

Ludwig Wittgenstein's *Bemerkungen über die Grundlagen der Mathematik/Remarks on the Foundations of Mathematics* was first published in 1956, in a bilingual German/English edition. It consists of notes taken over several years, mostly between 1937 and 1944, that were meant to form part of *Philosophical Investigations*, his main later work; but they never reached a form satisfactory to Wittgenstein so that he did not include them in the end. Five years after his death, Wittgenstein's literary executors, G. H. v. Wright, R. Rhees, and G. E. M. Anscombe, selected some of these notes, edited them as a separate volume, and made it available to the general public. In 1967, the original volume was replaced by a second, significantly expanded edition.

Paul Bernays' "Betrachtungen zu Ludwig Wittgensteins 'Bemerkungen über die Grundlagen der Mathematik' " / "Comments on Ludwig Wittgenstein's *Remarks on the Foundations of Mathematics*" is a response to the first edition of Wittgenstein's book. This article originally appeared in the journal *Ratio*, in 1959, both in German and English (translator not specified). *Ratio* had been established two years earlier, as a successor to *Abhandlungen der Fries'schen Schule*, with Julius Kraft as its main editor. Its volumes were all published in parallel German and English editions. The English version of Bernays' article was subsequently reprinted in *Philosophy of Mathematics. Selected Readings*, edited by P. Benacerraf and H. Putnam. More precisely, it appeared in the first edition of that collection, published in 1964, as part of a more comprehensive section on Wittgenstein (Part Four). In the second edition, published in 1983, that entire section was omitted and replaced by more recent work.

From the early 1930s on, Wittgenstein wrote and lectured extensively on the foundations of mathematics. None of his corresponding notes were published initially, although their content was disseminated more informally, in Cambridge (where he was teaching), in Vienna (where he had conversations with members of the Vienna Circle), and elsewhere. Wittgenstein's views quickly attracted considerable attention. They came to be seen as a radical alternative to the still dominant schools of logicism, formalism, and intuitionism. Later, after the publication of *Remarks on the Foundations of Mathematics*, a number of influential logicians and philosophers of mathematics felt compelled to respond to it in print, including Georg Kreisel, Michael Dummett—and Paul Bernays. Given the close association of both Bernays and Kraft with Leonard Nelson and the Friesian school, it was also natural for Bernays to be asked to review Wittgenstein's book for *Ratio*.

In his review, Bernays addresses a number of issues central to the philosophy of mathematics and logic, including the following: the foundations of geometry, arithmetic, and set theory; the finite and the infinite, the denu-

merable and the non-denumerable; Frege's and Russell's reduction of arithmetic to logic; Dedekind's construction of the real numbers as cuts; Hilbert's demand for consistency proofs; Gödel's incompleteness theorems; Brouwer's intuitionism; strict finitist ideas; the broader issues of reduction, foundations, formalization, and the general role of logic. Along the way, Bernays touches on related views by Aristotle, Leibniz, Kant, Hegel, Mach, Husserl, Weyl, and Carnap. He also indicates his own position on topics such as the opposition between constructivism and a "Platonist-existential" viewpoint, the notion of a *characteristica universalis*, and even the general philosophical issue of human freedom. The article closes with a brief, but striking suggestion to look at mathematics as the "theoretical phenomenology of structures".

Bernays does find some value in Wittgenstein's remarks, e.g., in connection with motivating finitist and constructivist views and, especially, with countering dogmatic attitudes concerning reductions in mathematics. Mostly, however, his reaction is critical. Among his general criticisms are: that Wittgenstein attempts to play down the role of thought too much, in a manner reminiscent of behaviorism; that he is holding on to a strict and problematic separation of the linguistic and the factual; that he recognizes only one kind of factuality, that of the empirical; and that he treats mathematics too narrowly as a collection of techniques with simple practical uses. In addition, he formulates some more specific criticisms concerning, e.g., Wittgenstein's discussion of geometric axioms, of the role of formalization in mathematics, and of Dedekind's and Gödel's proof procedures. Finally—and most interestingly in connection with Bernays himself—these criticisms lead him to brief statements of his own contrasting views.