External Financing and the Role of Financial Frictions over the Business Cycle: Measurement and Theory

Online Appendix

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A Data Appendix

A.1 Aggregate Data Sources

A.1.1 U.S. Flow of Funds

We use quarterly data from the flow of funds seasonally adjusted annual rates table F.103 covering Nonfinancial Corporate Business. We define Available Funds as the sum of "Total Internal Funds + IVA" (FA106000105.Q) and "Net Dividends" (FA106121075.Q). We define Investment as "Capital Expenditures" (FA105050005.Q). We normalize Available Funds and Investment by "Gross Value Added of Nonfinancial Corporate Business" which we obtain from the Bureau of Economic Analysis, Table 1.14 ("Gross Value Added of Domestic Corporate Business in Current Dollars and Gross Value Added of Non-Financial Domestic Corporate Business in Current and Chained Dollars") Line 17.

To construct aggregate Debt-to-Total Assets, we use annual data from the flow of funds balance sheet table B.103 covering Nonfinancial Corporate Business. We define Debt as "Liabilities" (FL104190005.Q) and Total Assets as "Assets" (FL102000005.Q).

A.1.2 U.K. Flow of Funds

We use quarterly data from the UK Economic Accounts, tables 1.1.1 ("National Accounts Aggregates"), 3.3.3 ("Income and Capital Accounts: Private non-financial corporations: Allocation of Primary Income Account"), 3.3.4 ("Income and Capital Accounts: Private non-financial corporations: Secondary Distribution of Income Account") and 3.3.7 ("Income and Capital Accounts: Private non-financial corporations: Accumulation Accounts"). We define Available Funds as the sum of "Gross Disposable Income" (RPKZ, Table 3.3.4)) and "Dividend Payments" (RVFT, Table 3.3.3). We define Investment as the sum of "Gross Fixed Capital Formation" (ROAW, Table 3.3.7), "Changes in Inventories" (DLQY, Table 3.3.7) and "Acquisitions less disposals of non-produced non-financial assets" (RQBW, Table 3.3.7). We normalize Available Funds and Investment by "Gross National Income at market prices" (ABMZ, Table 1.1.1).

A.2 Cross-Sectional Data Sources

For all cross-sectional data sources, we exclude firms in the following industries according to 4-digit SIC classifications: Postal Services (4300-4399), Utilities (4900-4999), Finance, Insurance and Real Estate (6000-6999), and Other Government ($\geqslant$9000).

A.2.1 U.S. Compustat

We obtain data on U.S. publicly traded firms from the Compustat Monthly Updates – Fundamentals Annual File, North America from WRDS. We restrict attention to firms in Compustat which report Consolidated statements and standardized data, are in the domestic population source, and are active. We use the Historical Standard Industrial Classiciation to classify firms into industries.

For each firm, we define Available Funds as "Operating Activities – Net Cash Flow" (annual data item 308) if the firm reports its statement of cash flows using format code 7 (annual data item 318). Otherwise, we define Available Funds as "Funds from Operations – Total" (annual data item 110). We define Investment as the sum of "Capital Expenditures (Statement of Cash Flows)" (annual data item 128) and "Acquisitions (Statement of Cash Flows)" (annual data item 129) less "Sale of Property, Plant and Equipment (Statement of Cash Flows)" (annual data item 107).

We define net debt as "Liabilities – Total" (annual data item 181) minus "Cash and Short-Term Investments" (annual data item 1) and "Receivables – Total" (annual data item 2). We use "Assets – Total" (annual data item 6) for total assets. We then define a firm's return on assets in period t as Available Funds in period t divided by Total Assets in period $t-1$. Similarly, we define investment to total assets in period t as investment in period t divided by total assets in period t and net debt to total assets as net debt in period t divided by total assets in period t.

We drop firms who do not report available funds or capital expenditures. We code missing values for acquisitions, sale of property, plant and equipment, cash and short-term investments, and receivables as 0 unless the firm reports a missing value or combined data item for these objects in which case we drop the entire firm-year observation. Finally, we restrict attention to firms with positive assets, liabilities, and sales.

A.2.2 U.K. Compustat

We obtain data on U.K. publicly traded firms from the Compustat Monthly Updates – Fundamentals Annual File, Global from WRDS. We restrict attention to firms in Compustat which report Consolidated statements and standardized data, report their location as Great Britain, report their financial statements in British Pounds, and are active. We use the Historical Standard Industrial Classification to classify firms into industries.

For each firm, we define Available Funds as "Operating Activities – Net Cash" (data item G692). We define Investment as the sum of "Capital Expenditures" (data item G676) and "Acquisitions" (data item G681) or "Acquisitions and Disposals - Net Cash Flow" (mnemonic ACQDISN) less “Proceeds from Sale of Fixed Assets” (mnemonic PSFIX) or “Sale of Tangible Fixed Assets” (mnemonic STFIXA).

We define net debt as “Liabilities – Total” (data item G171) minus “Cash and Cash Equivlanets - Increase (Decrease)” (data item G684) and “Accounts Receivables/Debtors – Total” (data item G629). We use “Assets – Total” (data item G107) for total assets.

As with public firms in the U.S., we then define a firm’s return on assets in period t as Available Funds in period t divided by Total Assets in period $t-1$. Similarly, we define investment to total assets in period t as investment in period t divided by total assets in period t and net debt to total assets as net debt in period t divided by total assets in period t. Additionally. we exclude firm-year observations which do not report available funds (data item G692) or capital expenditures (data item G676). We code missing values for acquisitions, sale of property, plant and equipment, cash and short-term investments, and receivables as 0 unless the firm reports a missing value or combined data item for these objects in which case we drop the entire firm-year observation. Finally, we restrict attention to firms with positive assets, liabilities, and sales.

A.2.3 U.K. Amadeus

We obtain data on U.K. privately held firms from the Amadeus database created by the Bureau van Dijk and maintained by WRDS. We obtain data from all firm size datasets (Very Large, Large, Medium, and Small). We restrict attention to firms located in Great Britain with stated company type as "Private Limited Company" and which are not publicly quoted (variable QUOTED is "No"). Further, we restrict attention to those firms reporting positive total assets (mnemonic TOAS), positive fixed assets (mnemonic FIAS), positive total liabilities where total liabilities is defined as the sum of Current (mnemonic CULI) and Non-Current (mnemonic NCLI) Liabilities. We require firm’s balance sheet to add up correctly (TOAS is equal to total shareholders’ funds and liabilities, TSHF).

For a given firm-year observation, we construct available funds as the sum of “Profit (loss) for Period)" (mnemonic PL) and “Depreciation” (mnemonic DEPR). We construct investment in year t as tangible fixed assets (mnemonic TFAS) at end of year (in year t) minus lagged tangible fixed assets from year $t-1$ plus depreciate declared in year t. As with public firms in the U.S. and the U.K, we then define a firm’s return on assets in period t as Available Funds in period t divided by Total Assets in period $t-1$. Similarly, we define investment to total assets in period t as investment in period t divided by total assets in period t and net debt to total assets as net debt in period t divided by total assets in period t. Lastly, we exclude firms for which lagged total assets, available funds, or investment (or any component of investment) are not available.

B Sensitivity Analysis of Empirical Findings

In this Appendix, we perform a subsample analysis of the use of external funds.

B.1 Industry Subsample Analysis

The first source of heterogeneity we consider is the difference in industry composition. One possibility is that private firms are concentrated more heavily in high external financing industries. This is not the case in our sample. While we do find differences in the industry composition of our public and private firm sample, we also find that within each industry, private firms use more external funds as a share of private firm investment than do public firms.

Table 1 reports each industry's average share of investment and use of external funds by company type (i.e., private or public). Patterns of investment shares by industry are broadly similar between public and private firms. Not surprisingly, the services sector accounts for a greater deal of investment in private firms, while the manufacturing sector accounts for a larger share of investment by public firms.

	Investment Share		**Use of Ext. Fin.**	
Industry	Private	Public	Private	Public
Agriculture	0.58%	0.05%	0.39%	0.01%
Construction	-1.32%	0.17%	9.61%	1.10%
Manufacturing	19.53%	34.71%	12.93%	7.28%
Mining	17.68%	2.21%	5.85%	0.84%
Retail Trade	10.31%	18.74%	5.78%	2.00%
Services	30.64%	8.85%	26.89%	2.21%
Transportation	17.39%	35.19%	16.99%	4.26%
Wholesale Trade	5.20%	1.03%	3.20%	0.53%
Total	100%	100%	81.64%	18.23%

Table 1: Share of public or private investment and use of external funds by industry in the United Kingdom.

Table 1 clearly illustrates that total use of external funds by private firms in each industry as a share of total private investment is larger than the analogous number for public firms. In other words, the contribution to the total use of external funds by private firms from each industry is substantially larger than that from public firms at the broad industry classification level.

Figures 1 and 2 reveal the differences in investment shares and external financing by listing status and industry and clearly show that the differences between public and private firms persist over time.

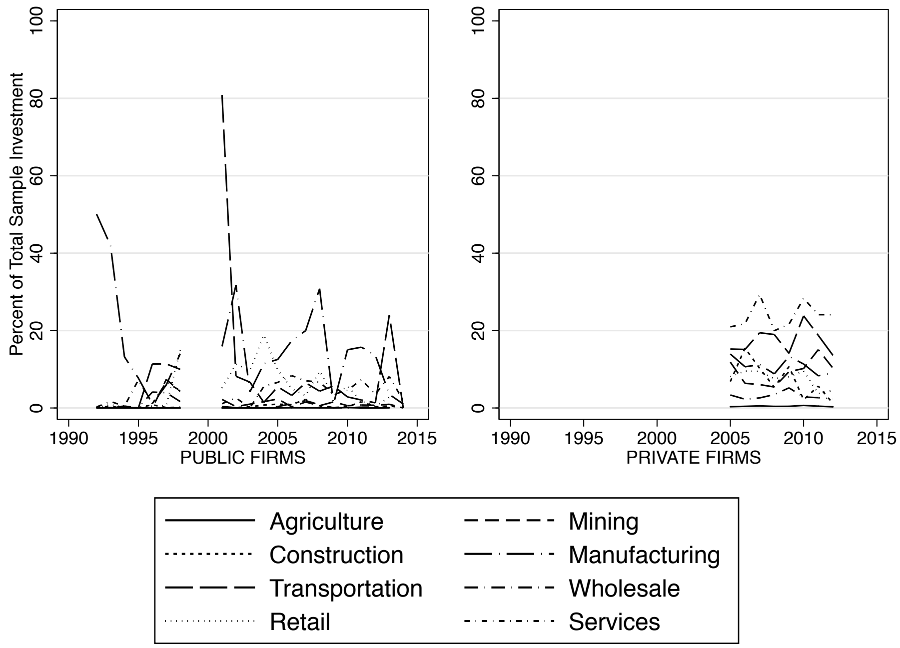

Figure 1: Investment Share by Industry and Listing Status

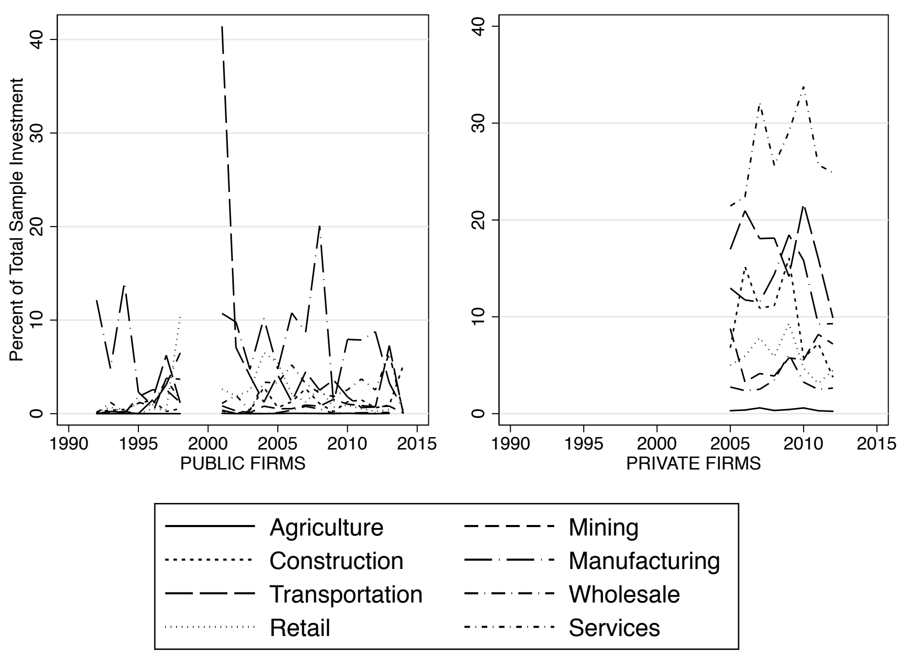

Figure 2: External Financing by Industry and Listing Status

B.2 Size Subsample Analysis

The second source of heterogeneity we consider is firm size. It is possible that dividing our sample into public and private firms is simply capturing differences in size, because we expect those firms that typically become public to be among the larger private firms. While public firms are typically larger than private firms, we show that large private firms use more external funds (as a share of total private investment) than do comparably sized large public firms.

With the understanding that private firms are typically smaller than public firms, we now seek to construct comparable samples by size and compare the use of external firms of similarly sized public and private firms. To do so, for each year we partition our sample of public firms into quartiles by total assets.[1] This partition yields three thresholds per year, which we then use to partition our sample of privately held firms. As a result, if a single private company is in the largest quartile, for example, and if that single firm were to become publicly traded, it would be in the top quartile among public firms. Table 2 reports sample moments of total assets, sales, and investment to compare the relative sizes of public and private firms in these quartiles.

	N		**Total Assets**		**Sales**		**Investment**	
Quartile	Private	Public	Private	Public	Private	Public	Private	Public
Q1	631,028	2,519	1.11	9.19	2.00	12.23	0.04	0.28
Q2	45,596	2,506	24.53	43.04	38.08	55.84	0.94	1.87
Q3	20,709	2,507	115.46	181.21	143.59	237.52	4.49	9.15
Q4	6,135	2,508	1,512.20	4,095.51	1,103.15	3,378.74	46.00	190.12

Table 2: Sample Statistics for Public and Private firms by Public Firm Asset Quartiles in the United Kingdom. **N**: Number of Firm-Year Observations. Total Assets, Sales, Investment reported in Millions of Pounds.

Within each public or private sample, for each quartile, we measure total use of external funds and normalize by total sample investment in that year. Thus, for each quartile, we find the contribution of that quartile's use of external funds to the overall sample's use of external financing.

Table 3 reports the time series average of each quartile's investment share, each quartile's use of external funds, and the investment share of those firms using external funds for public and private firms. Figure 3 displays these measures for each quartile over time.

[1] We have also repeated this exercise using deciles of the firm size distribution and obtained similar findings. Details are available upon request.

	Investment Share		Investment Share External Financing		Firms Using Ext. Funds	
Quartile	Private	Public	Private	Public	Private	Public
Q1	6.03%	0.18%	8.25%	0.44%	4.60%	0.13%
Q2	9.83%	1.27%	9.69%	0.93%	7.31%	0.78%
Q3	21.55%	5.25%	17.93%	2.19%	16.12%	2.82%
Q4	62.59%	93.34%	45.76%	14.55%	51.57%	32.40%

Table 3: Time series average of investment share, use of external funds, and investment share of firms using external funds for public and private firms by public firm asset quartiles in the United Kingdom.

We draw two conclusions from Table 3. First, private firms at all asset classes use substantially more external funds, as a share of aggregate private investment, than do the largest public firms. Second, among private firms, those firms that use external funds also undertake the majority of investment. For example, in the largest quartile, firms using external funds undertake over 50% of all private firm investment, while all firms in the largest quartile account for roughly 62% of aggregate private firm investment. Figure 4 demonstrates that over the entire sample period where both public and private firm data are available, the differences in use of external funds by size are prevalent and sizable.[2]

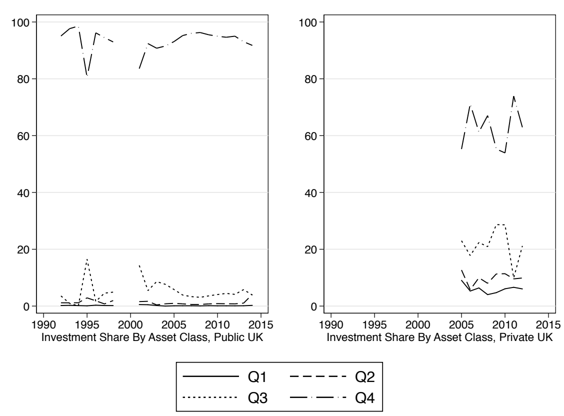

Figure 3: Investment Shares by Asset Quartile and Listing Status

To summarize, we have documented a large degree of heterogeneity in external financing across firms. This finding suggests that the role of financial markets in reallocating capital across firms is important. In addition, we have established that private

[2]We have also examined differences in use of external funds by measures of firm-level growth rates. Within each quartile of firm growth, we also find that private firms use substantially more external funds for investment. Details on this finding are available upon request.

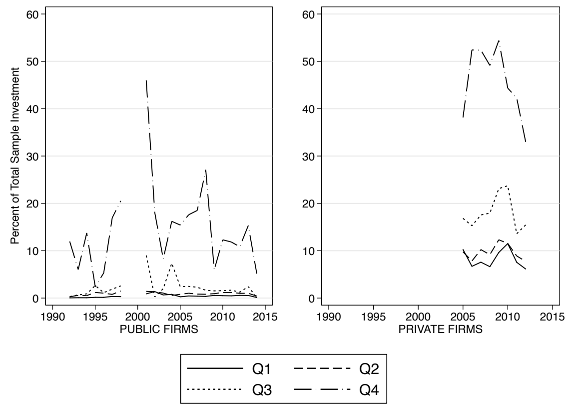

Figure 4: External Financing by Asset Quartile and Listing Status

firms use more external funds for investment than do public firms. This difference holds even when controlling for size, industry, and profitability, and is an order of magnitude larger than the difference arising from size and industry. Thus, the role of financial markets in reallocating capital across private firms is particularly important. We interpret the difference in the use of external funds between public and private firms as arising from differences in these firms' ability to diversify idiosyncratic shocks to firm-level productivity.

C Proofs of Theoretical Results

C.1 Equilibrium Definition and Production and Saving Decisions of Privately Held Firms

In this section, we define a recursive, competitive equilibrium and provide a characterization of the privately held firm's optimal capital, labor, and savings decisions.

Definition 1. A stationary recursive competitive equilibrium consists of prices $p_i(a,z), r, w$, aggregate output Q, distributions $G_l(a,z), G_u(a,z)$, value functions $V_l(a,z), V_u(a,z)$, policy functions for firms $\left(d_i(a,z), a'_i(a,z), k_i(a,z), l_i(b,k,z), I_i(b,k,z)\right)_{i\in l,u}$, households' asset holding A^h, and policy functions for households, $C_h(A), L_h(A), A'(A)$ such that

1. Given aggregate output, the wage, the interest rate, and the inverse demand curve of the final good producer, $(d_u(a,z), a'_u(a,z), k(a,z), l_u(a,z))$ and $V_u(a,z)$ solve the problem of a private firm given by

$$
\begin{aligned}
V_u(a,z) &= \max_{d,a',k,l,I} \log(d) + \beta(1-\zeta)\int_{z'} V_u(a',z')d\Psi(z'|z) && (1)\\
&\quad \text{subject to} \\
d + a' &\leqslant p_u(a,z)z\left(k^{\alpha}l^{1-\alpha}\right)^{\eta} I^{1-\eta} - wl - I - (r+\delta)k \\
&\quad +(1+r)a && (2)\\
k &\leqslant \lambda a \\
p_u(a,z) &= Q^{\frac{1}{\rho}}\left(z\left(k^{\alpha}l^{1-\alpha}\right)^{\eta} I^{1-\eta}\right)^{-\frac{1}{\rho}}
\end{aligned}
$$

2. Given aggregate output, the wage, the interest rate, and the inverse demand curve of the final good producer, $\left(d_l(a,z), a'_l(a,z), k_l(a,z), l_l(a,z), I_l(a,z)\right)$ and $V_l(a,z)$ solve the problem of a public firm given by

$$
\begin{aligned}
V_l(a,z) &= \max_{d,a',k,l,I} d + \frac{1}{1+r}E_{z'}V_l(a',z') && (3)\\
&\quad \text{subject to} \\
d + a' &\leqslant p_l(a,z)z\left(k^{\alpha}l^{1-\alpha}\right)^{\eta} I^{1-\eta} - wl - I - (r+\delta)k \\
&\quad +(1+r)a && (4)\\
k &\leqslant \lambda a \\
p_l(a,z) &= Q^{\frac{1}{\rho}}\left(z\left(k^{\alpha}l^{1-\alpha}\right)^{\eta} I^{1-\eta}\right)^{-\frac{1}{\rho}}
\end{aligned}
$$

subject to (4), (5), and (6) in the main text;

3. Given dividend payments of public firms and the wage rate, household consumption and labor supply solve

$$\begin{aligned}
V^h(A) = & \quad \max U(C_h, L_h) + \beta V^h\left(A'\right) \\
& \quad \text{subject to} \\
C_h + A' \; = & \quad wL_h + (1-s)\int d(a,z)G_l(da,dz) + (1+r)A
\end{aligned}$$

4. Markets clear:

   (a) Final goods:

$$C_h + n_u \int_{a,z} d_u(a,z)G_u(da,dz) + K - (1-\delta)K = Q - \sum_{i=l,u} n_i \int_{a,z} I_i(a,z)G_i(da,dz), \tag{5}$$

   where n_i is the measure of firms of type i firms (either public or private) and K is the aggregate capital stock given by

$$K = \sum_{i=l,u} n_i \int_{a,z} k_i(a,z)G_i(da,dz)$$

   (b) Intermediate goods:

$$q_i(a,z) = z\left(k_i(a,z)^{\alpha} l_i(a,z)^{1-\alpha}\right)^{\eta} I_i(a,z)^{1-\eta}, \text{ for } i = l, n \tag{6}$$

   (c) Labor:

$$L_h = \sum_{i=l,u} n_i \int_{a,z} l_i(a,z)G_i(da,dz). \tag{7}$$

The distributions G_l and G_u are stationary:

$$G_i(\mathcal{A},\mathcal{Z}) \;=\; \int_{a,z} \mathbf{1}\left\{a_i'(a,z) \in \mathcal{A}\right\} \Psi(\mathcal{Z}|z)\, G_i(da,dz). \tag{8}$$

The problem of a surviving privately held firm can be written recursively as

$$
\begin{aligned}
& V_u(a,z) = \max_{c,a',k,l} u(c) + \beta\zeta E[V_u(a',z')|z] \\
& \text{s.t.} \\
& c + a' \leqslant p(a,z) z^{\frac{1}{\rho-1}} \left(k^\alpha l^{1-\alpha}\right)^\eta I^{1-\eta} - wl - (r+\delta)k - I + (1+r)a \\
& p(a,z) = Q^{\frac{1}{\rho}} \left(z^{\frac{1}{\rho-1}} \left(k^\alpha l^{1-\alpha}\right)^\eta I^{1-\eta}\right)^{-\frac{1}{\rho}} \\
& k \leqslant \lambda a \\
& a_0 \text{ given}
\end{aligned}
$$

The decision of how much capital to install and how much labor to hire is a static one. We therefore can use the results from our static model, namely Lemma (2) to define profits of a privately held firm, which we denote $\Pi_u(a,z;w,r,Q)$. Then the problem of an private firm can be simplified to a consumption and savings problem written as

$$
\begin{aligned}
V^u(a,z) &= \max_{c,a'} u(c) + \beta\zeta E\left[V^u(a',z')|z\right] \\
& \quad \text{s.t.} \\
c + a' &\leqslant \Pi_u(a,z;w,r,Q) + (1+r)a
\end{aligned}
$$

As in Aiyagari (1994), the only intertemporal effect of the borrowing constraint comes from distorting the savings decisions. The nature of the borrowing constraint ensures that this happens in a smooth way. Thus, the optimal savings decision can be solved as in Aiyagari (1994), except for the extra term that captures the effect of savings on the next period's profit function. We then have the following lemma, in which we suppress the dependence of Π_u on equilibrium parameters w, r, and Q and write it only as $\Pi_u(a,z)$.

Lemma 1. *For a private firm, the optimal asset position policy is given by a function $a'(a,z)$ that satisfies*

$$
\begin{aligned}
& u'(\Pi(a,z) + (1+r)a - a'(a,z)) \\
= \;& \beta\zeta E\left\{\left[\Pi_a^U(a'(a,z),z') + (1+r)\right] u'(\Pi(a'(a,z),z') + (1+r)a'(a,z) - a'(a'(a,z),z'))|z\right\}
\end{aligned}
$$

C.2 Equilibrium Results Concerning Publicly Firms

We now prove Proposition 1 from the paper.

Proposition 1. *Suppose that in a stationary equilibrium, the interest rate is positive and productivity is bounded above. Then the collateral constraint never binds for any public firm. In addition, the amount of external funds used by public firms to finance investment is indeterminate.*

Proof of Proposition 1. Recall that public firms solve

$$\begin{aligned}
&\max E_0 \sum_t \beta^t d_t \\
&d_t + a_{t+1} \leqslant f(z_t, k_t) + (1+r)a_t \\
&k_t \leqslant \lambda_t a_t \\
&d_t \geqslant 0
\end{aligned}$$

or

$$\max E_0 \sum_t \beta^t \left[(f_t(z_t, k_t) + (1+r)a_t - a_{t+1})(1+\eta(z^t)) + \mu(z^t)\left(\lambda a_t - k_t\right) \right].$$

The optimality conditions are

$$\begin{aligned}
f'(z_t, k_t) &= \mu(z^t) \\
(1+\eta(z^t)) &= \beta E_t \left[(1+r)\left(1+\eta(z_{t+1}, z^t)\right) + \mu(z_{t+1}, z^t)\lambda \right]
\end{aligned}$$

where the last may be re-written using $\beta(1+r) = 1$:

$$\eta(z^t) = \beta E_t \left[(1+r)\eta(z_{t+1}, z^t) + \mu(z_{t+1}, z^t)\lambda \right].$$

Thus, if $\eta(z_{t+1}, z^t)$ or $\mu(z_{t+1}, z^t)$ are positive for any z_{t+1} with strictly positive probability following z^t then $\eta(z^t) = 0$ and

$$a_{t+1}(z^t) = f(z_t, k_t) + (1+r)a_t.$$

Since $f \geqslant 0$ and $1 + r > 1$, the firm's assets grow (strictly) as long as constraints ever bind in the future.

Next, the optimal unconstrained capital scale satisfies

$$f'(z_t, k_t) = 0.$$

Let the optimal scale be defined as

$$k^*(z) = \left(f'\right)^{-1}(0, z).$$

It is easy to show that $k^*(z)$ is increasing in z. Suppose $z \in [\underline{z}, \bar{z}]$. Then, if $a_t \geqslant \lambda k^*(\bar{z})$, the constraint does not bind currently and choosing any $a_{t+1} \geqslant a_t$ will ensure that the constraint never binds again in the future. Suppose at time s the firm's assets satisfy $a_s \geqslant \lambda k^*(z)$. In this case, the firm solves

$$\max E_s \sum_{t \geqslant s} \beta^{t-s} \left[(f_t(z_t, k_t) + (1+r)a_t - a_{t+1}) \right],$$

and since $\beta(1+r) = 1$ this simplifies to

$$E_s \sum_{t \geqslant s} \beta^{t-s} \left[(f_t(z_t, k_t) \right] + a_t(1+r).$$

Note then, in the steady state, that no publicly held firms will be constrained (assets are increasing until the constraint never binds and then are indeterminate above $\bar{a}$). Then, investment for any firm is simply

$$k^*(z_{t+1}) - (1-\delta)k^*(z_t).$$

Thus, investment is bounded above by

$$k^*(\bar{z}) - (1-\delta)k^*(\underline{z}).$$

However, available funds, in the steady state, are not pinned down since they are equal to

$$f(z_t, k^*(z_t)) - r(k^*(z_t) - a_t).$$

Since the firm is indifferent between any $a_t \geqslant \bar{a}$, there are a continuum of equilibria with different steady state asset holdings of publicly held firms each corresponding to different amounts of external financing.

C.3 Analysis of Non-financial Linkages in a Static Version of our Benchmark Model

In this section, we develop a static version of our model in order to establish the main economic mechanism of the model. We use the static model to illustrate how shocks to the collateral constraint affects both firms for whom the constraint binds and also those for whom the constraint does not bind. We derive a sufficient condition on the model parameters under which a tightening of the collateral constraint leads all firms to reduce output in equilibrium, justifying Proposition 2 in the paper.

Consider a static economy populated by a continuum of intermediate good firms, a representative final good producer, and a representative household.

Intermediate Good Producers. Each intermediate good firm $i \in [0,1]$ has an asset level a_i and productivity z_i. Moreover, (a_i, z_i) is distributed according to $F(a,z)$. An intermediate good firm with productivity z_i and assets a_i, rents labor, l, and capital, k, rents out its assets a_i and purchases an amount of the final good, I, to be used as an input to production according to the production function

$$q_i = z_i^{\frac{1}{\rho-1}} \left(k^{\alpha} l^{1-\alpha}\right)^{\eta} I^{1-\eta}.$$

Each firm may rent capital up to a multiple of the value of the firm's assets. Specifically, we impose a *collateral constraint* so that the amount of capital rented by a firm with asset level a_i is bounded by λa_i where $\lambda \geqslant 1$. One can rationalize this type of constraint by a model of moral hazard or limited enforcement. In line with the rest of the literature, we impose this constraint and do not provide a formal micro foundation for it.

Final Good Producer. The final good producer uses a bundle of inputs purchased from intermediate good firms and takes their prices as given. Given a bundle $\{q_i\}_{i\in[0,1]}$, the final good producer uses the following Dixit-Stiglitz production function:

$$Q = \left[\int_0^1 q_i^{\frac{\rho-1}{\rho}} dF(i)\right]^{\frac{\rho}{\rho-1}}$$

with $\rho > 1$.

Households. We assume that there is a representative households who buys the final good and provides labor to intermediate good producers. Following Greenwood et al. (1988), we assume that household preferences are given by

$$U\left(c - \psi \frac{l^{1+\frac{1}{\varepsilon}}}{1+\frac{1}{\varepsilon}}\right).$$

Markets. We assume that the labor market is competitive at wage level w. As for capital market, we assume that the economy is small and open. That is, there exist suppliers of capital that have deep pockets and inelastically supply capital at a given interest rate r. This is an assumption that simplifies the analysis; under this assumption, when we perform comparative statics, we do not need to consider general equilibrium effects that arise from changes in the interest rate for capital. Similar analysis can be done in a closed economy.

Moreover, we assume that there is monopolistic competition across intermediate good firms and prices are given by p_i. The final good producer takes these prices as given and intermediate good producers take the demand function for intermediate output as given. We normalize the price of final good to 1.

Given the above market structure, the final good producer's maximization problem is given by

$$\max_{q_i} \left[\int_0^1 q_i^{\frac{\rho-1}{\rho}} dF(i)\right]^{\frac{\rho}{\rho-1}} - \int_0^1 p_i q_i dF(i).$$

The resulting demand for intermediate good i is given by

$$q_i^{-\frac{1}{\rho}} Q^{\frac{1}{\rho}} = p_i.$$

Given this demand function, each intermediate good firm maximizes its profit subject to its collateral constraint:

$$\pi_i = \max_{k,l,I,p_i} p_i z_i^{\frac{1}{\rho-1}} \left(k^{\alpha} l^{1-\alpha}\right)^{\eta} I^{1-\eta} - wl - rk - I + ra_i \tag{9}$$

subject to

$$\begin{aligned} p_i &= Q^{\frac{1}{\rho}} \left(z_i^{\frac{1}{\rho-1}} \left(k^{\alpha} l^{1-\alpha}\right)^{\eta} I^{1-\eta}\right)^{-\frac{1}{\rho}}, \\ k &\leqslant \lambda a_i. \end{aligned}$$

We say that a firm is *financially constrained* if the collateral constraint is binding for a firm in equilibrium and is *financially unconstrained* otherwise.

To complete the definition of competitive equilibrium, we need to specify the household's optimization problem as well as market clearing conditions. The representative household's optimization problem is given by

$$\max_{c,L} U\left(c - \psi \frac{L^{1+\frac{1}{\varepsilon}}}{1+\frac{1}{\varepsilon}}\right)$$

subject to

$$c \leqslant wL + \int_0^1 \pi_i dF(i).$$

Labor market and product market clearing are given by

$$\begin{aligned} \int_0^1 l_i dF(i) &= L \\ c + \int_i I_i dF(i) &= Q. \end{aligned}$$

Hence a competitive equilibrium of this economy is given by $\left\{\{k_i, l_i, I_i, p_i\}_{i\in[0,1]}, c, L, Q, w\right\}$ that satisfies the above conditions.

Because of monopolistic competition, the revenue function of the firm exhibits decreasing returns to scale. As a result, for every z, there is an unconstrained optimal scale, which is increasing in z. Not surprisingly, then, every z, there is a threshold in assets, say $a^*(z)$ such that firms with assets and productivity (a, z) with $a \geqslant a^*(z)$ are financially unconstrained and if $a < a^*(z)$ the firm is financially constrained. We state this result along with optimal capital, labor, and intermediate input decisions for firms in the following lemma (the proof is omitted).

Lemma 2. *For every z, there exists $a^*(z)$ such that for $a \geqslant a^*(z)$, $k(a,z) \leqslant \lambda a$ and for $a < a^*(z), k = \lambda a$. Furthermore, $a^*(z)$ satisfies*

$$a^*(z) = \frac{1}{\lambda}\left[\nu(1-\eta)\right]^{\frac{(1-\eta)\nu}{1-\nu}} \left(\frac{\alpha\eta\nu}{\hat{r}}\right)^{1+\frac{\alpha\eta\nu}{1-\nu}} \left(\frac{\nu(1-\alpha)\eta}{w}\right)^{\frac{(1-\alpha)\eta\nu}{1-\nu}} Qz.$$

If $a \geqslant a^*(z)$ then

$$k(a,z) = \left[\nu(1-\eta)\right]^{\frac{(1-\eta)\nu}{1-\nu}} \left(\frac{\alpha\eta\nu}{\hat{r}}\right)^{1+\frac{\alpha\eta\nu}{1-\nu}} \left(\frac{\nu(1-\alpha)\eta}{w}\right)^{\frac{(1-\alpha)\eta\nu}{1-\nu}} Qz$$

if $a < a^*(z)$ then $k(a,z) = \lambda a$. Finally,

$$\begin{aligned} l &= \left(\frac{\nu(1-\alpha)\eta}{w}\right)^{\frac{1-(1-\eta)\nu}{1-(1-\alpha\eta)\nu}} \left(\nu(1-\eta)\right)^{\frac{(1-\eta)\nu}{1-(1-\alpha\eta)\nu}} (Qz)^{\frac{(1-\nu)}{1-(1-\alpha\eta)\nu}} k^{\frac{\alpha\eta\nu}{1-(1-\alpha\eta)\nu}} \\ I &= \left[\nu(1-\eta)(Qz)^{1-\nu}\left(k^{\alpha} l^{1-\alpha}\right)^{\eta\nu}\right]^{\frac{1}{1-(1-\eta)\nu}} \end{aligned}$$

where $\nu = 1 - \frac{1}{\rho}$.

Given the decisions of firms along with optimal labor supply of households, we can characterize equilibrium output, Q and the wage rate, w using the production function of the final good producer and the labor market clearing conditions. Given these equilibrium values, we can show that the equilibrium wage rate is increasing in the collateral constraint parameter λ. This final result follows because a relaxing of the collateral constraint induced by an increase in λ must increase aggregate capital demand and therefore labor demand causing the wage to rise. We have the following lemma.

Lemma 3. *Any competitive equilibrium must satisfy* $Q = \frac{1}{(1-\alpha)\eta\left(1-\frac{1}{\rho}\right)}\psi^{-\varepsilon}w^{1+\varepsilon}$. *Moreover, the equilibrium wage, w, is increasing in λ.*

Proof. Given capital, labor and intermediate input demand, we can construct aggregate excess output and labor as functions of prices r, w, and output Q. Specifically, let $A^* = \{(a, z) : a \geqslant a^*(z)\}$

$$Q^{\nu} = \int q^{\nu} G(da, dz) \tag{10}$$

$$\begin{aligned} &= \left(\frac{\nu(1-\alpha)\eta}{w}\right)^{\frac{\nu\eta(1-\alpha)}{1-\nu}} (\nu(1-\eta))^{\frac{\nu(1-\eta)}{1-\nu}} \left(\frac{\alpha\eta\nu}{\hat{r}}\right)^{\frac{\alpha\eta\nu}{1-\nu}} Q^{\nu} \int_{(a,z)\in A^*} zG(da, dz) \\ &+ \left(\frac{\nu(1-\alpha)\eta}{w}\right)^{\frac{\nu\eta(1-\alpha)}{1-(1-\alpha\eta)\nu}} (\nu(1-\eta))^{\frac{\nu(1-\eta)}{1-(1-\alpha\eta)\nu}} Q^{\frac{\nu(1-\nu)(1-\alpha\eta)}{1-(1-\alpha\eta)\nu}} \lambda^{\frac{\alpha\eta\nu}{1-(1-\alpha\eta)\nu}} \times \\ &\int_{(a,z)\notin A^*} z^{\frac{(1-\nu)}{1-(1-\alpha\eta)\nu}} a^{\frac{\alpha\eta\nu}{1-(1-\alpha\eta)\nu}} G(da, dz). \end{aligned} \tag{11}$$

Labor Demand is just a function of output. We have

$$l = \left(\frac{(1-\alpha)\eta\nu}{w}\right) Q^{1-\nu} q^{\nu}.$$

Thus

$$\int lG(da, dz) = \left(\frac{(1-\alpha)\eta\nu}{w}\right) Q^{1-\nu} \int q^{\nu} G(da, dz).$$

Household labor supply given our assumed (GHH) form for household preferences is just $\psi^{-\varepsilon}w^{\varepsilon}$.Thus labor market clearing is

$$\psi^{-\varepsilon}w^{\varepsilon} = \left(\frac{(1-\alpha)\eta\nu}{w}\right) Q$$

so that aggregate output in equilibrium satisfies

$$Q = \psi^{-\varepsilon}((1-\alpha)\eta\nu)^{-1}w^{1+\varepsilon}.$$

Re-write Q from (10) as

$$
\begin{aligned}
1 \;=\;& \left(\frac{\nu(1-\alpha)\eta}{w}\right)^{\frac{\nu\eta(1-\alpha)}{1-\nu}} (\nu(1-\eta))^{\frac{\nu(1-\eta)}{1-\nu}} \left(\frac{\alpha\eta\nu}{\hat{r}}\right)^{\frac{\alpha\eta\nu}{1-\nu}} \int_{(a,z)\in A^*} zG(da,dz) \qquad (12)\\
&+\left(\frac{\nu(1-\alpha)\eta}{w}\right)^{\frac{\nu\eta(1-\alpha)}{1-(1-\alpha\eta)\nu}} (\nu(1-\eta))^{\frac{\nu(1-\eta)}{1-(1-\alpha\eta)\nu}} Q^{\frac{-\alpha\eta\nu}{1-(1-\alpha\eta)\nu}} \lambda^{\frac{\alpha\eta\nu}{1-(1-\alpha\eta)\nu}} \times \qquad (13)\\
&\int_{(a,z)\notin A^*} z^{\frac{(1-\nu)}{1-(1-\alpha\eta)\nu}} a^{\frac{\alpha\eta\nu}{1-(1-\alpha\eta)\nu}} G(da,dz).
\end{aligned}
$$

Analyzing the derivative with respect to λ, we have

$$
\begin{aligned}
0 \;=\;& \left(\frac{\nu(1-\alpha)\eta}{w}\right)^{\frac{\nu\eta(1-\alpha)}{1-\nu}} (\nu(1-\eta))^{\frac{\nu(1-\eta)}{1-\nu}} \left(\frac{\alpha\eta\nu}{\hat{r}}\right)^{\frac{\alpha\eta\nu}{1-\nu}} \int_{(a,z)\in A^*} zG(da,dz)\\
&\times\left[-\frac{\nu\eta(1-\alpha)}{1-\nu}\frac{1}{w}\frac{dw}{d\lambda}\right]\\
&+\left(\frac{\nu(1-\alpha)\eta}{w}\right)^{\frac{\nu\eta(1-\alpha)}{1-(1-\alpha\eta)\nu}} (\nu(1-\eta))^{\frac{\nu(1-\eta)}{1-(1-\alpha\eta)\nu}} Q^{\frac{-\alpha\eta\nu}{1-(1-\alpha\eta)\nu}} \lambda^{\frac{\alpha\eta\nu}{1-(1-\alpha\eta)\nu}}\\
&\times\int_{(a,z)\notin A^*} z^{\frac{(1-\nu)}{1-(1-\alpha\eta)\nu}} a^{\frac{\alpha\eta\nu}{1-(1-\alpha\eta)\nu}} G(da,dz)\\
&\times\left[-\frac{\nu\eta(1-\alpha)}{1-(1-\alpha\eta)\nu}\frac{1}{w}\frac{dw}{d\lambda}-\frac{\alpha\eta\nu}{1-(1-\alpha\eta)\nu}\frac{1}{Q}\frac{dQ}{d\lambda}+\frac{\alpha\eta\nu}{1-(1-\alpha\eta)\nu}\frac{1}{\lambda}\right].
\end{aligned}
$$

Suppose $\frac{dw}{d\lambda} \leqslant 0$. Then $\frac{dQ}{d\lambda} \leqslant 0$. Then we must have

$$
-\frac{\nu\eta(1-\alpha)}{1-(1-\alpha\eta)\nu}\frac{1}{w}\frac{dw}{d\lambda}-\frac{\alpha\eta\nu}{1-(1-\alpha\eta)\nu}\frac{1}{Q}\frac{dQ}{d\lambda}+\frac{\alpha\eta\nu}{1-(1-\alpha\eta)\nu}\frac{1}{\lambda} \leqslant 0
$$

and since

$$
\frac{\alpha\eta\nu}{1-(1-\alpha\eta)\nu}\frac{1}{\lambda} > 0
$$

we have

$$
\frac{\nu\eta(1-\alpha)}{1-(1-\alpha\eta)\nu}\frac{1}{w}\frac{dw}{d\lambda}+\frac{\alpha\eta\nu}{1-(1-\alpha\eta)\nu}\frac{1}{Q}\frac{dQ}{d\lambda} > 0
$$

but the coefficients are all positive so this is a contradiction. As a result, the wage must be increasing in λ. □

We are now ready to state our necessary and sufficient condition for a tightening of the collateral constraint to cause both financially constrained and financially unconstrained firms to decrease output. Using the optimal production decisions of firms, we can show that a financially unconstrained firm's output satisfies

$$\begin{aligned} q_i &= \left(\frac{\nu(1-\alpha)\eta}{w}\right)^{\frac{\eta(1-\alpha)}{1-\nu}} (\nu(1-\eta))^{\frac{1-\eta}{1-\nu}} \left(\frac{\alpha\eta\nu}{\hat{r}}\right)^{\frac{\alpha\eta}{1-\nu}} Qz^{\frac{1}{\nu}} \\ &= \kappa w^{-\frac{\eta(1-\alpha)}{1-\nu}} Q \end{aligned}$$

A one percent increase in w causes Q to increase by $1+\varepsilon$ percent and $w^{-\frac{\eta(1-\alpha)}{1-\nu}}$ to decrease by $\frac{\eta(1-\alpha)}{1-\nu}$. Hence, output of financially unconstrained firms is increasing in w or λ if and only if $1+\varepsilon \geqslant \eta\rho(1-\alpha)$. We have then proved Proposition 2 from the paper, which is re-stated here for convenience.

Proposition 2. *Suppose there exists a positive measure set of constrained firms. If* $1+\varepsilon \geqslant \eta\rho(1-\alpha)$*, then output of all firms is increasing in the collateral constraint parameter,* λ.

C.4 Analysis of the Importance of the Use of External Funds in a Stylized Version of our Benchmark Model

In this section, we analyze a version of our model with perfect competition, perfect substitutes, and an i.i.d. process for firm-level productivity. Specifically, we assume that in every period, each firm has a probability π of having productivity equal to 1 and probability $1-\pi$ of having probability equal to 0. We solve analytically for the equilibrium and the amount of external financing used by firms in the model. We then compare the effect of changes in the collateral constraint parameter, λ, across economies with different probabilities of high productivity, π.

In particular, for each π-economy, we choose the collateral constraint parameter, $\lambda(\pi)$ so that the aggregate debt-to-assets ratio in the model is the same across all π-economies. We show that even though the debt-to-asset ratio is held constant, the amount of external financing is decreasing the probability of receiving a high productivity shock. Then, for each π-economy, we compare steady state wealth in the $\lambda(\pi)$ economy to that in the economy when $\lambda = 1$, in other words, the autarkic version of that economy. We show that the difference in steady state wealth between the $\lambda(\pi)$, high debt economy, and the $\lambda = 1$, no debt economy monotonically decreasing in the probability of receiving a high productivity shock.

Model and Solution

In this simplified version of our model, firms are identical, produce a homogeneous final output good, and the process for firm-level productivity is given by

$$z_t = \begin{cases} 1 & \text{with prob } \pi \\ 0 & \text{with prob } 1-\pi \end{cases}$$

where the shocks are independent and identically drawn across firms and time. We assume a small open economy with a fixed interest rate that satisfies $0 \leqslant r \leqslant \frac{1}{\beta} - 1$.

The problem of a firm in any period can be written recursively as

$$V(a,z) = \max \ln(c) + \beta E V(a', z')$$

subject to

$$c + a' \leqslant (1+r)a + \max_{k \leqslant \lambda a, l} z k^{\alpha} l^{1-\alpha} - wl - (r+\delta)k$$

Clearly a firm with $z = 0$ chooses $k = 0, l = 0$. It is straightforward to show that profits for a firm with $z = 1$ are given by

$$\left[\alpha\left(\frac{1-\alpha}{w}\right)^{\frac{1-\alpha}{\alpha}} - (r+\delta)\right]\lambda a.$$

Using this result, we may write the recursive problem of the firm as

$$V(a,z) = \max \ln(c) + \beta E V(a', z')$$

s.t.

$$c + a' \leqslant \left\{ z\lambda\left[\alpha\left(\frac{1-\alpha}{w}\right)^{\frac{1-\alpha}{\alpha}} - (r+\delta)\right] + (1+r)\right\} a.$$

Given our assumed form of preferences along with i.i.d. shock process for productivity, we immediately have that the savings functions are linear in asset holdings and given by

$$\begin{aligned} a'(a,1) &= \beta\left\{\lambda\left[\alpha\left(\frac{1-\alpha}{w}\right)^{\frac{1-\alpha}{\alpha}} - (r+\delta)\right] + (1+r)\right\} a \qquad (14) \\ a'(a,0) &= \beta(1+r)a. \end{aligned}$$

The law of motion for assets in a steady state equilibrium yields the equilibrium wage rate which must satisfy

$$1 = \beta(1+r) + \beta\pi\lambda\left[\alpha\left(\frac{1-\alpha}{w}\right)^{\frac{1-\alpha}{\alpha}} - (r+\delta)\right].$$

Labor market clearing (with aggregate labor normalized to 1), then, defines steady state wealth:

$$\bar{A}(\lambda,\pi) = \left[\frac{\alpha\beta(\pi\lambda)^{\alpha}}{1-\beta(1+r)+\beta(r+\delta)\pi\lambda}\right]^{\frac{1}{1-\alpha}}.$$

We now turn to analyzing the amount of external financing firms rely on as well as the amount of aggregate debt and assets.

External Financing

First, as in our quantitative model, we define available funds and investment by rewriting the budget

$$c_t + a_{t+1} = z_t k_t^{\alpha} l_t^{1-\alpha} - w l_t - (r+\delta)k_t + (1+r)a_t$$

of a firm with an explicit definition of debt $b_t = k_t - a_t$. We then have

$$c_t + k_{t+1} - (1-\delta)k_t = z_t^{\alpha} k_t^{\alpha} l_t^{1-\alpha} - w l_t - r b_t + b_{t+1} - b_t.$$

We then define available funds, debt, and investment as

$$\begin{aligned} AF_t &= k_t^{\alpha} l_t^{1-\alpha} - w l_t - r b_t \\ b_t &= k_t - a_t \\ X_t &= k_{t+1} - (1-\delta)k_t \end{aligned}$$

Available funds for specific firms depends on their asset holdings and their productivity in any period. All of the derivations are included below. Available funds for firms with assets a_t and productivity z_t satisfy

$$AF_t(a_t, z_t) = \begin{cases} a_t\left[\frac{1-\beta(1+r)}{\beta\pi} + \lambda\delta + r\right] & \text{if } z_t = \bar{z} \\ a_t r & \text{if } z_t = 0 \end{cases}$$

Investment, of course, depends on productivity and assets in period $t+1$ since these factors determine the amount of capital a firm uses in period $t+1$. Since assets in period $t+1$ are functions of assets and productivity in period t we may define investment as functions only of a_t, z_t and z_{t+1}. We have

$$X_t(a_t, z_t, z_{t+1}) = \begin{cases} a_t\lambda\left[\delta + \frac{1-\pi}{\pi}(1-\beta(1+r))\right] & \text{if } z_t = \bar{z}, z_{t+1} = \bar{z} \\ -a_t(1-\delta)\lambda & \text{if } z_t = \bar{z}, z_{t+1} = 0 \\ a_t\lambda\beta(1+r) & \text{if } z_t = 0, z_{t+1} = \bar{z} \\ 0 & \text{if } z_t = 0, z_{t+1} = 0 \end{cases}.$$

To aid us in defining external financing, it is useful to define the amount of excess available funds a firm has for investment.[3] One may show that external funds, $AF - X$, for each firm satisfy

$$AF - X = \begin{cases} a_t\left[r + (1-\beta(1+r))\left[\frac{1-\beta\lambda(1-\pi)}{\beta\pi}\right]\right] & \text{if } z_t = \bar{z}, z_{t+1} = \bar{z} \\ a_t\left[\frac{1-\beta(1+r)}{\beta\pi} + \lambda + r\right] & \text{if } z_t = \bar{z}, z_{t+1} = 0 \\ a_t\left[r(1-\lambda\beta) - \lambda\beta\right] & \text{if } z_t = 0, z_{t+1} = \bar{z} \\ a_t r & \text{if } z_t = 0, z_{t+1} = 0 \end{cases}$$

We use this expression to get a sense of which firms are likely to rely on external financing. Clearly unproductive firms in period $t+1$ will not typically rely on outside funds since both firms have 0 or negative investment. Typically, the firm that switches from unproductive in period t to productive in period $t+1$ ($z_t = 0, z_{t+1} = \bar{z}$) will rely on outside funds since that firm has low available funds in period t but a high amount of investment (when λ is sufficiently large). Finally the firm that is productive in two consecutive periods ($z_t = \bar{z}, z_{t+1} = \bar{z}$) will typically not rely on external funds since that firm's available funds are large in period t, however this is sensitive to the choice of λ since, as λ becomes large, even though the firm has high available funds, the amount the firm invests grows as well.

Before turning to the effects of changes in λ, we point out that in the aggregate, independent of the collateral constraint and the probability of being productive, in the aggregate firms can self finance all of their investment. To see this, notice that in the aggregate, investment is simply

$$\delta\pi\lambda\bar{A}$$

[3] Details on deriving external financing in this model are available upon request.

as productive firms are maintaining the capital stock, and available funds are given by

$$\bar{A}\left[r+\frac{1-\beta(1+r)}{\beta}+\pi\lambda\delta\right]$$

Thus, in the aggregate, firms can self-finance all of their investment as the aggregate excess is given by

$$\bar{A}\left[r+\frac{1-\beta(1+r)}{\beta}\right]$$

Finally, we have the aggregate debt-to-asset ratio:

$$\frac{\pi(\lambda-1)}{\pi\lambda+1-\pi}. \tag{15}$$

To see this final result, note that debt is just $k-a$ for firms with $k \geqslant a$. The only firms with $k \geqslant a$ are those with $z=1$. Hence, aggregate debt is simply $\pi\bar{A}(\lambda-1)$. Total assets, however, is not simply wealth, or $\bar{A}$. Total assets are capital installed by firms with $z=1$ and assets of firms with $z=0$ since these firms, in effect, have claims to financial assets. Hence, total assets are given by $\pi\lambda\bar{A}+(1-\pi)\bar{A}$. Note that the aggregate debt-to-asset ratio for any π varies from 0 to 1 as λ varies from 1 to ∞.

Relating External Financing to the Importance of Financial Markets

Consider the following exercise. For any π, choose λ so that the debt-to-asset ratio is constant (same amount of aggregate debt relative to assets in every π economy). Then, consider the difference in steady state wealth when $\lambda=\lambda(\pi)$ and when $\lambda=1$ (or when debt-to-assets falls from the constant level to 0). I do this because the metric is easier to analyze (with respect to π) than is the derivative of steady state wealth. We have the following proposition.

Proposition 3. *Suppose* $0<r<\frac{1}{\beta}-1$. *Let* $\pi\in[\underline{\pi},\bar{\pi}]$ *and define* $\lambda(\pi)$ *such that the debt-to-asset ratio in the* π*-economy with parameter* $\lambda(\pi)$ *is equal to* $\bar{B}$. *If for all* π, $\frac{1}{\beta}<\lambda(\pi)<\frac{1}{\beta(1-\pi)}$ *then external financing is decreasing in* π *and* $\log(\bar{A}(\lambda(\pi),\pi)-\log(\bar{A}(1),\pi)$ *is decreasing in* π. *(The result is the same for output).*

Proof. The assumptions of the proposition ensure that the only firm relying on external funds for investment is the firm switching from unproductive to productive. Formally, these assumptions place bounds on π for a given $\bar{B}$. To see this, using the definition of debt-to-assets in equation (15), we have that

$$\lambda(\pi) = \frac{\bar{B}}{\pi(1-\bar{B})} + 1. \tag{16}$$

Since $\lambda(\pi)$ is decreasing in π, we can replace the assumption on $\lambda(\pi)$ by ensuring that

$$\frac{1}{\beta} \leqslant \frac{\bar{B}}{\bar{\pi}(1-\bar{B})} + 1$$

and

$$\frac{\bar{B}}{\underline{\pi}(1-\bar{B})} + 1 \leqslant \frac{1}{\beta(1-\underline{\pi})}$$

It can be shown that these conditions are consistent with $\bar{\pi} > \underline{\pi}$.

Recall the definitions of external financing for the $(z_t = 0, z_{t+1} = \bar{z})$ and the $(z_t = \bar{z}, z_{t+1} = \bar{z})$ firms:

$$\begin{aligned} &a_t\left[r(1-\lambda\beta) - \lambda\beta\right] \quad \text{if } z_t = 0, z_{t+1} = \bar{z} \\ &a_t\left[r + (1-\beta(1+r))\left[\tfrac{1-\beta\lambda(1-\pi)}{\beta\pi}\right]\right] \quad \text{if } z_t = \bar{z}, z_{t+1} = \bar{z} \end{aligned}$$

Since $0 \geqslant 1-\lambda\beta$, it must be that firm switching from unproductive to productive $(z_t = 0, z_{t+1} = \bar{z})$ uses external funds for investment and since $1-\beta\lambda(\pi)(1-\pi) \geqslant 0$ the firm that is productive for two consecutive periods does not use external funds for investment. Therefore, our statistic on the amount of external funds used for investment satisfies

$$\begin{aligned} &\frac{\pi(1-\pi)\left[\beta(1+r)\lambda - r\right]}{\pi\delta\lambda} \\ = &\frac{(1-\pi)}{\delta}\left[\beta(1+r) - \frac{r}{\lambda}\right] \end{aligned}$$

Using the definition of λ in (16), we have

$$\frac{1}{\lambda} = \frac{\pi(1-\bar{B})}{\bar{B} + \pi(1-\bar{B})}$$

so that our external financing statistic satisfies

$$\begin{aligned} &\frac{(1-\pi)}{\delta}\left[\beta(1+r) - \frac{r\pi(1-\bar{B})}{\bar{B} + \pi(1-\bar{B})}\right] \\ = &\frac{(1-\pi)}{\delta}\left[\beta(1+r) - \frac{r}{\frac{\bar{B}}{\pi(1-\bar{B})} + 1}\right] \end{aligned}$$

As a result, we immediately see that our statistic is decreasing in π. Consider now the definition of steady state wealth. We have

$$\bar{A}(\lambda,\pi) = \left[\frac{\alpha\beta\,(\pi\lambda)^{\alpha}}{1-\beta(1+r)+\beta(r+\delta)\pi\lambda}\right]^{\frac{1}{1-\alpha}}$$

and

$$\bar{A}(1,\pi) = \left[\frac{\alpha\beta\pi^{\alpha}}{1-\beta(1+r)+\beta(r+\delta)\pi}\right]^{\frac{1}{1-\alpha}}$$

Then (recalling that $\pi\lambda = \frac{\bar{B}}{1-\bar{B}} + \pi$)

$$\begin{aligned}
& (1-\alpha)\left[\log(\bar{A}(\lambda(\pi),\pi)) - \log(\bar{A}(1,\pi))\right] \\
= \;& \alpha\log\left(1+\frac{\bar{B}}{(1-\bar{B})}\frac{1}{\pi}\right) - \log\left(1+\frac{\bar{B}}{1-\bar{B}}\frac{1}{\frac{1-\beta(1+r)}{\beta(r+\delta)}+\pi}\right)
\end{aligned}$$

Analyzing this equation, let $c = \frac{\bar{B}}{1-\bar{B}}$, $d = \frac{1-\beta(1+r)}{\beta(r+\delta)}$. Then I claim that

$$f(\pi) = \log\left(1+c\pi^{-1}\right) - \log(1+c(d+\pi)^{-1})$$

is decreasing in π. To see this, we have

$$\begin{aligned}
f'(\pi) &= \frac{-c\pi^{-2}}{1+c\pi^{-1}} - \frac{-c(d+\pi)^{-2}}{1+c(d+\pi)^{-1}} \\
&= c\left[(d+\pi)^{-2}(1+c(d+\pi)^{-1})^{-1} - \pi^{-2}(1+c\pi^{-1})^{-1}\right]
\end{aligned}$$

And we must have

$$(d+\pi)^{-2}(1+c(d+\pi)^{-1})^{-1} \leqslant \pi^{-2}(1+c\pi^{-1})^{-1}$$

since

$$\begin{aligned}
\pi^2 + c\pi &\leqslant (d+\pi)^2 + c\,(d+\pi) \\
0 &\leqslant d^2 + 2d\pi + cd
\end{aligned}$$

And we have that $\beta(1+r) \leqslant 1$ so that $d \geqslant 0$ and $c \geqslant 0$. □

Output is given by

$$\begin{aligned}
Y(\pi,\lambda) &= \left(\frac{1-\alpha}{w}\right)^{\frac{1-\alpha}{\alpha}} \pi\lambda\bar{A}(\pi,\lambda) \\
&= \left(\pi\lambda\bar{A}(\pi,\lambda)\right)^{\alpha} \\
&= \left[\frac{\alpha\beta\pi\lambda}{1-\beta(1+r)+\beta(r+\delta)\pi\lambda}\right]^{\frac{\alpha}{1-\alpha}} \\
&= \left[\frac{\alpha\beta}{(1-\beta(1+r))\,(\pi\lambda)^{-1}+\beta(r+\delta)}\right]^{\frac{\alpha}{1-\alpha}}
\end{aligned}$$

Then

$$\begin{aligned}
&\frac{1-\alpha}{\alpha}\left[\log Y(\pi,\lambda(\pi)) - \log(Y(\pi,1))\right] \\
= \ & \log\left(\frac{1}{\pi}(1-\beta(1+r)+\beta(r+\delta)\right) - \log\left(\frac{1}{\pi\lambda}(1-\beta(1+r))+\beta(r+\delta)\right) \\
= \ & \log\left(\frac{1}{\pi}c_1+c_2\right) - \log\left(\frac{1}{\bar{c}+\pi}c_1+c_2\right)
\end{aligned}$$

Differentiating with respect to π we have

$$\begin{aligned}
&\frac{-c_1\pi^{-2}}{\pi^{-1}c_1+c_2} - \frac{-c_1(\bar{c}+\pi)^{-2}}{(\bar{c}+\pi)^{-1}c_1+c_2} \\
= \ & c_1\left[\frac{1}{(\bar{c}+\pi)\,c_1+(\bar{c}+\pi)^2c_2} - \frac{1}{\pi c_1+\pi^2c_2}\right]
\end{aligned}$$

which must be negative.

D Sensitivity Analysis of Quantitative Model Results

In this section, we study the sensitivity of our quantitative exercise to changes in various parameters. We focus on examining how much gross value added responds to financial shocks in our model under different assumptions on the elasticity of substitution across goods or the probability of exit faced by private firms.

D.1 Non-Financial Trade Linkages

Trade linkages in our model arising from the imperfect substitutability of goods in the economy play a critical role in the steady state. In the absence of these trade linkages, private firms would not produce output, because their binding collateral constraint makes them effectively less productive. Here, we compare our results to those we obtain using an elasticity of substitution across goods of $\rho = 10$ in line with estimates typically used in the macro literature (see Basu and Fernald (1997) and Golosov and Lucas (2007) for examples). We find that changing this elasticity has little impact on the aggregate consequences of financial shocks but has important consequences for the responses of private and public firms.

Holding the remaining model parameters fixed, an increase in the elasticity of substitution across goods reduces the profit margins of firms, as the markups from monopolistic competition are much smaller (10% with $\rho = 10$ as opposed to roughly 30%). Consequently, in the steady state, firms use less debt (the debt-to-asset ratio is roughly 0.3 instead of 0.49) and less external financing (0.74 instead of 0.82).

In this economy, we find that the impact of the same financial shock we imposed above on gross value added is much larger, roughly -0.8% on impact. Under this parameterization, the financial shock is essentially larger because it induces a larger change in the aggregate debt-to-asset ratio. The direct effect of the shock on the sales of private firms is larger—private firm sales fall by roughly 8%—but the indirect general equilibrium effect, which tends to increase the output of public firms, is also larger (public firm sales rise by roughly 4%). In this parameterization, the sales of public firms remain above trend in all periods following the financial shock. We also find that the effect of the shock is more persistent. This last finding is driven by the fact that with smaller profit margins, it takes private firm owners longer to accumulate wealth lost (relative to the steady state) following the financial shock.

We conclude that trade linkages are essential for ensuring that both types of firms produce in the steady state and that unconstrained public firms reduce their sales following a financial shock (at least in the medium run when $\rho = 4$); however, these specific trade linkages appear less critical for understanding the aggregate response of the economy to a financial shock.

D.2 Exit Risk

Uninsurable exit risk faced by private firms in our model, captured by the parameter ζ, plays an important role in determining the strength of the precautionary motives of the owners of private firms. Here, we show that a reduction in ζ, holding the remaining model parameters fixed, reduces the response of GDP in our model to a financial shock. However, this change also causes our model to yield counterfactual predictions for financial flows. When we recalibrate our model given a lower value of ζ, we obtain similar results as in our benchmark analysis.

Consider modifying our benchmark calibration by only lowering the parameter ζ from 0.1 to 0.05 and then imposing the same 30% negative shock to the collateral constraint parameter that we analyzed above. Such an impulse yields only a 0.1% decline in gross value added.

The magnitude of the impact is only one quarter of that in our benchmark calibration. In this sense, our findings are sensitive to our choice of the exit risk faced by private firms. However, in the steady state, this model yields counterfactual implications for both the use of external funds by private firms and their aggregate indebtedness. Reducing the exit risk of private firms makes the owners of these firms effectively more patient and thus strengthens their precautionary motives. This parameterized model implies that the amount of debt used by private firms falls from 0.49 to 0.14. Interestingly, the use of external funds for investment actually rises in the steady state, as private firms have more relaxed collateral constraints. These constraints are more relaxed in equilibrium because these firms endogenously accumulate more net worth.

We have also recalibrated our model under the assumption that $\zeta = 0.05$ and subjected the economy to an aggregate tightening of the collateral constraint that induces a decline of one standard deviation in the debt-to-asset ratio.[4] To induce the same decline in the aggregate debt-to-total-assets ratio under this alternative calibration, we must impose a larger financial shock—roughly a 40% decline in λ versus 30% in our benchmark calibration with both private and public firms. This shock induces an over 0.2% decline in gross value added, which is smaller than our benchmark calibration though more than twice as large than the value we obtain when we only adjust the exit risk of private firms.

We conclude that matching the use of external funds, independent of the amount of

[4]Details of this calibration are available upon request.

exit risk imposed in the model, plays a key role in determining the responsiveness of the economy to financial shocks. One interpretation of this finding is that in an economy in which financial shocks are forecastable, the likelihood of financial shocks would directly impact the precautionary motives of entrepreneurs.However, such a change in the model also impacts the use of external funds by private firms; we suspect if such a model matched patterns of external financing in the data, the model would obtain effects of financial shocks with similar magnitudes to our benchmark findings.

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