b(y_l, z_h, v) = \bar{q}/(\beta(\pi_h - \pi_l))$. Suppose on some measureable set of v, the optimal contract satisfies $p_2^b(y_l, y_h, v) > \bar{q}/(\beta(\pi_h - \pi_l))$. Then we construct an alternative incentive feasible contract which dominates this conjectured optimum. In particular, we choose $\hat{p}_2^b(y_l, z_h, v) = \bar{q}/(\beta(\pi_h - \pi_l))$. Given that $E_{\pi_h}\hat{p}_2^b(y_l, z_2, v) < E_{\pi_h} p_2^b(y_l, z_2, v)$, the right hand side of (A1) is reduced. Thus, we may reduce $p_1^b(y_h)$ as in the previous proof and strictly increase depositors welfare, yielding the necessary contradiction.

4. **Proof that constraint (A1) holds with equality.** Ignoring the banker's participation constraint, if equation (A1) is slack, then payments to depositors can be increased in the same incentive compatible fashion as above following high first period returns. As a consequence, in an optimal contract, this equation must hold with equality.

A2. Proof of Proposition 1

Here, we demonstrate that optimal contracts under commitment have the feature that $x(y_l, v) = 0$ and $x(y_h, v) = 1$. To see that $x(y_l, v) = 0$, substitute for $p_1^b(y_h)$ from (8) into (9). Grouping the terms that depend on $x(y_l, v)$, we have

$$\int_v x(y_l, v) \left[(1 - \pi_h) \left(-I + \sum_i v_i p_2^i(y_l, v) \right) - \pi_h \frac{\pi_l \bar{q}}{\pi_h - \pi_l} \right] dG(v). \tag{A2}$$

Since for all v,

$$\sum_i v_i p_2^i(v) \le \bar{v} \sum_i p_2^i(v) \le \bar{v} \left(I + \pi_h z_h - \frac{\pi_h \bar{q}}{\beta(\pi_h - \pi_l)} \right)$$

the term in brackets multiplying $x(y_l, v)$ in (A2) is (weakly) smaller than

$$\left[(1 - \pi_h) \left(-I + \bar{v} \left(I + \pi_h z_h - \frac{\pi_h \bar{q}}{\beta(\pi_h - \pi_l)} \right) \right) - \pi_h \frac{\pi_l \bar{q}}{\pi_h - \pi_l} \right]$$

which is negative under the assumption in (10). Obviously, for $x(y_l, v) = 0$, any transfers which are incentive compatible have the feature that $p_1 n^i(y_l, v)$ is a constant, which is also optimal since in period 1 depositors' valuations are identically equal to 1.

Next we show that that optimal contracts under commitment satisfy $x(y_h, v) = 1$ for all v. Using Lemma 1, the payment to the banker following high returns in period 1 satisfies

$$p_1^b(y_h) = \frac{\bar{q}}{\pi_h - \pi_l} - \int_v x(v) \left[\beta \pi_h p_2^b(y_h, z_h, v) - \bar{q} \right] dG(v).$$

The problem of choosing optimal payments to the depositors along the outcome where $y_1 =$

y_h is then given by

$$\max \sum_i \int \left\{ x(v) \left[p^i_{1c}(v) + v_i p^i_2(v) \right] + (1 - x(v)) p^i_{1n} \right\} dG(v)$$

subject to

$$\int_{v_{-i}} \left[x(v_i, v_{-i}) p^i_{1c}(v_i, v_{-i}) + (1 - x(v_i, v_{-i})) p^i_{1n}(v_i, v_{-i}) \right] dG_{-i}(v_{-i}) + v_i \int x(v_i, v_{-i}) p^i_2(v_i, v_{-i}) dG_{-i}(v_{-i})$$
$$\geq \int_{v_{-i}} \left[x(\hat{v}_i, v_{-i}) \left(p^i_{1c}(\hat{v}_i, v_{-i}) + v_i p^i_2(\hat{v}_i, v_{-i}) \right) + (1 - x(\hat{v}_i, v_{-i})) p^i_{1n}(\hat{v}_i, v_{-i}) \right] dG_{-i}(v_{-i})$$
$$\int_{v_{-i}} \left[x(v_i, v_{-i}) p^i_{1c}(v_i, v_{-i}) + (1 - x(v_i, v_{-i})) p^i_{1n}(v_i, v_{-i}) \right] dG_{-i}(v_{-i}) + v_i \int x(v_i, v_{-i}) p^i_2(v_i, v_{-i}) dG_{-i}(v_{-i})$$
$$\geq \frac{I}{N}$$
$$\sum_i p^i_{1c}(v) \leq y_h - \frac{\bar{q}}{\pi_h - \pi_l} + \int_v x(v) \left[\beta \pi_h p^b_2(y_h, z_h, v) - \bar{q} \right] dG(v)$$
$$\sum_i p^i_{1n}(v) \leq I + y_h - \frac{\bar{q}}{\pi_h - \pi_l} + \int_v x(v) \left[\beta \pi_h p^b_2(y_h, z_h, v) - \bar{q} \right] dG(v)$$
$$\sum_i p^i_2(v) \leq I + \rho y_h + \pi_h \left(z_h - p^b_2(y_h, z_h, v) \right)$$
$$p^b_2(y_h, z_h, v) \geq \frac{\bar{q}}{\beta(\pi_h - \pi_l)}$$

where we have dropped y_1 from the functions for notational convenience in this appendix.

We will focus on a relaxed problem which satisfies the budget constraints in expectation (over v), and then show we satisfy the budget constraints ex post. Towards this end, we multiply the first and third resource constraint by $x(v)$, the second by $(1 - x(v))$ and integrate. We have

$$\sum_i \int_v x(v) p^i_{1c}(v) dG(v) \leq \left[y_h - \frac{\bar{q}}{\pi_h - \pi_l} + \int_v x(v) \left[\beta \pi_h p^b_2(y_h, z_h, v) - \bar{q} \right] dG(v) \right] \int x(v) dG(v)$$
$$\sum_i \int (1 - x(v)) p^i_{1n}(v) dG(v)$$
$$\leq \left[I + y_h - \frac{\bar{q}}{\pi_h - \pi_l} + \int_v x(v) \left[\beta \pi_h p^b_2(y_h, z_h, v) - \bar{q} \right] dG(v) \right] \int (1 - x(v)) dG(v)$$
$$\int_v x(v) \sum_i p^i_2(v) dG(v) \leq \left[I + \rho y_h + \pi_h \left(z_h - p^b_2(y_h, z_h, v) \right) \right] \int_v x(v) dG(v)$$

If we add the first two, we have

$$\sum_i \int_v \left[x(v) p^i_{1c}(v) + (1 - x(v)) p^i_{1n}(v) \right] dG(v) \leq I + y_h - \frac{\bar{q}}{\pi_h - \pi_l} + \int_v x(v) \left[\beta \pi_h p^b_2(y_h, z_h, v) - \bar{q} - I \right] dG(v)$$

So the planning problem simplifies (in ex-ante terms) to

$$\max \sum_i \int \left[x(v)p_{1c}^i(v) + (1-x(v))p_{1n}^i(v)\right] dG(v) + \sum_i \int x(v) v_i p_2^i(v) dG(v)$$

subject to

$$\int_{v_{-i}} \left[x(v_i, v_{-i})p_{1c}^i(v_i, v_{-i}) + (1-x(v_i, v_{-i}))p_{1n}^i(v_i, v_{-i})\right] dG_{-i}(v_{-i}) + v_i \int x(v_i, v_{-i})p_2^i(v_i, v_{-i})dG_{-i}(v_{-i})$$
$$\geq \int_{v_{-i}} \left[x(\hat{v}_i, v_{-i})\left(p_{1c}^i(\hat{v}_i, v_{-i}) + v_i p_2^i(\hat{v}_i, v_{-i})\right) + (1-x(\hat{v}_i, v_{-i}))p_{1n}^i(\hat{v}_i, v_{-i})\right] dG_{-i}(v_{-i})$$
$$\sum_i \int_v \left[x(v)p_{1c}^i(v) + (1-x(v))p_{1n}^i(v)\right] dG(v) \leq I + y_h - \frac{\bar{q}}{\pi_h - \pi_l} + \int_v x(v)\left[\beta \pi_h p_2^b(y_h, z_h, v) - \bar{q} - I\right] dG(v)$$
$$\int_v x(v) \sum_i p_2^i(v) dG(v) \leq \left[I + \rho y_h + \pi_h\left(z_h - p_2^b(y_h, z_h, v)\right)\right] \int_v x(v) dG(v)$$
$$p_2^b(y_h, z_h, v) \geq \frac{\bar{q}}{\beta(\pi_h - \pi_l)}$$

along with the participation constraint. Let

$$\begin{aligned}
\zeta_i(v_i) &= \int_{v_{-i}} \left[x(v_i, v_{-i})p_{1c}^i(v_i, v_{-i}) + (1-x(v_i, v_{-i}))p_{1n}^i(v_i, v_{-i})\right] dG_{-i}(v_{-i}) \\
\rho_i(v_i) &= \int x(v_i, v_{-i})p_2^i(v_i, v_{-i})dG_{-i}(v_{-i})
\end{aligned}$$

so that

$$u_i(v_i) = \zeta_i(v_i) + v_i \rho_i(v_i).$$

Then the participation constraints can be written compactly as $u_i(v_i) \geq I/N$.

It is straightforward, following Myerson (1981) to show that the incentive constraints imply that $\rho_i(v_i)$ is increasing and

$$u_i(v_i) = u_i\left(\underline{v}\right) + \int_{\underline{v}}^{v_i} \rho_i(z) dz.$$

Straightforward calculus then yields

$$\sum_i \int u_i(v_i) dG_i(v_i) = \sum_i u_i(\underline{v}) + \sum_i \int_{v_i} \frac{1 - G_i(v_i)}{g_i(v_i)} \rho_i(v_i) dG_i(v_i)$$

and

$$\sum_i \int_{v_i} \zeta_i(v_i) dG_i(v_i) = \sum_i u_i(\underline{v}) + \sum_i \int_{v_i} \left[\frac{1 - G_i(v_i)}{g_i(v_i)} - v_i\right] \rho_i(v_i) dG_i(v_i)$$

Substituting this last equality into the ex ante period 1 resource constraint, we obtain that any incentive compatible allocation satisfies

$$\sum_i u_i(\underline{v}) + \sum_i \int_{v_i} \left[\frac{1-G_i(v_i)}{g_i(v_i)} - v_i\right] \rho_i(v_i) dG_i(v_i)$$
$$\leq I + y_h - \frac{\bar{q}}{\pi_h - \pi_l} + \int_v x(v) \left[\beta \pi_h p_2^b(y_h, z_h, v) - \bar{q} - I\right] dG(v) \tag{A3}$$

It is straightforward, following Mailath and Postlewaite (1990) to prove these conditions are also sufficient. The proof of sufficiency follows by construction where

$$\begin{aligned} x(v)p_{1c}^i(v) + (1-x(v))p_{1n}^i(v) &= \frac{I}{N} - \frac{1}{N-1}\sum_{j\neq i}\int_{v_j} \rho_j(v_j)\left(v_j - \frac{1-G_j(v_j)}{g_j(v_j)}\right) dG_j(v_j) \\ &\quad - v_i\rho_i(v_i) + \int_{\underline{v}}^{v_i} \rho_i(z)dz + \frac{1}{N-1}\sum_{j\neq i}\left[v_j\rho_j(v_j) - \int_{\underline{v}}^{v_j}\rho_j(z)dz\right]. \end{aligned}$$

Using this transfer function, one can show that $u_i(v_i) = \frac{I}{N} + \int_{\underline{v}}^{v_i} \rho_i(z)dz$ which immediately implies $x, p_{1c}^i, p_{1n}^i, p_2^i$ satisfy incentive compatibility and the participation constraint. Moreover, using the fact that x, p_2^i satisfy (A3) straightforward substitutions show that $x, p_{1c}^i, p_{1n}^i, p_2^i$ also satisfy the resource constraints. Hence, it suffices to consider the following problem

$$\max_{x,\rho_i(v_i)} \sum_i u_i\left(\underline{v}\right) + \sum_i \int_{v_i} \rho_i(v_i)\frac{1-G_i(v_i)}{g_i(v_i)} dG_i(v_i)$$

subject to

$$\sum_i u_i\left(\underline{v}\right) + \sum_i \int_{v_i} \left[\frac{1-G_i(v_i)}{g_i(v_i)} - v_i\right] \rho_i(v_i) dG_i(v_i)$$
$$\leq I + y_h - \frac{\bar{q}}{\pi_h - \pi_l} + \int_v x(v) \left[\beta \pi_h p_2^b(y_h, z_h, v) - \bar{q} - I\right] dG(v) \tag{A4}$$

$$0 \leq \left[I + \rho y_h + \pi_h\left(z_h - p_2^b(y_h, z_h, v)\right)\right] \int_v x(v)dG(v) - \sum_i \int_{v_i} \rho_i(v_i) dG_i(v_i) \tag{A5}$$

$$p_2^b(y_h, z_h, v) \geq \frac{\bar{q}}{\beta(\pi_h - \pi_l)} \tag{A6}$$

$\rho_i(v_i)$ non-decreasing.

Now, suppose on some set v where $x(v) > 0$, the solution has $p_2^b(y_h, z_h, v) > \bar{q}/\left(\beta(\pi_h - \pi_l)\right)$. Consider perturbing the contract by setting $\hat{p}_2^b(y_h, z_h, v) = p_2^b(y_h, z_h, v) - \frac{\varepsilon}{\pi_h}$ for some ε sufficiently small so that $\hat{p}_2^b(y_h, z_h, v)$ still satisfies the banker's incentive constraint, (A6). Then, set $\hat{\rho}_i(v_i) = \rho_i(v_i) + \frac{\varepsilon}{N}\int_v x(v)dG(v)$. Since $\rho_i(v_i)$ is non-decreasing, so is $\hat{\rho}_i(v_i)$. Moreover, this perturbation satisfies the period 2 resource constraint (A5) by construction. Lastly, since $\beta < \underline{v}$, we claim this perturbed contract also satisfies the implementability constraint

(A4). To see this, note that

$$
\begin{aligned}
& I + y_h - \frac{\bar{q}}{\pi_h - \pi_l} + \int_v x(v)\left[\beta\pi_h \hat{p}_2^b(y_h, z_h, v) - \bar{q} - I\right] dG(v) + \sum_i \int_{v_i} \left[v_i - \frac{1 - G_i(v_i)}{g_i(v_i)}\right] \hat{\rho}_i(v_i) dG_i(v_i) \\
= \; & I + y_h - \frac{\bar{q}}{\pi_h - \pi_l} + \int_v x(v)\left[\beta\pi_h p_2^b(y_h, z_h, v) - \bar{q} - I\right] dG(v) + \sum_i \int_{v_i} \left[v_i - \frac{1 - G_i(v_i)}{g_i(v_i)}\right] \rho_i(v_i) dG_i(v_i) \\
& -\varepsilon\beta \int_v x(v) dG(v) + \frac{\varepsilon}{N} \int_v x(v) dG(v) \sum_i \int_{v_i} \left[v_i - \frac{1 - G_i(v_i)}{g_i(v_i)}\right] dG_i(v_i) \\
= \; & I + y_h - \frac{\bar{q}}{\pi_h - \pi_l} + \int_v x(v)\left[\beta\pi_h p_2^b(y_h, z_h, v) - \bar{q} - I\right] dG(v) + \sum_i \int_{v_i} \left[v_i - \frac{1 - G_i(v_i)}{g_i(v_i)}\right] \rho_i(v_i) dG_i(v_i) \\
& +\varepsilon\left(\underline{v} - \beta\right) \int_v x(v) dG(v) \\
\geq \; & \sum_i u_i\left(\underline{v}\right) + \varepsilon\left(\underline{v} - \beta\right) \int_v x(v) dG(v) \\
\geq \; & \sum_i u_i(\underline{v}).
\end{aligned}
$$

Hence, this pertubred allocation satisfies the constraints but strictly increases welfare. We then know that at an optimal contract, $p_2^b(y_h, z_h, v) = \bar{q}/\left(\beta(\pi_h - \pi_l)\right)$. Consequently, the constraints (A4) and (A5) can be written compactly as

$$
0 \leq y_h - \frac{\bar{q}}{\pi_h - \pi_l} + \left[\frac{\pi_l \bar{q}}{\pi_h - \pi_l} - I\right] \int_v x(v) dG(v) + \sum_i \int_{v_i} \left[v_i - \frac{1 - G_i(v_i)}{g_i(v_i)}\right] \rho_i(v_i) dG_i(v_i) \tag{A7}
$$

$$
0 \leq \left[I + \rho y_h + \pi_h \left(z_h - \frac{\bar{q}}{\beta\left(\pi_h - \pi_l\right)}\right)\right] \int_v x(v) dG(v) - \sum_i \int_{v_i} \rho_i(v_i) dG_i(v_i) \tag{A8}
$$

where we have used $u_i(\underline{v}) = I/N$ and cancelled like terms in (A4).

Suppose now for contradiction that $\int_v x(v) dG(v) < 1$. We claim by increasing the average value of $x(v)$, we can strictly increase the objective. Consider an alternative contract in which $\hat{x}(v) = 1$ for all v and

$$
\hat{\rho}_i(v_i) = \rho_i(v_i) + \left[I + \rho y_h + \pi_h \left(z_h - \frac{\bar{q}}{\beta\left(\pi_h - \pi_l\right)}\right)\right] \frac{1}{N} \left(1 - \int_v x(v) dG(v)\right).
$$

Clearly, if ρ_i is increasing, then so is $\hat{\rho}_i$. By construction, this choice of $\hat{x}, \hat{\rho}_i$ satisfies the second constraint, (A7). To see that $\hat{x}, \hat{\rho}_i$ satisfies the first constraint, (**??**), observe

$$
\begin{aligned}
& y_h - \frac{\bar{q}}{\pi_h - \pi_l} + \left[\frac{\pi_l \bar{q}}{\pi_h - \pi_l} - I\right] \int_v \hat{x}(v) dG(v) + \sum_i \int_{v_i} \left[v_i - \frac{1 - G_i(v_i)}{g_i(v_i)}\right] \hat{\rho}_i(v_i) dG_i(v_i) \\
&= y_h - \frac{\bar{q}}{\pi_h - \pi_l} + \left[\frac{\pi_l \bar{q}}{\pi_h - \pi_l} - I\right] \int_v x(v) dG(v) + \sum_i \int_{v_i} \left[v_i - \frac{1 - G_i(v_i)}{g_i(v_i)}\right] \rho_i(v_i) dG_i(v_i) \\
&\quad + \left(1 - \int_v x(v) dG(v)\right) \left(\frac{\pi_l \bar{q}}{\pi_h - \pi_l} - I\right) \\
&\quad + \left(1 - \int_v x(v) dG(v)\right) \left[I + \rho y_h + \pi_h \left(z_h - \frac{\bar{q}}{\beta\left(\pi_h - \pi_l\right)}\right)\right] \frac{1}{N} \sum_i \int_{v_i} \left[v_i - \frac{1 - G_i(v_i)}{g_i(v_i)}\right] dG_i(v_i) \\
&= y_h - \frac{\bar{q}}{\pi_h - \pi_l} + \left[\frac{\pi_l \bar{q}}{\pi_h - \pi_l} - I\right] \int_v x(v) dG(v) + \sum_i \int_{v_i} \left[v_i - \frac{1 - G_i(v_i)}{g_i(v_i)}\right] \rho_i(v_i) dG_i(v_i) \\
&\quad + \left(1 - \int_v x(v) dG(v)\right) \left[\frac{\pi_l \bar{q}}{\pi_h - \pi_l} - I + \underline{v} \left[I + \rho y_h + \pi_h \left(z_h - \frac{\bar{q}}{\beta\left(\pi_h - \pi_l\right)}\right)\right]\right] \\
&\geq y_h - \frac{\bar{q}}{\pi_h - \pi_l} + \left[\frac{\pi_l \bar{q}}{\pi_h - \pi_l} - I\right] \int_v x(v) dG(v) + \sum_i \int_{v_i} \left[v_i - \frac{1 - G_i(v_i)}{g_i(v_i)}\right] \rho_i(v_i) dG_i(v_i) \\
&\geq 0
\end{aligned}
$$

where the second-to-last inequality follows from the assumption that

$$
\underline{v} \left[I + \rho y_h + \pi_h \left(z_h - \frac{\bar{q}}{\beta\left(\pi_h - \pi_l\right)}\right)\right] > I.
$$

Hence, the optimum has $\int_v x(v) dG(v) = 1$ so that $x(v) = 1$ a.e. We conclude that in the optimal contract, $x(y_l, v) = 0$ and $x(y_h, v) = 1$ for all v.

A3. Proof of Proposition 2

We prove that as $N \to \infty$, the probability that $x(y_l, v) = 1$ is ex post efficient tends to 1. Consider the problem of maximizing depositor welfare given the incentive constraint of the banker. This problem is given by

$$
\max \sum_i \int_v x(v) \left[-I + v_i p_2^i(v)\right] dG(v)
$$

subject to

$$
\sum_i p_2^i(v) \leq I + \pi_h z_h - \frac{\pi_h \bar{q}}{\beta(\pi_h - \pi_l)}
$$

and $p_2^i(v) \geq I/(v_i N)$ where we have nested the banker's incentive constraint in the above resource constraint. Clearly, efficiency dictates that $x(v) = 1$ if and only if

$$
\frac{I}{N} \sum_i \frac{1}{v_i} \leq I + \pi_h - \frac{\pi_h \bar{q}}{\beta(\pi_h - \pi_l)}.
$$

Under the assumption of the proposition that

$$IE\left[\frac{1}{v_i}\right] < I + \pi_h z_h - \frac{\pi_h \bar{q}}{\beta(\pi_h - \pi_l)},$$

by a law of large numbers, the result follows.

A4. Proof of Lemma 4

Define

$$\begin{aligned}
\zeta_i(v_i) &= \int_{v_{-i}} x(v_i, v_{-i}) p_2^i(v_i, v_{-i}) G_{-i}(dv_{-i}) \\
\rho_i(v_i) &= \int_{v_{-i}} x(v_i, v_{-i}) G_{-i}(dv_{-i})
\end{aligned}$$

Then

$$u_i(v_i) = -\frac{I}{N}\rho_i(v_i) + v_i \zeta_i(v_i). \tag{A9}$$

Adding and subtracting $\rho_i(\hat{v}_i)I/(N\hat{v}_i)$ to the incentive constraint implies

$$\frac{1}{v_i}u_i(v_i) \geq \frac{1}{\hat{v}_i}u_i(\hat{v}_i) + \rho_i(\hat{v}_i)\frac{I}{N}\left[\frac{1}{\hat{v}_i} - \frac{1}{v_i}\right]$$

and similarly

$$\frac{1}{\hat{v}_i}u_i(\hat{v}_i) \geq \frac{1}{v_i}u_i(v_i) + \rho_i(v_i)\frac{I}{N}\left[\frac{1}{v_i} - \frac{1}{\hat{v}_i}\right].$$

Combing these inequalities, we obtain

$$\begin{aligned}
\rho_i(v_i)\frac{I}{N}\frac{v_i - \hat{v}_i}{\hat{v}_i v_i} &\geq \frac{1}{v_i}u_i(v_i) - \frac{1}{\hat{v}_i}u_i(\hat{v}_i) \geq \rho_i(\hat{v}_i)\frac{I}{N}\frac{v_i - \hat{v}_i}{\hat{v}_i v_i} \\
\frac{\rho_i(v_i)}{\hat{v}_i v_i}\frac{I}{N} &\geq \frac{\frac{1}{v_i}u_i(v_i) - \frac{1}{\hat{v}_i}u_i(\hat{v}_i)}{v_i - \hat{v}_i} \geq \frac{\rho_i(\hat{v}_i)}{\hat{v}_i v_i}\frac{I}{N}
\end{aligned}$$

For $v_i > \hat{v}_i$, this implies that $\rho_i(v_i)$ is increasing in v_i. Taking limits as $\hat{v}_i \to v_i$, we have

$$\frac{1}{v_i}\rho_i(v_i)\frac{I}{N} = u_i'(v_i) - \frac{1}{v_i}u_i(v_i).$$

By solving the differential equation we, obtain the integral form of the local incentive constraint given

$$u_i(v_i) = v_i\left[\frac{u_i(\underline{v})}{\underline{v}} + \frac{I}{N}\int_{\underline{v}}^{v_i}\frac{1}{z^2}\rho_i(z)dz\right] \tag{A10}$$

This concludes the "If" portion of the proof. The "Only if" portion follows standard arguments.

A5. Proof of Lemma 5

We prove the following "If" statement: Suppose (p_2^i, x) satisfy the depositor's incentive constraints, depositors participation constraints, and the period 2 resource constraint. Then (21) is satisfied and $\rho_i(v_i)$ is increasing for all i. To see this result, recall by Lemma 4 that we obtain immediately that ρ_i is increasing. Next, recall the resource constraint

$$\sum_i \int_v x(v)p_2^i(v)G(dv) \leq \left[I + \rho y_l + E_{\pi_h} z_2 - \frac{\pi_h \bar{q}}{\beta(\pi_h - \pi_l)}\right] \int_v x(v)G(dv)$$

Using the definition of $\zeta_i(v_i)$, we can express expected payments to depositor i in period 2 as $\int_v x(v)p_2^i(v)G(dv) = \int_{v_i} \zeta_i(v_i)G_i(dv_i)$. We now expand the term $\zeta_i(v_i)$ using results from the proof of Lemma 4. First, note from equation (A9), the expected value of ζ_i can be written as

$$\int_{v_i} \zeta_i(v_i)G_i(dv_i) \quad = \quad \int_{v_i} \left[\frac{u_i(v_i)}{v_i} + \frac{I}{Nv_i}\rho_i(v_i)\right] G_i(dv_i).$$

Expanding $u_i(v_i)/v_i$ using equation (A10), we then have

$$\int_{v_i} \zeta_i(v_i)G_i(dv_i) \quad = \quad \int_{v_i} \left[\frac{u_i(\underline{v})}{\underline{v}} + \frac{I}{N}\int_{\underline{v}}^{v_i} \frac{1}{z^2}\rho_i(z)dz + \frac{I}{Nv_i}\rho_i(v_i)\right] G_i(dv_i).$$

The above expression can be simplified with straightforward calculus to

$$\int_{v_i} \zeta_i(v_i)G_i(dv_i) \quad = \quad \frac{u_i(\underline{v})}{\underline{v}} + \frac{I}{N}\left[\int_{v_i} \rho_i(v_i)\left[\frac{1 - G_i(v_i)}{v_i^2 g_i(v_i)} + \frac{1}{v_i}\right] G_i(dv_i)\right].$$

Summing over i and combining with the resource constraint yields the desired result.

The "Only if" can be demonstrated using a transfer scheme similar to that considered in Mailath and Postlewaite (1990).

A6. Proof of Lemma 3

We follow the proof in Mailath and Postlewaite (1990). In particular, consider solving the auxilliary problem given by

$$\max \int_v x(v)G(dv)$$

subject to

$$\int_v x(v)\left[I + \pi_h z_h - \pi_h \frac{\bar{q}}{\beta(\pi_h - \pi_l)} - \frac{I}{N}\sum_i \left[\frac{1 - G_i(v_i)}{v_i^2 g_i(v_i)} + \frac{1}{v_i}\right]\right] G(dv) \geq 0$$

and the continuation rule $x(v)$ is such that

$$\rho_i(v_i) = \int_{v_{-i}} x(v_i, v_{-i})G_{-i}(dv_{-i})$$

is increasing. We call the solution to this problem the maximal continuation rule. Ignoring the monotinicity constraint, the maximal continuation rule has the property that

$$x(v) = 1 \Leftrightarrow I + \pi_h z_h - \frac{\pi_h \bar{q}}{\beta(\pi_h - \pi_l)} \geq \frac{I}{N} \sum_i \left[\frac{1 - G_i(v_i)}{v_i^2 g_i(v_i)} + \frac{1}{v_i} \right]$$

Since $(1 - G_i(v_i))/g_i(v_i)$ is decreasing, it is immediate that this maximal continuation rule satisfies the monotonicity constraint. Forming the lagrangian, we have that $x(v) = 1$ if and only if the condition above is satisfied when modified by incorporating the inverse of the lagrange multiplier on the implementability constraint. An argument from Mailath and Postlewaite (1990) ensures that the lagrange muliplier converges to ∞ as $N \to \infty$ so that this term vanishes in the limit. Then, the term multiplying I, by a law of large numbers, converges to

$$E \left[\frac{1 - G_i(v_i)}{v_i^2 g_i(v_i)} + \frac{1}{v_i} \right] = \frac{1}{\underline{v}}$$

Thus, as $N \to \infty$, the RHS converges to $\frac{I}{\underline{v}}$ and $\underline{v} \left(I + \pi_h z_h - \frac{\pi_h \bar{q}}{\beta(\pi_h - \pi_l)} \right) < I$. Therefore, $x(v) \to 0$ for all v. Formally, the assumptions of the lemma coincide with those in Mailath and Postlewaite (1990) and the result follows from an application of their Theorem 2. This completes the proof.

A7. A Decentralization of the Single Bank Model

In this Appendix, we formalize the idea that optimal contracts in the single bank model can be attained by way of short term debt contracts by presenting a decentralized version of the benchmark economy. Technology and preferences are identical to the benchmark economy. We allow the banker to offer short-term contracts in period 0 and period 1. These contracts offer a gross rate of return $R_1(y_1)/I$ in period 1 and $R_2(y_1, z_2)/I$ in period 2. In the special case we consider below, we will show $R_1(y_1)$ is independent of y_1 but $R_2(y_1, z_2)$ will depend on the returns to the banker's loan in period 1.

The timing is as follows. The banker offers contracts for sale in period 0. Each depositor chooses how many period 0 contracts to purchase. If the banker sells I contracts, then the banker undertakes investment and chooses an effort level e_0. If I contracts are not sold, any purchased contracts are rebated to depositors (this last assumption rules out the possibility of no-lending equilibria when outcomes where the depositors lend to the bank are indeed feasible). In period 1, the banker's loan return, y_1 is realized and the short term returns, $R_1(y_1)/I$, are paid to the depositors. Each depositor then realizes their private discount factor v_i. Next, the banker offers new contracts with gross rate of return $R_2(y_1, z_2)/I$. Again, each depositor chooses how many contracts to purchase and, if the banker sells I contracts, then investment is undertaken and the banker chooses an effort level e_1. In period 2, the banker's loan yields return $I + \rho y_1 + z_2$ and returns are paid to depositors.

We focus on equilibria in which the banker chooses the terms of the contract to maximize ex ante depositor value. This focus is motivated by the idea that there are multiple bankers in time 0 competing for depositors. In period 1, however, the banker in period 0 is the only banker which can generate returns from period 1 to 2, so we must assume that the banker can commit to a sequence of short-term interest rates. A *competitive equilibrium* is a set of time- and state-contingent returns $(R_1(y_1), R_2(y_1, y_2))$ and a number of contracts purchased by each depositor such that no alternative contract earns more ex ante value for depositors and depositors' purchases are optimal.

We show that competitive equilibrium with short-term debt contracts necessarily feature liquidations after low first period returns. We go on to show, under a stronger assumption on the technologies, that the competitive equilibrium with short-term debt exactly replicates optimal commitment outcomes.

First, consider the best debt contracts the banker can offer depositors following low returns in period 1 and assume first that $R_1(y_l) = I/N$, The banker chooses rates of return R_h and R_l to maximize the payout to depositors under the expectation of their future valuation subject to a constraint that the banker must want to choose high effort and a resource constraint. That is, the banker solves the following problem

$$\max_{R_l, R_h} \frac{\tilde{v}}{I} \left(\pi_h R_h + (1 - \pi_h) R_l\right)$$

subject to

$$\beta \pi_h \left(z_h - R_h\right) + \beta(1 - \pi_h)\left(z_l - R_l\right) - \bar{q} \geq \beta \pi_l \left(z_h - R_h\right) + \beta(1 - \pi_l)\left(z_l - R_l\right)$$

and

$$R_i \leq I + z_i.$$

Given $z_l = 0$, the incentive constraint can be simplified to

$$z_h - R_h \geq \frac{\bar{q}}{\beta\left(\pi_h - \pi_l\right)} - R_l.$$

Clearly, then, the banker chooses $R_l = I$ and $R_h = I + z_h - \bar{q}/(\beta(\pi_h - \pi_l))$.

We now study the depositor's decision of how many such contracts to purchase and show that under Assumption 1, the banker will not raise sufficient resources to continue. Let d denote the number of debt contracts an individual depositor purchases. The problem of a depositor can then be written as

$$\max_{c_1, c_{2h}, c_{2l}} c_1 + v_i \left(\pi_h c_{2h} + (1 - \pi_h) c_{2l}\right)$$

subject to $c_1 + d \leq I/N$ and $c_{2j} = dR_j/I$.

$$\begin{aligned} c_1 + d &\leq I/N \\ c_{2j} &= \frac{d}{I} R_j. \end{aligned}$$

Substituting the constraints, it is immediate that the depositor will choose $d = I/N$ and purchase I/N contracts if and only if

$$v_i\left[I+\pi_h\left(z_h-\frac{\bar{q}}{\beta(\pi_h-\pi_l)}\right)\right]>I. \tag{A11}$$

Since continuation requires all depositors to roll their debt over, for a large number of depositors, the probability that at least one depositor has a valuation which violates (A11) is high. Indeed, as $N\to\infty$, the probability that a depositor has a low enough discount factor such that the depositor would not purchase this contract tends to 1 which implies such contracts feature liquidation with probability 1. To simplify the problem, we now make one additional assumption on technologies.

ASSUMPTION 4. Loan returns in period 1 satisfy $y_h=\bar{q}(1-\pi_l)/(\pi_h-\pi_l)$.

This assumption implies that there are no excess returns in period 1 when the bank yields high returns after compensating the banker. In other words, in the optimal contract with commitment, the payout to depositors in period 1 is zero for each depositor. The primary advantage of this formulation of the problem is to simplify the incentive constraints of the depositors following high returns in period 1. In this case, since the continuation contract has zero payments in period 1 to depositors, the incentive compatibility of the depositors requires payments in period 2 to be independent of their reported discount factor. That is, the optimal contract has the feature that $p^i_{1c}(y_h,v)=0$ and $p^i_2(y_h,v)$ is constant. Under limited commitment, this implies the optimal contract has $p^i_1(y_1)=I/N$ for $y_1=y_l$ and $y_1=y_h$. Under constant transfers to depositors following high returns, it is immediate that the decentralized economy with short-term debt yields identical outcomes to the optimal contract under commitment. The equilibrium returns in this economy satisfy $R_1(y_h)=R_1(y_l)=I/N$ and $R_2(y_h,z_l)=I+\rho y_h$, $R_2(y_h,z_h)=I+\rho y_h+z_h-\bar{q}/(\beta(\pi_h-\pi_l))$. Given this structure of returns, it is clear the optimum can be implemented with non-contingent debt contracts as long as depositors may claim an equal share of the returns if low returns are realized in period 2. We have the proved the following proposition.

A8. Formal Analysis of the Multiple Bank Economies

Here we formulate the problem of designing optimal contracts for the multiple bank economies studied in Section 3. We begin by describing the problem under full commitment. We prove that optimal contracts the replica economy feature the same liquidation rule as in the benchmark analysis. We then prove an equivalence between the replica economy and the correlated return economy with greater return risk when there is full commitment to contracts. Next, we describe our notion of limited commitment in this multiple bank model. We prove that the enforcement constraints are tightest when transfers made prior to when the depostiors realize their future discount factors are largest. We show that in an economy with independent returns across banks, even when risk is increased, optimal commitment contracts are not enforceable. We finally show, using a straightforward adaptation of the arguments above that optimal commitment contracts are enforceable in the correlated return economy. This proves that welfare is strictly higher in the correlated return economy when there is limited commitment to contracts.

A1. Optimal Contracts under Full Commitment in Multiple Bank Economies
Consider first the problem of designing optimal contracts for either the replica economy or the correlated return economy under full commitment. We introduce notation to account for the richer history structure. In particular, let $h_1 = \left(y_1^A, y_1^B\right)$ and $h_2 = \left(z_2^A, z_2^B\right)$ with the convention that $z_2^j = \emptyset$ if bank j is not continued. Similarly, let the effort choice of bank j be denoted $e_1^j = \emptyset$ if bank j is not continued. Then, in period 2, the probability of outcome h_2 is given by $\pi(h_2|e_1^A, e_2^A)$.In both the replica and correlated economy, we assume $\pi(z_h, \emptyset|e_1^j, \emptyset) = \pi_h$ if $e_1^j = \pi_h$ so that if only one bank is continued, the loan return process is identical to the benchmark economy. Moreover, we index the continuation rule for each bank as $x^j(h_1, v)$. As in the paper, we begin by characterizing constraints on the optimal contract.

The period 1 resource constraint, for any history h_1 satisfies

$$\sum_i p_1^i(v; h_1) \leq 2I + y_1^A + y_1^B - p_1^A(h_1) - p_1^B(h_1) - I\left[x^A(v; h_1) + x^B(v; h_1)\right].$$

The resource constraint in period 2 for payments to the depositors, taken in expecation over realized period two returns is given by

$$\begin{aligned}
\sum_i p_2^i(v; h_1) \quad \leq \quad & I\left[x^A(v; h_1) + x^B(v; h_1)\right] + x^A(v; h_1)\rho y_1^A + x^B(v; h_1)\rho y_1^B \\
& + x^A(v; h_1)x^B(v; h_1)\sum_{h_2}\pi\left(h_2|e_1^A, e_2^B\right)\left(z_2^A + z_2^B - p_2^{bA}(h_2; h_1) - p_2^{bB}(h_2; h_1)\right) \\
& + x^A(v; h_1)(1 - x^B(v; h_1))\sum_{h_2}\pi\left(h_2|e_1^A, \emptyset\right)\left(z_2^A - p_2^{bA}(h_2; h_1)\right) \\
& + (1 - x^A(v; h_1))x^B(v; h_1)\sum_{h_2}\pi\left(h_2|\emptyset, e_1^B\right)\left(z_2^B - p_2^{bB}(h_2; h_1)\right).
\end{aligned}$$

Contracts are incentive compatible for bankers if, conditional on continuing bank j, the period 1 effort constraint is satisfied

$$\beta\sum_{h_2}\pi(h_2|e_h, e_1^{-j})p_2^{bj}(h_2; v, h_1) - \bar{q} \geq \beta\sum_{h_2}\pi(h_2|e_l, e_1^{-j})p_2^{bj}(h_2; v, h_1),$$

and, given that effort will be induced from period 1 onwards, the following period 0 effort constraint must also be satisfied:

$$\begin{aligned}
& \sum_{h_1}\pi(h_1|e_h, e_0^{-j})\left[p_1^{bj}(h_1) + \int_v x^j(v; h_1)\left[\beta\sum_{h_2}\pi(h_2|e_h, e_1^{-j}(v; h_1))p_2^{bj}(h_2; v, h_1) - \bar{q}\right]dG(v)\right] - \bar{q} \\
\geq \; & \sum_{h_1}\pi(h_1|e_l, e_0^{-j})\left[p_1^{bj}(h_1) + \int_v x^j(v; h_1)\left[\beta\sum_{h_2}\pi(h_2|e_h, e_1^{-j}(v; h_1))p_2^{bj}(h_2; v, h_1) - \bar{q}\right]dG(v)\right].
\end{aligned}$$

Incentive compatibility for each depositor after any history can then be expressed simply as

$$\int_{v_{-i}} \left[p_1^i(v_i, v_{-i}; h_1) + v_i p_2^i(v_i, v_{-i}; h_1)\right] dG_{-i}(v_{-i}) \geq \max_{\hat{v}_i} \int_{v_{-i}} \left[p_1^i(\hat{v}_i, v_{-i}; h_1) + v_i p_2^i(\hat{v}_i, v_{-i}; h_1)\right] dG_{-i}(v_{-i}).$$

The ex ante participation constraints of the depositors are given by

$$\sum_{h_1} \pi(h_1 | e_0^A, e_0^B) \int_v \left[p_1^i(v; h_1) + v_i p_2^i(v; h_1)\right] dG(v) \geq \frac{I}{N}.$$

An optimal contract under commitment is an incentive feasible contract which maximizes aggregate depositor welfare given by

$$\sum_{h_1} \pi(h_1 | e_0^A, e_0^B) \sum_{i=1}^{2N} \int_v \left[p_1^i(v; h_1) + v_i p_2^i(v; h_1)\right] dG(v).$$

Optimal Contracts in the Replica Economy

Consider the case of the replica economy with independent bank loan returns. Here we prove that the optimal continuation rule has the feature that $x^j(v; h_1) = 1$ whenever h_1 is such that $y_1^j = y_h$ and $x^j(v; h_1) = 0$ whenever h_1 is such that $y_1^j = y_l$. In other words, the continuation rule replicates the optimal continuation for a single banker independent of the returns to the other banker.

In this replica economy, the banker's IC constraints become

$$\sum_{z_2} \pi\left(z_2 | e_1^{-j}\right) p_2^{bj}(z_h, z_2; v, h_1) \geq \frac{\bar{q}}{\beta\left(\pi_h - \pi_l\right)} + \sum_{z_2} \pi\left(z_2 | e_1^{-j}\right) p_2^{bj}(z_l, z_2; v, h_1)$$

and

$$\sum_{y_1^{-j}} \pi(y_1^{-j} | e_0^{-j}) \left[p_1^{bj}(y_h^j, y_1^{-j}) + \int_v x^j(v; y_h^j, y_1^{-j}) \left[\beta \sum_{h_2} \pi(h_2 | e_h, e_1^{-j}(v; y_h^j, y_1^{-j})) p_2^{bj}(h_2; v, y_h^j, y_1^{-j}) - \bar{q}\right] dG(v)\right] - \bar{q}$$

$$\geq \sum_{y_1^{-j}} \pi(y_1^{-j} | e_l, e_0^{-j}) \left[p_1^{bj}(y_l^j, y_1^{-j}) + \int_v x^j(v; y_l^j, y_1^{-j}) \left[\beta \sum_{h_2} \pi(h_2 | e_h, e_1^{-j}(v; y_l^j, y_1^{-j})) p_2^{bj}(h_2; v, y_l^j, y_1^{-j}) - \bar{q}\right] dG(v)\right].$$

Let $t_2^{bj}(z_2^j; v, h_1) = \sum_{z_2} \pi\left(z_2 | e_1^{-j}\right) p_2^{bj}(z_2^j, z_2; v, h_1)$, $t_1^{bj}(y_1^j) = \sum_{y_1^{-j}} \pi(y_1^{-j} | e_0^{-j}) p_1^{bj}(y_1^j, y_1^{-j})$,then the constraints simplify to

$$t_2^{bj}(z_h; v, h_1) \geq \frac{\bar{q}}{\beta\left(\pi_h - \pi_l\right)} + t_2^{bj}(z_l; v, h_1)$$

$$t_1^{bj}(y_h) + \int_v \sum_{y_1^{-j}} \pi\left(y_1^{-j} | e_0^{-j}\right) x^j(v; y_h^j, y_1^{-j}) \left[\beta E_{\pi_h} t_2^{bj}(z_2; v, y_h^{-j}, y_1^{-j}) - \bar{q}\right] dG(v)$$

$$\geq \frac{\bar{q}}{\pi_h - \pi_l} + t_1^{bj}(y_l) + \int_v \sum_{y_1^{-j}} \pi\left(y_1^{-j} | e_0^{-j}\right) x^j(y_l^j; y_1^{-j}, v) \left[\beta E_{\pi_h} t_2^{bj}(z_2; v, y_l^{-j}, y_1^{-j}) - \bar{q}\right] dG(v)$$

Now consider the resources constraints. The period 2 resource constraints can be simplifed as

$$\begin{aligned}\sum_i p_2^i(v;h_1) \quad &\leq \quad x^a(v;h_1)\left[I+\rho y_1^A + E_{\pi_h} z_2^A - E_{\pi_h} t_2^{bA}(z_2;v,h_1)\right] \\ &\qquad +x^b(v;h_1)\left[I+\rho y_1^B + E_{\pi_h} z_2^B - E_{\pi_h} t_2^{bB}(z_2;v,h_1)\right].\end{aligned}$$

The period 1 resource constraints simplify to

$$\sum_i p_1^i(v;h_1) \leq 2I + y_1^A + y_1^B - p_1^{bA}(h_1) - p_1^{bB}(h_1) - I\left[x^A(v;h_1) + x^B(v;h_1)\right].$$

The value of a contract is then given by

$$\sum_{h_1} \pi(h_1;e_0^A,e_0^B)\left[\sum_i p_1^i(v;h_1) + v_i p_2^i(v;h_1)\right]$$

As in the single bank model, it is immediate that if bank j obtains low returns in period 1 and low returns in period 2, then the banker receives zero compensation in period 2 if the bank is continued. Moreover, following low returns in period 1 and high returns in period 2, the banker receives the minimum payment necessary to maintain the incentive constraint. In other words, we have

$$t_2^{bj}(z_l;v,y_1^j = y_l, y_1^{-j}) = 0$$

and

$$t_2^{bj}(z_h;v,y_1^j = y_l, y_1^{-j}) = \frac{\bar{q}}{\beta(\pi_h - \pi_l)}.$$

Also, clearly $p_1^{bj}(y_1^j = y_l, y_l^{-j}) = 0$ and, as above, we may restrict attention to contracts in which banker j receives a payment in period 2 only if the banker obtains high returns. That is, without loss we may set $t_2^{bj}(z_l;y_1^j = y_h, y_1^{-j}) = 0$.

Much as in the single bank model, holding the bankers' payments fixed, we may rewrite the incentive constraints of the depositors and eliminate period 1 transfers. Given our formulation, we may define

$$\rho_i(v_i;h_1) = \int_{v_{-i}} p_2^i(v_i,v_{-i};h_1)dG_{-i}(v_{-i}).$$

Then, incentive compatibility for depositors may be written as $u_i(v_i) = u_i(\underline{v}) + \int_{\underline{v}}^{v_i} \rho_i(z)dz$ along with the requirement that $\rho_i(v_i)$ is increasing in v_i for all i. Using this formulation, the optimal contract can be obtained by solving the following problem

$$\max \sum_{h_1} \pi(h_1) \sum_i \left[u_i(\underline{v};h_1) + \int_{v_i} \frac{1-G_i(v_i)}{g_i(v_i)} \rho_i(v_i;h_1)dG_i(v_i)\right]$$

subject to

$$2I + y_1^A + y_1^B - p_1^A(h_1) - p_1^B(h_1) - I \int_v \left[x^A(v;h_1) + x^B(v;h_1) \right] dG(v)$$
$$+ \sum_i \int_{v_i} \left[v_i - \frac{1-G_i(v_i)}{g_i(v_i)} \right] \rho_i(v_i;h_1) dG_i(v_i) \geq \sum_i u_i(\underline{v};h_1) \tag{A12}$$

$$\int_v x^A(v;h_1) \left[I + \rho y_1^A + \pi_h z_h - E_{\pi_h} t_2^{bA}(z_2;v,h_1) \right] dG(v)$$
$$+ \int_v x^B(v;h_1) \left[I + \rho y_1^B + \pi_h z_h - E_{\pi_h} t_2^{bB}(z_2;v,h_1) \right] dG(v) \geq \sum_i \int_{v_i} \rho_i(v_i;h_1) dG_i(v_i) \tag{A13}$$

$$\pi_h p_1^{bj}(y_h,y_h) + \pi_h \int_v x^j(v;y_h,y_h) \left[\beta \pi_h t_2^{bj}(z_h;v,y_h,y_h) - \bar{q} \right] dG(v)$$
$$+ (1-\pi_h) p_1^{bj}(y_h,y_l) + (1-\pi_h) \int_v x^j(v;y_h,y_l) \left[\beta \pi_h t_2^{bj}(z_h;v,y_h,y_l) - \bar{q} \right] dG(v)$$
$$\geq \frac{\bar{q}}{(\pi_h - \pi_l)} + \pi_h \frac{\pi_l \bar{q}}{\pi_h - \pi_l} \int_v x^j(v;y_l,y_h) dG(v) + (1-\pi_h) \frac{\pi_l \bar{q}}{\pi_h - \pi_l} \int_v x^j(v;y_l,y_l) dG(v)$$
$$t_2^{bj}(z_h;v,h_1) \geq \frac{\bar{q}}{\beta(\pi_h - \pi_l)}.$$

Exactly as in the proof of Proposition 1, the assumption that $\beta < \underline{v}$ ensures $t_2^{bj}(z_h;v,h_1) = \bar{q}/(\beta(\pi_h - \pi_l))$ for all j, v, h_1. Moreover, the fact that $\bar{v}(I + \rho y_h + \pi_h z_h - \pi_h \bar{q}/(\beta(\pi_h - \pi_l))) > I$ also ensures that $x^j(v;h_1) = 1$ whenever h_1 is such that $y_1^j = y_h$ as in the proof of Proposition 1. Hence, it only remains to prove that $x^j(v;h_1) = 0$ if h_1 is such that $y_1^j = y_l$.

To see this, first consider the history $h_1 = (y_l, y_l)$. Suppose by way of contradiction that a contract has $\int_v x^A(v;h_1)dG(v) > 0$ or $\int_v x^B(v;h_1)dG(v) > 0$. In the state h_1, the maximal value attainable by this contract is given by

$$\int_v \left[\sum_{i=1}^{2N} p_1^i(v;y_l,y_l) + v_i p_2^i(v;y_l,y_l) \right] dG(v) \leq$$
$$2I + 2y_l + \left[-I + \bar{v} \left(I + \rho y_l + \pi_h z_h - \pi_h \frac{\bar{q}}{\beta(\pi_h - \pi_l)} \right) \right] \int_v \left[x^A(v;y_l,y_l) + x^B(v;y_l,y_l) \right] dG(v)$$

where the right hand side is obtained using the fact that $p_1^{bj}(y_l,y_l) = 0$ for $j = A, B$ and we substitute the contract which gives all period 2 returns (net of the bankers' rents) to the most patient depositor.

We prove existene of a contract which satisfies all of the constraints and raises ex ante welfare. This contract has the feature that $x^j(v;y_l,y_l) = 0$. The fact implies a potential loss in the history $h_1 = (y_l, y_l)$ int he amount

$$\left[-I + \bar{v} \left(I + \rho y_l + \pi_h z_h - \pi_h \frac{\bar{q}}{\beta(\pi_h - \pi_l)} \right) \right] \int_v \left[x^A(v;y_l,y_l) + x^B(v;y_l,y_l) \right] dG(v).$$

We then perturb the payments to banker j following the history $\left(y_1^j = y_l, y_1^{-j} = y_h\right)$. In particular, we choose the period 1 transfers to the bankers

$$\begin{aligned}\hat{p}_1^{bA}(y_h, y_l) &= p_1^{bA}(y_h, y_l) - \frac{\pi_l \bar{q}}{\pi_h - \pi_l} \int_v x^A(y_l, y_l, v) dG(v) \\ \hat{p}_1^{bB}(y_l, y_h) &= p_1^{bB}(y_l, y_h) - \frac{\pi_l \bar{q}}{\pi_h - \pi_l} \int_v x^B(y_l, y_l, v) dG(v).\end{aligned}$$

Consider banker j's period 0 incentive constraint. This constraint under the pertubed allocation must satisfy

$$\begin{aligned}&\pi_h \hat{p}_1^{bj}(y_h, y_h) + \pi_h \frac{\pi_l \bar{q}}{\pi_h - \pi_l} \int_v x^j(v; y_h, y_h) dG(v) \\ &+ (1-\pi_h)\hat{p}_1^{bj}(y_h, y_l) + (1-\pi_h)\frac{\pi_l \bar{q}}{\pi_h - \pi_l} \int_v x^j(v; y_h, y_l) dG(v) \geq \frac{\bar{q}}{(\pi_h - \pi_l)} + \pi_h \frac{\pi_l \bar{q}}{\pi_h - \pi_l} \int_v x^j(v; y_l, y_h) dG(v)\end{aligned}$$

which, substiting $\hat{p}_1^{bj}(y_h, y_l)$ is satisfied by construction. This perturbation also relaxes the period 1 implementability constraint (A12) by the amount $\frac{\pi_l \bar{q}}{\pi_h - \pi_l} \int_v x^A(y_l, y_l, v) dG(v)$ in the history (y_h, y_l) and by the amount $\frac{\pi_l \bar{q}}{\pi_h - \pi_l} \int_v x^B(y_l, y_l, v) dG(v)$ in the history (y_l, y_h). Relaxing these allows larger lowest utility levels to be allocated. That is, in these histories, we may increase $\sum_i u_i\left(\underline{v}; h_1\right)$ by these stated amounts.

Overall, then, this perturbation leads to a chance in expected welfare given by

$$\begin{aligned}&\pi_h(1-\pi_h)\frac{\pi_l \bar{q}}{\pi_h - \pi_l} \int_v x^A(y_l, y_l, v) dG(v) + (1-\pi_h)\pi_h \frac{\pi_l \bar{q}}{\pi_h - \pi_l} \int_v x^B(y_l, y_l, v) dG(v) \\ &-(1-\pi_h)^2 \left[-I + \bar{v}\left(I + \rho y_l + \pi_h z_h - \pi_h \frac{\bar{q}}{\beta(\pi_h - \pi_l)}\right)\right] \int_v \left[x^A(v; y_l, y_l) + x^B(v; y_l, y_l)\right] dG(v).\end{aligned}$$

Under the assumptions of the single bank model, namely that

$$\pi_h \frac{\pi_l \bar{q}}{\pi_h - \pi_l} - (1-\pi_h)\left[-I + \bar{v}\left(I + \rho y_l + \pi_h z_h - \pi_h \frac{\bar{q}}{\beta(\pi_h - \pi_l)}\right)\right] > 0$$

we have $x^j(v; y_l, y_l) = 0$ for all j, v.

Next, suppose $\int_v x^j(v; y_h, y_l) dG(v) > 0$. We again construct a perturbation which strictly raises welfare. Following this history, consider the problem of designing optimal transfers to maximize depositor utility. This problem is given by

$$\max \sum_i \left[u_i(\underline{v}; y_h, y_l) + \int_{v_i} \frac{1 - G_i(v_i)}{g_i(v_i)} \rho_i(v_i; y_h, y_l) dG_i(v_i)\right]$$

subject to

$$\sum_i u_i(\underline{v}; y_h, y_l) \leq$$

$$I + y_h + y_l - p_1^A(y_h, y_l) - I \int_v x^B(v; y_h, y_l) dG(v) + \sum_i \int_{v_i} \left[v_i - \frac{1 - G_i(v_i)}{g_i(v_i)} \right] \rho_i(v_i; y_h, y_l) dG_i(v_i)$$

$$\sum_i \int_{v_i} \rho_i(v_i; y_h, y_l) dG_i(v_i) \leq$$

$$\left[I + \rho y_h + \pi_h z_h - \frac{\pi_h \bar{q}}{\beta(\pi_h - \pi_l)} \right] + \left[I + \rho y_l + \pi_h z_h - \frac{\pi_h \bar{q}}{\beta(\pi_h - \pi_l)} \right] \int_v x^B(v; y_h, y_l) dG(v)$$

where $\rho_i(v_i)$ must be non-decreasing. Here, we are focusing on the optimal choice of transfers, and we study the consequences in terms of depositors' utility of lowering the probability of continuing bank B. Of course, changes in $x^B(v; y_h, y_l)$ will change the period 0 incentive constraint of banker j and allow for different payments in different histories.

Take an arbitrary contract with $\int_v x^B(v; y_h, y_l) dG(v) > 0$. We seek to place an upper bound the losses associated with an alternative contract in which $\int_v x^B(v; y_h, y_l) dG(v) = 0$. To do so, we will construct an alternative, incentive-feasible KOtransfer rule and compare the utility associated with this new contract. Let

$$Y_{2k} = I + \rho y_k + \pi_h z_h - \frac{\pi_h \bar{q}}{\beta(\pi_h - \pi_l)}$$

so that Y_{2h} indicates the expected gross return from continuing bank A following high returns and Y_{2l} denotes the expected gross return from continuing bank B following low returns in period 1.Let $\hat{x}^B(v; y_h, y_l) = 0$, and let

$$\hat{\rho}_i(v_i; y_h, y_l) = \frac{Y_{2h}}{Y_{2h} + Y_{2l} \int_v x^B(v; y_h, y_l) dG(v)} \rho_i(v_i; y_h, y_l).$$

By construction, $\hat{\rho}_i$ is non-decreasing and satisfies the period 2 resource constraint (when $\int_v \hat{x}^B(v; y_h, y_l) dG(v) = 0$). Next, we will choose

$$\hat{u}_i(\underline{v}; y_h, y_l) = u_i(\underline{v}; y_h, y_l) - \frac{1}{2N} (\bar{v} Y_{2l} - I) \int_v x^B(v; y_h, y_l) dG(v).$$

In principle, this may not be feasible if it causes a net decrease in depositor utility since it might violate the participation constraint. However, since we will argue this perturbation, when joined with changes in the contract for other histories raises depositor welfare, we can ignore this effect. First, note that this perturbed allocation satisfies the period 1 implementability constraint in this history since

$$
\begin{aligned}
& I + y_h + y_l - p_1^A(y_h, y_l) - I\int_v \hat{x}^B(v; y_h, y_l)dG(v) + \sum_i \int_{v_i}\left[v_i - \frac{1 - G_i(v_i)}{g_i(v_i)}\right]\hat{\rho}_i(v_i; y_h, y_l)dG_i(v_i) \\
&= I + y_h + y_l - p_1^A(y_h, y_l) + \frac{Y_{2h}}{Y_{2h} + Y_{2l}\int_v x^B(v; y_h, y_l)dG(v)}\sum_i \int_{v_i}\left[v_i - \frac{1 - G_i(v_i)}{g_i(v_i)}\right]\rho_i(v_i; y_h, y_l)dG_i(v_i) \\
&= I + y_h + y_l - p_1^A(y_h, y_l) - I\int_v x^B(v; y_h, y_l)dG(v) + \sum_i \int_{v_i}\left[v_i - \frac{1 - G_i(v_i)}{g_i(v_i)}\right]\rho_i(v_i; y_h, y_l)dG_i(v_i) \\
&+ I\int_v x^B(v; y_h, y_l)dG(v) - \frac{Y_{2l}\int_v x^B(v; y_h, y_l)dG(v)}{Y_{2h} + Y_{2l}\int_v x^B(v; y_h, y_l)dG(v)}\sum_i \int_{v_i}\left[v_i - \frac{1 - G_i(v_i)}{g_i(v_i)}\right]\rho_i(v_i; y_h, y_l)dG_i(v_i) \\
&\geq \sum_i u_i(\underline{v}; y_h, y_l) + (I - \bar{v}Y_{2l})\int_v x^B(v; y_h, y_l)dG(v) \\
&= \sum_i \hat{u}_i(\underline{v}; y_h, y_l)
\end{aligned}
$$

In terms of depositors' utility, the costs associated with this perturbation are given by

$$
\begin{aligned}
& \sum_i\left[u_i\left(\underline{v}; y_h, y_l\right) + \int_{v_i}\frac{1 - G_i(v_i)}{g_i(v_i)}\rho_i(v_i)dG_i(v_i)\right] - \sum_i\left[\hat{u}_i\left(\underline{v}; y_h, y_l\right) + \int_{v_i}\frac{1 - G_i(v_i)}{g_i(v_i)}\hat{\rho}_i(v_i)dG_i(v_i)\right] \\
=\ & (\bar{v}Y_{2l} - I)\int_v x^B(v; y_h, y_l)dG(v) + \frac{Y_{2l}\int_v x^B(v; y_h, y_l)dG(v)}{Y_{2h} + Y_{2l}\int_v x^B(v; y_h, y_l)dG(v)}\sum_i \int_{v_i}\frac{1 - G_i(v_i)}{g_i(v_i)}\rho_i(v_i; y_h, y_l)dG_i(v_i) \\
\leq\ & \left[\left(\bar{v} + \frac{1}{g_i\left(\underline{v}\right)}\right)Y_{2l} - I\right]\int_v x^B(v; y_h, y_l)dG(v)
\end{aligned}
$$

where the inequality follows from the fact that $(1-G_i(v_i))/g_i(v_i)$ is assumed to be decreasing and from substituting the period 2 resource constraint associated with the original contract, $\rho_i(v_i)$.

As in the case when both banks earn low returns, by choosing the contract to liquidate bank B following the history (y_h, y_l), it is feasible to reduce $p_1^{bB}(y_h, y_h)$ by $\pi_l\bar{q}/\left(\pi_h - \pi_l\right)\int_v x^B(v; y_h, y_l)dG(v)$ and increase the resulting payout to depositors. In expecation, the benefit of this perturbation is bounded below by

$$
\pi_h\pi_h\pi_l\bar{q}/\left(\pi_h - \pi_l\right)\int_v x^B(v; y_h, y_l)dG(v) - \pi_h(1-\pi_h)\left[\left(\bar{v} + \frac{1}{g_i\left(\underline{v}\right)}\right)Y_{2l} - I\right]\int_v x^B(v; y_h, y_l)dG(v).
$$

Hence, as long as

$$
\pi_h\frac{\pi_l\bar{q}}{\pi_h - \pi_l} - (1 - \pi_h)\left[\left(\bar{v} + \frac{1}{g_i\left(\underline{v}\right)}\right)Y_{2l} - I\right] > 0
$$

the optimal contract features $\int_v x^B(v; y_h, y_l)dG(v) = 0$.The proof that $\int_v x^A(v; y_l, y_h)dG(v) = 0$ is identical. This completes our analysis of optimal continuation rules in the replica economy under full commitment.

Optimal Contracts in the Correlated Returns Economy

Consider next the incentive constraints of the bankers when returns are perfectly correlated. Here, we analyze an equilibrium in which both bankers expect the other banker to exert high effort. The incentive constraints require each banker to choose to exert high effort under this expectation. In this economy, the banker's incentive constraints are given by

$$p_2^{bj}(z_h, z_h; v, h_1) \geq \frac{\bar{q}}{\beta\left(\pi_h - \pi_l\right)} + p_2^{bj}(z_l, z_l; v, h_1)$$

and

$$\begin{aligned} & p_1^{bj}(y_h^j, y_h^{-j}) + \int_v x^j(v; y_h^j, y_h^{-j}) \left[\beta E_{\pi_h} p_2^{bj}(h_2; v, y_h^j, y_h^{-j}) - \bar{q}\right] dG(v) - \bar{q} \\ \geq \ & p_1^{bj}(y_l^j, y_l^{-j}) + \int_v x^j(v; y_l^j, y_l^{-j}) \left[\beta E_{\pi_h} p_2^{bj}(h_2; v, y_l^j, y_1^{-j}) - \bar{q}\right] dG(v). \end{aligned}$$

Since with correlated loan returns, we assume that $\hat{y}_l < y_l$ and $\hat{y}_h > y_h$, clearly the same conditions needed to ensure $x\left(y_h, y_h\right) = 1$ and $x\left(y_l, y_l\right) = 0$ continue to hold under these loan return distributions. Hence, it only remains to prove that we may choose $\hat{y}_h$ so that the optimal contract under commitment in this correlated return economy yields the same expected welfare to depositors as the replica economy. We prove this result by continuity. Specifically, we prove that the value of the optimal contract under full commitment, which we denote W^{IID} lies in some interval $\left[\underline{W}^{IID}, \bar{W}^{IID}\right]$. We then prove that for any $\hat{y}_h$, the value of the correlated economy, denoted $W^C(\hat{y}_h)$, lies in some other interval $\left[\underline{W}^C(\hat{y}_h), \bar{W}^C(\hat{y}_h)\right]$. We then prove that there are thresholds $\underline{y}$ and $\bar{y}$ such that for $\hat{y}_h \leq \underline{y}$ we have $\bar{W}^C(\hat{y}_h) \leq \underline{W}^{IID}$ and for $\hat{y}_h \geq \bar{y}$ we have $\underline{W}^C(\hat{y}_h) \geq \bar{W}^{IID}$. Hence, by continuity, there is some $\hat{y}_h \in \left[\underline{y}, \bar{y}\right]$ such that $W^C(\hat{y}_h) = W^{IID}$.

First, consider the value of the optimal contract with independent returns. We construct a lower bound with a feasible contract. This contract sets payments to banker j which is independent of the returns to bank $-j$ and has constant transfers to the depositors. Specifically, $p_1^{bj}(y_h, y_1^{-j}) = (1-\pi_l)\bar{q}/(\pi_h - \pi_l)$. By construction, these payments satisfy the period 0 effort constraint of the banker. Then, transfers in every history are simply an equal share of the resource constraints. This contract is trivially incentive compatible and has value

$$\begin{aligned} \underline{W}^{IID} =& 2\pi_h^2 \left[y_h - \frac{(1-\pi_l)\bar{q}}{\pi_h - \pi_l} + \left(I + \rho y_h + E_{\pi_h} z_2 - \frac{\pi_h \bar{q}}{\beta\left(\pi_h - \pi_l\right)}\right)\tilde{v}\right] \\ & + 2\pi_h(1-\pi_h)\left[I + y_h - \frac{(1-\pi_l)\bar{q}}{\pi_h - \pi_l} + \left(I + \rho y_h + E_{\pi_h} z_2 - \frac{\pi_h \bar{q}}{\beta\left(\pi_h - \pi_l\right)}\right)\tilde{v}\right] + 2(1-\pi_h)^2 I \\ =& 2\pi_h \left[y_h - \frac{(1-\pi_l)\bar{q}}{\pi_h - \pi_l} + \left(I + \rho y_h + E_{\pi_h} z_2 - \frac{\pi_h \bar{q}}{\beta\left(\pi_h - \pi_l\right)}\right)\tilde{v}\right] + 2(1-\pi_h) I. \end{aligned} \tag{A14}$$

Obviously, since $\left(I + \rho y_h + E_{\pi_h} z_2 - \frac{\pi_h \bar{q}}{\beta(\pi_h - \pi_l)}\right)\tilde{v} \geq I$, this contract also satisfies the participation constraints of the depositors. We construct a (non-binding) upper bound in which the payments to the bankers are all zero and in which all period 2 returns are allocated to the most patient depositor. While this contract is not incentive compatible, it clearly yields a larger value than any incentive-feasible contract. The value of this contract is given by

$$\bar{W}^{IID} = 2\pi_h \left[y_h + \left(I + \rho y_h + E_{\pi_h} z_2 - \frac{\pi_h \bar{q}}{\beta\left(\pi_h - \pi_l\right)} \right) \bar{v} \right] + 2(1-\pi_h)I. \tag{A15}$$

Next, for any $\hat{y}_h$ we construct similar bounds for the value of the optimal contract under commitment when returns are perfectly correlated. The lower bound assumes payments to the bankers in period 1 equal to $(1-\pi_l)\bar{q}/(\pi_h - \pi_l)$ when both banks earn high returns. The lower bound contract also assumes constant transfers. The value of this contract is given by

$$\underline{W}^C(\hat{y}_h) = 2\pi_h \left[\hat{y}_h - \frac{(1-\pi_l)\bar{q}}{\pi_h - \pi_l} + \left(I + \rho \hat{y}_h + E_{\pi_h} z_2 - \frac{\pi_h \bar{q}}{\beta\left(\pi_h - \pi_l\right)} \right) \tilde{v} \right] + (1-\pi_h)I. \tag{A16}$$

In obtaining this result, we have used the fact that $\hat{y}_l = -I/2$. Then, we obtain an upper bound by assuming all period 2 returns are allocated to the most patient depositor. This contract has value

$$\bar{W}^C(\hat{y}_h) = 2\pi_h \left[\hat{y}_h - \frac{(1-\pi_l)\bar{q}}{\pi_h - \pi_l} + \left(I + \rho \hat{y}_h + E_{\pi_h} z_2 - \frac{\pi_h \bar{q}}{\beta\left(\pi_h - \pi_l\right)} \right) \bar{v} \right] + (1-\pi_h)I. \tag{A17}$$

Next, we construct the values, $\underline{y}$ and $\bar{y}$. It is straightforward using the formulas (A14)-(A17) to show that for $\hat{y}_h \leq \underline{y}$ where

$$\underline{y} = \frac{1}{1+\rho\bar{v}} \left[y_h \left(1+\rho\tilde{v}\right) - \left(I + +E_{\pi_h} z_2 - \frac{\pi_h \bar{q}}{\beta(\pi_h - \pi_l)} \right) (\bar{v} - Ev) \right] < y_h$$

we have $\bar{W}^C(\hat{y}_h) \leq \underline{W}^{IID}$ and for $\hat{y}_h \geq \bar{y}$ where

$$\bar{y} = \frac{1}{1+\rho Ev} \left[y_h \left(1+\rho\bar{v}\right) + \frac{(1-\pi_h)I}{2\pi_h} + \frac{1-\pi_l}{\pi_h - \pi_l}\bar{q} + \left(I + E_{\pi_h} z_2 - \frac{\pi_h \bar{q}}{\beta\left(\pi_h - \pi_l\right)} \right) (\bar{v} - Ev) \right] > y_h$$

we have $\underline{W}^C(\hat{y}_h) \geq \bar{W}^{IID}$. Hence, by continuity, there is some $\hat{y}_h \in \left[\underline{y}, \bar{y}\right]$ such that $W^C(\hat{y}_h) = W^{IID}$.

A2. Optimal Contracts in Multibank Economies with Limited Commitment

We now introduce our complete notion of limited commitment in the multiple bank economy. We think of an economy in which the first N depositors invest solely in bank A and the second N depositors invest solely in bank B. As in Section 2B, we introduce early payments. We assume these payments are symmetric across initial depositors in a single bank but may vary across the depositors in the two banks in the sense that they may depend on the returns realized only in an indivdiual depositor's bank. We denote these early payments $p_1^i(y_1)$ with the restriction that for $i \leq N$, $p_1^i(y_1^A)$ is a function only of the returns to bank A and for $i \geq N+1$, we have $p_1^i(y_1^B)$ so that early payments to the period 0 depositors of bank B only depend on bank $B's$ returns. Once discount factors are realized, we allow all $2N$ depositors to attempt to design new contract which improe ex ante welfare of all of the depositors. Similarly, the continuation component of the optimal contract under limited commitment may effectively pool resources across all of the $2N$ depositors. A consequene of these assumptions is that under commitment, such restrictions on early payments would be irrelevant.

Exactly as in the single bank model, we say a contract is enforceable if there is no alternative incentive-feasible continuation contract which raises the expected welfare of the $2N$ depositors. For any history of returns to the two banks, h_1, let $w^i(h_1, v)$ denote the continuation utility to depositor i in history h_1 if the vector of realized discount factors (across all $2N$ depositors) is v. Incentive-feasible re-negotiation contracts (indexed with $\hat{}$'s), given a history h_1 and early payments $p_1^i(y_1)$ must sastisfy the following constraints

$$\sum_{i=1}^{2N} \hat{p}_1^i(h_1, v) \leq 2I + y_1^A + y_1^B - p_1^A(h_1) - p_1^B(h_1) - \sum_{i=1}^{2N} p_1^i(h_1) - \int_v \left[\hat{x}^A(v) + \hat{x}^B(v)\right] dG(v)$$

$$\sum_{i=1}^{2N} \hat{p}_2^i(h_1, v) \leq \left[I + \rho y_1^A + E_{\pi_h}\left(z_2 - \hat{p}_2^{bA}(h_2; v, h_1)\right)\right] \int_v \hat{x}^A(v) dG(v)$$

$$+ \left[I + \rho y_1^B + E_{\pi_h}\left(z_2 - \hat{p}_2^{bB}(h_2; v, h_1)\right)\right] \int_v \hat{x}^B(v) dG(v)$$

$$\int_{v_{-i}} \left[\hat{p}_1^i(h_1, v_i, v_{-i}) + v_i \hat{p}_2^i(h_1, v_i, v_{-i})\right] dG_{-i}(v_{-i}) \geq \int_{v_{-i}} \left[\hat{p}_1^i(h_1, \hat{v}_i, v_{-i}) + v_i \hat{p}_2^i(h_1, \hat{v}_i, v_{-i})\right] dG_{-i}(v_{-i})$$

$$\hat{p}_1^i(h_1, v) + p_1^i(h_1) \geq 0$$

along with the effort constraint of the bankers.

In the paper, we consider whether the optimal contract is enforceable when early payments are equal to the entirety of each bank's returns. We showed that with independent returns, under some conditions, this is not the case, but with correlated returns, the optimal commitment contract is enforceable. Here, we simply demonstrate that these results suffice for the indepedent return case. That is we prove that for any other choice of early payments, the optimal commitment contract remains not enforceable. In the paper, we proved that when $p_1^i(y_1) = \bar{p}_1^i(y_1) = (I + y_1 - p_1^{bj}(h_1))/N$ (with $j = A$ for $i = 1, \ldots, N$ and $j = B$ for $i = N+1, \ldots, 2N$) there exist re-negotiation contracts which strictly improve ex ante continuation welfare of the depositors. Let the period 1 continuation payments in the associated dominating re-negotiation contract be denoted by $\tilde{p}_1^i(h_1, v)$. Suppose an arbitrary status quo contract has $p_1^i(y_1) = \bar{p}_1^i(y_1) - \varepsilon^i$ for $\varepsilon^i > 0$. Then let $\hat{p}_1^i(h_1, v) = \tilde{p}_1^i(h_1, v) + \varepsilon_i$. Obviously, this re-negotiation contract is incentive compatible, resource feasible, and satisfies the limited liability constraints as long as the original re-negotiation contract is (which we proved exists in the paper). Moreover, since the original re-negotiation contract dominates the original status quo, the continuation welfare in the new re-negotiation contract must also dominate the status quo. Consequently, for any early payments in the replica economy, the optimal commitment contract is not enforeceable.