# The Maturity Structure of Inside Money*

Burton Hollifield† Ariel Zetlin-Jones‡

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**Abstract**

We develop a new rationale for banks to transform risk and maturity based on the role banks play in issuing liabilities that function like money. We show risk and maturity transformation improve the liquidity of banks' liabilities only when real investment opportunities are sufficiently risky. The competitive equilibrium typically features too little maturity transformation. In such cases, optimal policies encourage more maturity transformation suggesting policies that discourage maturity transformation can harm the liquidity of bank liabilities. In line with our theory, we find empirically that when banks reduce maturity transformation, their liabilities tend to provide fewer liquidity services.

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†Tepper School of Business, Carnegie Mellon University

‡Tepper School of Business, Carnegie Mellon University

# 1 Introduction

The 2007 Financial Crisis sparked renewed interest in implementing policies that restrict how banks transform risk and maturity on their balance sheets. For example, in 2014, U.S. financial regulators instituted a Liquidity Coverage Ratio (LCR) that requires banks to hold sufficient short-term, liquid assets relative to their short-term liabilities in an effort to reduce the likelihood of early liability redemptions (Diamond and Kashyap (2016)).

The standard theoretical rationales for policies that restrict transformation are built upon Diamond and Dybvig (1983) in which banks transform maturity to provide insurance against households' idiosyncratic liquidity risk. See Farhi et al. (2009) and Stein (2012) for leading examples. In the standard models, balance sheet transformation leaves banks exposed to the risk of runs and fire-sales and motivates policies to reduce transformation. These theories, however, are silent on other economic functions of banks. In particular, as described by Calomiris and Gorton (1991), in these theories, "Banks liabilities do not circulate as a medium of exchange ... so there is no sense in which demand deposits function like money."

We develop a new theory in which the key economic role of banks is to issue liabilities that function like money. In some cases, banks will transform risk and maturity on their balance sheets. One might conjecture that policies that immunize banks against bank runs would make their liabilities safer and, therefore, a better medium of exchange. The opposite is true in our model. In contrast to models with fire-sales and bank runs, the competitive equilibrium in our model has less maturity transformation than socially optimal—regulation should aim to increase maturity transformation rather than to reduce it.

Our goal is to show how using bank liabilities as inside money can affect socially efficient and equilibrium bank risk and maturity transformation. Whether or not policies that reduce or increase maturity transformation are useful in practice depends on the relative importance of fire-sales and run risk relative to the importance of the banks' role for transactional services.

Our theory describes a novel tradeoff for policymakers between managing the risk of bank runs and managing the liquidity of bank liabilities. Regulatory policies such as the LCR can reduce the liquidity of bank liabilities. We provide some suggestive evidence consistent with this prediction. Exploiting geographic variation in the location of banking activities, we document that regulatory policies implemented since the crisis are associated with reduced liquidity measured by the velocity of bank liabilities.

We model the banks' ability to provide transactions services using a search-theoretic approach. Heterogeneous households trade in frictional decentralized markets following Lagos and Wright (2005). As in Kocherlakota (1998), household anonymity and inability to enforce private credit arrangements leads to household demand liquidity. That liquidity comes from inside money issued by risk neutral agents we refer to as banks following Cavalcanti and Wallace (1999) and Gu et al. (2013). Decentralized trade is facilitated by inside money backed by the banks' underlying real investments with stochastic cash flows and by banks' own endowments.

Following Kiyotaki and Wright (1989), Rocheteau (2011), Lester et al. (2012), and Nosal and Rocheteau (2013), liquidity premia may arise with exogenous asset specific liquidity constraints, informational asymmetries, or asset liquidation costs. We assume scarce enough outside money for inside money—bank liabilities—to be the only assets used to facilitate trade. Banks issuing inside money cannot fully commit to long-term promises as in Calomiris and Kahn (1991) so they must retain sufficient inside equity in on-going investments. Costly early liquidation lets banks change the timing of cash flows, allowing them to pay liabilities with shorter-term payoffs than their investments.

When long-term cash flows are risky enough, socially efficient allocations have risk transformation since households' liquidity constraints make households risk averse to bank liabilities' cash flows. It is efficient for banks to use their inside equity to insure households by issuing liabilities with less risky payoffs than bank investments in order to provide risk transformation.

In states with low enough investment returns, risk transformation without maturity transformation may require banks to promise to transfer more of their equity than they can credibly commit to do. Early liquidation relaxes the banks' commitment problem allowing them to provide households better insurance. With commitment problems, maturity transformation can be an efficient way to improve risk transformation.

Other authors study how much balance sheet transformation banks provide. Jacklin (1987) and Farhi et al. (2009) show that when households can trade bank liabilities, banks provide no maturity transformation in the presence of idiosyncratic liquidity risk without policy. We study outcomes when banks insure households against aggregate not idiosyncratic risk.

In several theories of banking with fire-sales, bank assets are priced ex post in spot markets leading to a pecuniary externality on the asset side of the bank balance sheet causing banks to issue too much short-term debt in Stein (2012), and issue too much total credit in Lorenzoni (2008). We can interpret the early liquidation costs as a fire-sale in our model, but early liquidation itself is not a source of inefficiency. Inefficiency arises because banks do not internalize the impact of their liability issues on aggregate liquidity premia. The externality in our model is on the liability side of the bank balance sheet, rather than on the asset side.

Our finding that banks issue too little short-term debt differs from Brunnermeier and Oehmke (2013)'s finding that banks who cannot commit to a debt maturity structure issue too much short-term debt. Banks in their model don't provide insurance against idiosyncratic or aggregate risk. DeAngelo and Stulz (2015) and Gale and Gottardi (2017) examine efficiency of bank leverage in a model where bank liabilities serve as inside money. We instead examine efficiency of bank liability risk and maturity, while holding fixed bank leverage.

When constrained efficient allocations feature maturity transformation our model, policies which require banks to issue liabilities with minimum expected short term payout improve on competitive equilibrium allocations. Such policies conflict with the types of liquidity management policies implemented in the wake of Basel III, such as the Liquidity Coverage Ratio requiring banks hold sufficient liquid assets to cover expected short-term net outlays during a 30-day stress period. LCRs incentive banks to minimize expected short term outlays. That banks should be incentivized to issue liabilities with large enough short term payouts is a novel cost of the liquidity management policies implemented after Basel III.

## 2 The Model

In this section we develop a tractable model where the key economic role of banks is to provide a medium of exchange. To do so, we adapt a standard, money-search framework (as in Lagos and

Wright (2005) or Rocheteau and Wright (2005)) to a finite horizon. The money-search framework is a useful device that allows us to endogenously determine the value of liabilities with different maturity and risk profiles as media of exchange in contrast to imposing an ad hoc cash-in-advance constraint. In Appendix E, we develop a reduced form version of our model with an ad hoc cash-in-advance constraint that induces the same equilibrium and efficient allocations as our model.

The model economy is populated by two types of of individuals: *banks* and *households*. Our model is set up so that banks issue liabilities subject to limited commitment and use the proceeds from their issuance along with their endowment of equity to finance real investments. Households, who buy claims issued by banks trade periodically in frictional markets where a medium of exchange—households' holdings of bank liabilities—is required for trade.

To be specific, there are three periods, 0, 1 and 2. Period 1 and period 2 are split into two sub-periods, a decentralized market sub-period followed by a centralized market sub-period. Period 0 features only a centralized market sub-period. In each decentralized, frictional market, households will require a medium of exchange to engage in any trade of goods.

The model features an aggregate endowment of non-reproducible identical trees that which produce consumption goods in the centralized sub-periods of 2. These trees are the real investments banks will finance. We make the assumption that trees are long-term assets—they pay consumption goods only in period 2—for simplicity only. This assumption allows us to highlight the role banks play in transforming maturity on their balance sheet. At the start of (the decentralized sub-period of) period 1, banks and households all observe a common, random public signal $\omega \in \{\omega_l, \omega_h\} \equiv \Omega$ that determines the consumption produced by each tree. Let $\gamma(\omega_i) \equiv \text{Prob}\,(\omega = \omega_i)$.

## 2.1 Banks

A large number of identical banks are each initially endowed with $K^B \geq 0$ units of trees. Banks issue liabilities to households to fund the purchase of additional trees. Each risk neutral bank values consumption of goods during the centralized market sub-periods at time $t = 1, 2$ with zero discount rate between periods. The bank chooses state and time contingent consumption plan $c^B \equiv \{c_t^B(\omega)\}$.

Each bank also has the ability to transform the maturity of their assets by way of a liquidation technology (as in Diamond and Dybvig (1983)). Specifically, banks may liquidate part of their trees in the centralized sub-period of period 1. We let $L(\omega) \in [0,1]$ denote the fraction of the bank's tees liquidated in the first period in state $\omega$. Liquidation is costly so that when banks obtain consumption goods from their trees in period 1, they receive only a portion of what the trees would deliver in period 2. The liquidation cost is $1-\kappa$ with $0 < \kappa < 1$.

Lastly, we assume that banks are subject to limited commitment which may inhibit their ability to issue liabilities with arbitrary promised payouts. We capture this limited commitment by assuming that banks may *abscond* or walk away with a portion $\xi \in [0,1]$ of their capital between period 0 and 1 or between period 1 and 2.[1]

Suppose that the bank does not abscond. For an initial holdings of trees, $I > 0$ and a state-contingent liquidation plan $L(\omega) \in [0,1]$ the project has stochastic cash flows

$$\begin{aligned} &\text{period 1,} \quad I\kappa L(\omega)z(\omega), \\ &\text{period 2,} \quad I[1-L(\omega)]z(\omega), \end{aligned} \tag{1}$$

where $z(\omega_h) \geq z(\omega_l) \geq 0$ are the stochastic levels of period 2 consumption output per tree.

If the bank absconds with its trees between period 0 and period 1 it consumes the cash flows from the trees and receives an expected payoff of

$$\xi I \sum_{\omega \in \Omega} \gamma(\omega) z(\omega), \tag{2}$$

If the bank absconds with its trees between period 1 and period 2 it receives second period payoffs of

$$\xi I[1-L(\omega)]z(\omega). \tag{3}$$

[1] Alternatively, one could interpret the bank's payoffs from absconding as the bank's utility costs of managing the trees over time.

For simplicity, we assume that once the bank absconds with its trees, any trees remaining become worthless and that the bank cannot be forced to make any coupon payments.

The bank issues liabilities with coupon payments backed by the investment project cash flows. We normalize the number of liabilities issued to one. Let $d_t(\omega) \geq 0$ be the coupon payoffs per liability at time $t$. A liabilities issue $D$ is a vector of state and time contingent coupon payments:

$$D \equiv \{d_1(\omega_l), d_2(\omega_l), d_1(\omega_h), d_2(\omega_h)\}. \tag{4}$$

We let $D(\omega) = \{d_1(\omega), d_2(\omega)\}$ denote the time contingent coupons associated with liability issue $D$ in state $\omega$. We define period $t$ prices in units of the consumption good in the centralized sub-period of period t. Then, the function $p_0(D)$ is the initial price of the bank liabilities, $p_t(D(\omega))$ is the ex-coupon liability price in period $t$ and $p_t(D(\omega)) + d_t(\omega)$ the period $t$ cum-coupon liability price. For an investment and liquidation decision, ex-coupon liability prices depend only on the coupon process $D$. $p_0^k$ is the price of trees in period 0.

Banks purchase trees, decide on a liquidation strategy, and a state and date contingent coupon payment plan. The bank improves on allocations obtained by households alone because the bank has the unique ability to deliver on promises across periods. Specifically, households are unable to commit to deliver on any promises issue against any holdings of trees.

Each bank may issue liabilities with different payoff structures and commit to a feasible investment, liquidation and liability issuance strategy indexed by the coupon strategy. The model admits aggregation of banks—in equilibrium, all banks issue the same type of liabilities taking the pricing functions as given. The representative bank takes the pricing function as given and solves

$$\max_{\{I,L,D,c^B\}} \sum_{\omega\in\Omega} \gamma(\omega) \left[c_1^B(\omega) + c_2^B(\omega)\right], \tag{5}$$

subject to:

$$
\begin{aligned}
p_0^k I &\leq p_0^k K^B + p_0(D), && (6)\\
c_1^B(\omega) + d_1(\omega) &= \kappa L(\omega) I z(\omega), && (7)\\
c_2^B(\omega) + d_2(\omega) &= [1 - L(\omega)] I z(\omega), && (8)\\
c_t^B(\omega) &\geq 0, \quad t = 0, 1, 2, && (9)\\
c_2^B(\omega) &\geq [1 - L]\xi I z(\omega), && (10)\\
\sum_{\omega\in\Omega} \gamma(\omega)\left[c_1^B(\omega) + c_2^B(\omega)\right] &\geq \xi I \sum_{\omega\in\Omega} \gamma(\omega) z(\omega), && (11)\\
\sum_{\omega\in\Omega} \gamma(\omega)\left[c_1^B(\omega) + c_2^B(\omega)\right] &\geq K^B \sum_{\omega\in\Omega} \gamma(\omega) z(\omega). && (12)
\end{aligned}
$$

Inequality (6) is the bank's period 0 budget constraint, equations (7) and (8) are resource constraints and inequalities (9) are limited liability constraints. Inequalities (10) and (11) are limited commitment constraints ensuring that the bank does not abscond with trees between period 1 and period 2 in any state $\omega$, and between period 0 and period 1. Inequality (12) is the bank's ex-ante participation constraint. Define $\mathcal{D}$ as the set of coupons that can be issued by the bank satisfying the feasibility conditions (6)–(12).

## 2.2 Households

Households produce and consume general goods in centralized markets and trade a special good in decentralized markets subject to trading frictions. Households may purchase portfolios of liabilities issued by banks to facilitate trade in decentralized markets.

Households are either buyers or sellers. Each household knows if it is a buyer or a seller with their type fixed over period as in Rocheteau and Wright (2005).[2] The superscript $b$ denotes buyers and $s$ denotes sellers. There is measure 1 of buyers and measure 1 of sellers. Each type $i \in \{b, s\}$ household is initially endowed with $k^i$ units of trees and the aggregate stock of trees

[2] Alternatively, we could allow household types to vary as in Lagos and Wright (2005). Equilibrium asset prices would reflect similar liquidity premia as our model and yield similar results on liquidity and risk transformation.

initially held by households $K^H$

$$K^H \equiv k^b + 1k^s. \tag{13}$$

Let $q_t$ denote goods produced or consumed in the decentralized sub-period $t$, $x_t$ denote goods consumed in centralized sub-period $t$, and $y_t$ denote production of goods in centralized sub-period $t$. Buyers have period $t$ preferences

$$U_t^b\left(q_t, x_t, y_t\right) = u(q_t) + \left[v(x_t) - y_t\right], \tag{14}$$

and sellers have period $t$ preferences

$$U_t^s\left(q_t, x_t, y_t\right) = -c(q_t) + \left[v(x_t) - y_t\right]. \tag{15}$$

Buyers' and sellers' have concave utility $v$ from consuming the general good in centralized markets, linear disutility of labor in the centralized market, and do not discount utility over time.[3] Buyers enjoy utility $u(q_t)$ from consuming $q_t$ and sellers pay utility cost $c(q_t)$ from producing $q_t$ in the decentralized market. The gains from trade are $u(q_t) - c(q_t)$.

Buyers and sellers face matching fractions in decentralized markets. Let $\alpha > 0$ denote the probability that a buyer meets a seller. When a buyer and a seller meet in a decentralized market, they engage in proportional bargaining to determine the terms of trade. We interpret the case where $\alpha = 0$ as one in which bank liabilities are not accepted in decentralized trade; liquidity associated with any possible coupon would be zero. When $\alpha > 0$ then bank liabilities are accepted in (some) decentralized trade.

Since households observe the state $\omega$ at the beginning of period 1, there is no residual uncertainty about the liability payoffs after the beginning of period 1. As a result, the relevant aggregate state for a household is $D(\omega)$, the coupons associated with the liability issued by the representative bank. The idiosyncratic state of a household upon entering the centralized market in period $t \in \{1, 2\}$ is the number of the representative bank's liabilities the household owns, $a$,

[3] Rocheteau and Wright (2005) allow a discount rate of $\beta_d$ between the centralized and decentralized sub-periods as well as discounting over time. For simplicity, we abstract from discounting in our finite horizon model.

with cum-dividend value of $a \times [p_t(\omega) + d_t(\omega)]$.

A type $i \in \{b, s\}$ household solves[4]

$$W_t^i(a; D(\omega)) = \max_{x,y,a'} v(x) - y + V_{t+1}^i(a'; D(\omega)), \tag{16}$$

subject to:

$$x + a' p_t(D(\omega)) \leq y + a[p_t(D(\omega)) + d_t(\omega)], \tag{17}$$

where $V_{t+1}^i(a'; D(\omega))$ is the household value function entering the decentralized market in $t+1$.

In period $t = 0$ before $\omega$ is realized, the household sells capital and purchases liabilities from the representative bank; the relevant aggregate state for a household is the vector of liability coupons, $D$. The household's problem is

$$W_0^i(D) = \max_{x,y,a'} v(x) - y + \sum_{\omega \in \Omega} \gamma(\omega) V_1^i(a'; D(\omega)), \tag{18}$$

subject to:

$$x + a' p_0(D) \leq y + p_0^k k^i.$$

Let $\big(q_t(a^b, a^s; D(\omega)), m_t(a^b, a^s; D(\omega))\big)$ be the terms of trade in a meeting between a buyer and seller in the period $t$ decentralized market when the buyer owns $a^b$ liabilities and the seller owns $a^s$ liabilities, with $q_t$ the amount produced for the buyer and $m_t$ the amount of liabilities transferred from the buyer to the seller. The distributions over liabilities held at the start of the period $t$ decentralized market are $\Psi_t^i$ for $i \in \{b, s\}$. The time $t$ buyer value function entering the decentralized market is

$$V_t^b(a; D(\omega)) = \alpha \int_{a^s} \Big\{ u[q_t(a, a^s; D(\omega))] + W_t^b(a - m_t(a, a^s; D(\omega)); D(\omega)) \Big\} d\Psi_t^s(a^s) \\ + (1 - \alpha)\, W_t^b(a; D(\omega)), \tag{19}$$

[4]We could allow households to purchase trees in the centralized markets in periods 0, 1, or 2. Since we assume that households' direct claims to trees may not be used to facilitate trade in decentralized markets, it is without loss of generality to focus on equilibria where households do not them.

the seller value function is

$$V_t^s(a;D(\omega)) = \alpha \int_{a^b} \left\{ -c[q_t(a^b,a;D(\omega))] + W_t^s(a + m_t(a^b,a;D(\omega));D(\omega)) \right\} d\Psi_t^b(a^b) \\ + (1-\alpha)\, W_t^s(a;D(\omega)), \quad (20)$$

with $V_3^i(a;D(\omega)) = 0$, $i \in \{b,s\}$.

Terms of trade in decentralized meetings are determined by proportional bargaining with $\eta \in [0,1]$ the buyer's bargaining power.[5] In a match with liabilities $\left(a^b,a^s\right)$ in state $\omega$, the terms of trade $(q_t,m_t)$ solve

$$\max_{q_t,m_t} u(q_t) + \left[ W_t^b(a^b - m_t;D(\omega)) - W_t^b(a^b;D(\omega)) \right], \quad (21)$$

subject to:

$$m_t \le a^b, \quad (22)$$

$$u(q_t) + \left[ W_t^b(a^b - m_t;D(\omega)) - W_t^b(a^b;D(\omega)) \right] \\ = \frac{\eta}{1-\eta} \left[ -c(q_t) + (W_t^s(a^s + m_t;D(\omega)) - W_t^s(a^s;D(\omega)) \right]. \quad (23)$$

The trading constraint (22) assumes that the only medium of exchange available to households is their holdings of bank liabilities. Households may not use their own holdings of trees to facilitate trade in decentralized markets; we assume a limited commitment to deliver on assets pledged in decentralized markets.

Without the trading constraint (22), sellers would always produce the efficient level of output $q^*$ in decentralized meetings, where $u'(q^*) = c'(q^*)$. If buyers do not bring sufficient bank liabilities into meetings with sellers or if the bank liabilities are not valuable enough, then sellers may produce less than $q^*$ units of output.

Figure 1 summarizes the model's time-line.

[5] In Appendix E, we show that all of our results are robust to assuming that the price of the specialized good in the decentralized market is determined in a competitive market where buyers are subject to the trading constraint(22).

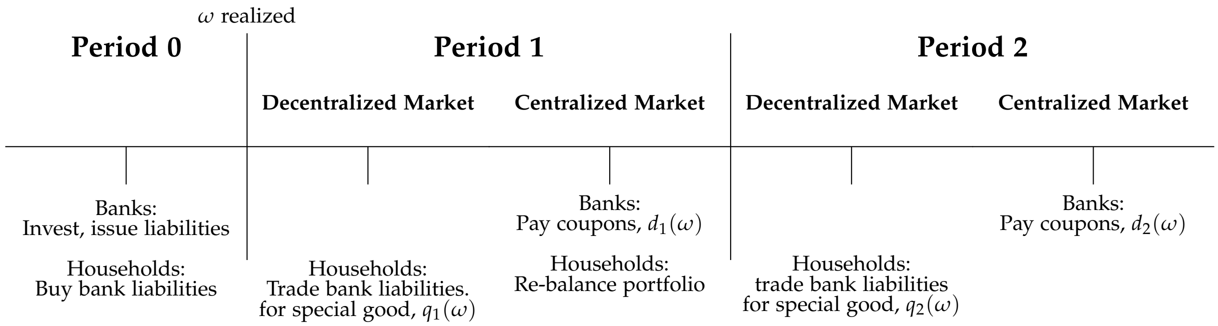


Figure 1: Timeline

# 3 Pareto Optimal Outcomes

Before turning to equilibrium outcomes, we first develop an efficient benchmark. This benchmark is useful for understanding the novel, efficiency reasons for banks to transform risk and maturity on their balance sheet in our model.

We characterize efficient allocations by solving the problem of a social planner who chooses banks' liability issuances and allocates resources to buyers and sellers subject to the decentralized trading frictions. Notice, the proportional bargaining constraints in the decentralized markets depend on the equilibrium price of bank liabilities. Since those liability prices themselves depend on the planner's choice of liability coupons, the planner internalizes how different liability issuances impact the amount of goods that each liability commands in decentralized markets.

**Characterizing Efficient Allocations Conditional on a Liability Issue.** For a given liability issuance, the remaining equilibrium prices and allocations resemble those in standard search-theoretic monetary economies as in Lagos and Wright (2005) and Rocheteau and Wright (2005). Quasi-linearity of households' preferences ensures that in any centralized market, a household's optimal choice of liabilities to purchase is independent of the liabilities they bring into the centralized market. As a result, the equilibrium distributions of liability holdings for buyers and sellers are degenerate. Following Rocheteau and Wright (2005), we characterize equilibrium in which in each centralized market the buyers purchase all of the bank's liabilities and use these liabilities to facilitate trade in the subsequent decentralized market. Since buyers and bank liabilities have same measure, each buyer holds 1 bank liability in any equilibrium. The buyers' marginal decision to hold bank liabilities determines the liabilities' equilibrium prices. Since equilibrium outcomes given a liability issuance are standard for this class of models, Appendix A describes the equilibrium; here we report terms of trade, asset prices, and the indirect utilities as these are instrumental in characterizing conditions under which it is socially efficient for banks to transform risk and maturity.

Recall that $q^*$ is the level of output that maximizes the static surplus in a meeting between a buyer and a seller—the efficient level of trade in a decentralized meeting. Let $d^*$ be

$$d^* = (1-\eta)u(q^*) + \eta c(q^*). \tag{24}$$

The threshold $d^*$ is the value of a bank liability that is sufficient to support efficient trade in decentralized markets when each buyer holds 1 unit of the liability. Define

$$\hat{q}_t\left(D(\omega)\right) \equiv \left(q \,|(1-\eta)u(q) + \eta c(q) = p_t(D(\omega)) + d_t\left(\omega\right)\right), \tag{25}$$

where $p_2(D(\omega)) = 0$. From the the quasi-linear preferences, equilibrium production in decentralized meetings is

$$q_t^{eq}(D(\omega)) = \begin{cases} q^*, & \text{if } p_t(D(\omega)) + d_t\left(\omega\right) \geq d^*, \\ \hat{q}_t\left(D(\omega)\right), & \text{else.} \end{cases} \tag{26}$$

Production in each period is efficient (and equal to $q^*$) when the cum-dividend liability price is high enough. Production is constrained (and below $q^*$) when the cum-dividend liability price is too low (below $d^*$). Note that $q_2^{eq}\left(D\left(\omega\right)\right)$ depends only on the period 2 coupon $d_2(\omega)$ while $q_1^{eq}\left(D\left(\omega\right)\right)$ depends directly on the period 1 coupon $d_1\left(\omega\right)$ and indirectly on the period 2 coupon $d_2\left(\omega\right)$ which influences the period 1 ex-coupon price.

The first period ex-coupon asset price is determined by the buyer's marginal decision to purchase bank liabilities in period 1. In state $\omega$, the price at which buyers are willing to purchase

1 unit of bank liabilities is

$$p_1\left(D(\omega)\right) = d_2\left(\omega\right) + \alpha\eta d_2\left(\omega\right)\frac{u'\left(q_2^{eq}\left(D(\omega)\right)\right) - c'\left(q_2^{eq}\left(D(\omega)\right)\right)}{(1-\eta)u'\left(q_2^{eq}\left(D(\omega)\right)\right) + \eta c'\left(q_2^{eq}\left(D(\omega)\right)\right)}, \tag{27}$$

The liability price reflects the discounted expected coupon plus a discounted liquidity premium. The liquidity premium is strictly positive only when decentralized trade is constrained or, when $d_2(\omega) < d^*$ so that $q_2^{eq}\left(D(\omega)\right) < q^*$.

Equation (27) holds in models with no decentralized trade and risk-neutral agents where the asset price is the discounted expected value of the coupons. Equation (27) also holds in monetary models where asset prices reflect not only their coupons but also their usefulness in relaxing trading frictions—see Lagos (2010) for example. The period 0 price of bank liabilities is

$$p_0(D) = \sum_{\omega}\gamma(\omega)\left[d_1(\omega) + p_1(D(\omega))\right]\left[1 + \alpha\eta\frac{u'\left(q_1^{eq}\left(D(\omega)\right)\right) - c'\left(q_1^{eq}\left(D(\omega)\right)\right)}{(1-\eta)u'\left(q_1^{eq}\left(D(\omega)\right)\right) + \eta c'\left(q_1^{eq}\left(D(\omega)\right)\right)}\right]. \tag{28}$$

In Appendix A, we show that the planner's welfare function may be written as

$$\max_{D\in\mathcal{D}} W_0^P(D) = (1+n)\,\bar{v} + \sum_{\omega}\gamma\left(\omega\right)\Big(U_1^P(D(\omega)) + U_2^P(D(\omega))\Big), \tag{29}$$

with $U_t^P\left(D\left(\omega\right)\right)$ the planner's period $t$ indirect welfare function,

$$U_t^P\left(D\left(\omega\right)\right) \equiv (1+n)\bar{v} + d_t\left(\omega\right) + \alpha\left[u\left(q_t^{eq}\left(D\left(\omega\right)\right)\right) - c\left(q_t^{eq}\left(D\left(\omega\right)\right)\right)\right], \tag{30}$$

and $\bar{v} = \max_x v\left(x\right) - x$. The welfare function in (30) sums the buyers' and sellers' utilities in the centralized and decentralized markets with $(1+n)\,\bar{v} + d_t\left(\omega\right)$ the households' welfare in the centralized market and the final term welfare in the decentralized market.

The period two indirect welfare function $U_2^P(D\left(\omega\right))$ in Equation (30) depends only on $d_2\left(\omega\right)$ and is concave in $d_2\left(\omega\right)$ near $d^*$. For values of $d_2$ below $d^*$, decentralized trade is constrained—surplus between buyers and sellers in decentralized markets is increasing in $d_2$ for such values. When $d_2 \geq d^*$, decentralized trade is efficient so surplus in decentralized meetings is independent of $d_2$ in this region. Below $d^*$, $U_2^P\left(D\left(\omega\right)\right)$ is increasing and concave sufficiently close to $d^*$.

The planner is risk averse with respect to coupon payments if in some states $d_2(\omega) < d^*$, and risk-neutral otherwise.

If $d_2(\omega) < d^*$, then trade is constrained in period 2 decentralized markets, and the liquidity premium in the liability price is strictly positive. A change in the period 2 cash flow in that state will impact the liquidity premium and, therefore, liability prices. Increasing the promised period one coupons through liquidation may increase the price and may prove useful for the planner to improve decentralized trade in period 1.

**Characterizing Efficient Liability Issuance.** Before characterizing efficient liability issuance, we provide explicit definitions of maturity and risk transformation in our model. The stochastic return on households' initial endowment of trees is $z(\omega) K^H$. The planner may transform risk by requiring banks to issue liabilities with less volatile coupon payments than the households' returns and may transform maturity by requiring banks to issue liabilities with period 1 coupon payments.

**Definition 1 (Risk Transformation).** *An allocation features* risk transformation *if the liability payoffs satisfy*

$$z(\omega_l) K^H < d_1(\omega_l) + d_2(\omega_l) \leq d_1(\omega_h) + d_2(\omega_h) < z(\omega_h) K^H. \tag{31}$$

Risk transformation occurs when liability payoffs are less volatile than the return on households' initial endowment of trees.

**Definition 2 (Maturity Transformation).** *An allocation features* maturity transformation *if for some* $\omega$, $d_1(\omega) > 0$.

Maturity transformation occurs when banks' liability payoffs are larger than bank assets without any liquidation and households' initial endowment of trees yield positive returns in period 1. Recall that in the absence of liquidation, households' trees yield no returns in period one.

Before characterizing efficient allocations, we make the following assumption regarding the limited commitment frictions.

**Assumption 1** (**Minimum Bank Capital**). *The endowments, $K^B$ and $K^H$ and the absconding parameter, $\xi$ satisfy*

$$\frac{K^B}{K^H + K^B} \geq \xi.$$

Assumption 1 places a lower bound on the endowment of banks' (inside) equity relative to the attractiveness of absconding with trees indexed by $\xi$. This assumption ensures that it is commitment-feasible for a bank to issue a claim with neither risk nor maturity transformation (e.g. a claim that delivers the same consumption as households' initial endowments of trees). We make this assumption so that the limited commitment friction alone does not induce maturity transformation as in Calomiris and Kahn (1991). We now state our main result while leaving a full characterization of efficient allocations in Appendix B.

**Proposition 1** (**Risk and maturity transformation**). *There exists $\underline{\xi} < K^B/(K^B + K^H)$, $\underline{\kappa} > 0$ and $\underline{z} > 0$ such that if $\xi \geq \underline{\xi}$, $\kappa \geq \underline{\kappa}$, and $z(\omega_l) < \underline{z}$, then efficient allocations feature both risk and maturity transformation.*

Proposition 1 describes conditions such that efficient liability issuance features both risk and maturity transformation. The Proposition reveals that maturity and risk transformation are features of efficient arrangements when (i) trees deliver sufficiently low consumption in period two in the low economic state summarized by low $z(\omega_l)$ and (ii) liquidation costs are small as summarized by high values of $\kappa$ and high values of $\xi$.

When $z(\omega_l)$ is small enough, claims backed by households' initial endowment of trees do not provide efficient liquidity. Such claims would not be valued enough to deliver efficient production in decentralized markets (either in period one or two). As a result, the concavity in the planner's welfare function (30) provides a rationale for risk transformation. The planner allocates a portion of the returns to banks' own inside equity to households in the low state thereby increasing coupons in the low state (and allocates a portion of the returns to households' initial tree endowments to banks in the hight state).

However, when $z(\omega_l)$ is sufficiently low, the amount of risk transformation the planner can undertake is limited by the commitment constraint of the banks. By transforming maturity—

liquidating trees—the planner can relax this constraint. Liquidation, however, entails direct resources costs at rate $(1-\kappa)$ as well as indirect costs that are sensitive to the absconding parameter, $\xi$. Proposition 1 asserts that when the direct costs of liquidation are small ($\kappa$ close to 1) and when the indirect costs are small (which happens when $\xi$ sufficiently large), then maturity transformation caused by liquidation is socially efficient. In other words, maturity transformation allows the planner to provide more efficient insurance against aggregate liquidity shocks.

We now briefly illustrate the proof of Proposition 1 so show these forces more clearly. Consider first the best allocation the planner may attain without liquidation. In Appendix B, we show this allocation satisfies the bank's limited commitment constraint with equality in state $\omega_l$:

$$d_2(\omega_l) = \left(K^H + K^B\right)(1-\xi)z(\omega_l)\,. \tag{32}$$

Suppose that $z(\omega_l)$ is sufficiently small so that (32) implies that $d_2(\omega_l) < d^*$. If the bank's limited commitment constraint were slack, then the planner could increase $d_2(\omega_l)$, decrease $d_2(\omega_h)$—allowing the planner to continue to satisfy the bank's ex ante participation constraint—and strictly raise households' welfare through an improvement in decentralized terms of trade. If $d_2(\omega_l)$ satisfies (32), $d_2(\omega_h)$ in the best allocation without liquidation may be obtained from the bank's ex ante participation constraint holding with equality when $d_1(\omega) = c_1^B(\omega) = 0$.

Consider a perturbation from the best allocation without liquidation to an allocation where $L(\omega_l) = \varepsilon > 0$ and where the bank's limited commitment constraint in period 2 state $\omega_l$ continues to hold with equality. By construction, the perturbation reduces the bank's consumption $c_2^B(\omega_l)$ so we must also raise $c_2^B(\omega_h)$ enough to satisfy the bank's participation constraint.Under the conditions of Proposition 1, the perturbation does not reduce $d_2(\omega_h)$ below $d^*$. The marginal impact on welfare from the perturbation is

$$\left(K^H + K^B\right) z(\omega_l)\,\gamma(\omega_l)\left\{U^P_{1,1_l}\kappa - \left(U^P_{1,2_l} + U^P_{2,2_l}\right)(1-\xi)\right\} \\ - \left(K^H + K^B\right) z(\omega_l)\,\gamma(\omega_h)\,\xi\frac{\gamma(\omega_l)}{\gamma(\omega_h)}\left\{U^P_{1,2_h} + U^P_{2,2_h}\right\}, \tag{33}$$

where $U^P_{t,i_j}$ is the derivative of $U^P_t$ with respect to $d_i(\omega_j)$ for $j = l, h$.

The perturbation raises period 1 liability payouts in the low state in period 1 but lowers all other liability payouts. The marginal adjustments to liability payouts all occur at rate $\left(K^H + K^B\right) z\left(\omega_l\right)$. The first line of (33) is the net impact of changes in liability payouts in the low state. The perturbation increases the period 1 coupon at rate $\kappa$ with marginal benefit $U^P_{1,1_l} > 1$.

The marginal benefit is larger than one in $\omega_l$ because $d_1\left(\omega_l, \varepsilon\right) + p_1\left(\omega_l, \varepsilon\right) < d^*$ and decentralized terms of trade are improved. The perturbation decreases the period 2 coupon at rate $\left(1 - \xi\right)$ with marginal cost $U^P_{1,2_l} + U^P_{2,2_l} > 1$. The marginal cost is larger than 1 because $d_2\left(\omega_l, \varepsilon\right) < 1$ and the reduction in $d_2\left(\omega_l, \varepsilon\right)$ reduces both second period decentralized terms of trade as well as first period terms of trade through its impact on period 1 liability price. If $\xi$ is large, the perturbation does not reduce period 2 coupon payments significantly.

The second line of (33) is the net impact of changes in liability payouts in $\omega_h$. The perturbation reduces the period 2 coupon at rate $\xi\gamma\left(\omega_l\right)/\gamma\left(\omega_h\right)$ with marginal cost $U^P_{1,2_h} + U^P_{2,2_h} = 1$. The marginal cost of this reduction is 1 because $d_2\left(\omega_h, \varepsilon\right) > d^*$. The perturbation reduces the period 2 coupon in order to compensate the bank for receiving lower consumption in period 2 in $\omega_l$. The planner is able to compensate the bank in $\omega_h$ where households' marginal value of coupon payments is low, therefore allowing the possibility for liquidation to be optimal. Simplifying (33), the marginal benefit of the perturbation is

$$\left(K^H + K^B\right) z\left(\omega_l\right) \gamma\left(\omega_l\right) \left\{U^P_{1,1_l}\kappa - \left(U^P_{1,2_l} + U^P_{2,2_l}\right)\left(1 - \xi\right) - \xi\right\}. \tag{34}$$

In Appendix B, we show that when the conditions of Proposition 1 hold, the perturbation yields a Pareto improvement: efficient liabilities feature maturity transformation.

Four conditions need to be satisfied for maturity transformation to be optimal. First, bank liabilities must circulate as a medium of exchange: $\alpha > 0$. Households are risk-averse to bank liability payouts only if liabilities serve as a medium of exchange. Since the benefit of liquidation is an improvement in smoothing payouts, households do not value liquidation when bank liabilities do not circulate. Second, there must be sufficient (downside) risk in the value of productive trees. When $z(\omega_l)$ is sufficiently high, claims directly backed by risk assets provide sufficient liquidity in all states to serve as a useful medium of exchange. Third, liquidation costs cannot be too large: $\kappa$ must be close enough to 1. An increase in $\kappa$ directly reduces the costs of liquidation making it a more attractive option. Fourth, $\xi$ must be large enough for the bank's limited commitment to be sufficiently binding. Maturity transformation is only efficient if it relaxes banks' limited commitment constraints and improves risk-sharing. We summarize these results in the following proposition that illustrates when efficient allocations feature only risk transformation or neither risk nor maturity transformation.

**Proposition 2 (Efficiency without Maturity Transformation).** *If $z(\omega_l) \geq d^*/K^H$, then efficient liability issuance features neither risk nor maturity transformation. Furthermore, there exists a threshold $\underline{z}_r \leq d^*/K^H$ such that if $z(\omega_l) \in (\underline{z}_r, d^*/K^H)$ and $E_0 z(\omega) \geq d^*/K^H$, then efficient liability issuance features risk transformation but no maturity transformation.*

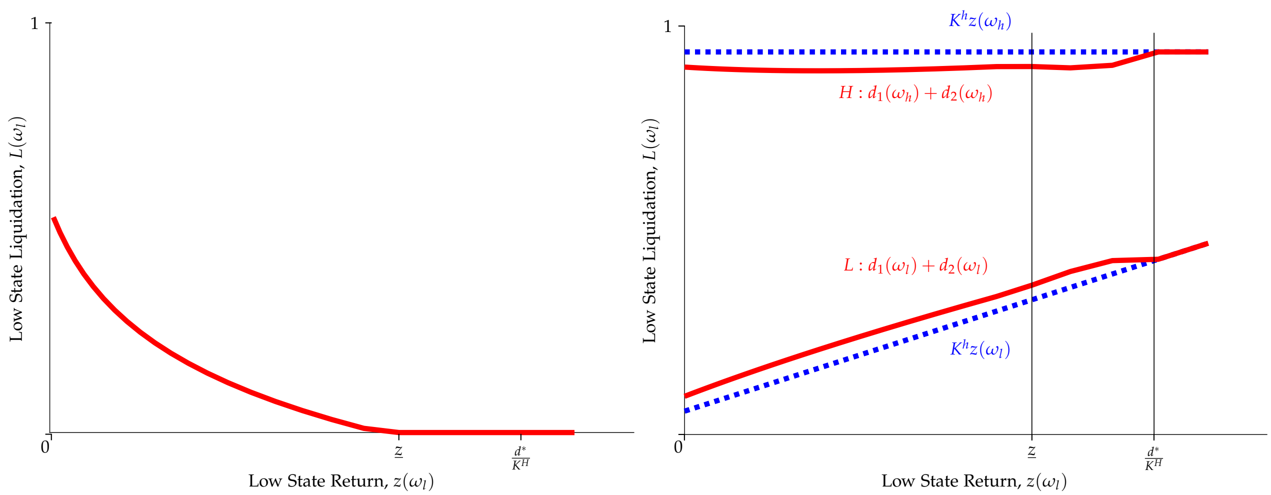


Figure 2: Constrained efficient liquidation in state $\omega_l$ (left-panel) and total state-contingent coupon payments (right-panel) for various values of $z(\omega_l)$ in a numerical example. The parameter assumptions are: $\alpha(n) = n, u(q) = [(q+z)^{(1-a)} - z^{(1-a)}]/(1-a)$ where $z = 10^{-5}$ and $a = 0.5$, $c(q) = q, \eta = 0.5, n = 0.5, \gamma(\omega_h) = \gamma(\omega_l) = 0.5$, $K^h = 20, K^B = 43.3, \xi = 0.65, \kappa = 0.98$.

Figure 2 illustrates features of constrained efficient allocations in a numerical example. The left panel of Figure 2 plots the constrained efficient liquidation level in state $\omega_l$. The solid lines in the right panel of Figure 2 plot total coupon payments $d_1(\omega) + d_2(\omega)$ for each state against

$z(\omega_l)$. The dashed lines plot the present discounted value of households' endowments of capital goods against $z(\omega_l)$.

If $z(\omega_l) \geq \underline{z}$, efficient allocations feature no liquidation and if $z(\omega_l) < \underline{z}$ liquidation is strictly positive. Once $z(\omega_l)$ falls below $d^*/K^H$, however, efficient allocations smooth coupon payments relative to the value of households' endowments so that $d_1(\omega_l) + d_2(\omega_l) > K^h z(\omega_l)$ and $d_1(\omega_h) + d_2(\omega_h) < K^h z(\omega_h)$. Figure 2 shows that as investments become less productive in state $\omega_l$, efficiency first calls for banks to engage in risk-transformation. As bank investments become even less productive, efficiency calls for banks to engage in maturity transformation to provide risk transformation.

Proposition 1 states that if there is enough risk in bank investment opportunities and if banks' limited commitment problem is severe enough, then liabilities which only feature risk transformation do not provide sufficient liquidity to households. Risk transformation is impeded by the bank's limited commitment constraint. Maturity transformation relaxes the bank's commitment constraint and allows for an improvement in risk transformation, making the households better off. Given the direct costs of liquidation and the severity of the bank's commitment problem, maturity transformation is socially optimal only when bank assets are risky enough.

Figure 3 illustrates the threshold $\underline{z}$ for various values of the match rate between buyers and sellers, $\alpha$ for high and low values of bank capital. To the left of each threshold, constrained efficient allocations feature maturity transformation while to the right of each threshold constrained efficient allocations feature no maturity transformation. Independent of the bank's capital level, the figure illustrates that maturity transformation is more likely to be a feature of constrained efficient allocations when $\alpha$ is high so that households expect to match in decentralized markets frequently. In such a case, the motive for the planner to smooth liquidity distortions is high leading to maturity transformation.

Figure 3 also demonstrates that maturity transformation is more likely to be a feature of constrained efficient allocations when bank capital is low so that the region of maturity transformation is larger. The result follows because the limited commitment constraints are more severe for banks with less capital and binding commitment constraints interact with the desire

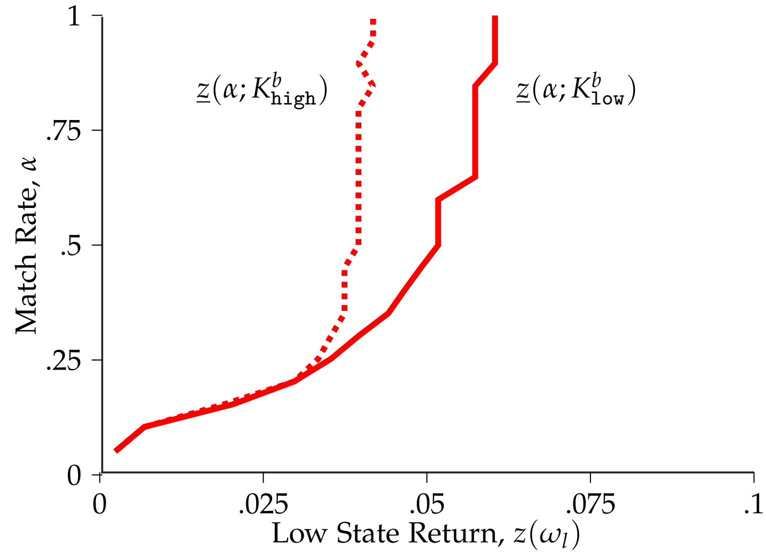


Figure 3: Numerical illustration of the threshold, $\underline{z}$, as a function of the match rate $\alpha$ for two given values of bank capital. The parameter assumptions are the same as in Figure 2 with $\alpha(n) = \alpha n$ for all $\alpha \in (0, 1)$, $K^B_{\mathtt{low}} = 43.3$ and $K^B_{\mathtt{high}} = 52$.

to smooth liquidity distortions imply maturity transformation.

## 4 Competitive Equilibrium Outcomes

In this section, we describe competitive outcomes and show that they do not coincide with Pareto Optimal outcomes. Appendix A describes the competitive equilibrium that obtains when all banks issue arbitrary liabilities $D \in \mathcal{D}$. Once banks issue their liabilities, equilibrium prices are determined by the trading decisions of the households. We consider sequential and symmetric subgame perfect equilibrium where each bank takes aggregate liability issues as given.

Computing a bank's optimization problem for alternative liability issues requires banks to know what proceeds they will receive from any possible liability issue. Let $D$ be the aggregate liability issue, $D^i$ the liability issue that the $i^{th}$ bank is considering and $p_0(D^i; D)$ bank $i$'s conjectured liability pricing for coupon vector $D^i$ given an aggregate liability issue $D$. To construct $p_0(D^i; D)$, note that in the symmetric equilibrium, each bank will issue the same liability. We re-write the period 0 price as a linear combination of the state-contingent coupon payments with weights resembling Arrow-Debreu prices. Define $\pi_t(\omega; D)$ as

$$\pi_t(\omega; D) = \gamma(\omega)\left[1 + \alpha\eta \frac{u'\left(q_t^{eq}\left(D(\omega)\right)\right) - c'\left(q_t^{eq}\left(D(\omega)\right)\right)}{(1-\eta)u'\left(q_t^{eq}\left(D(\omega)\right)\right) + \eta c'\left(q_t^{eq}\left(D(\omega)\right)\right)}\right]. \tag{35}$$

Using the Arrow-Debreu prices and from (28) and (27), the symmetric subgame equilibrium liability price is

$$p_0(D; D) = \sum_t \sum_\omega \pi_t(\omega; D) d_t(\omega). \tag{36}$$

For any alternative liability issue $D^i$, assume the liability price is

$$p_0(D^i; D) = \sum_t \sum_\omega \pi_t(\omega; D) d_t^i(\omega). \tag{37}$$

Banks understand that issuing liabilities with larger coupon payments will raise their revenues from issuance, but they don't perceive that their issuance decisions will impact households' willingness to purchase liabilities. In this sense, banks are price-takers.

Each competitive bank solves (5) using the conjectured pricing function in Equation (37) taking the Arrow-Debreu prices as given, subject to the bank's budget constraints in equations (6) to (10) and the bank's participation constraint (12). To reduce notation, we let $D^*$ be the equilibrium liability issue, and let $p_0(D^*)$ be the pricing function computed using the Arrow-Debreu prices evaluated at the equilibrium liability issue: $p_0(D^*) \equiv p_0(D^*; D^*)$.

When the planner's optimal allocation features no risk or maturity transformation, the competitive issue also features no risk or maturity transformation. In such situations, the equilibrium Arrow-Debreu prices satisfy $\pi_t(\omega; D) = \gamma(\omega)$. Such an economy has risk-neutral pricing and the planner's allocation is a competitive equilibrium. With risk-neutral pricing, there are no liquidity premia in liability prices. When the planner's optimal allocation features risk transformation but no maturity transformation, then liability prices do contain liquidity premia. Nonetheless, the planner's allocation is a competitive equilibrium.

**Proposition 3.** *If the efficient allocation does not feature any maturity transformation, then the efficient* *allocation is a competitive equilibrium.*

Suppose instead that the conditions of Proposition 1 are satisfied so that the planner's optimal allocation features maturity transformation in $\omega_l$. In this case, equilibrium allocations are inefficient, a result we state in the next Proposition.

**Proposition 4.** *Suppose the efficient allocation satisfies $L(\omega_l) > 0$. Then the efficient allocation cannot be implemented as a competitive equilibrium and the competitive equilibrium features less liquidation than the efficient allocation.*

To understand why the equilibrium is inefficient and features too little liquidation—that is, too little maturity transformation—let $D^*$ denote the planner's optimal coupon issuance. Note that in this case, the planner liquidates only in $\omega_l$. Let $\pi_t(\omega; D^*)$ denote the Arrow-Debreu prices which would obtain in an equilibrium consistent with the planner's allocation where, since liquidity is scarce in $\omega_l$, these prices feature a strictly positive liquidity premium in $\omega_l$ so that $\pi_t(\omega_l; D^*) > \gamma(\omega)$.

We show that at these implied prices, banks are able to increase their payoffs by deviating to an alternative allocation. Under the conditions of Proposition 1, bank optimality requires the bank receive no consumption in period 1 and, since the conjectured equilibrium prices feature a strictly positive liquidity premium in $\omega_l$, the bank's commitment constraint in $\omega_l$ must bind. With these binding constraints, the bank's optimality condition for liquidation satisfies

$$\kappa\pi_1(\omega_l; D^*) - (1 - \xi)\pi_2(\omega_l; D^*) - \xi\gamma(\omega_l) = 0. \tag{38}$$

The optimality condition in (38) reflects the impact of a marginal increase in liquidation on revenues raised through liability issuance of the bank less the forgone consumption in period 2 in $\omega_l$. A marginal increase in $L(\omega_l)$ given a stock of trees $I$ allows the bank to pay $I\kappa$ more coupons in period 1 in $\omega_l$ increasing revenues by $I\kappa\pi_1(\omega_l; D^*)$. The marginal increase requires the bank to pay $I(1 - \xi)$ fewer coupons in period 2 in $\omega_l$ because the binding commitment constraint reduces issuance revenues by $I(1 - \xi)\pi_2(\omega_l; D^*)$. The resulting marginal increase in liquidation requires an expected reduction in bank consumption in period 2 in $\omega_l$ of $I\xi\gamma(\omega_l)$.

Next, compare (38) to the optimality condition of liquidation in $\omega_l$ for the planner. Using the Arrow-Debreu prices, (34) is

$$\begin{aligned}\kappa\pi_1(\omega_l;D^*) - (1-\xi)\pi_2(\omega_l;D^*) - \xi\gamma(\omega_l) &= -\gamma(\omega_l)(1-\eta)(1-\kappa) \\ &+ \frac{d\pi_2(\omega_l;D^*)}{dq_2^{eq}} \frac{(1-\xi)d_2^*(\omega_l)\left[\pi_1(\omega_l;D^*) - \gamma(\omega_l)\right]}{\pi_1(\omega_l;D^*)\left[(1-\eta)u'\left(q_2^{eq}\left(D^*(\omega_l)\right)\right) + \eta c'\left(q_2^{eq}\left(D^*(\omega_l)\right)\right)\right]}. \end{aligned} \tag{39}$$

When $q_2^{eq}$ lies below $q^*$, an increase in $q_2^{eq}$ reduces the liquidity premium associated with bank liabilities and therefore decreases the Arrow-Debreu price: $d\pi_2(\omega_l;D^*)/dq_2^{eq} < 0$. Under the conditions of Proposition 1 $\pi_1(\omega_l;D^*) > \gamma(\omega_l)$, the second line of (39) is strictly negative. The efficient allocation does not satisfy bank optimality (38) since (39) implies the bank strictly prefers to reduce $L(\omega_l)$.

The difference between optimal liquidation for the bank in Equation (38) and efficient liquidation in Equation (39) shows why equilibrium allocations are inefficient. The Arrow-Debreu price in period 2 in the planner's allocation in $\omega_l$ is too high in the competitive equilibrium—it provides incentives for a single bank to increase period 2 coupon payments and increase issuance revenues and expected consumption faster than the resulting losses from revenues from the concomitant period 1 $\omega_l$ coupon issue.

The Arrow-Debreu price $\pi_2(\omega_l;D^*)$ is too high for two reasons. The first source of inefficiency is the first term on the right hand side of (34) and is proportional to $1-\eta$. The inefficiency is a standard bargaining inefficiency. Since bank liabilities are priced by buyer-type households, these buyers do not internalize the fact that by bringing more liabilities into decentralized markets they generate more surplus for the seller-type household they meet. The first source of inefficiency resembles those that arise in most models with bargaining (see Hosios (1990)) and as $\eta \to 1$, the first source of inefficiency vanishes.

The second source of inefficiency is the second term on the right hand side of (34) and reflects a pecuniary externality novel to our environment. While the planner internalizes how a change in liquidation impacts the liquidity premium and, therefore, the Arrow-Debreu price reflected by $d\pi_2(\omega_l;D^*)/dq_2^{eq}$, an individual bank does not internalize the effect. An individual bank is able to free-ride on the high liquidity premium associated with period 2 coupon issues in $\omega_l$ in the efficient allocation; the bank does not internalize that if all banks were to issue more period 2 coupon issuances, they would ultimately reduce the liquidity premium associated with period 2 coupons. In any competitive equilibrium, banks liquidate less than the efficient amount in $\omega_l$.

Figure 4 illustrates Proposition 4. The dashed red line is constrained efficient liquidation in state $\omega_l$, and the solid green line is liquidation in the competitive equilibrium.

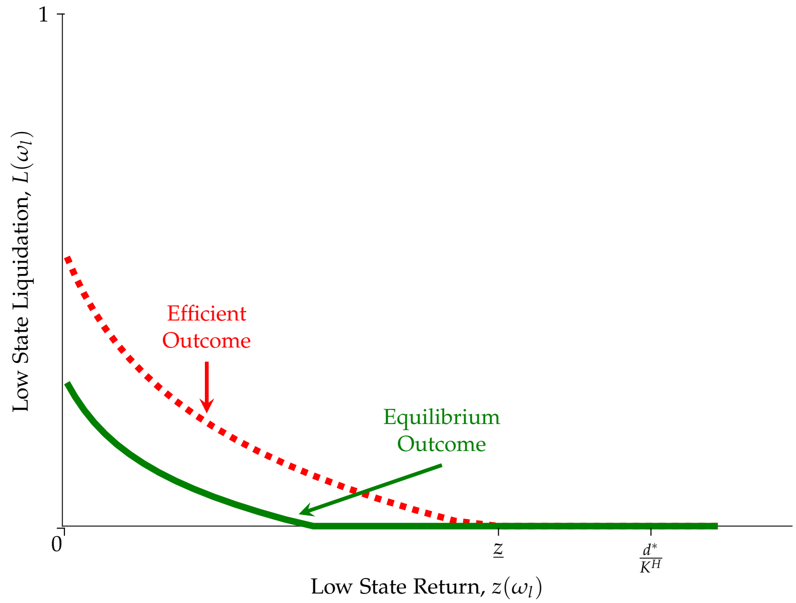


Figure 4: Constrained efficient liquidation and equilibrium liquidation in state $\omega_l$. The parameters are the same as in Figure 2

Proposition 4 shows that there is a role for regulative policy when $\alpha > 0$ and efficient allocations require banks to perform maturity transformation. In the absence of policy, banks issue liabilities which promise too many cash flows in period 2 and too few cash flows in period 1 in low-return states. Inside money issued by banks in the unregulated competitive equilibrium features too much risk since expected discounted cash flows are more volatile than in the efficient allocation.

The simplest policy which implements the constrained efficient allocations is a state-contingent liquidation floor. The bank's liquidation in state $\omega$ must satisfy $L(\omega) \geq \bar{L}(\omega)$, where $\bar{L}(\omega_h) = 0$, $\bar{L}(\omega_l) = L^*(\omega_l)$ with $L^*(\omega_l)$ the constrained efficient liquidation level in state $\omega_l$. Since banks have private incentives to reduce $L(\omega_l)$ below the threshold, the binding constraint in state $\omega_l$ ensures banks implement the constrained efficient level of maturity transformation.

We interpret the policy as a minimum expected short term payout policy for banks from the perspective of period 0. Such an optimal policy conflicts with liquidity policies implemented in the wake of Basel III.[6] The Liquidity Coverage Ratio introduced by U.S. financial regulators in 2014 requires that banks hold sufficient liquid assets to cover expected short-term net outlays during a 30-day stress period. Since banks may wish to minimize holdings of liquid assets generally with low returns, such policies incentive banks to minimize expected short term outlays. Our finding that banks must be incentivized to issue liabilities with large enough short term payouts suggests that the liquidity coverage ratio may impede on banks ability to create a stable, low-risk source of liquidity or means of payments to households.

# 5 Empirical Evidence

Our model predicts a negative relation between bank liquidity and money velocity. We provide suggestive empirical support for the negative relation using balance sheet items from the FDIC's Reports of Condition and Income (the Call Report), branch level deposit information from the FDIC's Summary of Deposits (the SOD), annual Consumption and GDP at the state level and GDP at the MSA level obtained from the Bureau of Economic Analysis.

Branch level deposits in the Summary of Deposits are the reported level of deposits for each bank branch as of June 30th for each year. We use each bank's reported levels of balance sheet items as of June 30th to calculate liquidity at the bank-level. Our measure of bank liquidity for each bank in each year is a simplified version of the main regulatory measure of bank liquidity, the Liquidity Coverage Ratio (LCR). The LCR is the ratio of liquid assets to expected net cash outflow over a 30 day period. Liquid assets are calculated as a weighted sum of assets on a bank's balance sheet, where the weights are reflective of the haircut required on sales of the particular

[6]See Ennis et al. (2011) for a discussion of revisions to capital requirements contained in Basel III and House et al. (2016) for a specific discussion of liquidity coverage ratios implemented in 2014 in the U.S.

asset in a period of low market liquidity. Bank $i$'s liquid assets in year $t$ are

$$\begin{aligned} \text{Liquid Asset}_{i,t} = {} & 1.00\,\text{US Treasury Securities}_{i,t} + 0.85\,\text{US Agency Securities}_{i,t} + \\ & 0.5\,(\text{Cash and Balances Due}_{i,t} + \text{Other Securities}_{i,t}), \end{aligned} \tag{40}$$

with

$$\text{Other Securities}_{i,t} = \text{Total Securities}_{i,t} - \text{US Treasury Securities}_{i,t} - \text{Agency Securities}_{i,t}. \tag{41}$$

Net cash outflow is the estimated cash outflow minus estimated cash inflow over 30 days in a period of low liquidity. Estimated inflows and outflows are calculated as a weighted sum of assets and of liabilities, where the weights are based on the expected inflow or outflow associated the each balance sheet item. Bank $i$'s inflow in year $t$ is:

$$\begin{aligned} \text{Inflow}_{i,t} = {} & 0.2\,\text{Interest Bearing Balances}_{i,t} + 0.05\,(\text{Securities}_{i,t} + \text{Net Loans and Leases}_{i,t}) \\ & + 0.5\,\text{Trade Assets}_{i,t} + 1.00\,\text{Fed Funds Sold and Reverse Repurchase Agreements}_{i,t}. \end{aligned} \tag{42}$$

Bank $i$'s outflow in year $t$ is:

$$\begin{aligned} \text{Outflow}_{i,t} = {} & 0.07\,\text{Deposits}_{i,t} + 0.40\,\text{Unused Commitments}_{i,t} + \\ & 0.50\,(\text{Trade Liabilities}_{i,t} + \text{Other Debt}_{i,t} + \text{Other Liabilities}_{i,t}) \\ & + 1.00\,\Big(\text{Derivatives}_{i,t} + \text{Fed Funds Purchased and Repurchase Agreements}_{i,t}\Big). \end{aligned} \tag{43}$$

The inflow used in the calculation is typically restricted so that there is a minimum net cash outflow. As in the regulatory calculation of the LCR, inflow is restricted to being 75 percent of cash inflow. The LCR for bank $i$ at time $t$ is:

$$\text{LCR}_{i,t} = \frac{\text{Liquid Assets}_{i,t}}{\text{Outflow}_{i,t} - \min\{\text{Inflow}_{i,t}, (0.75)\text{Outflow}_{i,t}\}} \times 100. \tag{44}$$

Our goal is to compare bank liquidity and money velocity, so we compute measures of bank liquidity, monetary aggregates, and economic activity. The BEA produces measures of GDP and consumption at the state level, and GDP at the MSA level. We compute measures of liquidity and monetary aggregates at the MSA and state level using a similar approach to Buchak et al. (2018) who use variation at the MSA level to study the relation between capital requirements, shadow banking and regulatory arbitrage. They construct MSA aggregates by computing weighted averages of bank variables, weighted by market share. We follow a similar approach to compute MSA level values of liquidity by aggregating liquidity across banks by bank deposit shares.

Given branch level data on deposits, we compute the total deposits of bank $i$ in region $r$ at time $t$ for all banks and all regions. From this, we proxy the monetary aggregate in region $r$ at time $t$ by total deposits via:

$$M_{r,t} = \sum_i \text{Deposits}_{i,r,t}. \tag{45}$$

Bank $i$'s market share in region $r$ at time $t$ is:

$$s_{i,r,t} = \frac{\text{Deposits}_{i,r,t}}{\sum_i \text{Deposits}_{i,r,t}}. \tag{46}$$

We compute bank liquidity in region $r$ at time $t$ as

$$\text{LCR}_{r,t} = \sum_i \left( s_{i,r,t} \times \text{LCR}_{i,t} \right). \tag{47}$$

The BEA produces state GDP on an annual and quarterly basis, MSA GDP on an annual basis, and state consumption on an annual basis. Given that the measures of liquidity and monetary aggregates are reported as of June 30th, we require a measure of economic activity at mid-year as well. For annual GDP and Consumption, our mid-year estimate is computed by taking 0.5 times GDP or consumption in year $t$ plus 0.5 times GDP or consumption in year $t-1$. For state-level GDP on a quarterly basis, we calculate a measure of mid-year activity as Q2 GDP in year $t$ plus Q1 GDP in year $t$ plus Q4 GDP in year $t-1$ plus Q3 GDP in year $t-1$. The quarterly-based measure of state GDP and the annual-based measure of state GDP have a correlation coefficient

0.99—the two measures yield approximately the same result.

Finally, given a particular measure of regional economic activity for region $r$ at time $t$, $Y_{r,t}$, money velocity is defined as

$$V_{r,t} = Y_{r,t}/M_{r,t}. \tag{48}$$

Table 1 reports summary statistics for the bank specific level variables. Our sample runs from 2003 to 2016. We have 66,546 bank-year observations for 7,134 banks. There is more variation in the Liquidity coverage ratios between banks than across time for the banks, although there is variation at both levels in our sample.

Table 1: Bank-Level Summary Statistics

| | Mean | SD | Min | Max | Between Bank SD | Within Bank SD |
|---|---|---|---|---|---|---|
| Securities | 0.202 | 0.150 | 0.0 | 1.0 | 0.141 | 0.065 |
| US Treasury Securities | 0.007 | 0.034 | 0.0 | 0.9 | 0.030 | 0.018 |
| US Agency Securities | 0.138 | 0.119 | 0.0 | 1.0 | 0.108 | 0.058 |
| Cash and Balances Due | 0.070 | 0.080 | -0.0 | 1.0 | 0.067 | 0.052 |
| Interest Bearing Balances | 0.042 | 0.076 | 0.0 | 1.0 | 0.062 | 0.050 |
| Repo Securities | 0.026 | 0.059 | 0.0 | 1.0 | 0.054 | 0.040 |
| Net Loans and Leases | 0.644 | 0.163 | 0.0 | 1.0 | 0.155 | 0.073 |
| Trade Assets | 0.001 | 0.010 | 0.0 | 0.7 | 0.008 | 0.006 |
| Deposits | 0.818 | 0.108 | 0.0 | 1.1 | 0.116 | 0.046 |
| Repo Liabilities | 0.015 | 0.043 | 0.0 | 0.9 | 0.041 | 0.022 |
| Trade Liabilities | 0.000 | 0.003 | 0.0 | 0.2 | 0.002 | 0.001 |
| Other Debt | 0.045 | 0.062 | 0.0 | 1.0 | 0.060 | 0.034 |
| Other Liabilities | 0.009 | 0.022 | 0.0 | 0.9 | 0.025 | 0.010 |
| Unused Commitments | 0.672 | 19.374 | 0.0 | 1900.5 | 31.714 | 3.954 |
| Derivatives | 0.095 | 4.374 | 0.0 | 537.4 | 4.872 | 0.927 |
| LCR High Quality Liquid Assets | 0.212 | 0.125 | 0.0 | 1.0 | 0.115 | 0.060 |
| LCR Net Cash Flow | 0.405 | 8.895 | 0.0 | 760.2 | 13.583 | 1.836 |
| Leverage | 0.113 | 0.071 | -0.1 | 1.0 | 0.078 | 0.029 |
| Liquidity Coverage Ratio | 4.665 | 13.526 | 0.0 | 1092.0 | 12.672 | 8.242 |
| Number of bank-year observations | 66546 | | | | | |
| Number of banks | 7134 | | | | | |
| Number of years | 14 | | | | | |

The variables presented as fraction of bank assets except for Leverage and LCR

The table reports summary statistics for the 7,134 banks in our sample.

We use use variation in Liquidity coverage ratios and money velocity to estimate the relation between changes in coverage ratios and money velocity. Table 2 reports summary statistics for our MSA level variables. On average, Velocity and the Liquidity Ratio drop in our sample, with less variation at the MSA level than across time (see the Within MSA SD in the table).

Table 2: MSA-Level Summary Statistics

| | Mean | SD | Min | Max | Between MSA SD | Within MSA SD |
|---|---|---|---|---|---|---|
| Velocity | 2.839 | 1.001 | 0.040 | 8.455 | 0.919 | 0.399 |
| Liquidity Ratio | 1.373 | 1.098 | 0.054 | 14.128 | 0.921 | 0.606 |
| Annual Change in Velocity | -0.036 | 0.258 | -4.213 | 4.213 | 0.055 | 0.252 |
| Annual Change in Liquidity | -0.054 | 0.420 | -4.706 | 4.861 | 0.074 | 0.413 |
| Leverage | 0.106 | 0.013 | 0.055 | 0.189 | 0.008 | 0.011 |
| Annual Change in Leverage | 0.001 | 0.008 | -0.062 | 0.064 | 0.001 | 0.007 |
| Lagged MSA GDP | 33742.019 | 95959.991 | 717.500 | 1575552.000 | 94878.087 | 14898.771 |
| Number of MSA-year observations | 5,329 | | | | | |
| Number of MSAs | 381.000 | | | | | |
| Number of years | 14 | | | | | |

The table reports summary statistics at the MSA level.

Our theory predicts a negative relationship between bank liquidity and money velocity, implying that a increases in bank liquidity should be associated with decreases in velocity. Figure 5 provides plots of the cross-sectional distributions of LCR's and velocity for 2002, 2008 and 2015. Liquidity coverage ratios have tended to increase over time, and velocity has increased and then decreased.

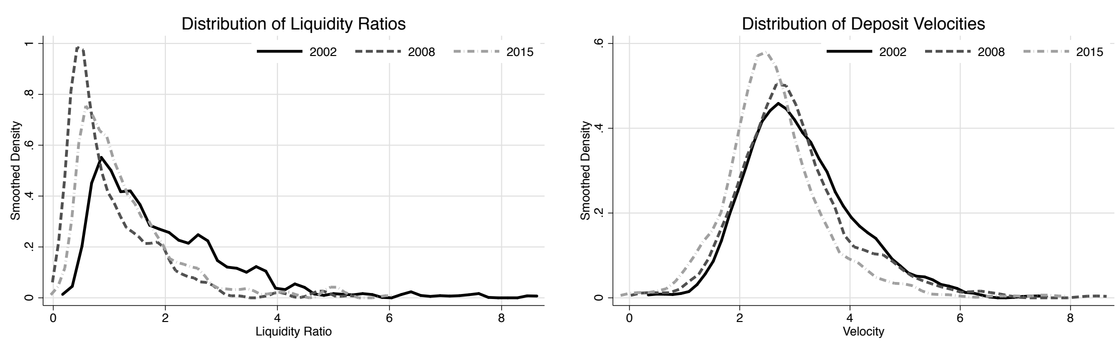


Figure 5: The left panel plots the cross-sectional distribution of LCR's across MSAs in 2002, 2008 and 2015. The right panel plots the cross-sectional of money velocities across MSA's in 2002, 2008 and 2015

Table 3 reports regression coefficients from a linear regression of Annual changes in MSA-level velocity against Annual changes in the Liquidity Coverage ration. The first column reports the univariate regression, and the remaining columns report the estimates including controls for changes in leverage, the level of leverage and lagged GDP at the MSA level. The point estimates on change in LCR is negative in all specifications.

Table 3: Simple OLS, No Fixed Effects

| | (1) | (2) | (3) | (4) | (5) |
|---|---|---|---|---|---|
| Annual Change in Liquidity | -0.0275* | -0.0305* | -0.0293* | -0.0281* | -0.0303* |
| | (0.0120) | (0.0119) | (0.0120) | (0.0120) | (0.0119) |
| Annual Change in Leverage | | 6.393*** | | | 6.708*** |
| | | (0.663) | | | (0.714) |
| Leverage | | | -0.953* | | 0.460 |
| | | | (0.378) | | (0.402) |
| Lagged MSA GDP | | | | -0.000000159*** | -0.000000160*** |
| | | | | (4.34e-08) | (4.28e-08) |
| Observations | 3425 | 3425 | 3425 | 3425 | 3425 |
| R2 | 0.00152 | 0.0280 | 0.00337 | 0.00541 | 0.0323 |
| Adjusted R2 | 0.00123 | 0.0274 | 0.00279 | 0.00483 | 0.0311 |

Standard errors in parentheses
* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

The Table reports OLS estimates regressing the annual Changes in Velocity at the MSA-level against changes in the liquidity coverage ratio and additional control variables. The control variables are the annual changes in leverage at the MSA-level, the level of leverage at the MSA level, and lagged MSA-level GDP.

The specifications in Table 3 do not control well for aggregate or state-level shocks. We report the results from specifications including time and state-level fixed effects in Table 4. Including fixed-effects reduces the economic and statistical significants of the coefficient on the change in LCR, but the point estimates still are negative.

The regression results reported in Tables 3 and 4 document a negative relation between changes in LCR and changes in Velocity at the MSA-level. Although we control for common shocks with the control variables and fixed effects, both the LCRs and Velocity are likely affected unobserved economic shocks. We allow for that possibility by using the MSA-level of liquidity in 2002 as a instrument for Liquidity Changes. Table 5 reports the estimates from an instrumental variables approach. The first column of the Table reports the first stage estimates and the second column reports the coefficient on Annual Changes in Liquidity. The point estimate of the coefficient is negative, although not statistically significant. We interpret the findings as weak evidence that increases in LCRs reduce velocity, consistent with the economic mechanism in our theory.

Table 4: Simple OLS including State and Time Fixed Effects

| | (1) | (2) | (3) | (4) | (5) | (6) |
|---|---|---|---|---|---|---|
| Annual Change in LCR | -0.0275* | -0.0134 | -0.00603 | -0.0131 | -0.0136 | -0.00637 |
| | (0.0120) | (0.0299) | (0.0296) | (0.0300) | (0.0300) | (0.0302) |
| Annual Change in Leverage | | | 5.566* | | | 5.445* |
| | | | (2.366) | | | (2.330) |
| Leverage | | | | -1.597 | | -0.223 |
| | | | | (1.519) | | (1.466) |
| Lagged MSA | | | | | -0.000000129 | -0.000000133 |
| | | | | | (7.18e-08) | (7.04e-08) |
| State FE | No | Yes | Yes | Yes | Yes | Yes |
| Time FE | No | Yes | Yes | Yes | Yes | Yes |
| Observations | 3425 | 3425 | 3425 | 3425 | 3425 | 3425 |
| R2 | 0.00152 | 0.117 | 0.135 | 0.120 | 0.119 | 0.137 |
| Adjusted R2 | 0.00123 | 0.102 | 0.119 | 0.105 | 0.104 | 0.121 |
| Within R2 | | 0.000350 | 0.0200 | 0.00394 | 0.00283 | 0.0226 |

Standard errors in parentheses
* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

The Table reports OLS estimates regressing the annual Changes in Velocity at the MSA-level against changes in the liquidity coverage ratio and additional control variables. The control variables are the annual changes in leverage at the MSA-level, the level of leverage at the MSA level, and lagged MSA-level GDP. The regressions include time and state-level fixed effects. Standard errors are clustered at the state and time level.

Table 5: IV Regression, Full Controls, All Fixed Effects

| | (1) Δ Liquidity | (2) Δ Velocity |
|---|---|---|
| Liquidity (2002) | -0.0212*** | |
| | (0.00509) | |
| Δ Liquidity | | -0.317 |
| | | (0.299) |
| Observations | 5329 | 5329 |
| Controls | Yes | Yes |
| Fixed Effects | State & Year | State & Year |
| $R^2$ | 0.00328 | -0.0982 |
| Fstat | 17.32 | |

Standard errors in parentheses
$^{*}$ $p < 0.05$, $^{**}$ $p < 0.01$, $^{***}$ $p < 0.001$

The Table reports IV estimates regressing the annual Changes in Velocity at the MSA-level against changes in the liquidity coverage ratio and additional control variables. The first column report the first-stage regression and the second column reports the coefficient on Annual Changes in Liquidity. The control variables are the annual changes in leverage at the MSA-level, the level of leverage at the MSA level, and lagged MSA-level GDP. The regressions include time and state-level fixed effects. Standard errors are clustered at the state and time level.

## 6 Conclusions

We develop a theory linking the usefulness of banks' liabilities as a medium of exchange to risk and maturity transformation in the presence of aggregate liquidity risk. Shortening the maturity of banks' liabilities only increases social surplus if such shortening also reduces the riskiness of long-term liabilities and banks face a binding commitment problem. When maturity transformation is socially efficient, aggregate long-term liquidity is scarce raising the relative price of long-term bank issuances. In the competitive equilibrium, banks issue too many long-term liabilities and perform too little maturity transformation relative to the social optimum. In our model, bank liabilities are backed by real assets–there is no maturity mismatch between the assets and liabilities. But even in the absence of roll-over risk, there is a social incentive for risk and maturity transformation. We provide empirical support for the mechanisms in our model.

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# A Equilibrium Characterization Given a Liability Issue

In this Appendix, we characterize equilibrium outcomes and asset prices for a given coupon issue.

**Definition 3 (Equilibrium).** *An* equilibrium *consists of an allocation for the bank* $\left(I, L, D, c^B\right)$*, households' value functions* $\{(W_t^i)_{t=0,1,2}, (V_t^i)_{t=1,2}\}_{i=s,b}$ *and policy functions* $\{(x_t^i, y_t^i, a_t^i)_{t=0,1,2}\}_{i=s,b}$*, terms of trade,* $\{(q_t, m_t)_{t=1,2}\}$*, and prices* $\{p_0^k, p_0(D), (p_t(D(\omega)))_{t=1,2})\}$ *such that*

1. *The bank's allocation solves the bank's problem (5) subject to* (6)—(12)*;*
2. *Given prices and value functions, the policy functions are optimal;*
3. *Given prices and policy functions, the value functions satisfies Equations (16), (19), and (20);*
4. *The terms of trade are the proportional bargaining solutions in Equations (21).*
5. *Goods, capital, and liabilities markets clear:*

$$x_0^b + nx_0^s = y_0^b + ny_0^s, \tag{A1}$$

$$x_t^b(\omega) + nx_t^s(\omega) = y_t^b(\omega) + ny_t^s(\omega) + d_t(\omega), \forall t, \omega, \tag{A2}$$

$$I = K^B + K^H, \tag{A3}$$

$$a_t^b(\omega) + na_t^s(\omega) = 1, \forall t, \omega. \tag{A4}$$

We proceed by backward induction. The ex-dividend price of liabilities in the centralized market of period 2 is zero: $p_2\left(D(\omega)\right) = 0$. Hence, the value functions for both buyers and sellers satisfy

$$W_2^i(a; D(\omega)) = ad_2\left(\omega\right) + \bar{v}, \tag{A5}$$

where $\bar{v} \equiv \max_x v(x) - x$.

In the decentralized market in period 2, in any match between a buyer and seller, the terms of trade, $q_2(a_2^b, a_2^s; D(\omega)), m_2(a_2^b, a_2^s; D(\omega))$ solve the proportional bargaining problem. Using the value function in equation (A5), note that for either a buyer or a seller, and for any number of liabilities exchanged, $m$, the net continuation surplus for the consumer is

$$W_2^i(a+m; D(\omega)) - W_2^i(a; D(\omega)) = (a+m)\, d_2\left(\omega\right) + \bar{v} - ad_2\left(\omega\right) - \bar{v} = md_2\left(\omega\right). \tag{A6}$$

Requiring buyers to receive total surplus equal to a fraction of the surplus of the seller is equivalent to requiring

$$u(q_2) - m_2 d_2(\omega) = \frac{\eta}{1-\eta}\left[-c(q_2) + m_2 d_2\left(\omega\right)\right], \tag{A7}$$

or

$$(1-\eta)u(q_2) + \eta c(q_2) = m_2 d_2\left(\omega\right). \tag{A8}$$

Hence, for a given amount of production $q_2$, the number of liabilities that must be transferred from the buyer to the seller is

$$m_2 = \frac{(1-\eta)u(q_2) + \eta c(q_2)}{d_2\left(\omega\right)}, \tag{A9}$$

Substituting this amount of liabilities exchanged into the surplus of the buyer, the production choice $q_2$ satisfies

$$\max_{q_2} \eta \left[u(q_2) - c(q_2)\right], \tag{A10}$$

subject to

$$(1-\eta)u(q_2) + \eta c(q_2) \leq d_2(\omega)\, a_2^b. \tag{A11}$$

$q_2$ and, therefore, $m_2$ is determined independently of $a_2^s$. Thus, the seller's asset holdings have no impact on the terms of trade,

$$q_2(a_2^b, a_2^s; D(\omega)) = q_2\left(a_2^b; D(\omega)\right), \text{and } m_2\left(a_2^b, a_2^s; D(\omega)\right) = m_2(a_2^b; D(\omega)). \tag{A12}$$

We now determine $q_2$. Recall that $q^*$ satisfies $u'(q^*) = c'(q^*)$. In a match between a buyer and a seller where the buyer has assets $a_2^b$ such that

$$a_2^b \geq \frac{1}{d_2(\omega)}\left[(1-\eta)u(q^*) + \eta c(q^*)\right], \tag{A13}$$

then $q_2(a_2^b; D(\omega)) = q^*$. Otherwise, the constraint in equation (A11) binds so that $q_2$ is determined by equation (A11) holding with equality. It also follows that the value functions $V_2^b$ and $V_2^s$ satisfy

$$V_2^b(a_2^b; D(\omega)) = \alpha\eta\left[u(q_2\left(a_2^b; D(\omega))\right) - c(q_2\left(a_2^b; D(\omega))\right)\right] + \left[a_2^b d_2(\omega) + \bar{v}\right], \tag{A14}$$

and

$$\begin{aligned} V_2^s(a_2^s; D(\omega)) = {} & \frac{\alpha}{n}(1-\eta)\int_{a_2^b}\left[u(q_2\left(a_2^b; D(\omega))\right) - c(q_2\left(a_2^b; D(\omega))\right)\right] d\Psi_2^b(a_2^b) \\ & + \left[a_2^s d_2(\omega) + \bar{v}\right]. \end{aligned} \tag{A15}$$

Next, we determine the value functions and asset price in the period 1 centralized market. Given the quasi-linearity of preferences in the centralized market, the problem of choosing asset holdings to carry into period 2 is independent of the number and value of the liabilities the consumer brings into the centralized market. The value function for either type of consumer is

$$W_1^i(a; D(\omega)) = (p_1(D(\omega)) + d_1(\omega))\, a + \bar{v} + \max_{a'} -p_1(D(\omega))a' + V_2^i(a'; D(\omega)). \tag{A16}$$

From (A15), the seller's value function $V_2^s$ is linear in $a'$ implying that the seller's optimal choice of $a'$ is bounded only if

$$p_1(D(\omega)) \geq d_2(\omega). \tag{A17}$$

Inequality (A17) holds in equilibrium with strict inequality so that all sellers choose $a_2^s = 0$ for all $\omega$. Consider the optimal choice of $a'$ for a buyer. Assuming an interior solution, the optimal choice for a buyer satisfies:

$$p_1(D(\omega)) = d_2(\omega) + \alpha\eta\left[u'(q_2(a'; D(\omega)) - c'(q_2(a', D(\omega))\right] \frac{dq_2(a'; D(\omega))}{da'} \tag{A18}$$

where

$$\frac{dq_2(a';D(\omega))}{da'} = \frac{d_2(\omega)}{(1-\eta)u'(q_2(a';D(\omega))) + \eta c'(q_2(a';D(\omega)))}. \tag{A19}$$

Under conditions on preferences and bargaining weights, $V_2^b(a_2^b, D(\omega))$ is strictly concave for $a_2^b \leq a^*$ where $a^*$ satisfies inequality (A13) with equality. This ensures a unique optimal choice of $a'$ for buyers so that $\Psi_2^b(a_2^b)$ is degenerate. We focus on equilibrium in which $a_2^b = 1$ implying that the asset price is

$$p_1(D(\omega)) = d_2(\omega)\left[1 + \alpha\eta \frac{u'(q_2(1;D(\omega)) - c'(q_2(1;D(\omega))}{(1-\eta)u'(q_2(1;D(\omega))) + \eta c'(q_2(1;D(\omega)))}\right]. \tag{A20}$$

We proceed iteratively to determine the period 1 decentralized market value functions as well as the period 0 centralized market value functions and the asset price $p_0$. The terms of trade are independent of the seller's holdings of liabilities and satisfy

$$q_1(a_1^b;D(\omega)) = \begin{cases} q^* & \text{if } a_1^b \geq a_1^* = [(1-\eta)u(q_1) + \eta c(q_1)] \,/\, (p_1(D(\omega)) + d_1(\omega)) \\ \hat{q}(a_1^b;D(\omega)) & \text{otherwise} \end{cases} \tag{A21}$$

where $\hat{q}(a_1^b;D(\omega))$ is the value of $q$ that satisfies

$$(1-\eta)u(q) + \eta c(q) = (p_1(D(\omega)) + d_1(\omega))\, a_1^b. \tag{A22}$$

Moreover, $m_1(a_1^b;D(\omega))$ is

$$m_1(a_1^b;D(\omega)) = \frac{(1-\eta)u(q_1(a_1^b;D(\omega))) + \eta c(q_1(a_1^b;D(\omega)))}{(p_1(\omega) + d_1(\omega))}. \tag{A23}$$

These terms of trade imply the value functions for buyers and sellers in the period 1 decentralized market are:

$$V_1^b(a_1^b;D(\omega)) = \alpha\eta\left[u\left(q_1\left(a_1^b;D(\omega)\right)\right) - c\left(q_1\left(a_1^b;D(\omega)\right)\right)\right] + W_1^b(a_1^b;D(\omega)), \tag{A24}$$

$$\begin{aligned} V_1^s(a_1^s;D(\omega)) = {} & \frac{\alpha}{n}(1-\eta)\int_{a_1^b}\left[u\left(q_1\left(a_1^b;D(\omega)\right)\right) - c\left(q_1\left(a_1^b;D(\omega)\right)\right)\right] d\Omega_1^b(a_1^b) \\ & + W_1^s(a_1^s;D(\omega)). \end{aligned} \tag{A25}$$

Buyers and sellers problems in the period 0 centralized market are

$$W_0^i(a) = p_0^k k^i + \bar{v} + \max_{a'} -p_0(D)a' + \sum_{\omega}\gamma(\omega)V_1^i(a';D(\omega)). \tag{A26}$$

To determine the period 0 asset price, note that the seller's demand for the asset is finite, when

$$p_0 \geq \sum_{s_1}(p_1(\omega) + d_1(\omega)), \tag{A27}$$

and at an interior solution for the buyer, we require that

$$p_0 = \sum_{\omega} \gamma(\omega) \frac{dV_1^b(a'; D(\omega))}{da'}. \tag{A28}$$

# B Efficiency Proofs

We assume that banks have large enough capital relative to households.

**Assumption 2.** *Endowments $K^H$, $K^B$ and the parameter $\xi$ satisfy*

$$\frac{K^B}{K^H + K^B} \geq \xi. \tag{B1}$$

Assumption 2 is a minimum capital requirement for the bank relative to households. We maintain Assumption 2 for the remainder of the paper.

Welfare for a given coupon issue is

$$W_0^P(D) = (1+n)\,\bar{v} + \sum_{\omega} \gamma(\omega) \left( U_1^P(D(\omega)) + U_2^P(D(\omega)) \right), \tag{B2}$$

with

$$U_t^P\left(D\left(\omega\right)\right) = (1+n)\bar{v} + d_t\left(\omega\right) + \alpha\left[u\left(q_t^{eq}\left(D\left(\omega\right)\right)\right) - c\left(q_t^{eq}\left(D\left(\omega\right)\right)\right)\right]. \tag{B3}$$

Before proving the propositions, we report two preliminary lemmas.

**Lemma 1 (No asset transformation).** *If $z\left(\omega_l\right) \geq \frac{d^*}{K^H}$, then efficient allocations feature neither risk nor maturity transformation.*

*Proof of Lemma 1.* The unconstrained optimal level of trade in decentralized markets satisfies $q_t^{eq}(D(\omega)) = q^*$. If this level of decentralized trade can be attained by a coupon issue which satisfies the planner's constraints and minimizes payments to the bank, that is,

$$\sum_{\omega} \gamma(\omega) c_t^B(\omega) = K^B \sum_{\omega} \gamma(\omega) z(\omega) \tag{B4}$$

then the allocation must be an efficient allocation.

Under the assumption of the lemma, the pass-through claim satisfies this property. By assumption, if $d_2(\omega) = K^H z(\omega)$, then $d_2(\omega) \geq d^*$ for $\omega = \omega_l, \omega_h$. Hence, the pass-through claim, $D(\omega) = \{0, K^H z(\omega_l), 0, K^H z(\omega_h)\}$ satisfies $q_t^{eq}(D(\omega)) = q^*$. Moreover, the commitment constraint in each state is satisfied since

$$c_2^B(\omega) = \left(K^H + K^B\right) z(\omega) - d_2(\omega) = K^B z(\omega) \geq \xi \left(K^H + K^B\right) z(\omega), \tag{B5}$$

where the final inequality follows from Assumption 2. ■

**Lemma 2 (Only Risk Transformation).** *There exists a threshold $\underline{z}_r \leq \frac{d^*}{K^H}$ such that if $\frac{d^*}{K^H} > z\left(\omega_l\right) \geq \underline{z}_R$, and $E_0\left[z\left(\omega\right)\right] \geq \frac{d^*}{K^H}$, then efficient allocations feature risk transformation and feature no maturity transformation.*

*Proof of Lemma 2.* We construct $d_t(\omega)$ such that $q_t^{eq}(D(\omega)) = q^*$ and

$$\sum_{\omega} \gamma(\omega) c_t^B(\omega) = K^B \sum_{\omega} \gamma(\omega) z(\omega). \tag{B6}$$

Since such an allocation attains the maximum of welfare subject to the resource feasibility and bank's participation constraints, the allocation is constrained efficient as long as it also satisfies the bank's limited commitment constraints. The pass-through claim does not attain this value since $d^* > K^H z(\omega_l)$.

Consider an allocation satisfying $d_2(\omega_l) = d^*$ and

$$d_2(\omega_h) = K^H z(\omega_h) - \frac{\gamma(\omega_l)}{\gamma(\omega_h)} \left[ d^* - K^H z(\omega_l) \right]. \tag{B7}$$

Under the assumptions of the Lemma, it follows that $d^* \leq d_2(\omega_h) < K^H z(\omega_h)$. By construction, the allocation satisfies the bank's participation constraint with equality, or $\sum_\omega \gamma(\omega) c_2^B(\omega) = K^B \sum_\omega \gamma(\omega) z(\omega)$. Moreover, it is straightforward to show that under the assumptions of the Lemma along with Assumption 2 that the commitment constraints of the bank are satisfied. ■

*Proof of Proposition 1.* Suppose

$$d^* > \left(K^H + K^B\right)(1 - \xi) z_{2l}. \tag{B8}$$

We guess and then verify that the commitment constraint is slack in the high state but binds in the low state. In this case, it is not commitment-feasible for the bank to choose $d_2(\omega_l) \geq d^*$. We start by characterizing the optimum taking $L(\omega) = 0$. Then we see if an increase in $L(\omega_l)$ can improve outcomes.

When $L(\omega_l) = 0$, it is immediate $d_2(\omega_l) = \left(K^H + K^B\right)(1 - \xi) z(\omega_l)$. To see this, suppose $d_2(\omega_l) < \left(K^H + K^B\right)(1 - \xi) z(\omega_l)$. Consider perturbing $d_2(\omega_l)$ to $d_2(\omega_l) + \varepsilon$ and $c_2^B(\omega_l)$ to $c_2^B(\omega_l) - \varepsilon$. Since

$$\begin{aligned} c_2^B(\omega_l) &= \left(K^H + K^B\right) z(\omega_l) - d_2(\omega_l) \\ &> \left(K^H + K^B\right) z(\omega_l) \xi \end{aligned} \tag{B9}$$

as long as

$$\varepsilon < c_2^B(\omega_l) - \left(K^H + K^B\right) z(\omega_l) \xi \tag{B10}$$

this perturbation will continue to satisfy the limited commitment constraint of the bank. Further, increase $c_2^B(\omega_h)$ by $\gamma(\omega_l)\varepsilon/\gamma(\omega_h)$ to ensure the bank's ex ante participation constraint is satisfied. This increase requires reducing $d_2(\omega_h)$ by the same amount.

To show this is feasible without reducing $d_2(\omega_h)$ below $d^*$, suppose that

$$K^H \sum_{\omega} \gamma(\omega) z(\omega) - \gamma(\omega_l) \left(K^H + K^B\right)(1 - \xi) z(\omega_l) > \gamma(\omega) d^*. \tag{B11}$$

Then for any allocation with $d_2(\omega_l) < \left(K^H + K^B\right)(1 - \xi) z(\omega_l)$ and

$$\sum_{\omega} \gamma(\omega) c_2^B(\omega) = K^B \sum_{\omega} \gamma(\omega) z(\omega), \tag{B12}$$

it must be the case that $d_2(\omega_h) > d^*$. As a consequence, the perturbation is feasible.

Now, consider the impact of this perturbation on welfare. Since $d_2(\omega_h) \geq d^*$, we have

$$U_{2,d_2}(\omega_h) = 1, U_{1,d_2}(\omega_h) = 0. \tag{B13}$$

Hence, the impact on ex ante welfare from this decrease in period 2 coupons is

$$-\gamma(\omega_h)\frac{\gamma(\omega_l)}{\gamma(\omega_h)}\varepsilon = -\gamma(\omega_l)\varepsilon. \tag{B14}$$

However, since $d_2(\omega_l) < d^*$,

$$U_{2,d_2}(\omega_l) > 1, U_{1,d_2}(\omega_l) > 0. \tag{B15}$$

Hence, the impact on ex ante welfare from the increase in period 2 coupon payments is

$$\gamma(\omega_l)\varepsilon\left(U_{2,d_2}(\omega_l) + U_{1,d_2}(\omega_l)\right) > \gamma(\omega_l)\varepsilon. \tag{B16}$$

So the overall effect of this perturbation must increase ex ante welfare. This proves that when $L(\omega_l) = 0$, $d_2(\omega_l) = \left(K^H + K^B\right)(1-\xi)z(\omega_l)$.

We now show that an allocation with $L(\omega_l) > 0$ improves welfare relative to the best allocation without liquidation. Consider a perturbed allocation with $L(\omega_l) = \epsilon$. Define the coupon payments in the perturbed allocation as

$$\begin{aligned}
d_1(\omega_l;\varepsilon) &= \kappa\left(K^H + K^B\right)z(\omega_l)\varepsilon & \text{(B17)}\\
d_1(\omega_h;\varepsilon) &= 0 & \text{(B18)}\\
d_2(\omega_l;\varepsilon) &= (1-\varepsilon)(1-\xi)\left(K^H + K^B\right)z(\omega_l) & \text{(B19)}\\
d_2(\omega_h;\varepsilon) &= d_2(\omega_h) - \varepsilon\xi\left(K^H + K^B\right)z(\omega_l)\frac{\gamma(\omega_l)}{\gamma(\omega_h)} & \text{(B20)}
\end{aligned}$$

By construction, this perturbed allocation leaves the bank's expected consumption unchanged, and, as long as $z(\omega_h)$ is sufficiently large, this perturbation will not reduce $d_2(\omega_h)$ below $d^*$.

For any $\varepsilon$, welfare satisfies

$$\sum_{\omega}\gamma(\omega)\left[U_1\left(d_1(\omega;\varepsilon), d_2(\omega;\varepsilon)\right) + U_2(d_2(\omega;\varepsilon))\right]. \tag{B21}$$

Hence, the impact of this perturbation is

$$\sum_{\omega}\gamma(\omega)\left[U_{1,d_1}(\omega)\frac{dd_1(\omega;\varepsilon)}{d\varepsilon} + U_{1,d_2}(\omega)\frac{dd_2(\omega;\varepsilon)}{d\varepsilon} + U_{2,d_2}(\omega)\frac{dd_2(\omega;\varepsilon)}{d\varepsilon}\right]. \tag{B22}$$

Because $U_{t,d_t}(\omega_h) = 1$ and $U_{1,d_2}(\omega_h) = 0$, we simplify the impact of this perturbation as

$$\gamma(\omega_l)\left(K^H + K^B\right)z(\omega_l)\left[U_{1,d_1}(\omega_l)\kappa - \left(U_{1,d_2}(\omega_l) + U_{2,d_2}(\omega_l)\right)(1-\xi) - \xi\right] \tag{B23}$$

We show that there exist thresholds $\underline{\kappa}$ and $\underline{\xi}$ such that as $z(\omega_l) \to 0$, the impact of this perturbation is strictly positive.

Consider the term in brackets in (B23). With a slight abuse of notation, let $q_t^{eq}(\varepsilon) = q_t^{eq}(d_1(\omega_l;\varepsilon), d_2(\omega_l;\varepsilon))$ and $p_1(d_1(\omega_l;\varepsilon), d_2(\omega_l;\varepsilon)) = p(\epsilon)$. Then, since

$$U_{1,d_1}(\omega_l) = 1 + \alpha\left[u'\left(q_1^{eq}(\varepsilon)\right) - c'\left(q_1^{eq}(\varepsilon)\right)\right]\frac{dq_1^{eq}(\varepsilon)}{dd_1(\omega_l;\varepsilon)} \tag{B24}$$

$$U_{2,d_1}(\omega_l) = \alpha\left[u'\left(q_1^{eq}(\varepsilon)\right) - c'\left(q_1^{eq}(\varepsilon)\right)\right]\frac{dq_1^{eq}(\varepsilon)}{dd_2(\omega_l;\varepsilon)} \tag{B25}$$

$$U_{2,d_2}(\omega_l) = 1 + \alpha\left[u'\left(q_2^{eq}(\varepsilon)\right) - c'\left(q_2^{eq}(\varepsilon)\right)\right]\frac{dq_2^{eq}(\varepsilon)}{dd_2(\omega_l;\varepsilon)} \tag{B26}$$

and

$$\frac{dq_1^{eq}(\varepsilon)}{dd_2(\omega_l;\varepsilon)} = \frac{dq_1^{eq}(\varepsilon)}{dd_1(\omega_l;\varepsilon)}\frac{dp_1(\varepsilon)}{dd_2(\omega_l;\varepsilon)}, \tag{B27}$$

this term in brackets simplifies to

$$\begin{aligned}&\alpha\left[u'\left(q_1^{eq}(\varepsilon)\right) - c'\left(q_1^{eq}(\varepsilon)\right)\right]\left[\kappa - (1-\xi)\frac{dp_1(\varepsilon)}{dd_2(\omega_l;\varepsilon)}\right]\frac{dq_1^{eq}(\varepsilon)}{dd_1(\omega_l;\varepsilon)} \\ &\qquad - \alpha\left[u'\left(q_2^{eq}(\varepsilon)\right) - c'\left(q_2^{eq}(\varepsilon)\right)\right]\frac{dq_2^{eq}(\varepsilon)}{dd_2(\omega_l;\varepsilon)}(1-\xi) - (1-\kappa).\end{aligned} \tag{B28}$$

Since

$$\frac{dq_t^{eq}(\varepsilon)}{dd_t(\omega_l;\varepsilon)} = \frac{1}{(1-\eta)u'\left(q_t^{eq}(\varepsilon)\right) + \eta c'\left(q_t^{eq}(\varepsilon)\right)}, \tag{B29}$$

and $\lim_{z(\omega_l)\to 0}\lim_{\varepsilon\to 0} q_t^{eq}(\varepsilon) = 0$, it follows that

$$\begin{aligned}&\lim_{z(\omega_l)\to 0}\lim_{\varepsilon\to 0}\alpha\left[u'\left(q_t^{eq}(\varepsilon)\right) - c'\left(q_t^{eq}(\varepsilon)\right)\right]\frac{dq_t^{eq}(\varepsilon)}{dd_t(\omega_l;\varepsilon)} \\ &= \lim_{z(\omega_l)\to 0}\lim_{\varepsilon\to 0}\alpha\frac{u'\left(q_t^{eq}(\varepsilon)\right) - c'\left(q_t^{eq}(\varepsilon)\right)}{(1-\eta)u'\left(q_t^{eq}(\varepsilon)\right) + \eta c'\left(q_t^{eq}(\varepsilon)\right)} \\ &= \frac{\alpha}{1-\eta}.\end{aligned} \tag{B30}$$

Similarly,

$$\lim_{z(\omega_l)\to 0}\lim_{\varepsilon\to 0}\frac{dp_1(\varepsilon)}{dd_2(\omega_l;\epsilon)} = 1 + \frac{\alpha\eta}{1-\eta} \tag{B31}$$

Then,

$$\begin{aligned}&\lim_{z_{2l}\to 0}\lim_{\varepsilon\to 0} U_{1,d_1}(\omega_l)\kappa - \left(U_{1,d_2}(\omega_l) + U_{2,d_2}(\omega_l)\right)(1-\xi) - \xi \\ &= \frac{\alpha}{1-\eta}\left[\kappa - (1-\xi)\left(2 + \frac{\alpha\eta}{1-\eta}\right)\right] - (1-\kappa)\end{aligned} \tag{B32}$$

If

$$\xi \geq \frac{1-\eta+\alpha\eta}{2(1-\eta)+\alpha\eta} = \underline{\xi}, \tag{B33}$$

where for Assumption 2 to be satisfied requires

$$\frac{K^B}{K^H+K^B} > \underline{\xi}, \tag{B34}$$

then there exists $\kappa < 1$ s.t. (B32) is strictly positive. Indeed, $\underline{\kappa}$ satisfies

$$\underline{\kappa}(\xi) \geq \frac{1-\eta}{1-\eta+\alpha}\left[1+(1-\xi)\left(2+\frac{\alpha\eta}{1-\eta}\right)\right]. \tag{B35}$$

Hence, if $\xi \geq \underline{\xi}$ and $\kappa \geq \underline{\kappa}(\xi)$, since (B32) is strictly positive, there must exist a threshold $\underline{z}$ such that for $z(\omega_l) < \underline{z}$, this perturbation strictly raises the value of the social planner implying that strictly positive liquidation—that is, $L(\omega_l) > 0$—is efficient. ■

# C Equilibrium Proofs

*Proof of Proposition 3.* From the proof of Lemma 1, the pass-through claim satisfies all the bank's constraints and mimimizes the bank's liability payoffs. At the resulting allocation, the liability has no liquidity premium. Using the Arrow-Debreu prices,

$$\pi_t(\omega_i; D) = \gamma(\omega_i),\ \ \omega_i \in \Omega,\ \ t \in \{1, 2\}. \tag{C1}$$

the pass-through claim is optimal for the bank so that the pass-through claim is a competitive equilibrium. A similar argument applies for the allocation in Lemma 2. ■

*Proof of Proposition 4.* The proof that an efficient allocation with $L(\omega_l) > 0$ is not an equilibrium is by contradiction. We begin by constructing the liability issue associated the efficient outcome when $L(\omega_l) > 0$. Since there is sufficient liquidity in the high state ($z(\omega_h)$ is sufficiently large), $L(\omega_h) = 0$ and it is immediate that $L(\omega_l) > 0$ only if the commitment constraint in the low state binds. For a given choice of liquidation, then, the efficient liability issue satisfies

$$d_1(\omega_l) = (K^H + K^B)\kappa z(\omega) L(\omega_l), \tag{C2}$$

$$d_1(\omega_h) = 0, \tag{C3}$$

$$d_2(\omega_l) = (K^H + K^B)(1-\xi) z(\omega_l)(1 - L(\omega_l)), \tag{C4}$$

$$\begin{aligned} d_2(\omega_h) &= (K^H + K^B) z(\omega_h) \\ &- \frac{1}{\gamma(\omega_h)}\left[K^B \sum_{\omega} \gamma(\omega) z(\omega) - \gamma(\omega_l)\xi(1 - L(\omega_l))(K^H + K^B) z(\omega_l)\right], \end{aligned} \tag{C5}$$

where the last equality results from the bank's participation constraint holding with equality.

The efficient level of $L(\omega_l)$ satisfies

$$\sum_{\omega} \gamma(\omega) \left[ \begin{array}{c} U_{1,d_1}(d_1(\omega), d_2(\omega)) \frac{dd_1(\omega)}{dL(\omega_l)} + U_{1,d_2}(d_1(\omega), d_2(\omega)) \frac{dd_2(\omega)}{dL(\omega_l)} \\ +U_{2,d_2}(d_2(\omega)) \frac{dd_2(\omega)}{dL(\omega_l)} \end{array} \right] = 0. \tag{C6}$$

Since there is sufficient liquidity in the high state, Conditions (C6) is

$$0 = -\xi + U_{1,d_1}(d_1(\omega_l), d_2(\omega_l))\kappa - (1-\xi)\left(U_{1,d_2}(d_1(\omega_l), d_2(\omega_l)) + U_{2,d_2}(d_2(\omega_l))\right). \tag{C7}$$

Let $D^*$ be the coupon issue defined by (C2)-(C5) when $L(\omega)$ satisfies (C7).

Define the function $H(q)$ as

$$H(q) \equiv \frac{u'(q) - c'(q)}{(1-\eta)u'(q) + \eta c'(q)}. \tag{C8}$$

Given $D^*(\omega)$, the market for liabilities in period 0 clears when the price of liabilities satisfy

$$\pi_1(\omega_h; D^*) = \gamma(\omega_h), \tag{C9}$$

$$\pi_2(\omega_h; D^*) = \gamma(\omega_h) \tag{C10}$$

$$\pi_1(\omega_l; D^*) = \gamma(\omega_l)\left[1 + \alpha\eta H(q_1^{eq}(d_1^*(\omega_l), d_2^*(\omega_l)))\right], \tag{C11}$$

$$\begin{aligned} \pi_2(\omega_l; D^*) = {} & \gamma(\omega_l)\left[1 + \alpha\eta H(q_1^{eq}(d_1^*(\omega_l), d_2^*(\omega_l)))\right] \\ & \times \left[1 + \alpha\eta H(q_2^{eq}(d_1^*(\omega_l), d_2^*(\omega_l)))\right]. \end{aligned} \tag{C12}$$

The period 0 budget constraint of a bank, then, is

$$I \leq K^B + \frac{1}{p_0^k} \sum_t \sum_{\omega} \pi_t(\omega; D^*) d_t(\omega). \tag{C13}$$

We construct a strictly profitable deviation for the bank from the efficient liability issue.

Take the Pareto allocation and consider the following perturbation:

$$\hat{L}(\omega_l) = L^*(\omega_l) - \epsilon, \tag{C14}$$

$$\hat{d}_1(\omega_l) = (K^H + K^B)\kappa z(\omega_l)\hat{L}(\omega_l), \tag{C15}$$

$$\hat{d}_2(\omega_l) = (K^H + K^B)(1-\xi)z(\omega_l)(1 - \hat{L}(\omega_l)), \tag{C16}$$

$$\hat{d}_1(\omega_h) = 0 \tag{C17}$$

$$\hat{d}_2(\omega_h) = d_2(\omega_h) + (K^H + K^B)\frac{\gamma_l}{\gamma_h}\xi\epsilon z(\omega_l). \tag{C18}$$

By construction, this perturbation has no impact on the bank's expected consumption since

$$\begin{aligned} \sum_{\omega} \gamma(\omega)[\hat{c}_1(\omega) + \hat{c}_2(\omega)] &= \sum_{\omega} \gamma(\omega)[c_1(\omega) + c_2(\omega)] \\ &\quad -\gamma_h(K^H + K^B)\frac{\gamma_l}{\gamma_h}\xi\epsilon z(\omega_l) + \gamma_l(K^H + K^B)\xi\epsilon z(\omega_l) \\ &= \sum_{\omega} \gamma(\omega)[c_1(\omega) + c_2(\omega)]. \end{aligned} \tag{C19}$$

However, consider how this perturbation impacts the revenues raised from issuing liabilities in the initial period. Revenues raised from the perturbed liability issuance are

$$\begin{aligned}\frac{1}{p_0^k}\sum_t\sum_\omega \pi_t^b(\omega;D^*)\hat{d}_t(\omega) = &\frac{1}{p_0^k}\sum_t\sum_\omega \pi_t^b(\omega;D^*)d_t^*(\omega)\\ &+\frac{1}{p_0^k}\epsilon\gamma_l(K^H+K^B)z(\omega_l)\\ &\times\left(\xi-[1+\alpha\eta H(q_1^*)]\,\kappa+[1+\alpha\eta H(q_2^*)]\,[1+\alpha\eta H(q_1^*)]\,(1-\xi)\right), \quad \text{(C20)}\end{aligned}$$

where with a slight abuse of notation we write $q_t^* = q_t^{eq}(d_1^*(\omega_l), d_2^*(\omega_l))$.

We now argue that

$$\xi-[1+\alpha\eta H(q_1^*)]\,\kappa+[1+\alpha\eta H(q_2^*)]\,[1+\alpha\eta H(q_1^*)]\,(1-\xi)>0. \quad \text{(C21)}$$

First, note that the left hand side of (C21) can be rewritten as

$$\begin{aligned}&\xi-[1+\alpha\eta H(q_1^*)]\,\kappa+[1+\alpha\eta H(q_2^*)]\,[1+\alpha\eta H(q_1^*)]\,(1-\xi)\\ &=\alpha\eta\left[\frac{1-\kappa}{\alpha\eta}-\kappa H(q_1^*)+(1-\xi)\left[H(q_1^*)+H(q_2^*)+\alpha\eta H(q_1^*)H(q_2^*)\right]\right]. \quad \text{(C22)}\end{aligned}$$

Next, (C7) which determines the efficient level of liquidation can be rewritten as

$$\begin{aligned}\xi = &\kappa\,(1+\alpha H(q_1^*))\\ &-(1-\xi)\left[\alpha H(q_1^*)\left[1+\alpha\eta H(q_2^*)+d_2^*(\omega_l)\alpha\eta G(q_2^*)\right]+1+\alpha H(q_2^*)\right], \quad \text{(C23)}\end{aligned}$$

where $d_2^*(\omega_l)$ is the efficient coupon and

$$G(q) \equiv \frac{c'(q)u''(q)-u'(q)c''(q)}{[(1-\eta)u'(q)+\eta c'(q)]^3}. \quad \text{(C24)}$$

Using (C23) to substitute for $\xi$ into the left-hand side of (C22) and re-arranging terms,

$$\begin{aligned}&\xi-[1+\alpha\eta H(q_1^*)]\,\kappa+[1+\alpha\eta H(q_2^*)]\,[1+\alpha\eta H(q_1^*)]\,(1-\xi)\\ &\quad=\alpha(1-\eta)\left[\kappa H(q_1^*)-(1-\xi)\left[H(q_1^*)+H(q_2^*)+\alpha\eta H(q_1^*)H(q_2^*)\right]\right]\\ &\qquad-(1-\xi)\alpha^2\eta H(q_1^*)d_2^*(\omega_l)G(q_2). \quad \text{(C25)}\end{aligned}$$

Combining (C22) and (C25) implies

$$\begin{aligned}&\kappa H(q_1^*)-(1-\xi)\left[H(q_1^*)+H(q_2^*)+\alpha\eta H(q_1^*)H(q_2^*)\right]\\ &\qquad=\frac{1-\kappa}{\alpha}+(1-\xi)\alpha\eta H(q_1^*)d_2^*(\omega_l)G(q_2). \quad \text{(C26)}\end{aligned}$$

It follows that

$$\xi - [1+\alpha\eta H(q_1^*)]\,\kappa + [1+\alpha\eta H(q_2^*)]\,[1+\alpha\eta H(q_1^*)]\,(1-\xi)$$
$$= \alpha\eta\left[\frac{(1-\kappa)(1-\eta)}{\alpha\eta} - (1-\xi)\alpha\eta H(q_1^*) d_2^*(\omega_l) G(q_2^*)\right] > 0. \quad \text{(C27)}$$

where the inequality follows because $H(q) \geq 0$ when $q < q^*$ and $G(q) \leq 0$. ■

# D Implementing the planner's solution with demand deposits

The planner's solution reported in the body of the manuscript assumes that banks issue long-term claims with the possiblity of paying dividends in both periods. Here we provide a brief description of how to implement the same allocation when banks instead issue demand deposits. In period 0, banks issue demand deposits to households in exchange for the household's trees.

Let the rates of return on demand deposits be $R_t(\omega)$, $t = 1, 2$ between $t$ and $t+1$. Each household who owns $a_0$ of demand deposits at time 0 has $a_0 R_1(\omega)$ redeemable demand deposts at time 1. A household who owns $a_1$ demand deposits at time 1 after any redemption has $a_1 R_2(\omega)$ deposits at time 2. Each demand deposit can be redeemed one-for-one with consumption from the bank. Any demand depost redeemed is then extinguished by the bank: If a household holding $X$ demand deposts at time 1 redeems $0 \leq Y \leq X$ demand deposts for consumption, then the household is left with $X - Y$ demand deposits, redeemable for $(X-Y)R_2(\omega)$ at time 2.

We show with the correct redemptions and gross rates of return that households are indifferent to redeeming or continuing to hold their claims at time 1. The allocation leads to the same allocation to be an equilibrium in the demand deposit economy as in the long-term claims economy. Recall that $p_0^k$ is the time 0 price of capital, $p_0(D)$ the time 0 price of bank claims, $p_t(D(\omega)$ the bank claim price at date $t$, and $d_t(\omega)$ the time $t$ dividends.

The proposed equilbrium is that the bank purcahases the household capital for $p_0(D)$ per unit of capital and issues $K^H p_0(\omega)$ units of time 0 demand deposits in total, meaning that each unit of capital at time 0 is worth $\frac{P_0(\omega)}{p_0^k}$ demand deposits. In aggregate, banks issue $p_0 K^H$ demand deposits of time 0 consumption at time zero. As in the long-term claim economy, we let $p_0^k$ denote the time 0 consumption value for a unit of capital. The total amount of time 0 deposits is

$$p_0(D) = K^H p_0^k. \quad \text{(D1)}$$

The gross rates of return offered by the banks are

$$R_1(\omega) = \frac{p_1(\omega) + d_1(\omega)}{p_0(D)}, \quad \text{(D2)}$$

and

$$R_2(\omega) = \frac{d_2(\omega)}{p_1(\omega)}. \quad \text{(D3)}$$

Each holder of $X$ deposits at time 1 and redeems $X \times \frac{d_1(\omega)}{p_1(\omega)+d_1(\omega)}$ demand deposits at time 1. In our proposed equilibrium, all deposit holders set $X = 1$.

We show that at this rates of return that households are indifferent to redeeming or continuing to hold their demand deposits at time 1 so will redeem the required quantity of deposits, and that the resulting payouts match those in the allocation with long-term claims. Suppose that a household with $p_1(\omega) + d_1(\omega)$ does redeem $d_1(\omega)$ claims. Such a household will be left with $p_1(\omega)$ claims that can be redeemed for

$$p_1(\omega)R_2(\omega) = p_1(\omega)\frac{d_2(\omega)}{p_1(\omega)} = d_2(\omega) \tag{D4}$$

units of consumption from the bank at time 2. That demand deposit will able to support the same allocation as equilibrium after time 1 as the long-term claims case. As a consequence, buyer will value the demand deposit at the liability as at the value in equation (27). By definition, buyers are indifferent between holding $p_1(\omega)$ units of the demand deposit or receiving $p_1(\omega)$ units of consumption at that price so are indifferent to holding $p_1(\omega) + d_1(\omega)$ demand deposits or redeeming $d_1(\omega)$ units of demand deposits while retaining $p_1(\omega)$ demand deposits.

If households redeem more that $d_1(\omega)$ demand deposits, banks are able to issue new demand deposits for the net redemptions equal to be $d_1(\omega)$ at the second period interest rate in equation (D3).

Applying similar logic to the initial price $p_0(\omega)$ and noting that the aggregate quantity of demand deposits is the same as the value of the long-term claims shows that initial the allocation is the same as in the long-term claims economy. Setting the demand deposit rate as in Equations (D2)–(D3), households holding $p_1(\omega) + d_1(\omega)$ demand deposits will redeem $d_1(\omega)$ units in the demand deposit economy. The resulting allocation is that same as in the long-term claims economy.

# E A Cash-in-Advance Model

We show the Propositions are modified replacing the search and matching friction in the decentralized markets with competitive markets subject to a cash-in-advance constraint.

The buyer's problem is:

$$\max_{x,y,q,a} v(x_0) - y_0 + \sum_{\omega \in \Omega} \gamma(\omega) \left\{ \sum_{t=1,2} u(q_t(\omega)) + v(x_t(\omega)) - y_t(\omega) \right\}, \tag{E1}$$

subject to:

$$\begin{aligned}
x_0 + a_1 p_0 &\leq y_0 + p_0^k k^b, && \text{(E2)}\\
x_1(\omega) + p_1(\omega)a_2(\omega) &\leq y_1(\omega) + \big(p_1(\omega) + d_1(\omega)\big)(a_1 - m_1(\omega)q_1(\omega)), && \text{(E3)}\\
x_2(\omega) &\leq y_2(\omega) + \big(p_2(\omega) + d_2(\omega)\big)(a_2(\omega) - m_2(\omega)q_2(\omega)), && \text{(E4)}\\
m_1(\omega)q_1(\omega) &\leq a_1, && \text{(E5)}\\
m_2(\omega)q_2(\omega) &\leq a_2(\omega). && \text{(E6)}
\end{aligned}$$

Constraints (E2)-(E4) are budget constraints in centralized market sub-periods while constraints (E5)-(E6) are cash-in-advance constraints in the frictional market subperiods.

The optimality conditions are

$$\gamma(\omega)u'(q_1(\omega)) = m_1(\omega)\{\gamma(\omega)(p_1(\omega)+d_1(\omega))+\lambda_1^m(\omega)\}, \quad \text{(E7)}$$
$$\gamma(\omega)u'(q_2(\omega)) = m_2(\omega)\{\gamma(\omega)(p_2(\omega)+d_2(\omega))+\lambda_2^m(\omega)\}, \quad \text{(E8)}$$
$$p_0 = \sum_{\omega\in\Omega}\{\gamma(\omega)(p_1(\omega)+d_1(\omega))+\lambda_1^m(\omega)\}, \quad \text{(E9)}$$
$$\gamma(\omega)p_1(\omega) = \gamma(\omega)(p_2(\omega)+d_2(\omega))+\lambda_2^m(\omega), \quad \text{(E10)}$$
$$\lambda_1^m(a_1-m_1(\omega)q_1(\omega)) = 0 \quad \text{with} \quad (\lambda_1^m\geq 0), \quad \text{(E11)}$$
$$\lambda_2^m(a_2-m_1(\omega)q_2(\omega)) = 0 \quad \text{with} \quad (\lambda_2^m\geq 0). \quad \text{(E12)}$$

A guess and verify approach shows that sellers acquire no liabilities in centralized market sub-periods. We write the seller's problem with this restriction. The seller's problem is:

$$\max_{x,y,q,a} v(x_0)-y_0+\sum_{\omega\in\Omega}\gamma(\omega)\left\{\sum_{t=1,2}-c(q_t(\omega))+v(x_t(\omega))-y_t(\omega)\right\}, \quad \text{(E13)}$$

subject to:

$$x_0 \leq y_0+p_0^k k^b, \quad \text{(E14)}$$
$$x_1(\omega) \leq y_1(\omega)+\big(p_1(\omega)+d_1(\omega)\big)m_1(\omega)q_1(\omega), \quad \text{(E15)}$$
$$x_2(\omega) \leq y_2(\omega)+\big(p_2(\omega)+d_2(\omega)\big)m_2(\omega)q_2(\omega). \quad \text{(E16)}$$

The first order conditions for the seller's problem imply:

$$c'(q_1(\omega)) = m_1(\omega)\{p_1(\omega)+d_1(\omega)\}, \quad \text{(E17)}$$
$$c'(q_2(\omega)) = m_2(\omega)\{p_2(\omega)+d_2(\omega)\}. \quad \text{(E18)}$$

Assuming $p_2=0$, and $a=1$, combining the buyers and sellers first order conditions gives the pricing equations:

$$p_0=\sum_{\omega\in\Omega}\gamma(\omega)\left\{\big[p_1(\omega)+d_1(\omega)\big]\left[1+\frac{u'(q_1(\omega))-c'(q_1(\omega))}{c'(q_1(\omega))}\right]\right\}, \quad \text{(E19)}$$

$$p_1(\omega)=d_2(\omega)\left[1+\frac{u'(q_2(\omega))-c'(q_2(\omega))}{c'(q_2(\omega))}\right], \quad \text{(E20)}$$

$$\big[u'(q_1(\omega))-c'(q_1(\omega))\big]\big[p_1(\omega)+d_1(\omega)-c'(q_1(\omega))q_1(\omega)\big]=0,\quad u'(q_1(\omega))\geq c'(q_1(\omega)), \quad \text{(E21)}$$

$$\big[u'(q_2(\omega))-c'(q_2(\omega))\big]\big[d_2(\omega)-c'(q_2(\omega))q_2(\omega)\big]=0,\quad u'(q_2(\omega))\geq c'(q_2(\omega)). \quad \text{(E22)}$$

Define

$$d^*\equiv c'(q^*)q^* \quad \text{(E23)}$$

The proofs of Lemma 1 and Lemma 2 directly apply to the cash-in-advance economy using the definition in (E23) for the value of $d^*$.

**Lemma 3 (No asset transformation with Cash-in-Advance).** *If $z(\omega_l)\geq\frac{d^*}{K^H}$, then efficient allocations feature neither risk nor maturity transformation.*

**Lemma 4 (Only Risk Transformation with Cash-in-Advance).** *There exists a threshold $\underline{z}_r \leq \frac{d^*}{K^H}$ such that if $\frac{d^*}{K^H} > z(\omega_l) \geq \underline{z}_R$, and $E_0[z(\omega)] \geq \frac{d^*}{K^H}$, then efficient allocations feature risk transformation and feature no maturity transformation.*

We now turn to the case with both liquidity and maturity transformation.

**Proposition 5 (Risk and maturity transformation with Cash-in-Advance).** *There exists $\underline{\xi} < K^B/(K^B + K^H)$, $\underline{\kappa} > 0$ and $\underline{z} > 0$ such that if $\xi \geq \underline{\xi}$, $\kappa \geq \underline{\kappa}$, and $z(\omega_l) < \underline{z}$, then efficient allocations feature both risk and maturity transformation.*

Equations (B7)–(B27) in the Proof to Proposition 1 continue to hold with $\alpha = 1$. Differentiating equations (E20)–(E22),

$$\frac{dq_t^{eq}(\epsilon)}{dd_t(\omega_l;\epsilon)} = \frac{1}{c''(q_t^{eq}(\epsilon)) + c'(q_t^{eq}(\epsilon))q_t^{eq}(\epsilon)}. \tag{E24}$$

If $\lim_{z(\omega_l)\to 0}\lim_{\epsilon\to 0} q_t^{eq}(\epsilon) = 0$ and using Equation (E24), the arguments in Equations (B28)-(B35) continue to hold in the Cash-in-Advance economy.

We now consider the efficiency of the competitive allocation.

**Lemma 5 (Efficiency).** *If the conditions of Lemma 5 or Lemma 4 are satisfied, competitive equilibrium allocations are efficient.*

If the conditions of Lemma 5 or Lemma 4 are satisfied, decentralized trading is unconstrained so that $q^{eq} = q^*$. Then, the implied Arrow-Debreu prices associated with the efficient allocation are:

$$\pi_t(\omega_i) = \gamma(\omega_i). \quad \omega_i \in \Omega, \quad t \in \{1,2\} \tag{E25}$$

so that liability issuance coinciding with the efficient issuance is a competitive equilibrium.

A version of Proposition 4 also holds in the cash-in-advance economy.

**Proposition 6 (Too little maturirity transformation with Cash-in-Advance).** *Suppose the efficient allocation satisfies $L(\omega_l) > 0$. Then the efficient allocation cannot be implemented as a competitive equilibrium and the competitive equilibrium features less liquidation than the efficient allocation.*

Similar to the proof for Proposition 4, we need to show that:

$$-(1-\xi)H(q_1^*)d_2^*(\omega_l)G(q_2^*) > 0, \tag{E26}$$

with

$$H(q) = \frac{u'(q) - c'(q)}{c'(q)} \quad \text{and} \quad G(q) = \frac{u''(q)c'(q) - c''(q)u'(q)}{c'(q)^2}. \tag{E27}$$

Condition (E26) holds because $H(q) \geq 0$ when $q < q^*$ and $G(q) \leq 0$. The remaining steps in the proof to Proposition 4 hold with $\alpha = \eta = 1$.