

Homework 4

1. Determine the units and counits of the following adjunctions.
 - (a) $(-) \times A \dashv (-)^A$
 - (b) $+ \dashv \Delta \dashv \times$
 - (c) $\exists \dashv * \dashv \forall$
2. Show that right adjoints preserve products and left adjoints preserve coproducts. (Hint: use Yoneda.)
3. Use the foregoing to infer that the following hold in any CCC with coproducts:
 - (a) $C \times (A + B) \cong C \times A + C \times B$
 - (b) $(A \times B)^C \cong A^C \times B^C$
4. (a) Show that in IFOL, one has $\forall y. \varphi(y) \wedge \psi(y) \dashv\vdash \forall y \varphi(y) \wedge \forall y \psi(y)$ as well as the dual for \exists and \vee .
 - (b) Show that \forall does *not* have a right adjoint, and that \exists does *not* have a left adjoint.