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External financing and the role of financial frictions over the business cycle: Measurement and theory

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ABSTRACT

Empirically, there is substantial cross-sectional variation in firms' use of external funds: roughly 80% of investment by privately held firms is financed externally, compared to 20% for publicly traded firms. In a model consistent with privately held and publicly traded firms' use of external funds, financial shocks generate only a modest response of output. This exercise casts doubt on the ability of financial shocks to generate significant economic fluctuations and emphasizes the role of non-financial linkages in understanding the importance of financial shocks.

1. Introduction

Financial markets have long been thought to play an important role in business-cycle fluctuations. The conventional view is that firms use external funds for investment, and shocks to financial markets disrupt the flow of funds among firms and households, thereby interrupting production. In evaluating the quantitative importance of this conventional view, a natural question to ask is to what extent firms actually use external funds for investment.

In this paper, we document empirical patterns of firms' use of external funds in the aggregate and in the cross section. Our analysis emphasizes the use of external funds by privately held firms, which may be particularly sensitive to disruptions in financial markets according to existing theory and evidence. We then show how to use this evidence to evaluate the quantitative importance of financial shocks in a canonical model of firm dynamics with financial frictions.

Empirically, the use of external funds for investment is particularly high among private firms—around 80% of investment—compared to 20% for publicly traded firms (henceforth called public firms). We then confront a model with both public and private firms that face financial frictions with our evidence on firms' use of external funds. A financial shock calibrated to induce an average fall in aggregate leverage generates a modest 0.45% decline in GDP when our model matches the shares of output produced by both public and private firms. In this sense, financial shocks face a challenge in generating large declines in aggregate economic activity.

Our empirical approach to determine how much firms use external funds for investment is to analyze the flow of funds into and out of firms. In particular, our measure of external financing is the difference between a firm's revenues after deducting interest, taxes, labor, and other business expenses, which we refer to as *available funds*, and that firm's capital expenditures, or *investment*. A firm with available funds less than its investment needs must use external funds. Conversely, a firm with available funds in excess of its investment needs generates an outflow of funds. This relationship gives us a

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sense of how much firms use external funds and, through the lens of a model with financial frictions, is informative about the effect of shocks to financial markets on aggregate outcomes.

Our evidence on the use of external funds is based on data from the United States and the United Kingdom. Evidence from aggregate data in both countries shows that available funds are almost always higher than investment. In this sense, in the aggregate, firms do not use *any* external funds to finance investment, or, put differently, in most periods, funds flow from firms to households, not from households to firms. Evidence from the cross section demonstrates that there are firms that use external funds for investment in certain periods and others that generate funds in excess of their investment needs. However, the extent to which firms use external funds for investment is very different among firms of different listing statuses. Specifically, in any year, for public firms, the total inflow of funds to firms that use external funds is roughly 20% of total public firms' investment. For private firms, the analogous number is roughly 80%. We document that this difference holds at the firm level even when controlling for industry and size. These observations suggest that disruptions in flow of funds among firms, and particularly among private firms, may play an important role in aggregate fluctuations.

To study these fluctuations, we develop a general equilibrium model where financial shocks, modeled as shocks to firms' collateral constraints, disrupt the flow of funds among firms. Our model features both public and private firms who differ in their ability to diversify idiosyncratic risk.[1] In addition, our model features trade linkages and general equilibrium effects. These features cause financial shocks to spill over from constrained firms to unconstrained firms, thereby amplifying the impact of these shocks.

Naturally, a shock that tightens the collateral constraints in our model is likely to lead to a larger decline in output when more firms have binding collateral constraints. Hence, disciplining the extent to which firms face binding collateral constraints is critical for the behavior of the economy in response to financial shocks. A contribution of our paper is to use evidence from the cross-sectional data on financial flows to discipline the tightness of the collateral constraints in the steady state of our model and, therefore, the quantitative response of the model economy to a financial shock.

Using our calibrated model, we examine the effect of a shock to firms' collateral constraints that generates a one standard deviation decline in aggregate leverage (upon impact). We find that this financial shock causes a decline in aggregate GDP of only roughly 0.45% on impact. The decline in output results from an increase in misallocation of capital caused by tighter collateral constraints. The impact on aggregate GDP is muted relative to the effect on private firms largely due to the response of public firms who, in equilibrium, do not face binding collateral constraints. On impact, the gross output of public firms rises, although this increase is dampened by the non-financial trade linkages in our model. We also consider a larger financial shock that generates the decline in aggregate leverage observed following the Great Recession in the United States. We show that the response of the economy to such a large financial shock is roughly a 1% decline in aggregate GDP.

We conclude that while financial shocks may have a significant impact on production decisions of private firms their impact on aggregate economic activity may be small. It is important to stress that our empirical findings alone do not imply that collateral constraints or financial shocks do not matter simply because few firms use external funds for investment. Rather, when one confronts a canonical model of firm dynamics with financial frictions with our evidence on firms' use of external funds, this model yields small responses to financial shocks.

The nature of our exercise is closest to that of Jermann and Quadrini (2012) and Khan and Thomas (2013). In a seminal paper, Jermann and Quadrini (2012) document the behavior of aggregate flows in the form of debt and equity over the business cycle between households and firms. We instead focus on the cross section of firms and identify the factors affecting external financing. More recently, Khan and Thomas (2013) show that in an economy with firm heterogeneity and irreversibilities, a large financial shock can generate a recession of the magnitude of the Great Recession. While our model has a similar flavor, we incorporate both public and private firms and discipline our model by matching patterns of external financing by these two sets of firms. We show that once our model matches these patterns, even a large financial shock generates at best a mild recession. In this sense, we conclude that it is a challenge for financial shocks to rationalize particularly large recessions in models that are consistent with patterns of external financing.

1.1. Related literature

Our paper is related to an extensive literature on the effect of financial frictions in macroeconomics, starting with Bernanke and Gertler (1989), Carlstrom and Fuerst (1997), Bernanke et al. (1999), and more recently Kiyotaki and Moore (2012) and Jermann and Quadrini (2012). The common goal of these works is to identify and understand the channels through which financial market disruptions affect economic activity and their quantitative importance. The aim of this paper is the same, but our approach differs since, to the best of our knowledge, our paper is the first to employ evidence on the use of external funds to discipline models of financial frictions.

Our empirical work on external financing is related to a literature in corporate finance that attempts to identify the extent to which firms face constraints in financing their investment (e.g., Fazzari et al., 1988 and Gilchrist and Himmelberg, 1995). Our approach differs by emphasizing the role financial markets play in firms' financing decisions (i.e., how much investment is financed using external funds). In this regard, our approach is closer to the one taken by Rajan and Zingales (1998) and Buera et al. (2011).

[1] The distinction between public and private firms in our model is motivated by evidence from Vissing-Jorgensen and Moskowitz (2002), who show that owners of private equity are less diversified than owners of public equity.

Our model of financial frictions is a natural extension of Evans and Jovanovic (1989) to dynamic environments.[2] The basic structure of the model is very similar to Gomes (2001). While Gomes (2001) models financial frictions as additional costs to external financing, we follow Evans and Jovanovic (1989) and assume that investment is bounded by a factor of net worth. More recently, Eisfeldt and Muir (2016) use an approach similar to that of Gomes (2001) in a model where firms can invest in low-yield liquid assets to measure the cost of external financing.

The paper is organized as follows: Section 2 documents evidence on firms' external financing behavior; Section 3 contains our model and theoretical analysis; Section 4 contains our main quantitative exercises; and Section 5 concludes the paper.

2. Evidence on the use of external financing

In this section, we present evidence on the use of external funds for investment in the aggregate and at firm level in the United States and in the United Kingdom and establish the following stylized facts:

1. In the aggregate, firms can self-finance the entirety of their investments.
2. Aggregate external funds used by private firms is substantially larger (as a share of their investment) than the aggregate external funds used by public firms.
3. At the firm level, private firms use more external funds than public firms when controlling for firm size, industry and investment.

2.1. Conceptual measurement of financial flows

We say an individual or a representative firm uses external funds if its capital expenditure exceeds its available funds, where available funds is simply a firm's revenue from operations and income from financial investments less cost of goods sold, taxes, and interest on its debt—sometimes referred to as net cash flows from operations. The use of external funds is defined as the difference between capital expenditures and available funds and is a measure of the net financial inflow to a firm.

If a firm's investment is less than its available funds, then funds flow out of the firm either to shareholders in the form of dividends or share repurchases, to debtholders in the form of debt repurchases, or as retained earnings (accumulation of financial assets). When funds flow out of the firm, we say the firm *generates external funds*. Conceptually, our approach treats dividends as a financial flow as opposed to a cost of operation—consistent with the view that the firm may adjust its dividends contemporaneously, while interest payments are fixed in advance.

If investment is larger than available funds, then funds flow into the firm and we say the firm *uses external funds*. If a firm uses external funds, it finances its capital expenditures either through issuing debt or equity, or by reducing its holdings of financial assets. While financial inflows measure use of external funds, an inflow or an outflow for a firm does not necessarily imply that the firm is financially constrained. For example, a firm might use external funds by selling some of its financial assets, which may not imply that the firm is constrained in any way. On the other hand, a firm's external funds may come from issuing debt, which may or may not be subject to constraints. Our approach is to simply measure the use of external funds by firms, and later we use a stylized model consistent with these measures to obtain implications on the quantitative impact of financial constraints.

In aggregate data, we measure the use of external funds as the difference between aggregate investment and aggregate available funds. Our preferred statistic of how individual firms rely on external funds for investment is given by

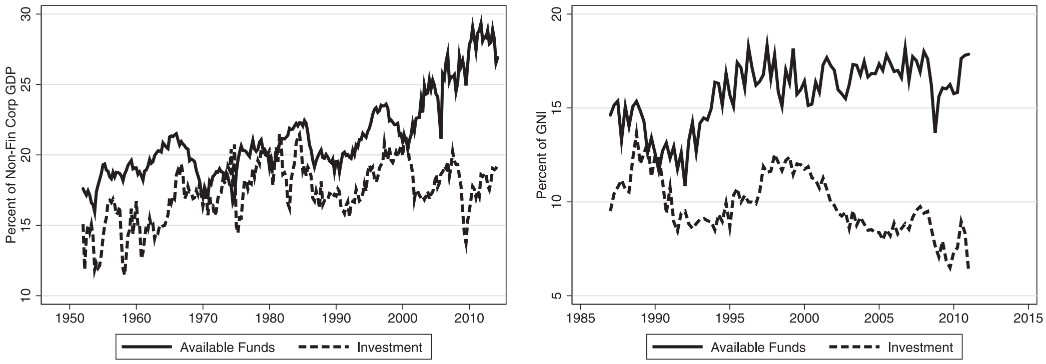
$$\frac{1}{T}\sum_{t=1}^{T}\frac{\sum_{i\in\mathcal{I}}(X_{it}-AF_{it})\mathbf{1}_{[X_{it}\geq AF_{it}]}}{\sum_{i\in\mathcal{I}}X_{it}}, \tag{1}$$

where $\mathcal{I}$ is the subset of firms with a given characteristic. The statistic in Eq. (1) represents the time series average of the total financial inflow to firms whose investment is greater than their available funds as a fraction of total investment, and it informs us about what fraction of aggregate investment must be financed externally.

2.2. Data description

We base our analysis on data from the United States and the United Kingdom. For the United States, our data sources include the Flow of Funds and Compustat. Our data sources for the United Kingdom include the U.K. Economic Accounts, Compustat Global, and Amadeus. Aggregate U.S. data are from the Flow of Funds. Our measure of aggregate available funds in the United States is the sum of internal funds and dividends of all non-financial corporate businesses. Note that we add dividends here because the definition of internal funds has excluded them. Our measure of aggregate investment is given by capital expenditures of non-financial corporate businesses.

[2] Our approach is also similar to an extensive literature that analyzes the effects of financial frictions on misallocation and total factor productivity (e.g., Midrigan and Xu, 2014 and Buera et al., 2011). Since we focus on business cycle frequency fluctuations and the importance of the role played by financial markets, our calibration is different in that we match evidence on external financing as well as the variance of debt-to-asset ratios, while the papers mentioned above focus on the dynamics of firm size.



(a) United States (b) United Kingdom

Fig. 1. Available funds and investment normalized by non-financial corporate business GDP in the United States (Panel A) and normalized by gross national income in the United Kingdom (Panel B).

Our source of aggregate data in the United Kingdom is the national economic accounts. We measure available funds as the sum of gross disposable income and dividends of private non-financial corporations. We measure investment as the sum of gross fixed capital formation, change in inventories, and acquisitions less disposals of valuables and non-financial, non-produced assets.

Our source of firm-level data on public firms in the United States and the United Kingdom is Compustat. In the United States, we focus on data from 1974 to 2013 for non-financial firms headquartered in the United States. In the United Kingdom, we focus on data from 1992 to 2013 for non-financial firms headquartered in the United Kingdom. After undertaking standard data cleaning, our Compustat sample in the United States consists of about 51,000 firm-year observations, with roughly 1400 firm-level observations in a typical year. Our Compustat U.K. sample consists of roughly 10,000 firm-year observations, with 550 firm-level observations in a typical year. We measure available funds as net cash flow from operating activities. Since we do not want to distinguish between physical investment in existing assets and acquisition of new assets, we define investment as the sum of capital expenditures and acquisitions less sale of property, plant, and equipment.

Our source of firm-level data on private firms is the Amadeus database. Amadeus contains financial information on more than 18 million private and public firms in Europe, with a focus on private firms. We restrict attention to the sample of data from 2005 to 2012 on private limited firms that are not quoted on a stock exchange and are located in the United Kingdom in that time interval. Our sample consists of over 980,000 firm-year observations, with roughly 100,000 firm-level observations in each year. We focus on data from the balance sheet and profit and loss accounts. Our measure of available funds is profit per period plus depreciation; our measure of capital expenditures is the change in tangible fixed assets plus depreciation.

2.3. Facts about financial flows and external financing

Fig. 1 depicts available funds and investment data for the United States and the United Kingdom. On average, available funds are roughly 1.25 times as large as investment in the United States and 1.6 times as large as investment in the United Kingdom. Moreover, in the United States, available funds exceed investment by roughly 3% on average over the entire sample. In this sense, the aggregate firm in the United States and in the United Kingdom does not rely on outside financing to fund investment.[3]

In the cross section, the aggregate use of external funds by public firms is 22% of aggregate investment (by public firms) in the United States and 18% in the United Kingdom.[4] In contrast, use of external funds by private firms is 82% of aggregate investment by private firms in the United Kingdom.[5] In Fig. 2, we plot the annual use of external funds in our data samples.[6] This figure shows that the different use of external funds by public and private firms is persistent over time.

The above evidence points to two types of heterogeneity in use of external funds in the cross section. First and in contrast with the aggregate evidence, there are indeed firms, both public and private, that use external funds for investment. Second,

[3] Even if one subtracts dividends from available funds, in most periods available funds less dividends are larger than investment in the aggregate. In this sense, even if aggregate dividends cannot be adjusted contemporaneously by firms, firms may not rely extensively on financial markets for external funds.

[4] The average reported for public firms in the United Kingdom excludes the years 1999 and 2000 because of a lack of data availability for acquisitions. Alternatively, if we ignore acquisitions, then over the entire public U.K. sample, roughly 8% of investment is externally financed.

[5] In an earlier sample of data, from 2001 to 2009, we find the use of external funds by private firms is 93% of aggregate investment by private firms in the United Kingdom. This finding suggests the difference between use of external funds for investment between public and private firms is not driven by outcomes during the Great Recession.

[6] We have experimented with winsorising our data sample at the 0.1% and 1% levels, and found little change in terms of the differential importance in the use of external funds for investment by private versus public firms. These experiments are available on request.

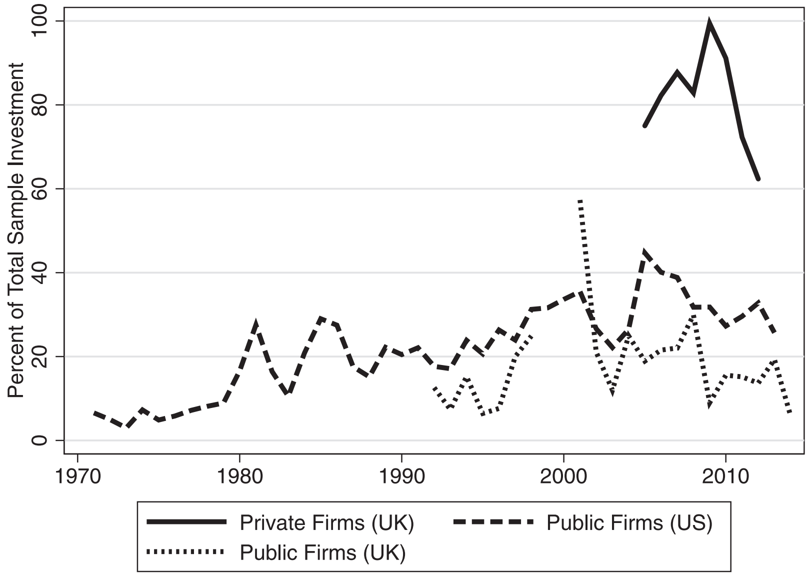


Fig. 2. Use of external funds by listing status and country.

our evidence also establishes that the extent to which firms use external funds for investment is very different across public and private firms. Since private firms finance a large fraction of investment using external funds and since private firms account for a substantial fraction of investment in our sample (61.3% of U.K. sample investment on average from 2005 to 2012), the response of private firms to financial market shocks may contribute significantly to the aggregate consequences of financial market shocks.[7]

Next, we examine the differences in use of external funds between private and public firms at the micro-level. To evaluate the importance of listing status on use of external funds, we estimate the following baseline specification using ordinary least squares regression:

$$\frac{X_{i,n,t} - AF_{i,n,t}}{X_t} = \beta \times \text{Private}_{i,n,t} + \rho_A A_{i,n,t}/X_t + \rho_X X_{i,n,t}/X_t + \alpha_n + \alpha_t + \epsilon_{i,n,t}. \quad (2)$$

The left-hand side of (2) is the use of external funds by firm i in industry n in year t expressed as a percentage of aggregate investment in year t. The variable $\text{Private}_{i,n,t}$ equals 1 if the firm is private and 0 otherwise. Variables $A_{i,n,t}$ and $X_{i,n,t}$ represent the firm's total assets and investment, respectively and α_n and α_t are industry and year fixed effects, respectively. Put into words, β captures the difference in external financing arising from the firms' listing status among firms within the same industry with similar assets and investment. Table 1 presents estimates on the importance of listing status for use of external funds. The reported t statistics are computed using robust standard errors to control for heteroskedasticity in the data.

Columns 1–3 report our findings when we examine only those firms using external funds in a given year. Our baseline specification, column 1, indicates that when controlling for size, industry, and investment, private firms use significantly more external funds than public firms. This column also indicates that firms undertaking more investment (in levels) use greater external funds. As a consequence, if we do not control for firm-level investment, then it appears that listing status has no impact on use of external funds as column 2 indicates. In column 3 we omit industry level fixed effects and our finding that private firms use more external funds for investment remains.

Columns 4 and 5 report our findings across the entire sample of public and private firms and they show again that private firms use more external funds than public firms. The significance of this difference is now greater, while investment no longer predicts use of external funds. We interpret this finding as suggesting that among firms that generate external funds (those firms with $AF_{i,n,t} > X_{i,n,t}$), public firms generate more.

To better understand these findings, Table 2 reports the time series average of the cross-sectional average and median of various measures of size for public and private firms. As expected, public firms are significantly larger in terms of assets, investment, and sales.

[7] Our estimate of the importance of investment by private firms is consistent with estimates from Asker et al. (2014), who find that U.S. private firms account for 54.5% of investment, as well as employment estimates from Davis et al. (2007), who find that private firm employment accounts for more than 70% of aggregate employment.

Table 1
Panel regression results for the regression specified in (2) and reported in percentages. Private equals 1 if the firm is private and 0 otherwise. Variables Assets and Investment represent the firm's total assets and investment, respectively, while Industry FE and Year FE indicate whether industry or year fixed effects are included in the regression model in each column. Reported *t* statistics are computed using robust standard errors.

	External financing/Aggregate investment				
	(1)	(2)	(3)	(4)	(5)
Private	0.004375**	0.0008169	0.004304**	0.03057***	0.02664***
	(3.28)	(0.41)	(3.17)	(6.32)	(4.14)
Assets	−0.008929	0.05788***	−0.008876	−0.01535*	−0.01410*
	(−1.39)	(7.85)	(−1.38)	(−2.05)	(−2.33)
Investment	0.7172***		0.7173***	0.09577	
	(12.35)		(12.36)	(0.85)	
Industry FE	Yes	Yes	No	Yes	Yes
Year FE	Yes	Yes	Yes	Yes	Yes
Observations	182,072	182,072	182,072	709,498	709,498
Adjusted R^2	0.687	0.313	0.687	0.132	0.126
Only Pos. Exfin	Yes	Yes	Yes	No	No

t statistics in parentheses * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$.

Table 2
Time series average of sample mean and median measures for public and private firms in the United Kingdom. Assets, investment, and sales reported in millions of pounds. I/A is the investment-to-total-Asset ratio. AF/A is cash generated from operations (available funds) relative to lagged total assets.

Company type	Total assets	Investment	Sales	I/A	AF/A
		Cross-sectional average			
Private	19.57	0.66	18.80	4.33	43.44
Public	1,191.48	53.15	1026.25	4.21	4.91
		Cross-sectional median			
Private	0.24	0.002	0.38	1.23	14.99
Public	115.86	2.66	126.71	3.07	10.42

Table 2 also reveals that the distinction between public and private firms in our sample does not necessarily capture simple differences in profitability or investment activity. Because Tobin's Q cannot be constructed for private firms, we have compared investment-to-asset ratios as well as cash return on assets for public and private firms.

We see that the median private firm undertakes investment relative to its total assets on the same order of magnitude as the median public firm. Moreover, the cash generated from operations relative to (lagged) total assets for the median private and public firm is roughly comparable; that is, relative to their size and over the entire sample, public and private firms undertake investment and generate available funds of roughly the same magnitude. This evidence suggests that private firms' extensive use of external funds does not reflect a lack of investment opportunities.

Our firm-level findings confirm our findings from aggregated data that private firms use more external financing than do public firms. Note, however, that our firm-level evidence considers the level of external financing, while our aggregated statistics consider external financing as a fraction of investment. The prevalence of firms with zero investment (i.e., roughly 23% of firms in our sample) impedes us from examining external financing as a fraction of investment at the firm level. We have attempted to address this concern by including controls for firm-level investment in our regression analysis. An alternative way to address this difference is to divide the data into subsamples and report the difference in means across subsamples. Appendix B contains such a subsample analysis and largely confirms our finding that private firms use more external financing as a share of investment, even among subsamples of firms with the same size and industry.

The behavior of external financing over the business cycle may also shed light on the importance of financial shocks. Unfortunately, since our panel data of private firms only span ten years, we are not able to examine the business cycle properties of private firm external financing. Nevertheless and for illustrative purposes, in Fig. 3, we present the behavior of external financing, investment, and income for public and private firms over the period 2006–2012 including the recession from the first quarter of 2008 to the second quarter of 2008 in the United Kingdom. We note that while external financing of private firms rises in the midst of the recession, in the aftermath it declines substantially, from roughly 100% of investment in 2009 to roughly 60% of investment in 2012, suggesting financial shocks may have played an important role, at least in understanding the slow recovery of the U.K. economy. We also note that while investment and earnings growth of public and private firms are correlated, they are not perfectly correlated, especially for investment.

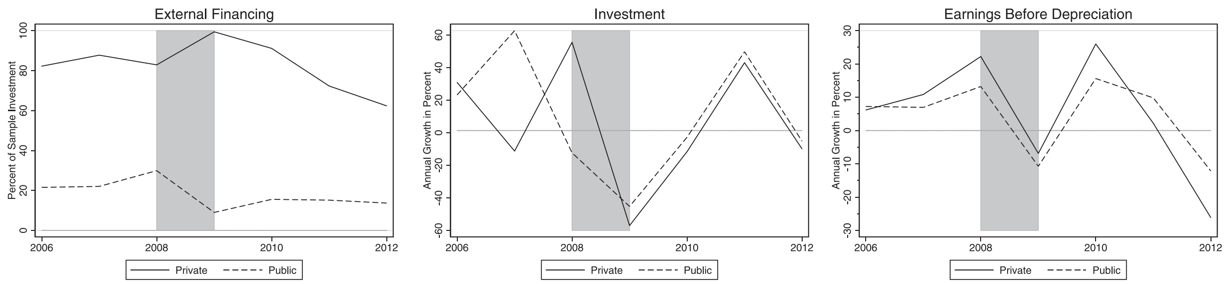


Fig. 3. Behavior of private and public external financing, investment growth, and earnings growth around 2009 recession in the U.K. Shaded bars in all three graphs indicate the recession.

3. A dynamic model of public and private firms

In this section, we develop a dynamic model of public and private firms and define a symmetric stationary equilibrium. Our model features typical elements of models of firm dynamics under financial constraints (as in Albuquerque and Hopenhayn, 2004; Buera et al., 2011, and Kiyotaki and Moore, 2012) while incorporating novel ingredients in the form of multiple productive sectors (public and private) and trade linkages (monopolistic competition and the input-output structure).

3.1. Model and equilibrium definition

Time is discrete, lasts forever, and is indexed by $t = 1, 2, \ldots$. Agents in the model include households, final good producers, and intermediate good producers. There are two types of households: entrepreneurs and workers. In the economy, there is a single consumption good, which is a composite good produced by a sector of competitive final good producers. We normalize the price of the final good to be 1 in each period. Final good producers aggregate the output of the intermediate good producers.

Each intermediate good producer has a monopoly in producing a differentiated output. There are two classes of intermediate good producers: private and public firms. We normalize the total measure of intermediate good-producing firms to be 1, and we assume there is a fixed measure of private firms, s. A firm's type is exogenously given and fixed for the lifetime of the firm. Let firms $i \in [0, s]$ denote the names of the s private firms and $i \in [s, 1]$ denote the names of the $1 - s$ public firms in any period. Firms exogenously exit at rate ζ. Upon exit, a firm is replaced by an otherwise identical firm endowed with the existing firm's assets. In our quantitative exercises, we consider a setting with only private firms (i.e., $s = 1$) and one with both private and public firms (i.e., $s < 1$), properly calibrated to match the data.

In our model, we distinguish between public and private firms by assuming that public firms are owned by and rebate dividends to diversified households. Private firms are owned by individual entrepreneurs for whom the cost of delayed dividend payments is forgone consumption.

In each period, a firm of either type can produce output according to the constant returns to scale production function, $y_{it} = z_{it}\left(k_{it}^{\alpha} l_{it}^{1-\alpha}\right)^{\eta} I_{it}^{1-\eta}$ where z_{it}, k_{it}, l_{it}, I_{it} are firm i's productivity, capital input, labor input, and intermediate input in period t, respectively. We assume that intermediate inputs are a perfect substitute for the final good. (See Basu, 1995 for an example of this type of input-output production structure.) We assume that the firm-level process for productivity follows a first-order Markov process governed by transition probability distribution $\Psi(z_{it}|z_{it-1})$.

There are a large number of competitive final good producers. Each of them can combine the output of the intermediate producers to create a composite final good according to the production function

$$Q_t = \left[\int_i q_{it}^{1-\frac{1}{\rho}} di\right]^{\frac{\rho}{\rho-1}}, \tag{3}$$

where q_{it} is the input of firm i in period t, and ρ is the elasticity of substitution across all goods in the economy. Perfect competition among final good producers ensures that we can focus on a representative firm that solves $\max_{Q, q_{it}} Q - \int_i p_{it} q_{it} di$ where Q is given by the production function (3) in each period. The final good producer's problem gives rise to an inverse demand curve for each intermediate good as a function of prices:

$$p_{it} = Q_t^{\frac{1}{\rho}} q_{it}^{\frac{-1}{\rho}}. \tag{4}$$

Intermediate good producers are local monopolists who face the inverse demand function in (4). We assume that all intermediate good producers have access to competitive labor markets; however, their access to capital markets is limited. This limit is proportional to their financial asset holding or net worth. In particular, capital rented, k_{it}, must satisfy

$$k_{it} \leq \lambda a_{it}, \tag{5}$$

where a_{it} is the total financial assets held by the firm. The financial assets held by the firm must satisfy the following budget constraint:

$$d_{it} + a_{it+1} \leq p_{it} z_{it} \left(k_{it}^{\alpha} l_{it}^{1-\alpha}\right)^{\eta} I_{it}^{1-\eta} - w_t l_{it} - I_{it} - (r_t + \delta) k_t + (1 + r_t) a_{it}. \tag{6}$$

The idea behind the financial friction is that firms can post their financial assets as collateral in order to rent capital. The variable λ determines the ease with which a firm can obtain capital using financial assets as collateral and represents the ability of the financial markets to reallocate capital across different firms.[8]

Each entrepreneur owns a private firm and faces incomplete markets in the sense that ownership of a private firm is concentrated and entrepreneurs cannot issue equity. Furthermore, we assume that bankruptcy is an extreme event in which a firm's entire stock of financial assets is reduced to zero. Critically, this bankruptcy risk is fully non-diversifiable. As a result, entrepreneurs' preferences are given by

$$E_j \sum_{t \geq j}^{\infty} (\beta(1-\zeta))^{t-j} \log(d_{it}) \tag{7}$$

for $i \in [0, s]$. Entrepreneurs maximize expected discounted lifetime utility (7) subject to the budget constraint described in (6), the collateral constraint (5), and the inverse demand (4) for their production.

Public firms have the same production opportunities as private firms. Since public firms are owned by diversified households, these firms can fully diversify the bankruptcy risk. As a result, public firms maximize the expected present discounted value of their dividends under the pricing kernel derived from household preferences, which, in the economy with no aggregate risk, is given by $M_t = \beta^t U_{Ct}$. The objective of a public firm is given by the expected present discounted value of dividends, or $E_0 \Sigma_t M_t d_{it}$. Similar to the owners of private firms, the owners of public firms face budget constraint (6), collateral constraint (5), and inverse demand (4) for their production. Furthermore, we assume that they cannot issue any equity (i.e., $d_{it} \geq 0$).

As mentioned above, one key difference between private and public firms is the ability of the owners of public firms to diversify the bankruptcy risk associated with a given firm. This implies that in each period, a fraction ζ of private firms exit. We assume that in each period, an equal measure of firms are born and draw assets and productivities from the joint stationary distribution of assets and productivities. That is, we assume that there are no social losses in the value of the assets from bankruptcy.

In each period, households decide how much to work, consume, and save in the risk-free bond. They maximize lifetime expected utility

$$E_0 \sum_{t=0}^{\infty} \beta^t U(C_{ht}, L_{ht}) \tag{8}$$

subject to the sequence of budget constraints

$$C_{ht} + A_{t+1}^h \leq w_t L_{ht} + (1 + r_t) A_t^h + \int_s^1 d_{it} di. \tag{9}$$

We assume that $U(C, L) = u(C - v(L))$, where $v(L) = \psi \epsilon l^{1+1/\varepsilon}/(1+\varepsilon)$ and $u'(0) = \infty$ as in Greenwood et al. (1988).[9] Note that since households face no constraint on their saving, their Euler equation implies that the pricing kernel is given by $M_t = \Pi_{s=0}^{t-1} \frac{1}{1+r_s}$.

The aggregate state of the economy in any period can be summarized by the distribution over net worth (a) and productivity (z), of private firms ($G_{u,\,t}(a, z)$), of public firms ($G_{l,\,t}(a, z)$), and household assets (A_t^h).

Since the definition of a recursive competitive equilibrium for this economy is standard, we relegate it to Appendix C, where we also provide a partial characterization of the optimal decision rules of firms.

By choosing to model all debt as intra-period, the optimal capital and labor decisions are purely static. As a result, the problem of a private firm can be simplified by two-stage budgeting. In any period, the firm chooses capital and labor; after making these decisions, the firm then decides how much to consume and save. The resulting consumption-savings decision is essentially the same as the one studied in Huggett (1993) or Aiyagari (1994). The key difference from those models is that savings affect future profits of the firm by potentially relaxing the collateral constraint in future periods.

Before discussing the theoretical and quantitative results from our model, it useful to demonstrate how to compute external financing in the model. Mechanically, the model generates simulated balance sheets and income statements for

[8] One way to rationalize the use of such collateral constraint is to assume that firms have access to competitive financial intermediaries who receive deposits and rent capital to firms, while lending to firms is not perfectly enforceable. After production, firms can choose to default on their loan from the financial intermediary and, with probability $1/\lambda$, they are able to retain their undepreciated capital stock. If a firm defaults, the financial intermediary seizes the financial wealth of the firm, but the loss of financial wealth is the only punishment firms face from defaulting on their debt obligations. This implies that the intermediary only lets the firm borrow at most λa_{it}.

[9] In formulating households' problem, we have assumed that households only have access to risk-free bond and cannot trade shares. Alternatively, we can let households trade shares of the public intermediate good firms via an intermediary.

each firm that can be analyzed exactly as in the data. To do so, rewrite the budget constraint in terms of physical capital k_{it} and debt $b_{it} = k_{it} - a_{it}$. With this notation, the budget constraint of the firm can be written as

$$d_{it} + x_{it} = p_{it} y_{it}(z_{it}, k_{it}, l_{it}) - w l_{it} - I_{it} + b_{it+1} - b_{it} - r_t b_{it} \tag{10}$$

At this point, one can construct available funds and investment exactly as in the data with $AF_{it} = p_{it} y_{it} - w_t l_{it} - I_{it} - r_t b_{it}$ and $x_{it} = k_{it+1} - (1 - \delta) k_{it}$. Since the state at t is given by (a_{it}, z_{it}), we may define investment x_{it} as a function of only (a_{it}, z_{it}) and z_{it+1}. External financing in the model can then be computed exactly as in the data (with a slight abuse of notation defining the distribution over a, z, z'):

$$\frac{\int_{a,z,z'} \left(x_t(a,z,z') - AF_t(a,z) \right) \mathbf{1}_{[X_t \geq AF_t]} dH(a,z,z')}{\int_{a,z,z'} x_t(a,z,z') dH(a,z,z')}. \tag{11}$$

3.2. Theoretical results

In this section, we describe theoretical results that shed light on the use of external funds by public firms in our model, and the general-equilibrium effects that arise from shocks to the collateral constraints and impact the choices of public firms.

First, because public firms discount at the same rate as households, in any equilibrium in which household consumption is stationary, public firms never face binding collateral constraints. Consequently, the amount of external funds used by public firms in our model is indeterminate.

To see this, note that if in a certain period a public firm is constrained (faces a binding collateral constraint) or it will be constrained with positive probability in the future, it is optimal to not pay dividends. This is because the value of funds inside the firm is more than the value of funds outside the firm. Furthermore, stationarity of households' consumption implies that $\beta(1 + r) = 1$ or $r > 0$. Hence, the financial assets of a constrained firm grow at a rate of at least r. If we assume that idiosyncratic productivity is bounded above, then the optimal scale of the firm is bounded above. Therefore, since financial assets for a constrained firm are growing at rate r, it must be that at some point in time, the amount of collateral available to the firm, λa, exceeds the maximum optimal scale and the firm becomes unconstrained. Thus, in a stationary equilibrium, all public firms must be unconstrained.

Given that public firms are unconstrained, the asset holdings of public firms in a stationary equilibrium are not determinate. Since these firms always operate at their optimal scale, indeterminacy of assets leads to an indeterminacy of available funds for the firm. We summarize this discussion in the following proposition:

Proposition 1. *Suppose that in a stationary equilibrium, the interest rate is positive and productivity is bounded above. Then the collateral constraint never binds for any public firm. In addition, the amount of external funds used by public firms to finance investment is indeterminate.*

Next, consider how a shock to the collateral constraint may impact unconstrained, public firms. We argue that general equilibrium effects lead to shocks to the collateral constraints to spill over to these unconstrained firms, and can dampen and even overturn their incentives to increase production in response to tightening of the constraints. To do so, in Appendix C, we analyze a static, partial equilibrium version of our model. We develop sufficient conditions for the equilibrium output of every firm to be decreasing in the tightness of the collateral constraints. The key parameters that determine the strength of these general equilibrium spillovers are the elasticity of household labor supply, ε, the elasticity of substitution across goods, ρ, the labor share parameter, α, and the intermediate input share parameter, η. We now state our main result from Appendix C.

Proposition 2. *Suppose that $\beta = 0$ and that there exists a positive measure of constrained firms. If $1 + \varepsilon \geq \eta\rho(1 - \alpha)$, then output of all firms is increasing in the parameter λ.*

The intuition for this result is as follows. A tightening of the collateral constraint causes constrained firms to reduce their demand for capital. Since capital and labor are complements in production, constrained firms also decrease their demand for labor causing the wage rate to fall and, in turn, triggering a decline in demand for the final good. Given our production function, the elasticity of output of an unconstrained firm with respect to the wage rate is simply $\eta\rho(1 - \alpha)$ (in our static version, we hold the interest rate fixed). If household preferences have the GHH form, the elasticity of aggregate demand with respect to the wage is just $1 + \varepsilon$. The condition $1 + \varepsilon > \eta\rho(1 - \alpha)$, simply ensures that the demand and intermediate input effects dominate the reduced marginal cost arising from the reduction in wage.

This result is useful for understanding our results in the dynamic economy below. If we assume a labor supply elasticity of 2.6, a labor share of 0.66, an input share of 0.5, and an elasticity of substitution of 4, then we should expect shocks to the collateral constraint to generate an aggregate recession where all firms decrease output (at least steady state to steady state).

4. Calibration and quantitative results

In this section, we calibrate the model and perform exercises intended to illustrate the contribution of changes in financial frictions faced by private firms to business cycle frequency fluctuations.

Table 3
Parameter values for the benchmark calibration.

Parameter	Value	Moment	Value
Calibrated parameters			
Collateral Constraint (λ)	6.98	External Financing	0.82
Persistence of Idio. TFP (ρ_z)	0.95	Debt-to-Total Assets	0.49
Std. of Idio. TFP (σ_z)	0.19	Dispersion in Net Debt-to-Assets	0.54
Disutility of Labor (ψ)	0.41	Aggregate Hours	0.3
Share of Private Firms (s)	0.41	Private Firms Share of Gross Output	0.4
Share of Intermediate Inputs (η)	0.43	Intermediate Input Share	0.43
Fixed parameters			
Discount Rate (β)	0.96		
Labor Supply Elasticity (ε)	2.6		
Elasticity of Substitution (ρ)	4		
Capital Share (α)	0.3		
Depreciation Rate (δ)	0.07		
Exit Risk of Private Firms (ζ)	0.10		

4.1. Calibration

We partition model parameters into those fixed in advance following estimates typically found in the macroeconomic literature and those estimated by matching model implied moments with the data (i.e., simulated method of moments).

The key parameters of the steady-state calibration are those governing the tightness of the collateral constraint, λ, and the process for idiosyncratic firm-level productivity. Firm-level productivity follows an AR(1) process, $\log z_{it} = \rho_z \log z_{it-1} + \sigma_z \varepsilon_{it}$ with $\varepsilon_{it} \sim N(0, 1)$. The parameters λ, ρ_z, and σ_z are jointly calibrated to match three moments in the data: the average aggregate debt to total assets in the U.S. economy since 1986, which is 0.49; the use of external funds for investment by U.K. private firms, which is 0.82; and the time series average of the standard deviation of size-weighted net debt-to-total-asset ratio of U.K. private firms, which is 0.54.

The measure of private firms, s, is chosen to ensure that private firms in the model account for 40% of gross output as they do in the data.[10] Additionally, the remaining model parameters—η, the parameter governing the intermediate input share in firm production, and ψ, the parameter capturing households' disutility of labor—are jointly calibrated to match the following moments: input's share of gross output is 43% in U.S. data (Jones, 2013), and aggregate hours account for 30% of households' time. Table 3 summarizes our model targets and the calibrated parameter values.

The bottom panel in Table 3 contains the set of fixed parameters. The discount rate is 0.96, targeting an annual real interest rate of 4%. The annual depreciation rate, δ, is 0.07, and the exit rate of private firms, ζ, is 10%, which implies a ten-year survival rate of 34%, consistent with estimates from Dunne et al. (1988). The parameter governing the capital share in the firm production function, α, is 0.3 and the elasticity of substitution across firms or goods is $\rho = 4$, in line with estimates from micro data evidence (see Burstein and Hellwig, 2008). Household preferences are given by $U(C, L) = \log\left(C - \psi\epsilon L^{1+\frac{1}{\varepsilon}}/(1+\epsilon)\right)$ and set the elasticity of labor supply, ε, to be 2.6.

Given the calibrated model, it is straightforward to determine the extent to which financial constraints bind in the steady state. Roughly 28% of firms face a binding collateral constraint in the steady state. The fact that a relatively small fraction of firms face a binding collateral constraint largely arises from the high value of the collateral constraint parameter, λ. The parameter value, $\lambda = 7$, is large in the sense that private firms are able to rent capital up to roughly seven times their net worth in any period. Put differently, this means that the amount of net debt outstanding for a given firm is at most equal to 86% of its capital stock. In our sample of private firms, the size-weighted net debt to fixed assets is on average 35%. However, in any given year, roughly 25% of firms have a net debt-to-fixed-asset ratio greater than 85%. In this sense, our value of the collateral constraint is not inconsistent with the fact that many firms appear able to borrow heavily against their physical assets.

Combined with the large collateral constraint parameter, our calibration selects parameter values with high levels of persistence in idiosyncratic productivity. These numbers are larger than those in Midrigan and Xu (2014) but also in line with those in Bachmann and Bayer (2014), who estimate the persistence of firm-level TFP to be 0.9675.

To more concretely evaluate our measure of persistence and the volatility of shocks in the idiosyncratic productivity process, we test the model by evaluating the volatility in firm-level labor, capital, and sales growth rates. Such a test has proven important in The literature on whether financial frictions can successfully explain cross-country differences in aggregate productivity, which has emphasized that it is critical to match estimates of firm-level volatility as opposed to the persistence of productivity alone in order to ensure a model economy is not artificially subjecting firms to financial constraints too frequently (see Midrigan and Xu, 2014).

[10] To arrive at this number, compare aggregating domestic (U.S.) gross output by public firms in Compustat to aggregate non-financial corporate gross output.

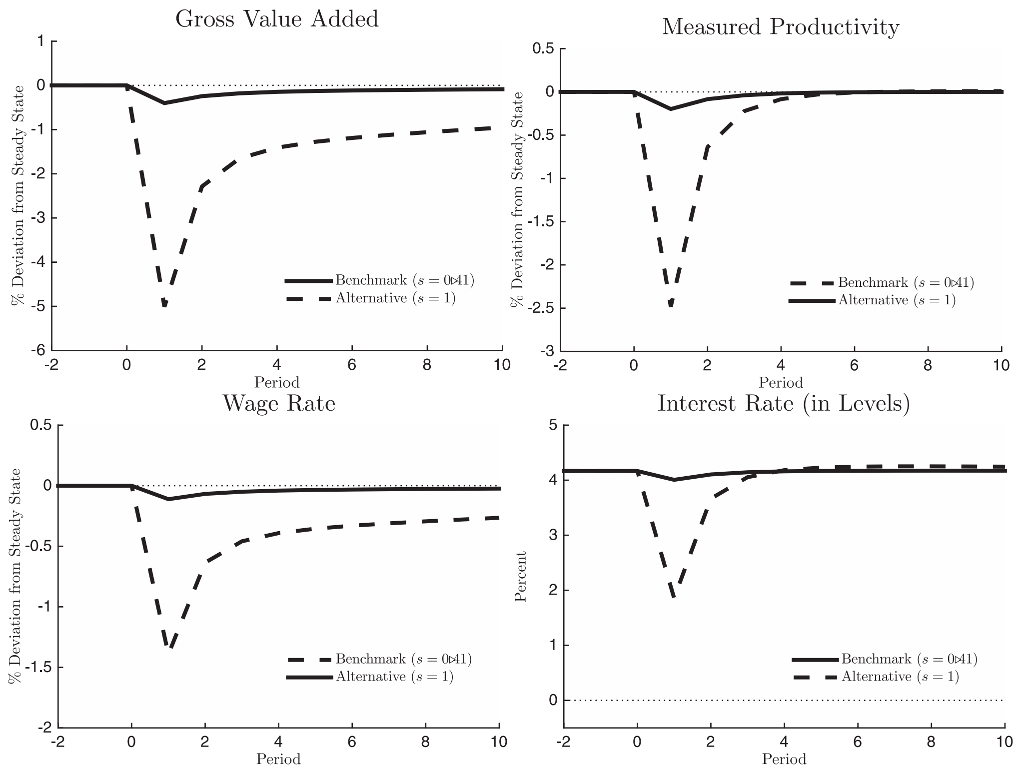


Fig. 4. Effects of a financial shock on macroeconomic outcomes in the model. The solid line in each graph represents impulse responses for the calibration reported in Table 3. The dashed line in each graph represents impulse responses for the same calibration reported in Table 3 but setting $s = 1$ so that the model features only private firms.

To address this issue, we compute the size-weighted standard deviation of employment, capital, or sales growth for private firms in our model as in Davis et al. (2007). In particular, let $g_{it} = (y_{it} - y_{it-1})/.5(y_{it} + y_{it-1})$ be the growth rate of y_{it} where y_{it} is employment, sales, or capital for firm i. For employment, let $\omega_{it} = (l_{it} + l_{it-1})/\sum_i (l_{it} + l_{it-1})$ be the share of employment in two subsequent periods for firm i while for capital and sales we let $\omega_{it} = a_{it}/\sum_i a_{it}$ be the share of total assets for firm i. In our model, the ω_{it}-weighted standard deviation of g_{it} is 0.47 for employment, 0.46 for capital, and 0.45 for sales, while in the data this value is 0.42 for employment (from Davis et al., 2007), 0.42 for capital (from our sample of private firms), and 0.52 for sales (again from our sample of private firms). Employment growth, capital growth and sales growth volatility in our model are thus roughly comparable to those found in the data.

4.2. Financial shocks

In our model, a financial shock is an aggregate shock to λ. We assume that this shock reduces the value of λ upon impact and the effect of this shock decays over time. The shock decays geometrically with a half-life of one year consistent with evidence on the duration of banking crises in advanced economies from Reinhart and Rogoff (2009).[11] The size of the initial impulse is chosen so that on impact, the shock generates a decline consistent with a one standard deviation in the aggregate debt-to-total-asset ratio in the United States since 1986. In our calibrated model, a roughly 32% decline in λ induces a one standard deviation decline in aggregate debt-to-total assets of roughly 3%. We then compute the impulse response path of the economy as it transitions back to the steady state (under perfect foresight).

Fig. 4 displays impulse responses of gross value added (output net of intermediate inputs), measured productivity, wages and the interest rate. This figure also shows the impulse response of the model under the same calibration when only private firms are included ($s = 1$).

Observe that the impulse to the economy generates a small decline in output on impact of around 0.45%. Among private firms, the financial shock generates a recession by inducing a greater degree of misallocation and therefore a decline in measured productivity relative to the steady-state outcome. This decline in productivity is initially driven because constrained firms in the model are now more constrained and must reduce their capital demand. Constrained firms are those firms that

[11] While the quantitative response of aggregate variables upon impact of the shock is roughly independent of the half-life, the endogenous persistence we document below varies slightly.

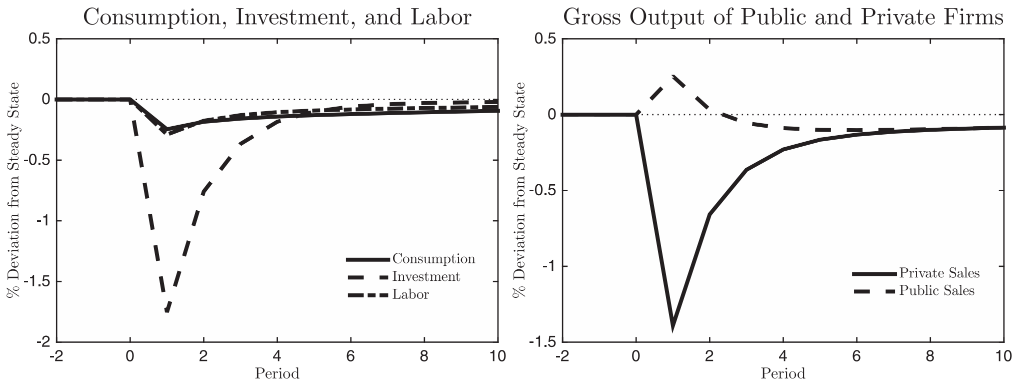


Fig. 5. Effects of a financial shock on consumption, investment, labor input (left panel) and gross output of public and private firms (right panel) for the model calibrated with parameters reported in Table 3.

experience a positive shock to idiosyncratic productivity either in the same period of or in those periods leading up to the financial shock. These high-productivity firms then grow more slowly along the impulse path than they would have in the steady state.

Additionally, a general-equilibrium response amplifies the direct impact of the financial shock. Since it causes a reduction in aggregate capital demand, the shock induces a downward movement in the real interest rate. This downward movement induces a response of unconstrained firms to increase their demand for capital. Unconstrained firms are those firms that experience a negative shock to idiosyncratic productivity either in the same period of or in those periods leading up to the financial shock and thus are, on average, low-productivity firms. These low-productivity firms then shrink more slowly along the impulse path than they would have in the steady state. Both the direct and indirect general equilibrium effects induce a decline in aggregate productivity and generate a decline in output among private firms.

Fig. 5 displays impulse responses of macro aggregates as well as the response of sales of public and private firms.

The financial shock causes consumption, employment, and investment to move in the same direction of output. The decline in employment is driven by the reduction in the wage rate given the fall in aggregate productivity (and by our assumption that households' labor supply decisions are independent of their wealth). Because the financial shock in the model generates a decline in aggregate productivity on impact, the initial decline in output coincides with a decline in investment and consumption.[12]

It is worth noting that the financial shock causes output of public firms to experience a modest rise on impact. The different initial response of sales by public and private firms is caused by the increased use of capital by public firms—none of which face binding collateral constraints—and the fact that in response to the 0.1% decline in the wage rate, aggregate labor does not decline dramatically. The response of public firms dampens the immediate consequences of the financial shock. Fig. 4 illustrates this dampening by also showing the response of the economy with only private firms to a financial shock in which output declines by around 5% upon impact. Not surprisingly, the economy with only private firms experiences a much larger decline (i.e., by a factor of 10) as the misallocation effect of this shock is large.

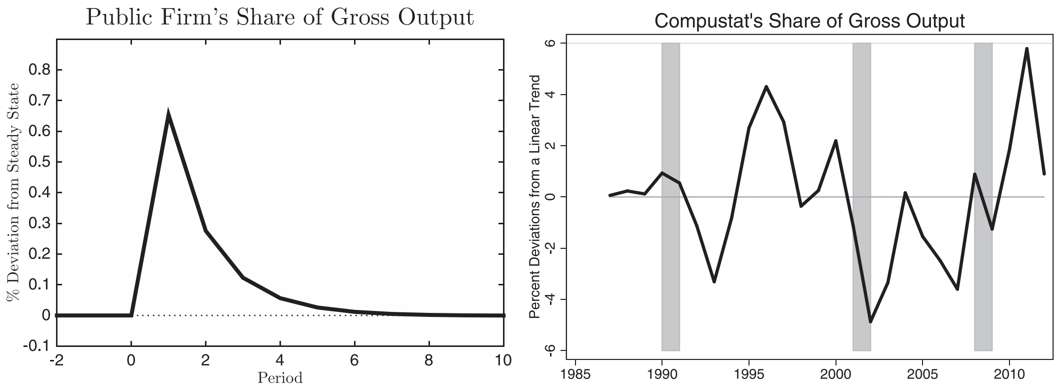
However, we also observe that the financial shock generates a persistent effect on output. Recall that our impulse to the collateral constraint recovers by half each year. However, output recovers by half after roughly two years, so the endogenous response of the economy is twice as persistent as the shock. This persistence is partly driven by the non-financial linkages in production among public and private firms, which ultimately induces a decline in sales by public firms. This decline reaches a minimum of roughly 0.1% below the steady state.

Since public firms face no financial constraints in equilibrium, in spite of the non-financial linkages, our model necessarily predicts a rise in the share of output among public firms following a financial shock. Fig. 6 plots the share of output by public firms in our model following a financial shock and the share of output accounted for by public firms in Compustat (plotted as deviations from a linear trend).

Observe that in the data, this share varies by roughly 6% but not only around business cycle dates. For example, although the share of output by Compustat firms rises from 2000 to 2001 and between 2007 and 2008, there are also significant movements (both increases and decreases) in the middle of business cycle recoveries. Interestingly, the data do suggest a large increase in the share of output produced by public firms in the midst of the financial crisis in late 2007 and during the 2000–2001 recession.

This evidence, while tentative, suggests that our model's prediction that the share of output accounted for by public firms should rise following a financial shock does not largely contradict the observations from Compustat. Our quantitative

[12] This co-movement finding is in contrast to models in which only new investment is affected by the financial constraint. In those types of models, a financial shock has no immediate impact on output and therefore a decline in investment coincides with an increase in consumption (e.g., Kiyotaki and Moore, 2012).



(a) Model

(b) Data

Fig. 6. Response of share of gross output accounted for by public firms in model (Panel A) and in data (Panel B: deviations from a linear trend of Compustat's share of Gross Output in U.S. data).

findings also suggest that non-financial linkages between public and private firms play an important role in understanding the aggregate consequences of financial shocks, suggesting that a model with richer non-financial linkages between firms may feature greater amplification of financial shocks (see Bigio and La'O, 2016).

4.3. The effects of productivity shocks

We next compare the effects of financial shocks to the effects of aggregate productivity shocks. We consider the transition dynamics that results from a purely unanticipated decline in aggregate productivity which slowly returns to steady state. Again, we fix the half-life of the impulse to one year. We choose the size of the productivity shock so that measured productivity in the model falls on impact by one standard deviation of the measured Solow residual in the United States, or, a 1% decline in measured productivity (at an annual rate). We also compare the effects of productivity shocks in a version of our model without collateral constraints.[13]

Fig. 7 displays the impulse path for measured productivity, gross value added, debt-to-total assets, and external financing following aggregate productivity or financial shocks. On impact, gross value added falls by 0.9% following a productivity shock. We conclude that financial shocks induce recessions that are roughly half as large as those caused by one standard deviation declines in aggregate productivity.

In our model, the effects of productivity shocks are dampened, primarily due to the monopoly distortions, which become less severe in response to a decline in aggregate productivity. We also find that our model, with or without collateral constraints, generates roughly the same effect on output. In this sense, in our economy, a fixed collateral constraint does not significantly amplify the effects of productivity shocks as found in earlier work by Kocherlakota (2000) and more recently Khan and Thomas (2013).

Fig. 7 also illustrates that leverage and external financing behave very differently in response to productivity and financial shocks. The debt-to-asset ratio does not move at all in response to a productivity shock and the use of external funds rises. In response to a decline in productivity for all firms, demand for debt by firms falls, since the level of capital they optimally want to install is lower. However, their demand for capital falls by roughly an equivalent amount, which causes the debt-to-asset ratio to remain roughly stable. The response of the use of external funds, on the other hand, is in the opposite direction of the response following a financial shock. When aggregate productivity declines, the collateral constraints for constrained firms are relaxed, since the unconstrained level of investment they want to undertake is lower. As a result, investment by constrained firms falls by less than the aggregate, inducing a rise in external financing.

4.4. The effects of a large financial shock

Motivated by the Great Recession, it is also useful to analyze the response of the model to a large shock to the collateral constraint. In particular, consider a decline in λ calibrated to match the peak-to-trough decline in the debt-to-asset ratio in the United States during the Great Recession of roughly 5%, or about two standard deviations. The response of gross value added to a large financial shock is roughly 1%, or about twice that of the benchmark financial shock.

An alternative exercise, which is closer in spirit to the analysis of Khan and Thomas (2013), is to impose a large enough shock to the collateral constraint to generate a 30% decline in the stock of debt. This exercise is intended to capture the

[13] Specifically, we analyze our model economy in the case where there are no private firms, but the process for idiosyncratic risk for public firms is the same as in our calibrated model above.

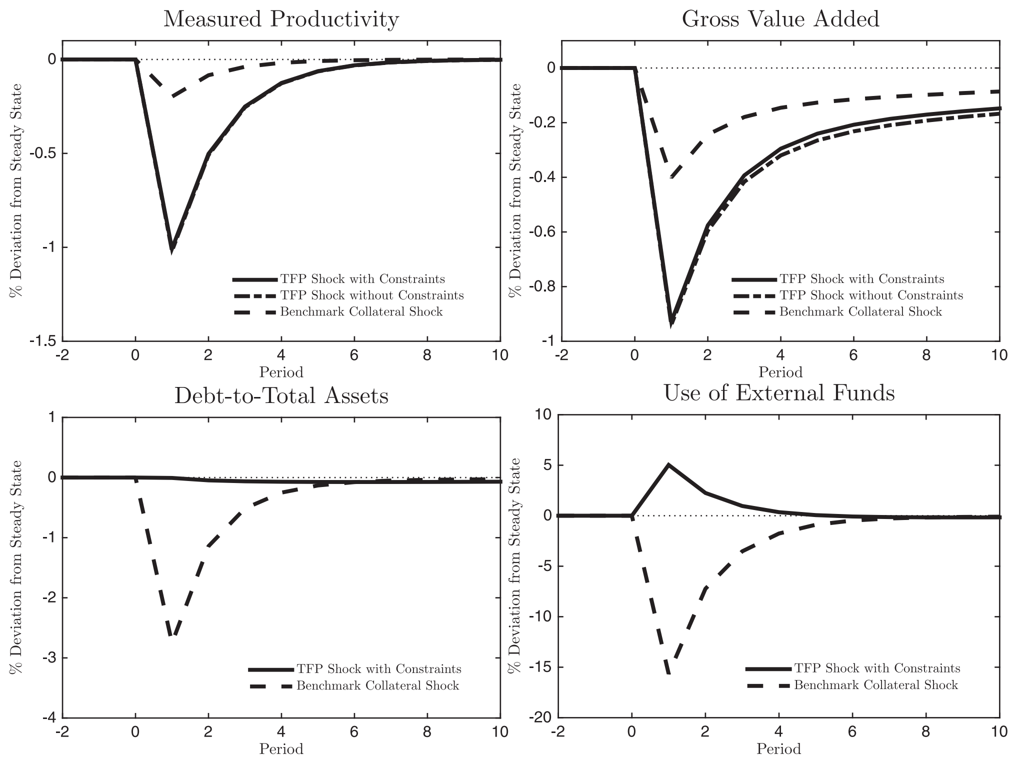


Fig. 7. Impulse response to a total factor productivity (TFP) shock in the model calibrated with parameters reported in Table 3. The solid line in each panel shows the response to a TFP shock. The dashed line shows the response to the financial shock from Fig. 4.

decline in the amount of outstanding commercial and industrial loans which occurred in the United States from 2008 to 2011. To induce such a decline in the stock of debt in the model, the value of firms' collateral must fall by roughly 20 percentage points.[14] This calibrated shock, which is roughly twice as large as the crisis simulation exercise above, causes aggregate GDP to fall by 2%, suggesting that with larger shocks, our model is capable of explaining a sizable percentage of the decline in GDP observed in 2007.

Summary of quantitative findings. First, our experiments show that a financial shock induces roughly half as large an impulse to the economy as a productivity shock but induces a persistent effect on macro aggregates. Moreover, financial shocks do not induce large recessions in the absence of additional amplification mechanisms. Second, we have shown that trade linkages enable financial shocks that directly impact private firms to spill over to public firms, at least in the medium run. In Appendix D, we perform sensitivity analysis with respect to the elasticity of substitution across goods (ρ) and show that while the aggregate consequences of financial shocks are not very sensitive to this parameter, the co-movement between public and private firms is.

Third, we have shown that fixed financial frictions do not amplify productivity shocks. This result depends critically on the precautionary motive of owners of private firms. Since uninsurable exit risk faced by private firms in our model plays an important role in determining the strength of the precautionary motives of the owners of private firms, in Appendix D we perform sensitivity analysis with respect to the exit risk parameter, ζ. We show that matching the use of external funds, independent of the amount of exit risk imposed in the model, plays a key role in determining the responsiveness of the economy to financial shocks. Similarly, if private firms perceived λ as a stochastic variable (holding other parameters fixed), this risk would increase their precautionary motives, dampen the response of the economy, but counterfactually reduce steady state use of external funds for investment. As a result, understanding sensitivity with respect to the risk of financial shocks requires re-calibrating the model with both idiosyncratic and aggregate shocks which is beyond the scope of this paper.

Fourth, our findings illustrate that the behavior of leverage and external financing in response to financial and productivity shocks is very different. It is not surprising that the time series variation in measures from aggregated financial flows

[14] Khan and Thomas (2013) require an 88 percentage point decline in the value of firms' collateral to generate this decline, which suggests that debt is much more sensitive in our model. This sensitivity is most likely due to the assumption that all capital is reversible and can be sold without a discount in our model.

may be useful in distinguishing between productivity and financial shocks. Given longer time series data on financial flows for private firms, these differences could be useful in undertaking such an analysis.

5. Conclusion

Our paper analyzes the extent to which firms use external financing and its implication for how financial shocks affect the economy. Our analysis employs data on firm-level financial flows to argue that private firms use external funds much more extensively than do public firms. In the context of our model, we have demonstrated that replicating patterns of external financing is important for understanding the macroeconomic response to financial shocks.

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Supplementary material

Supplementary material associated with this article can be found, in the online version, at 10.1016/j.jmoneco.2017.08.001.

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