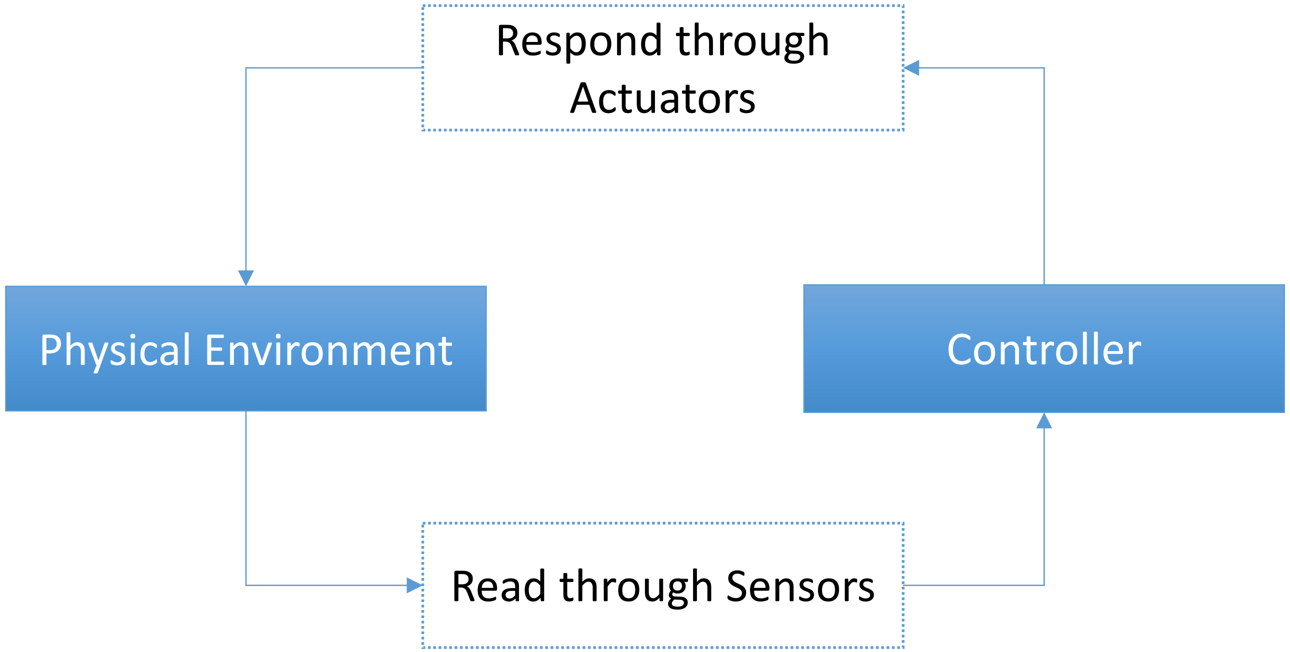
Robustness as Remedy for Model Checking Cyber-Physical Systems

Nima Roohi
University of Pennsylvania

Applications of Formal Methods to Control Theory and Dynamical Systems
June 23, 2018

Cyber-Physical Systems

What are they? Where they are?



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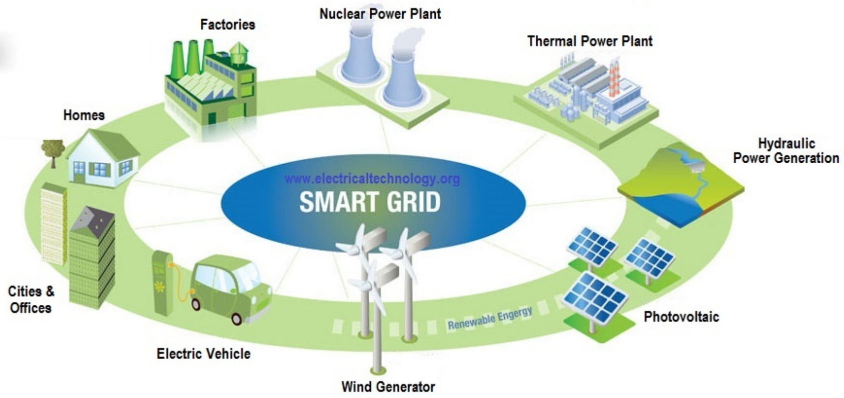
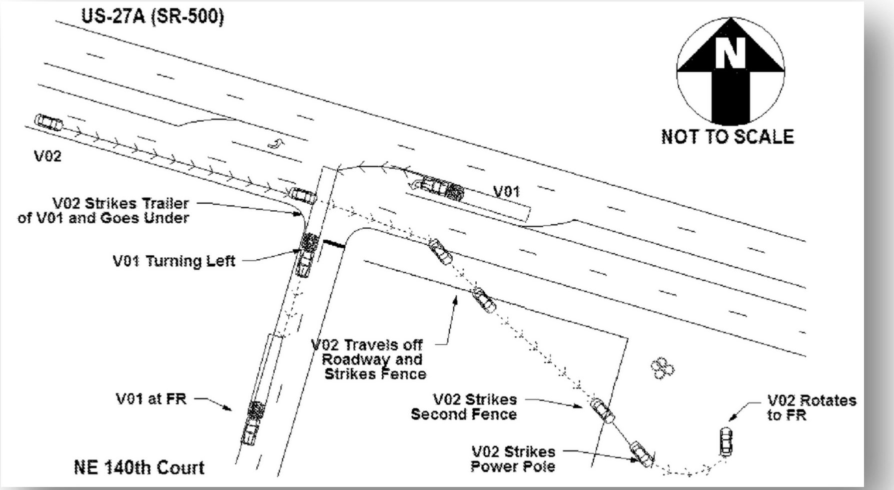
Respond through



Cyber-Physical Systems
What do we want?

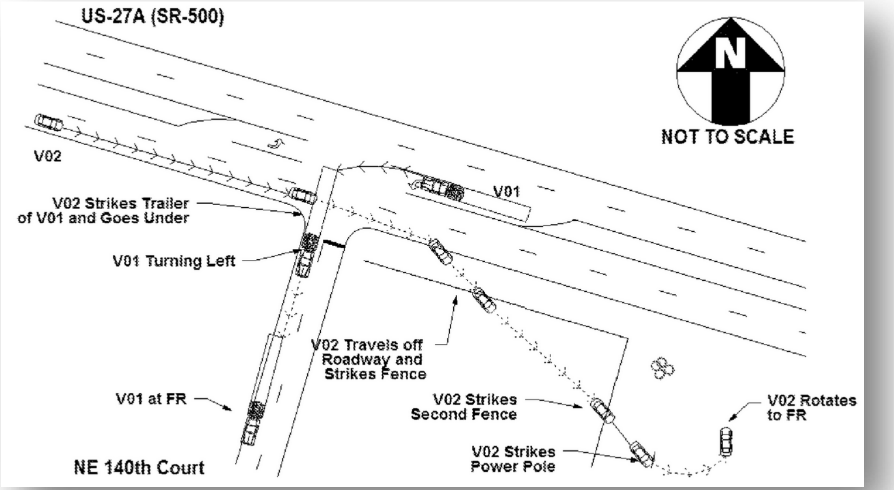
- Safety
  - Something bad never happens



Cyber-Physical Systems

What do we want?

- Safety
  - Something bad never happens
- Liveness
  - Something good will eventually happen



Cyber-Physical Systems
Formal Verification is a Necessity

- System failures are very expensive
  - Automakers recalled a record of 51.2 million vehicles over 868 separate recalls in 2015 for safety defects (USA TODAY January 21, 2016)
  - Study in University of Michigan shows self deriving cars has five times bigger accident rate (USA TODAY October 31, 2015)
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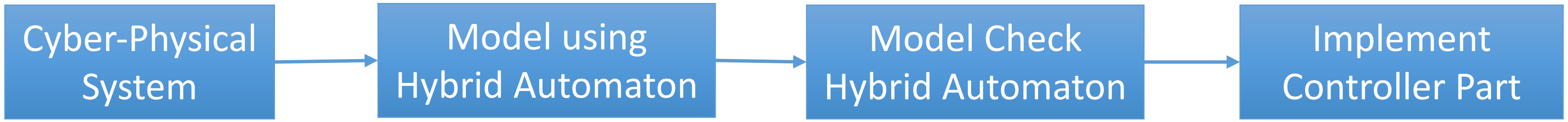
- Hybrid automata are used to model a cyber-physical system
  - Mathematical Model
  - Mathematical Proof

Formal Verification of Cyber-Physical Systems



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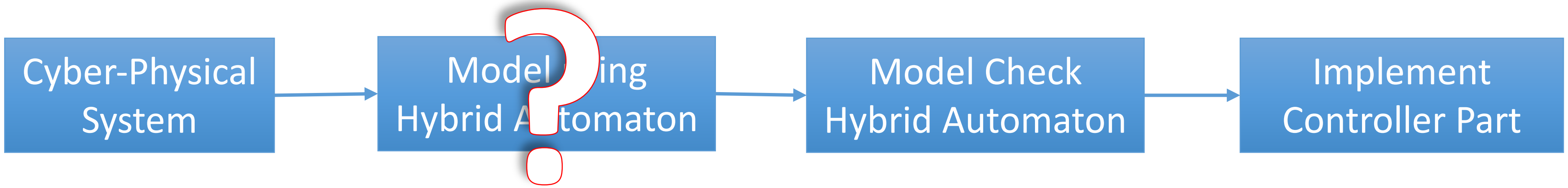

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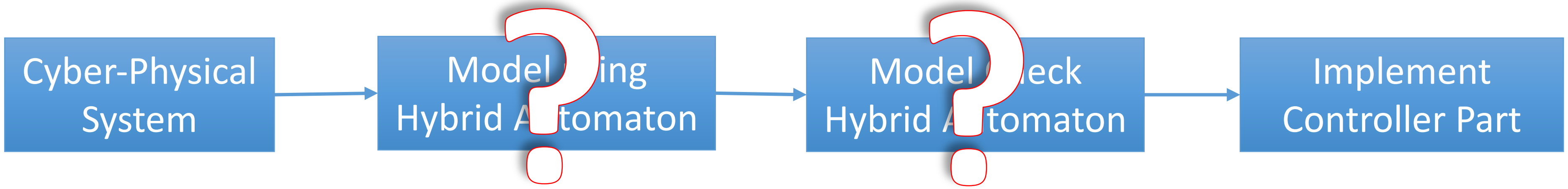

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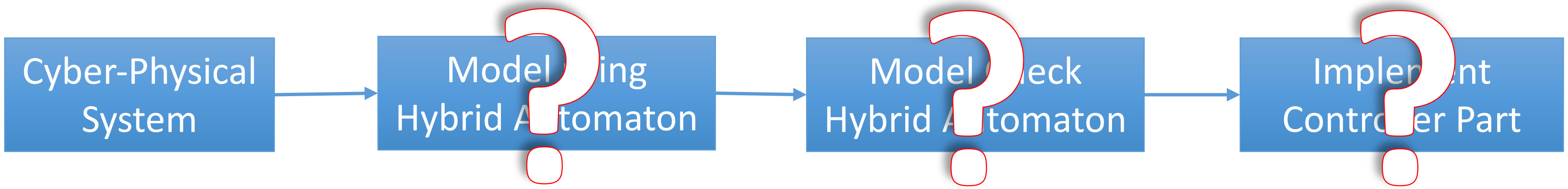

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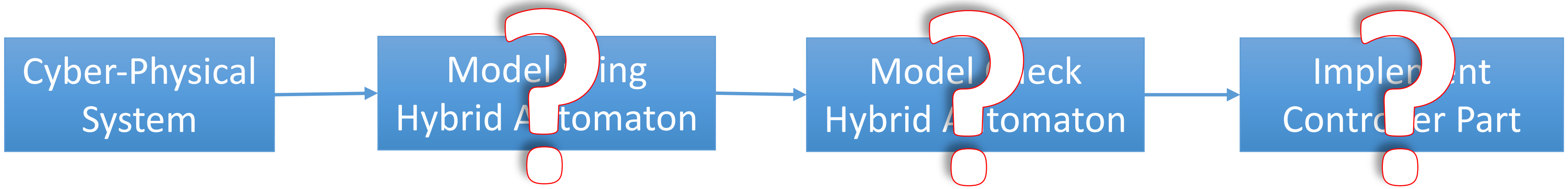

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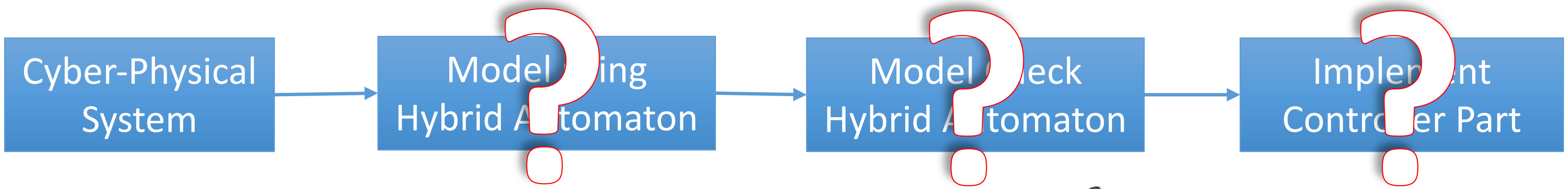

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Abstraction Helps

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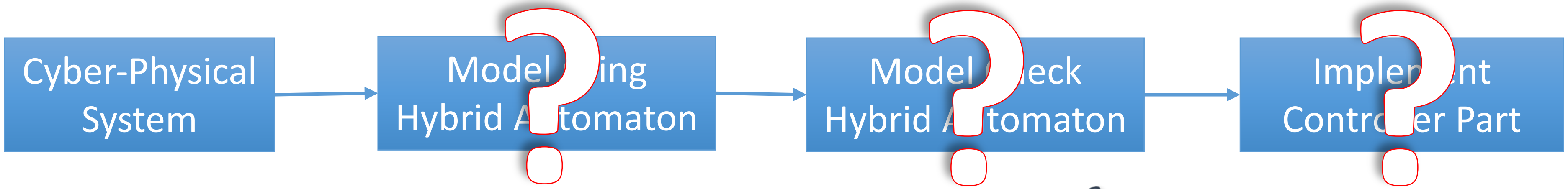

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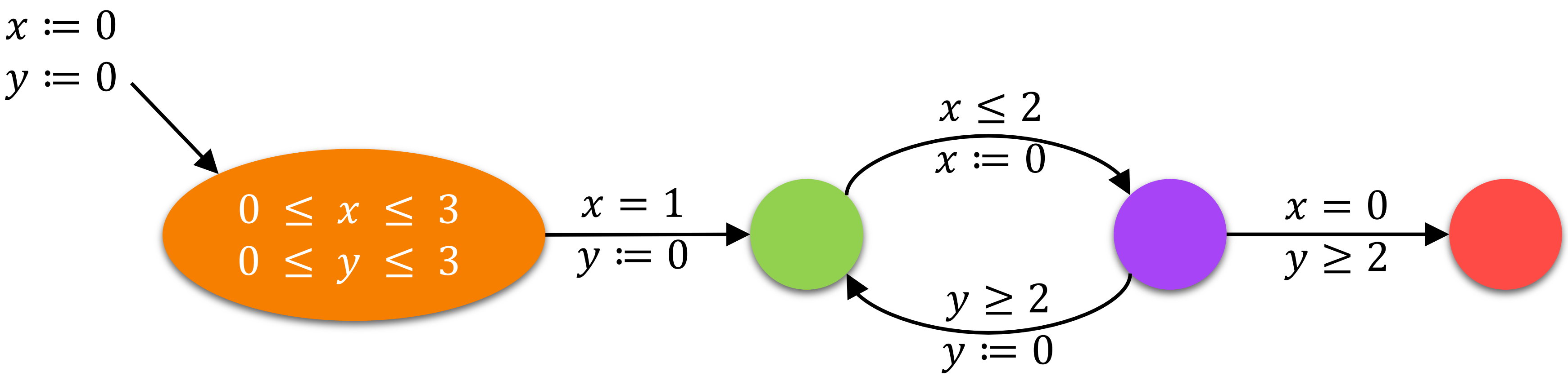
Robust Model Checking of Timed Automata

HSCC 2017

Timed Automata



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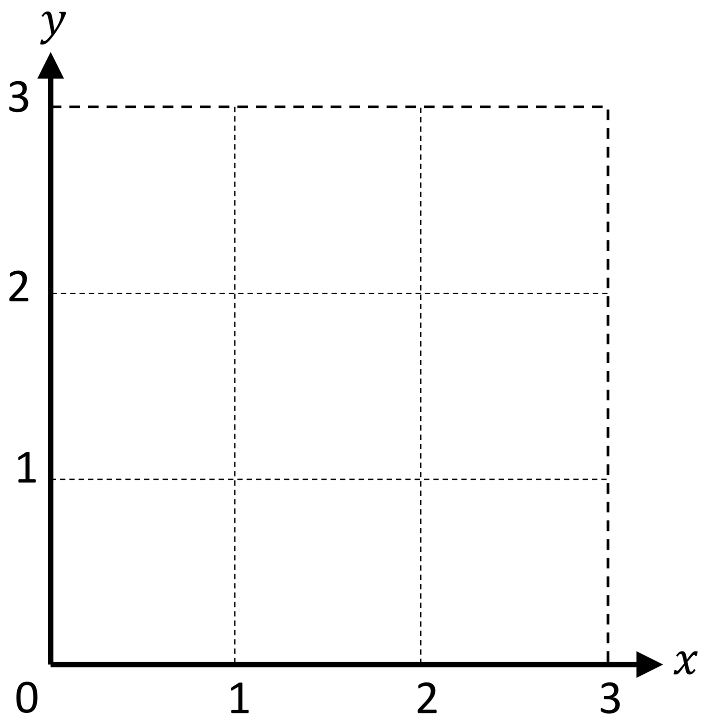

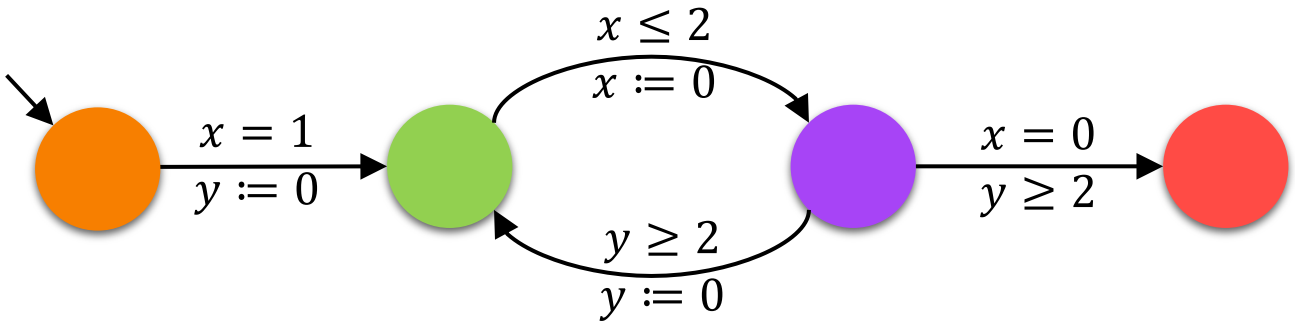

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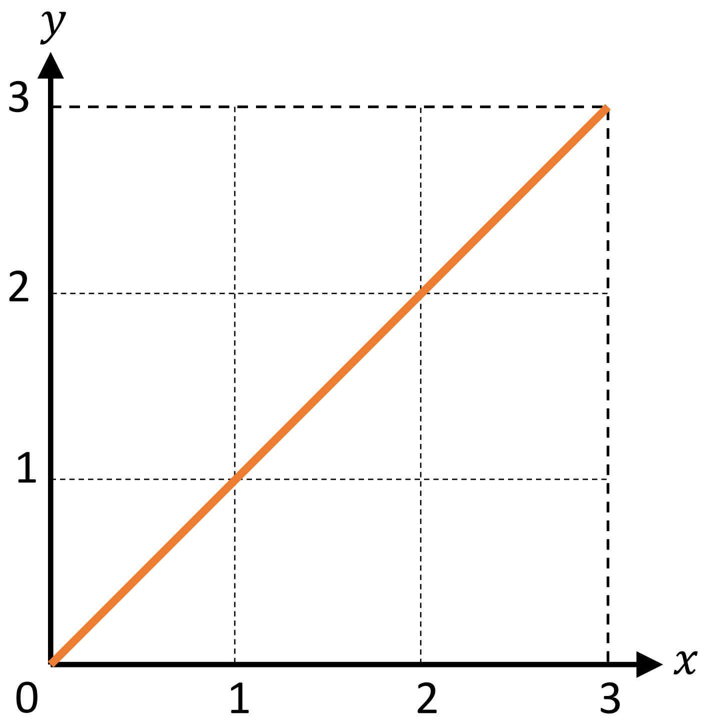

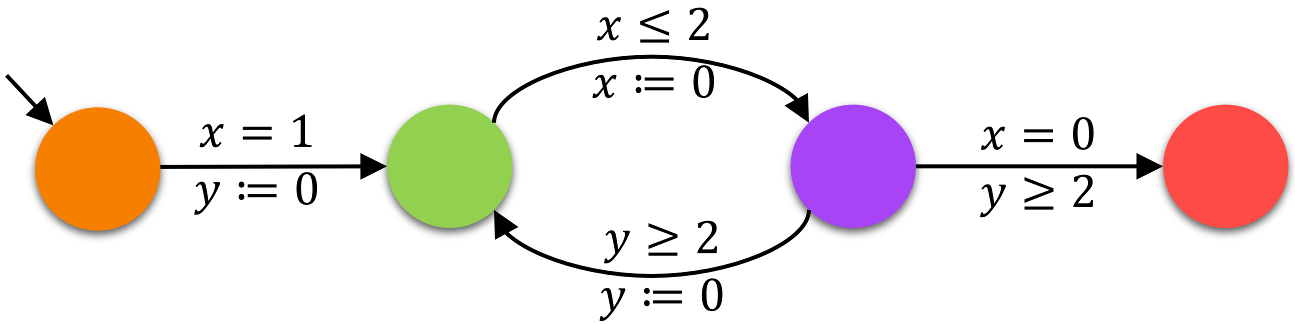

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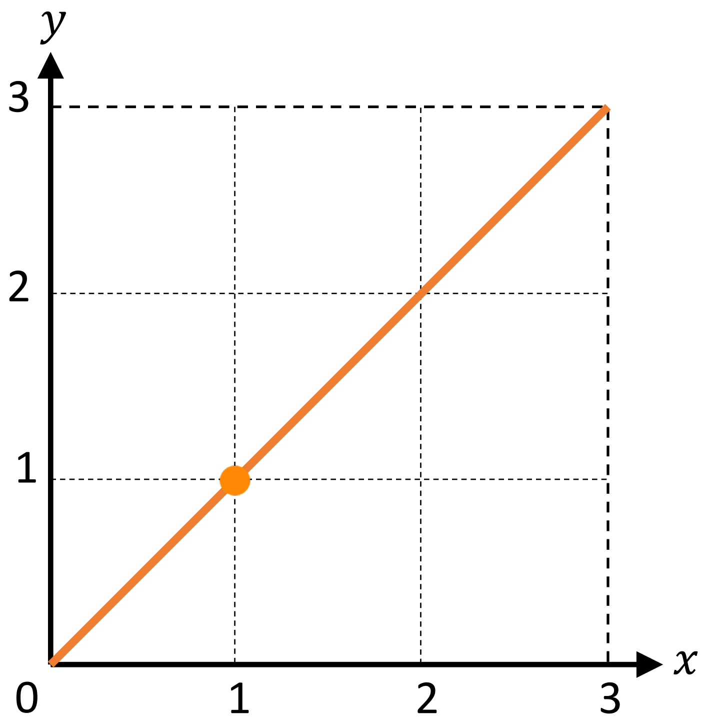

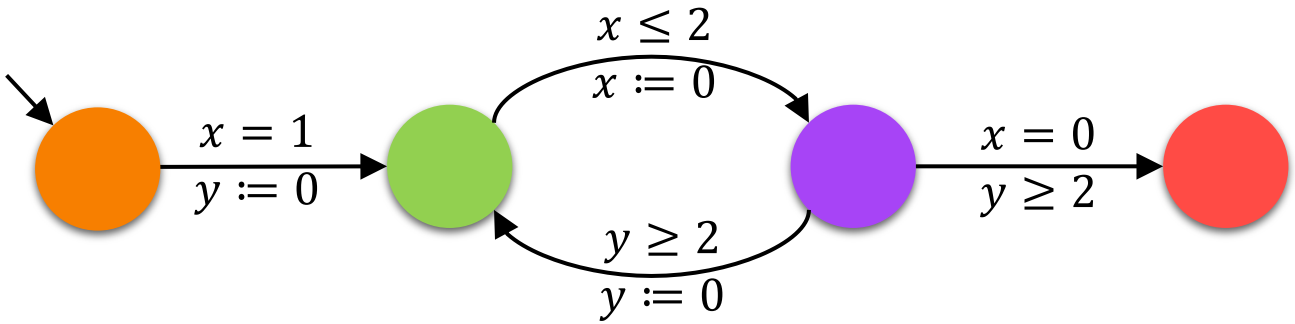

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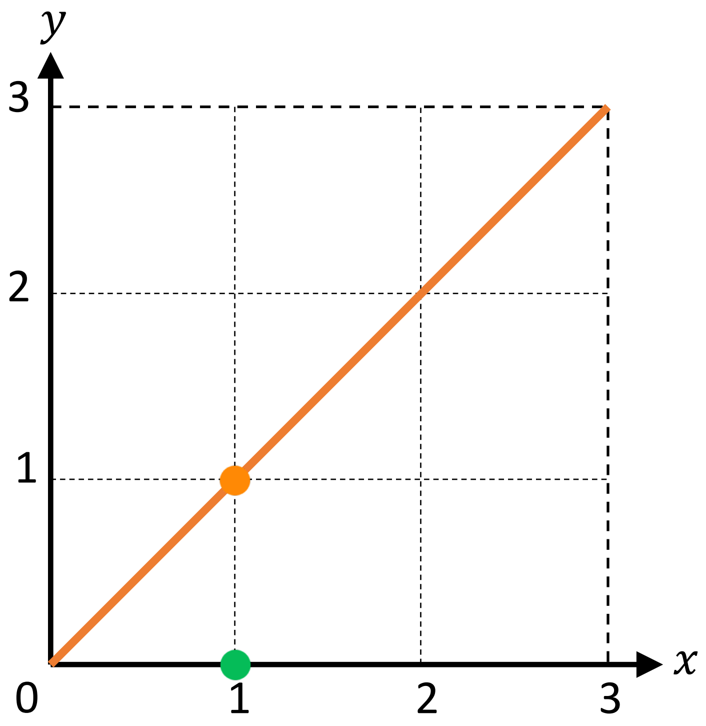

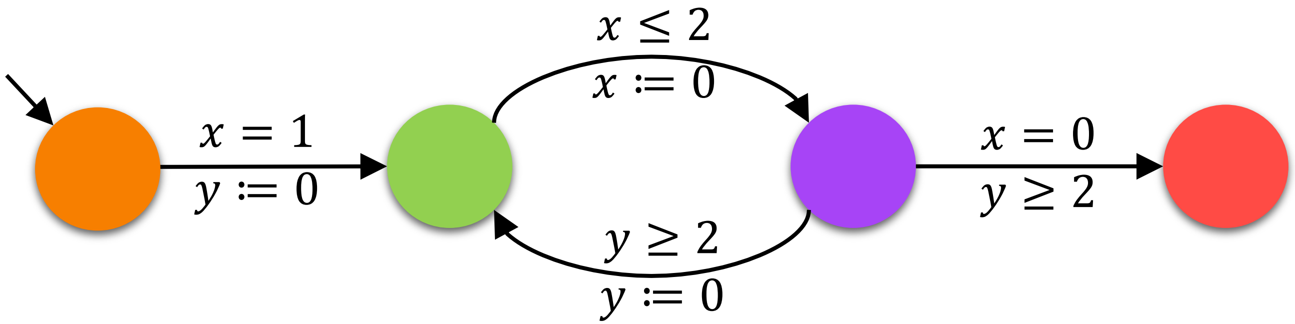

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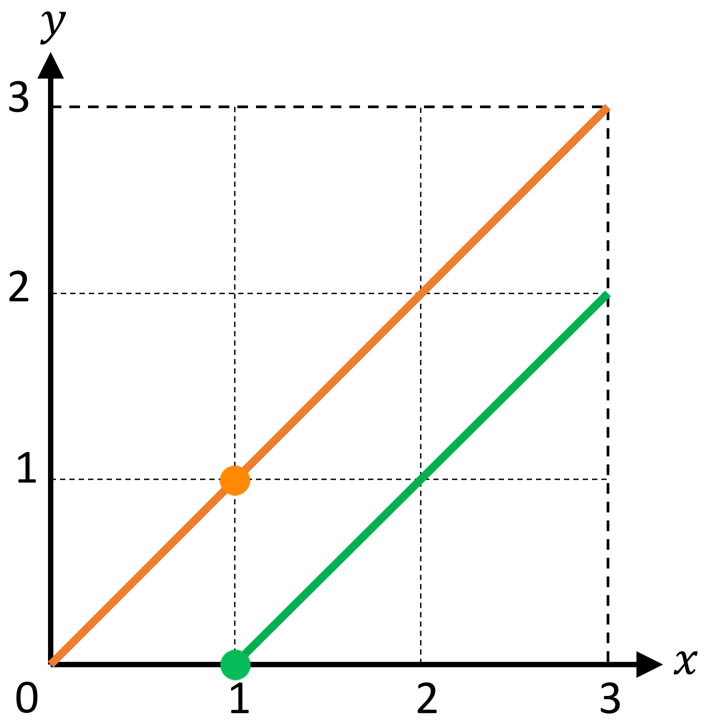

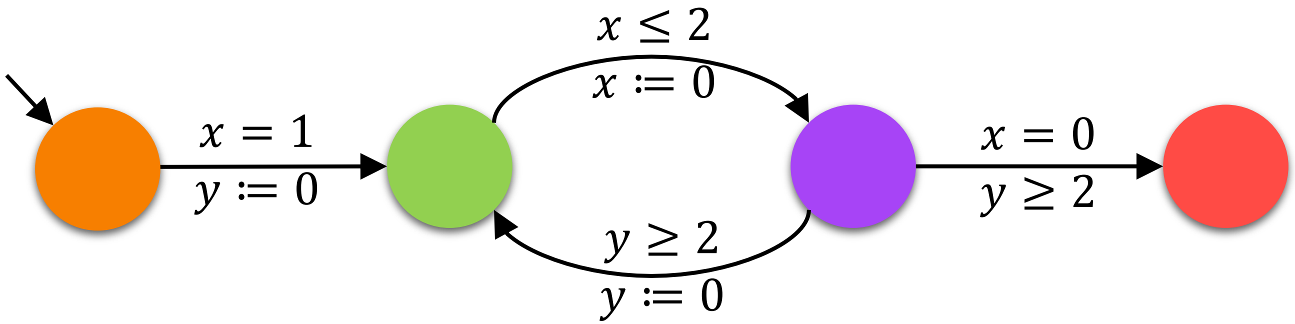

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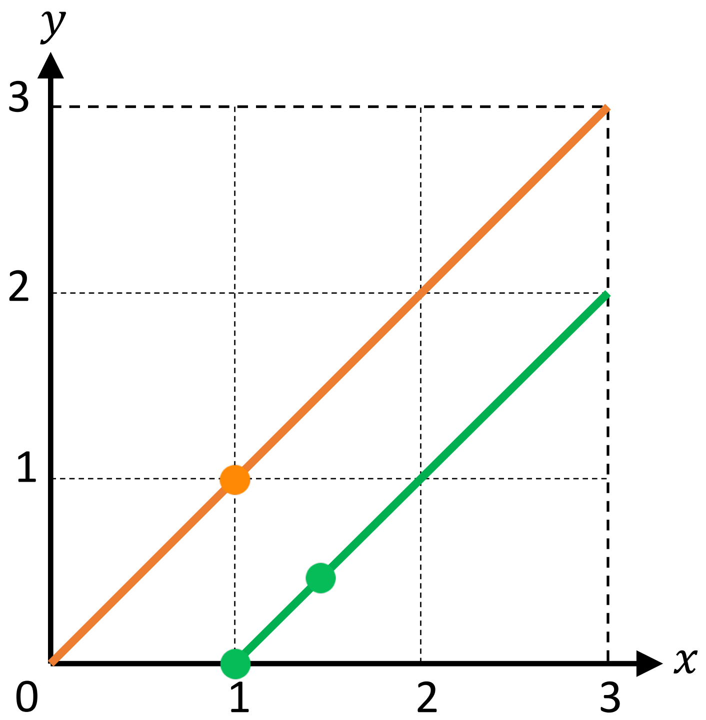

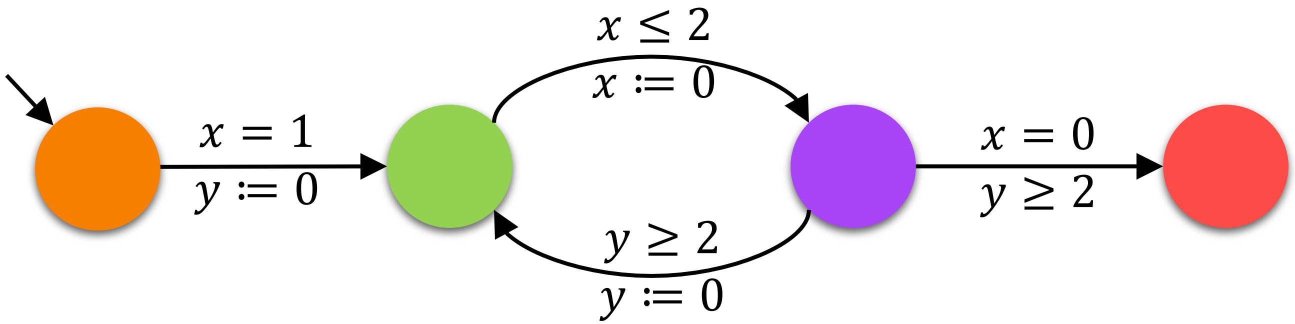

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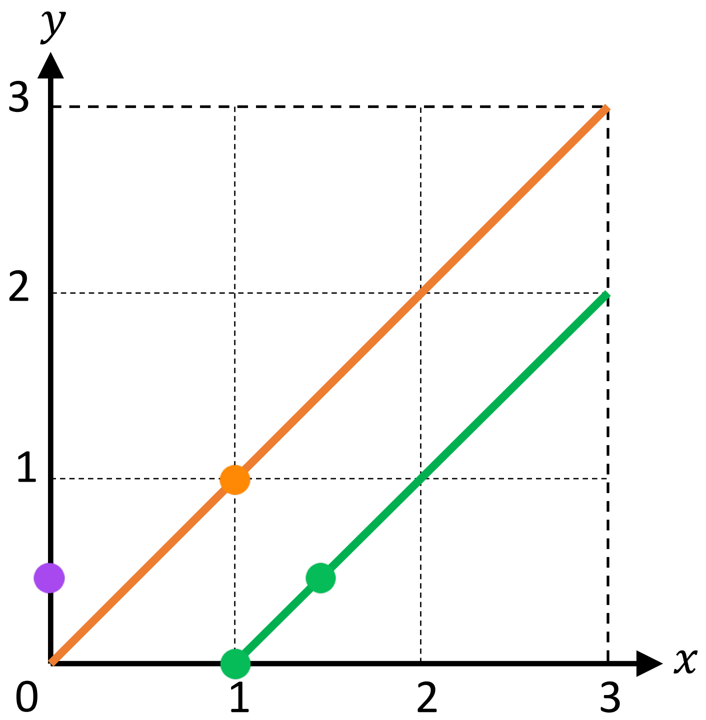

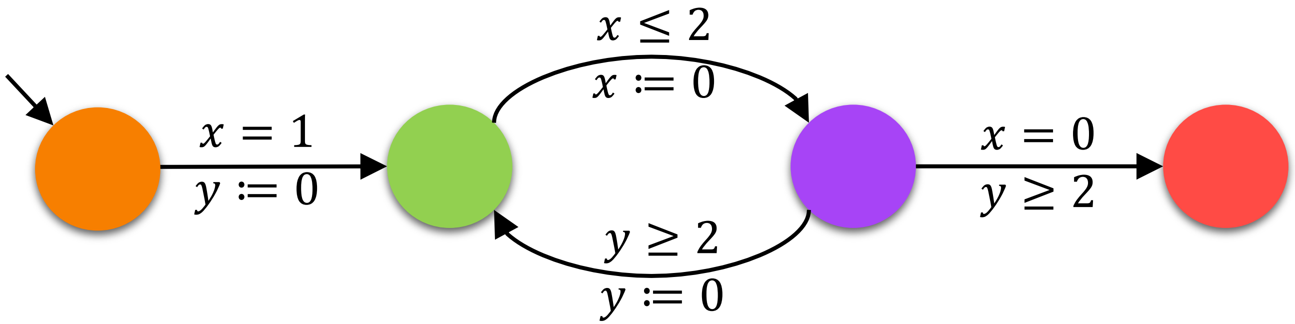

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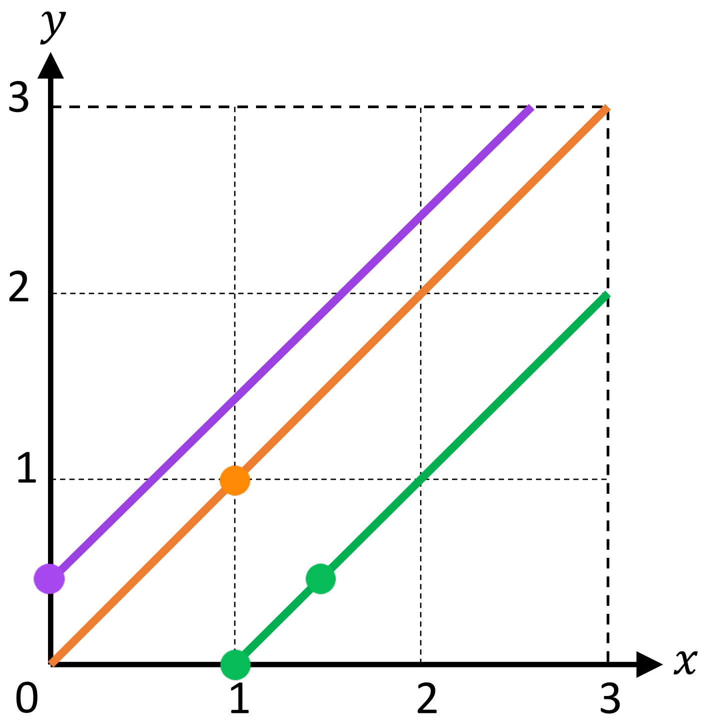

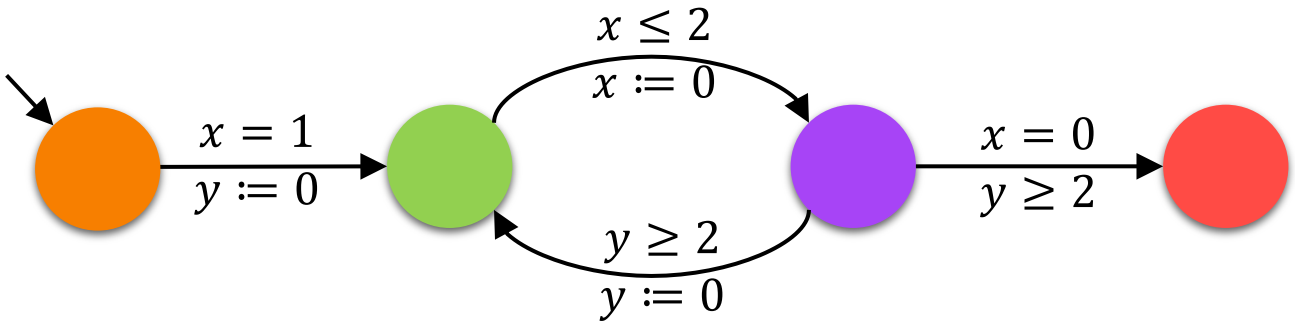

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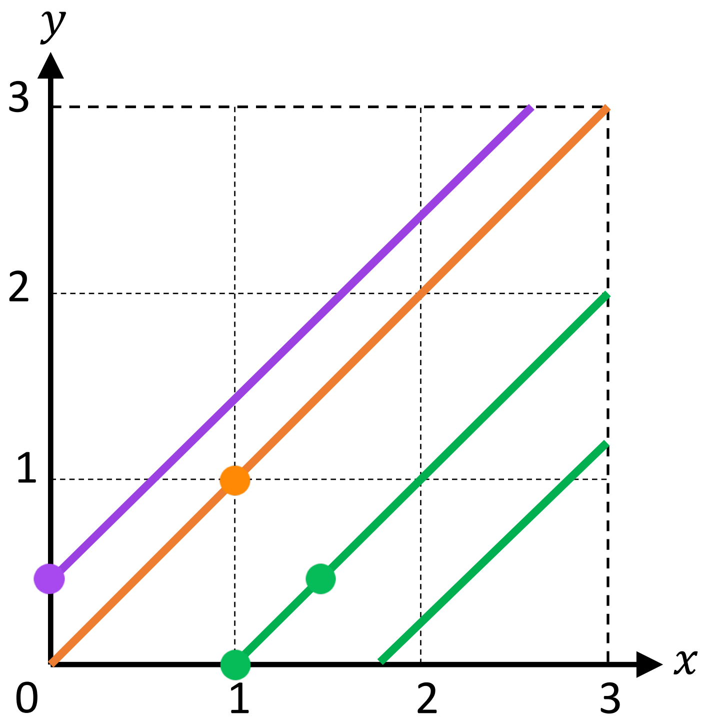

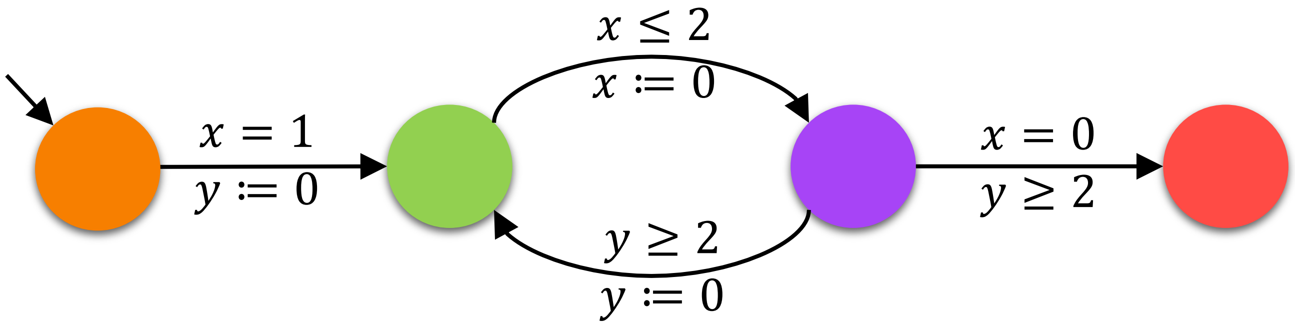

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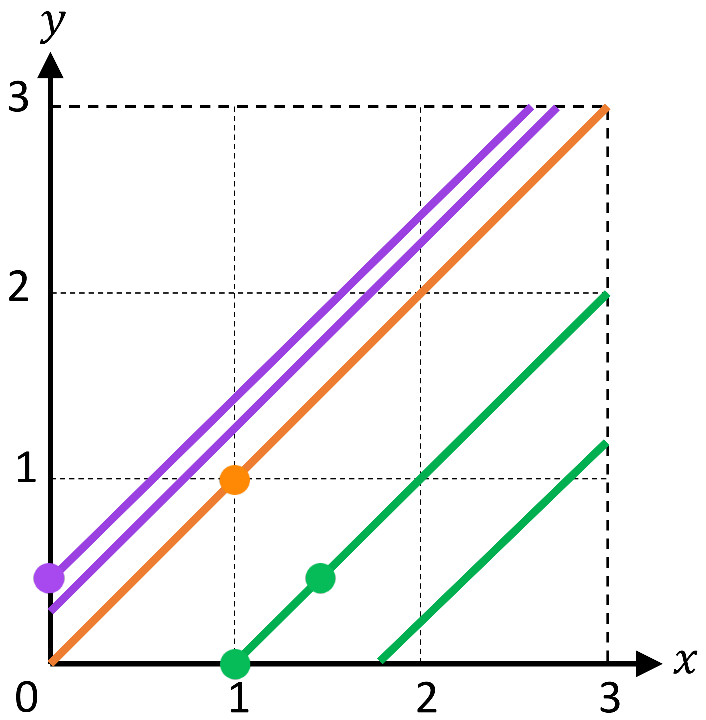

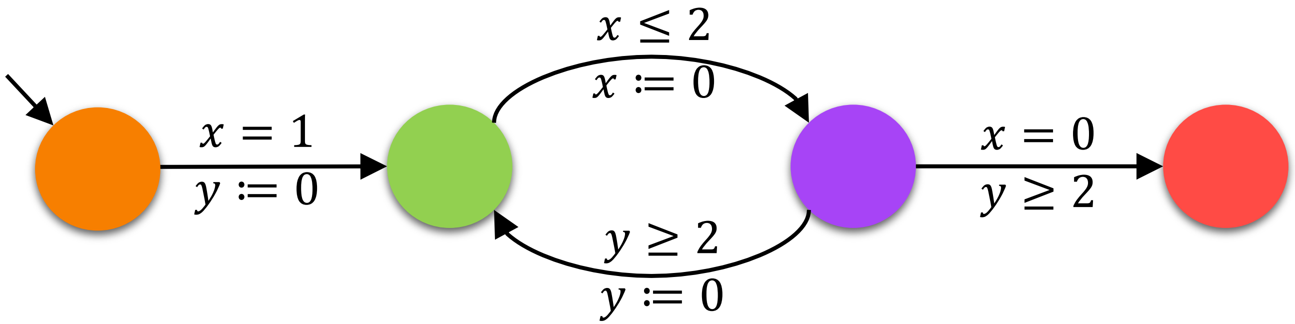

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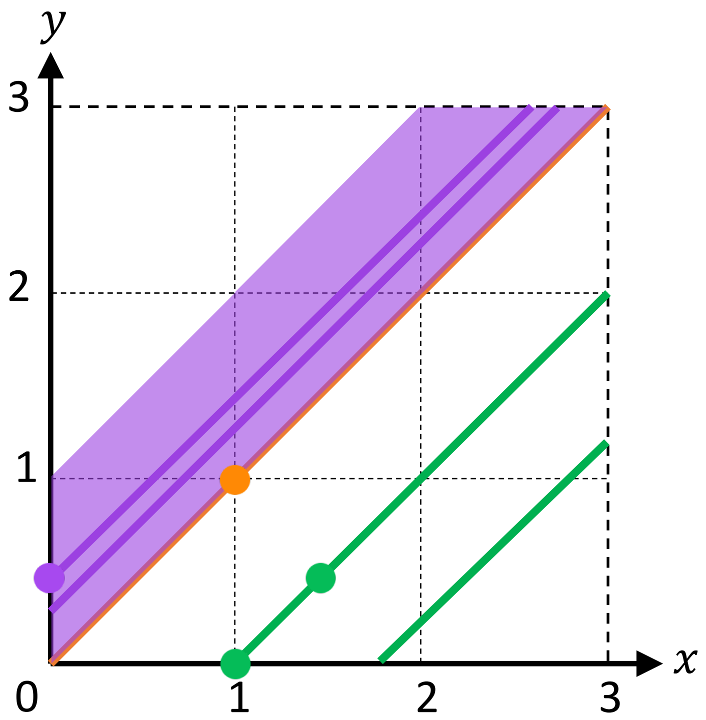

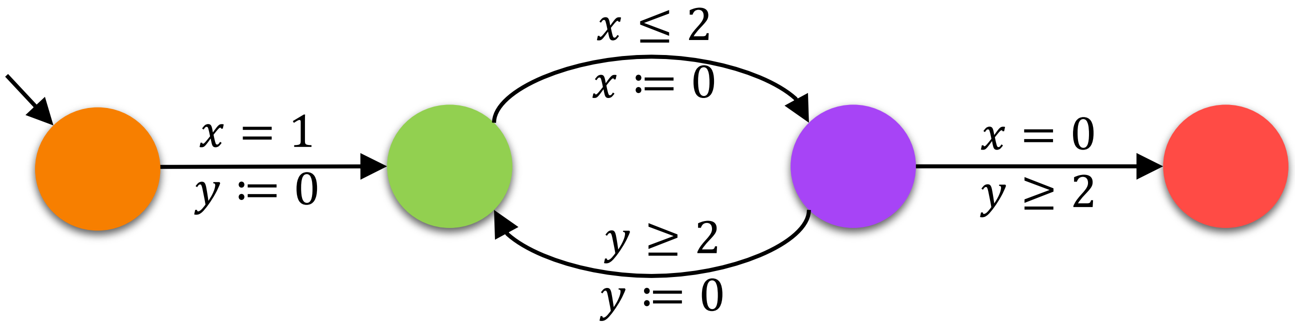

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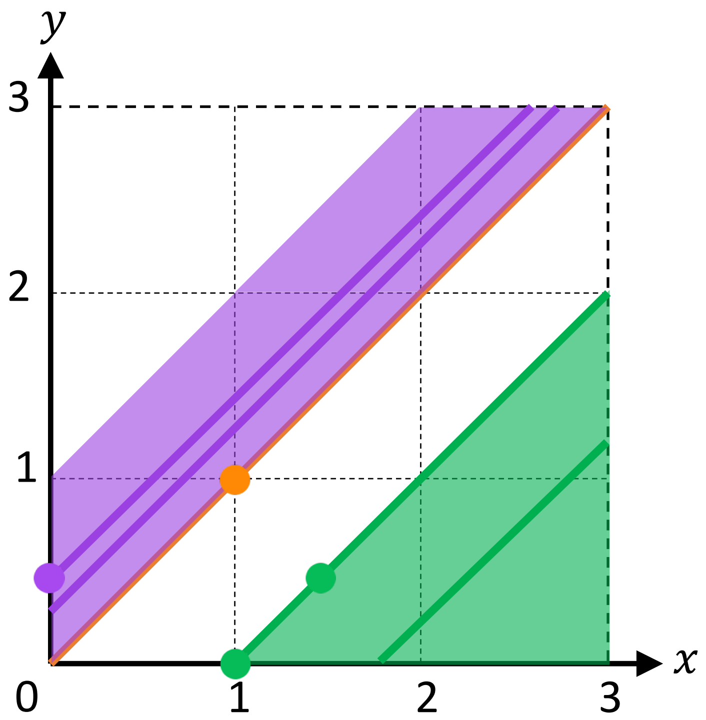

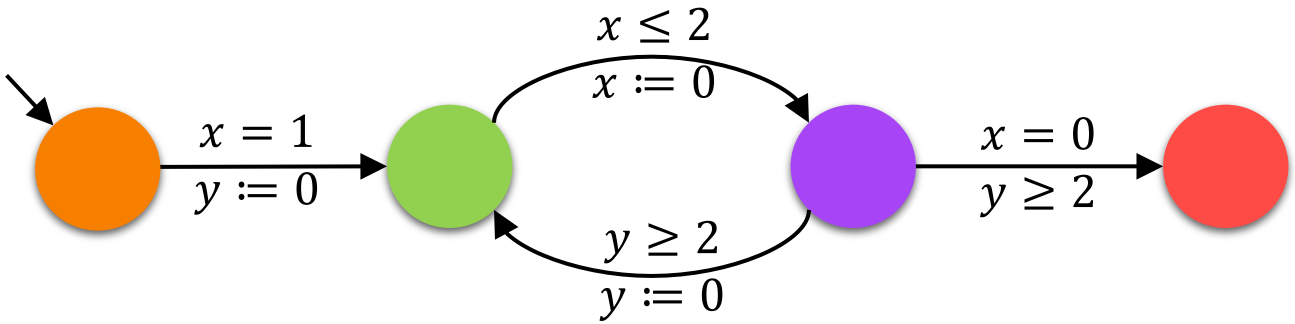

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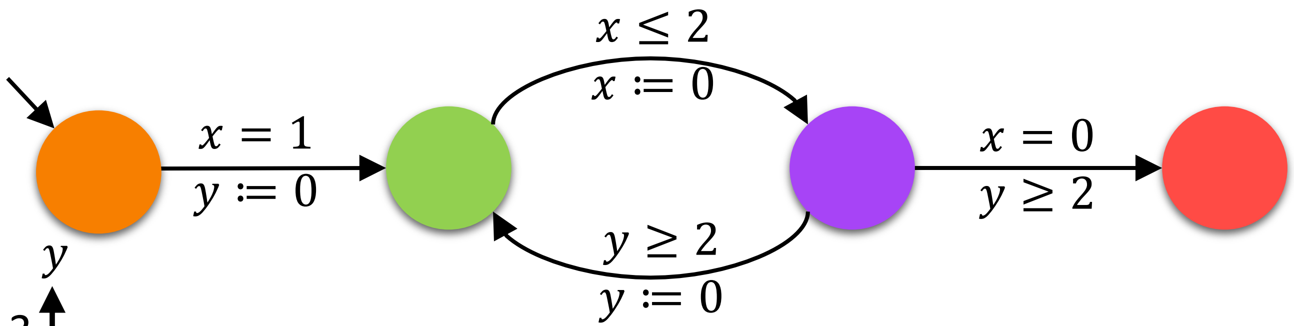

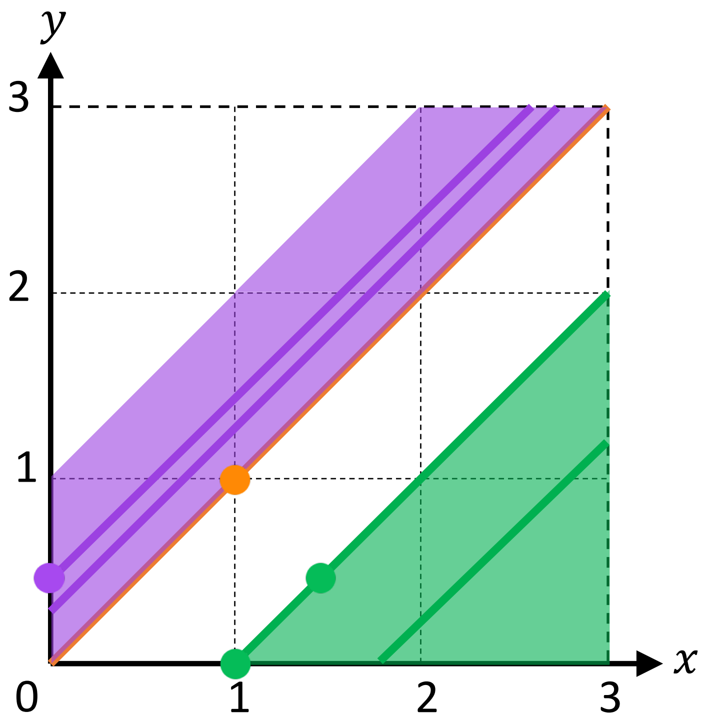

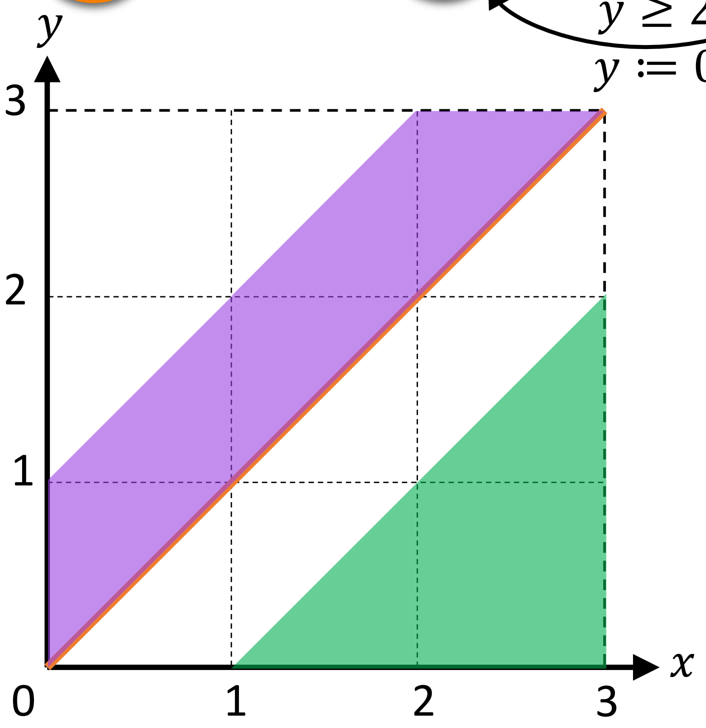

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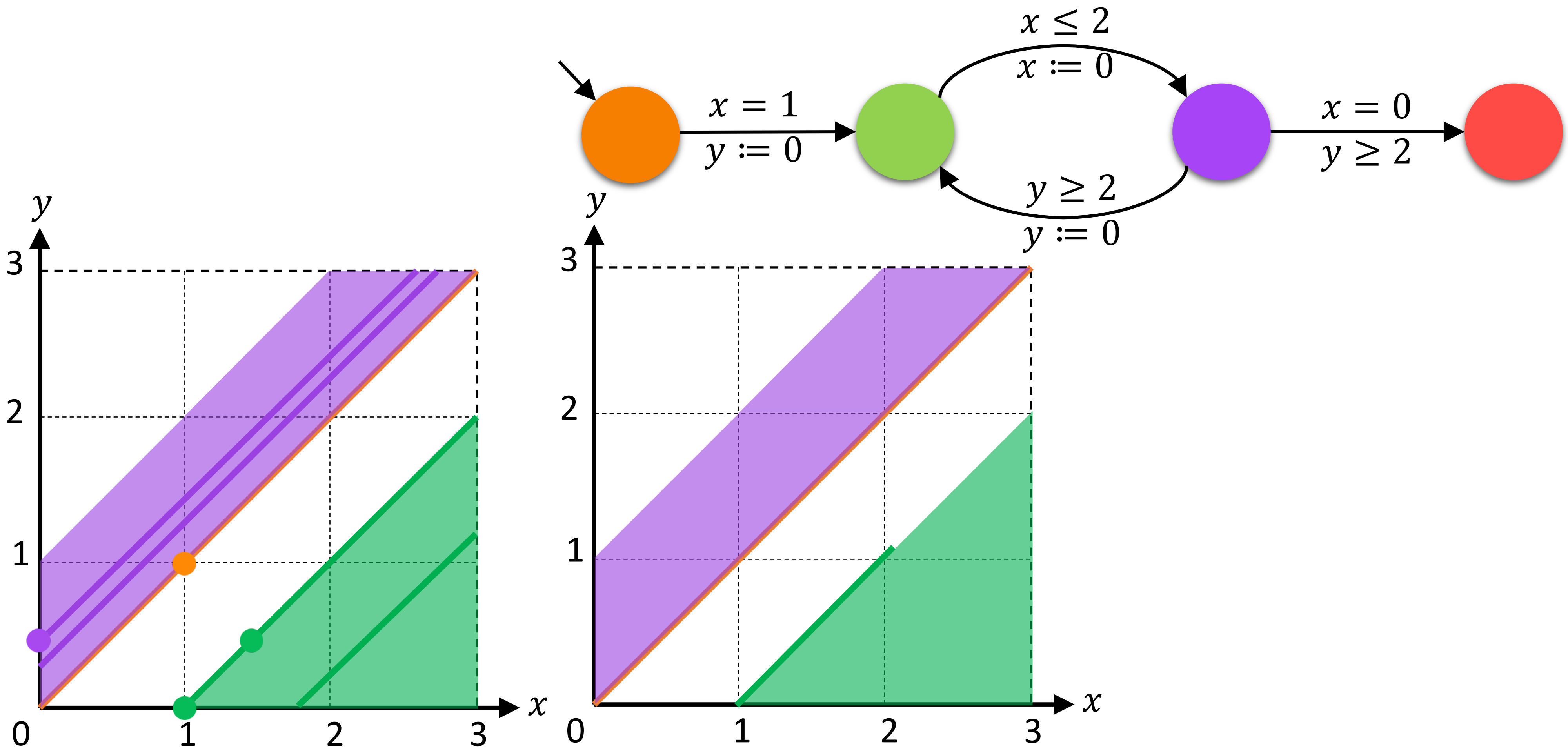

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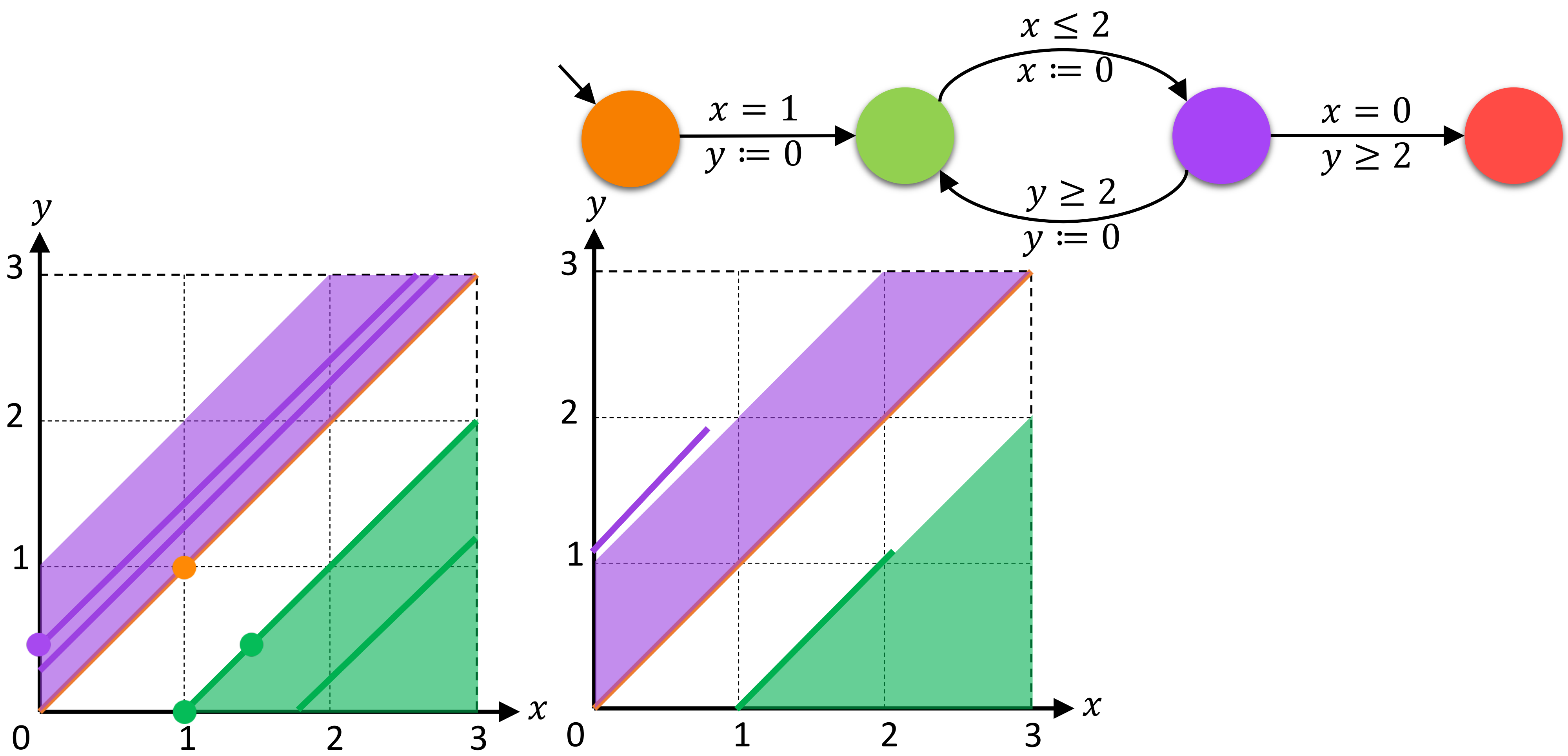

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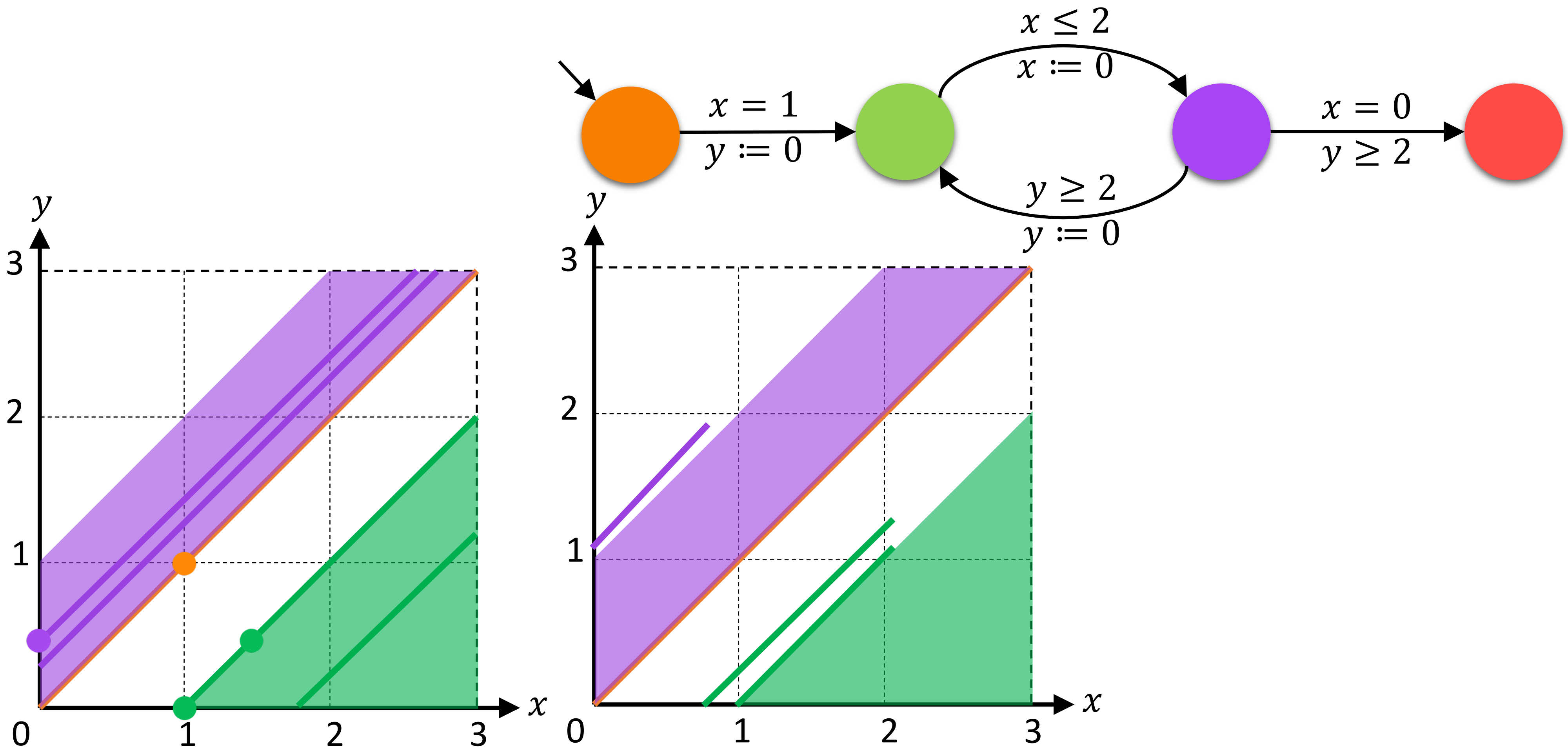

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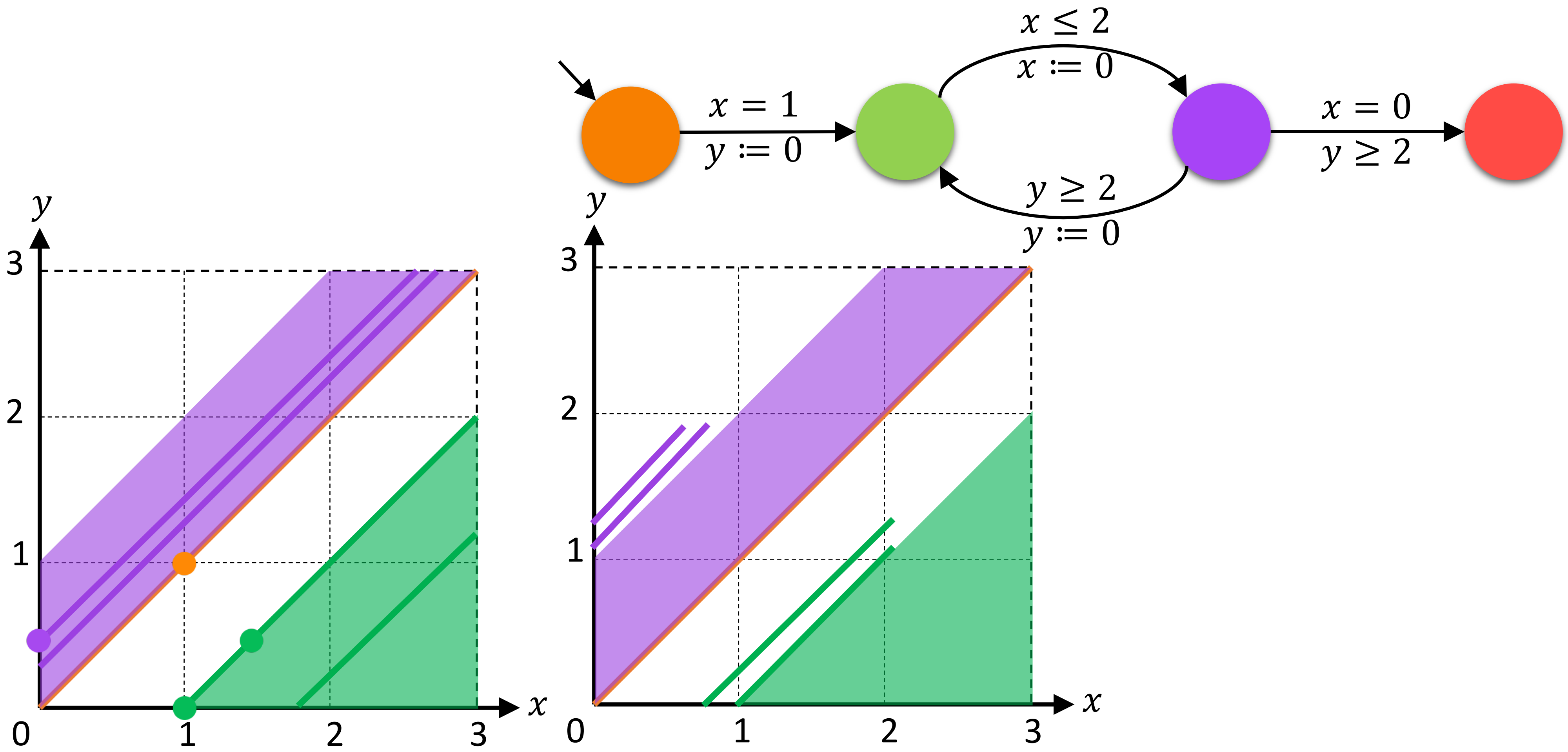

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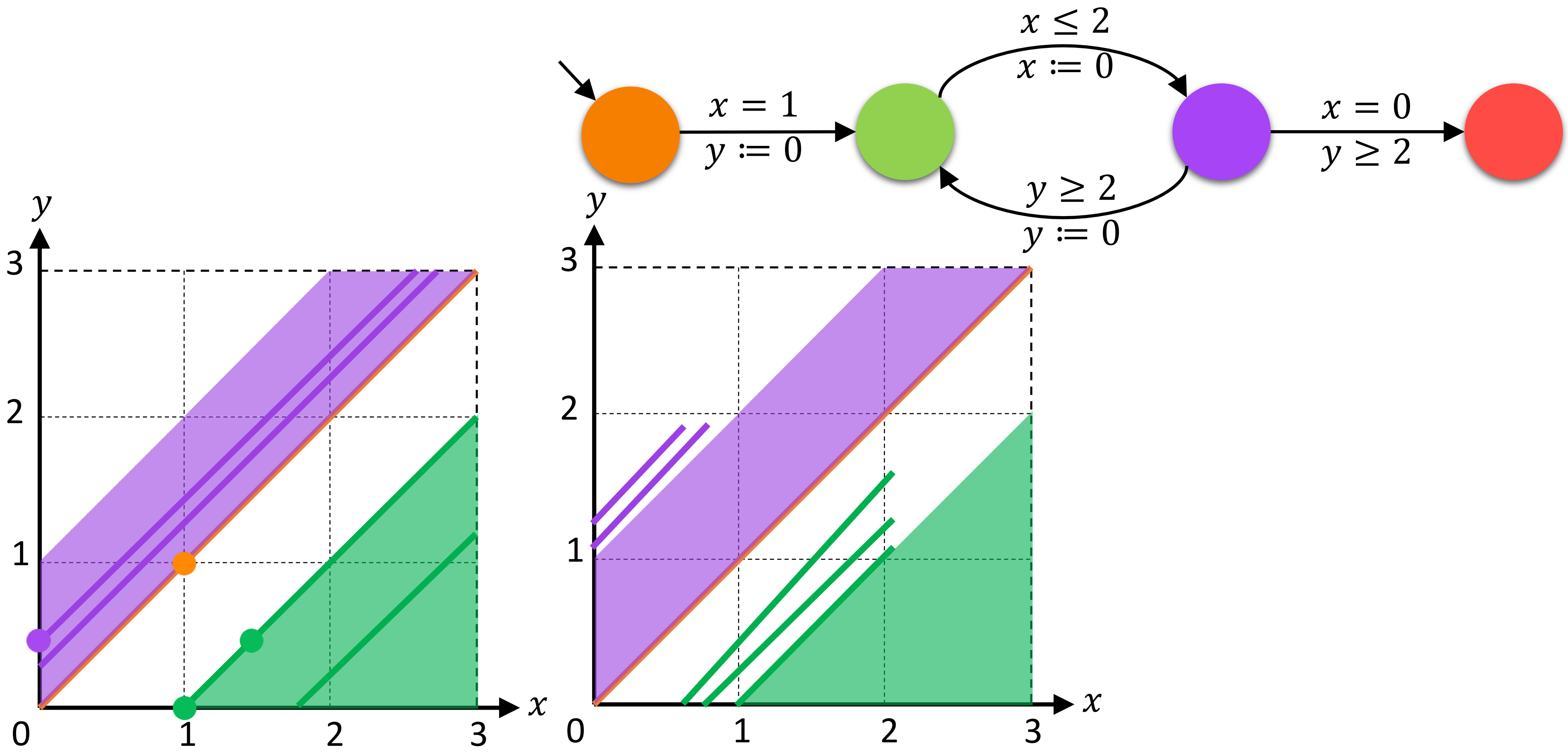

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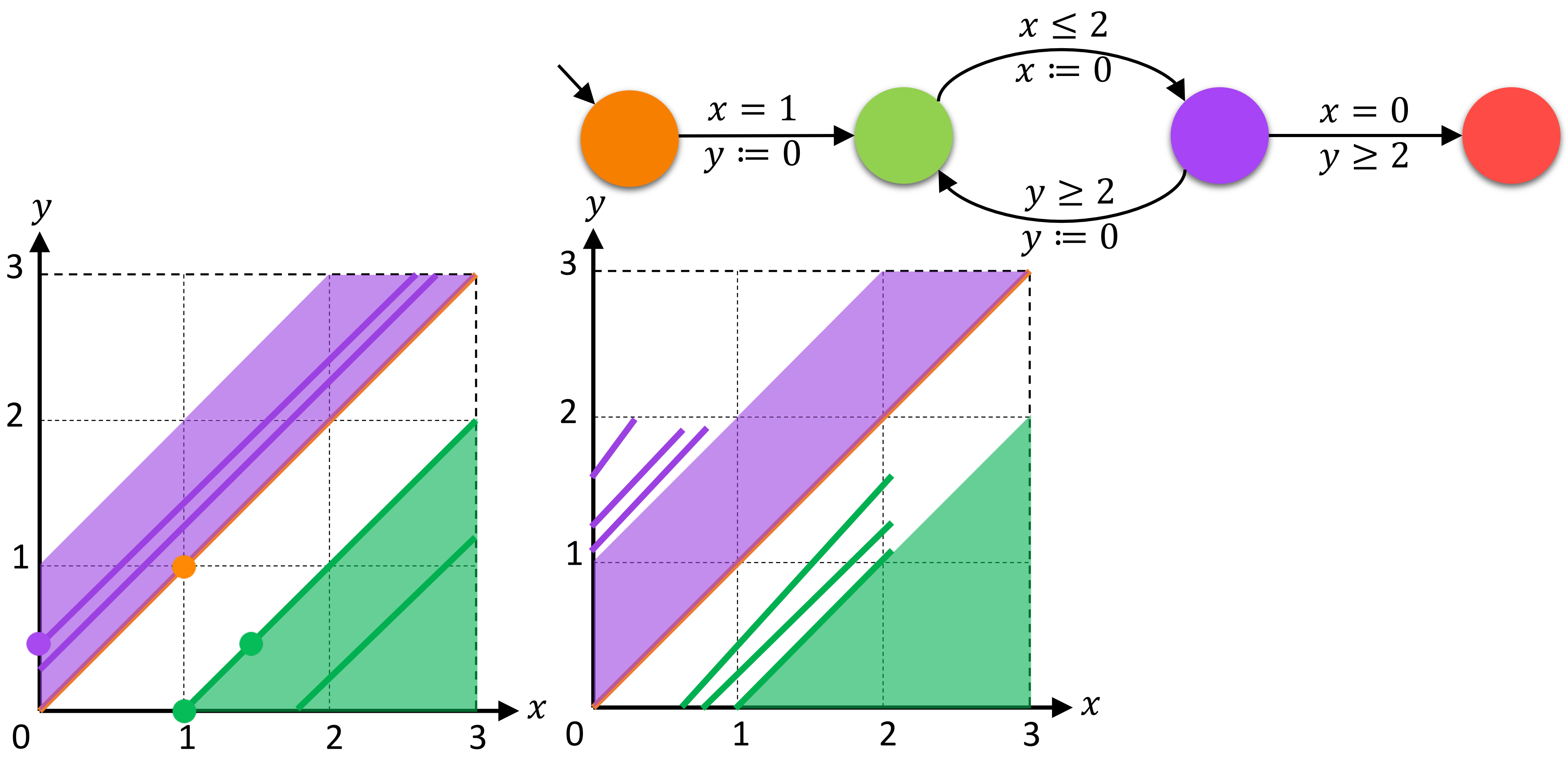

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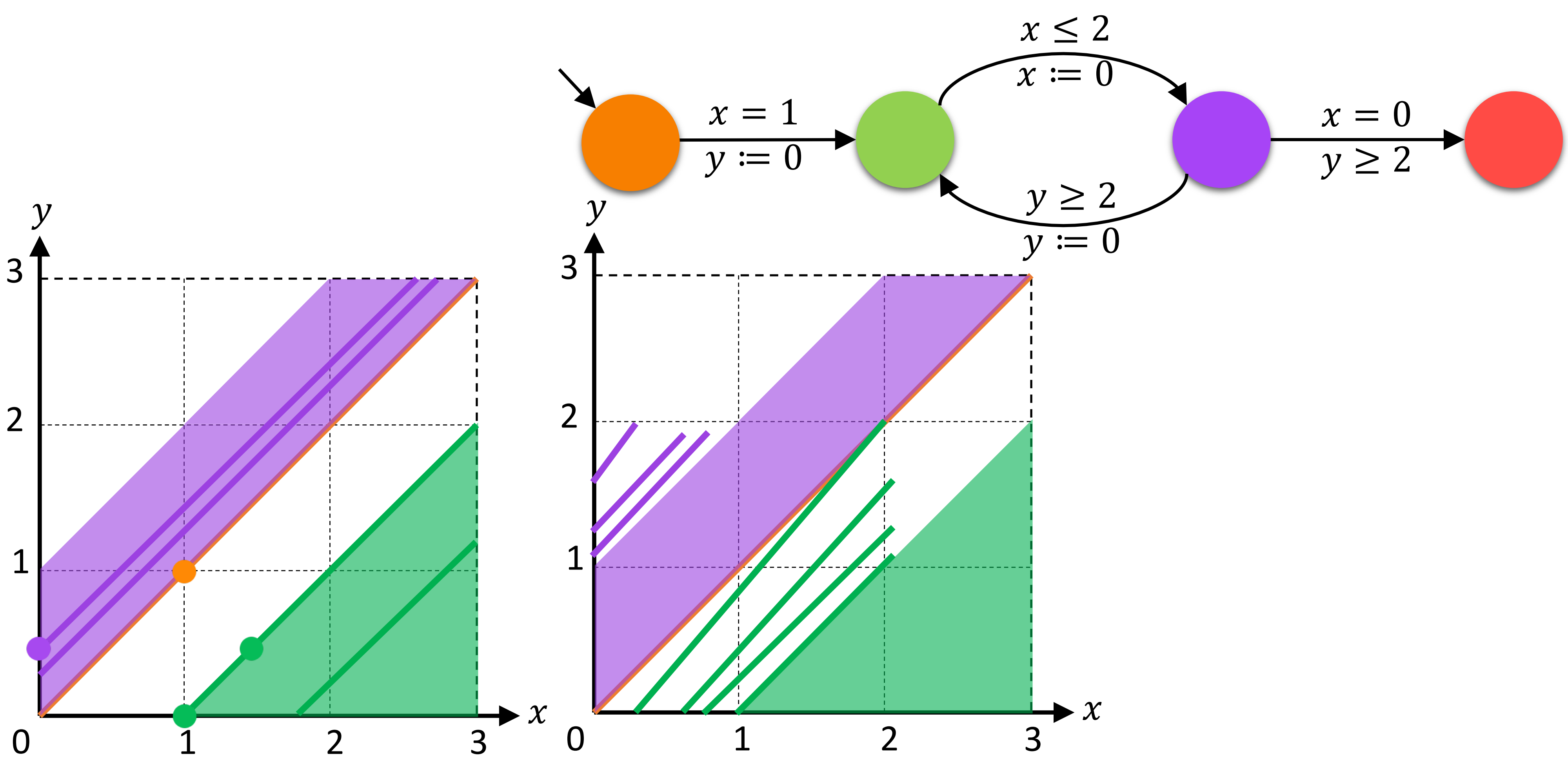

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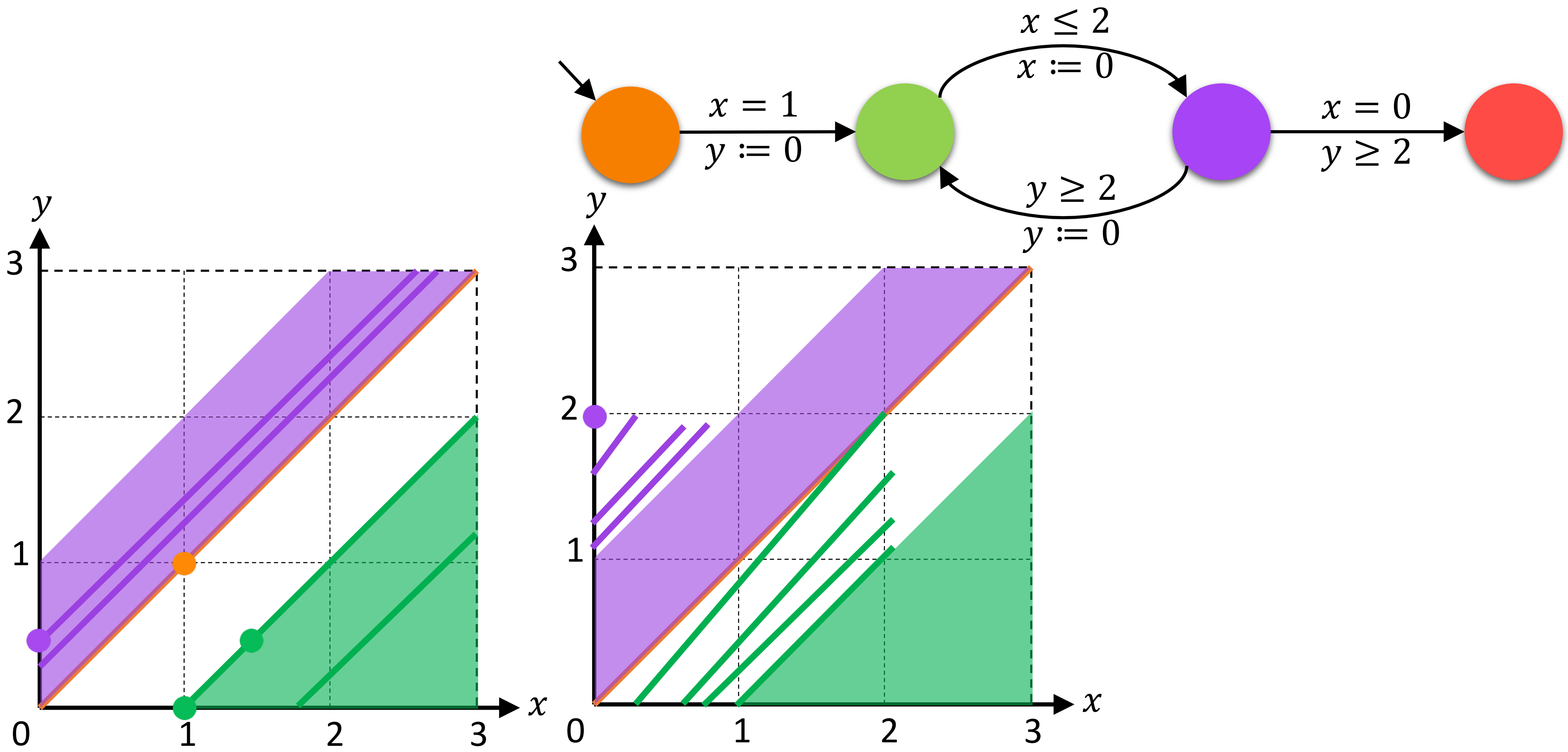

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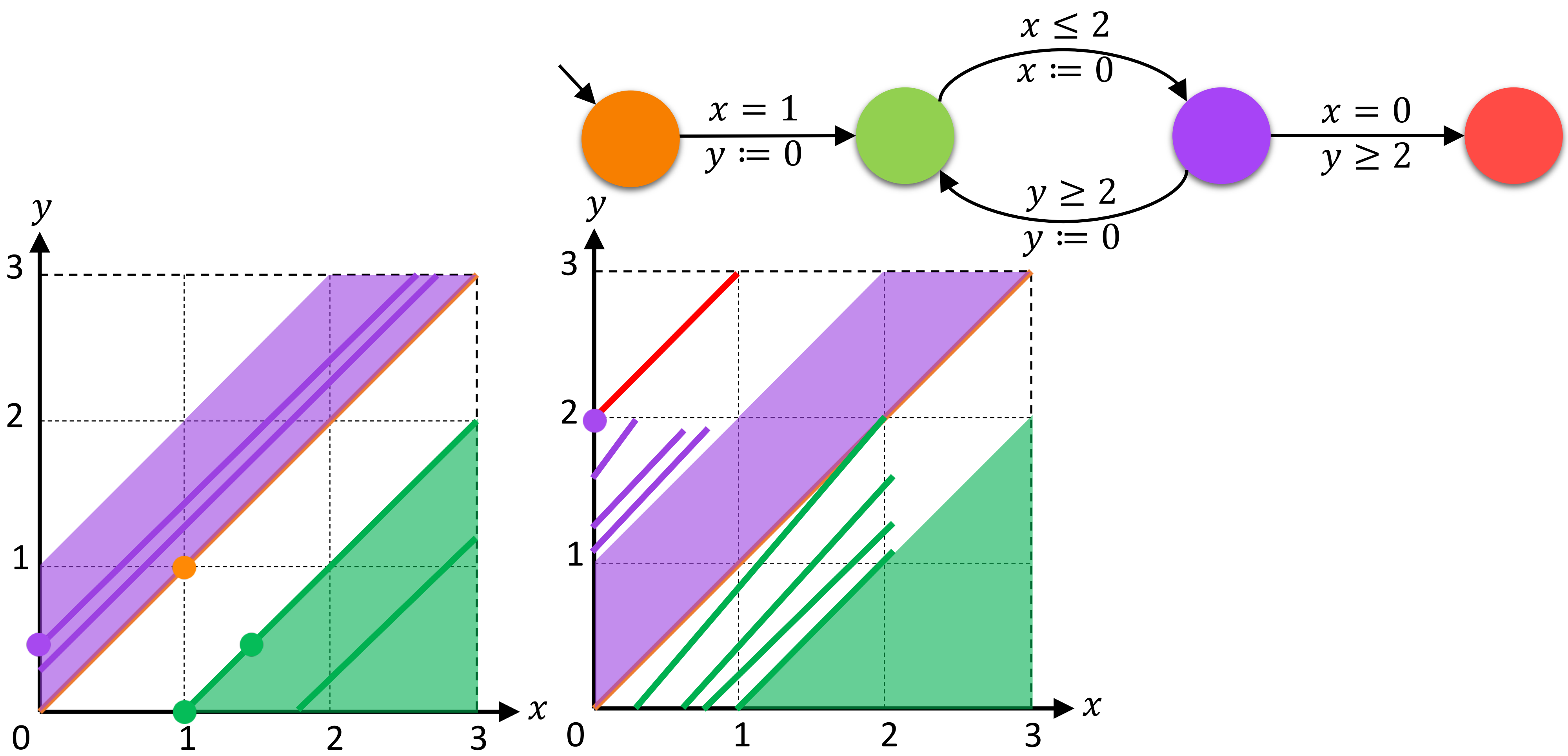

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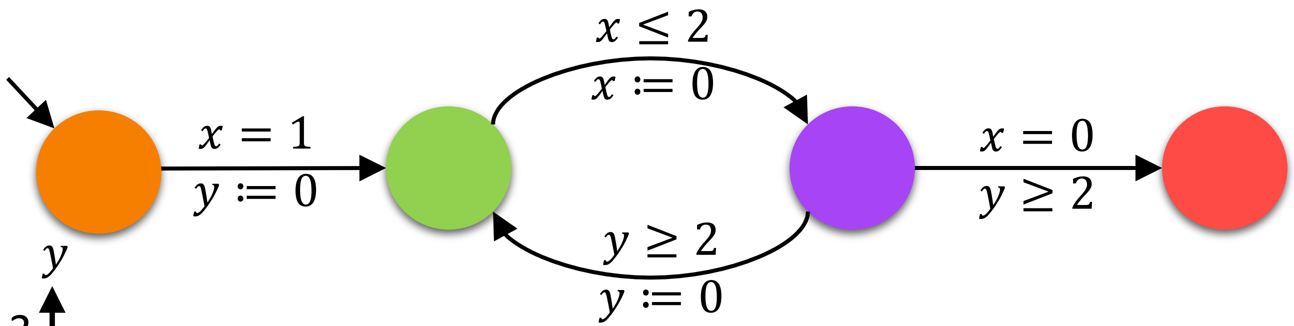

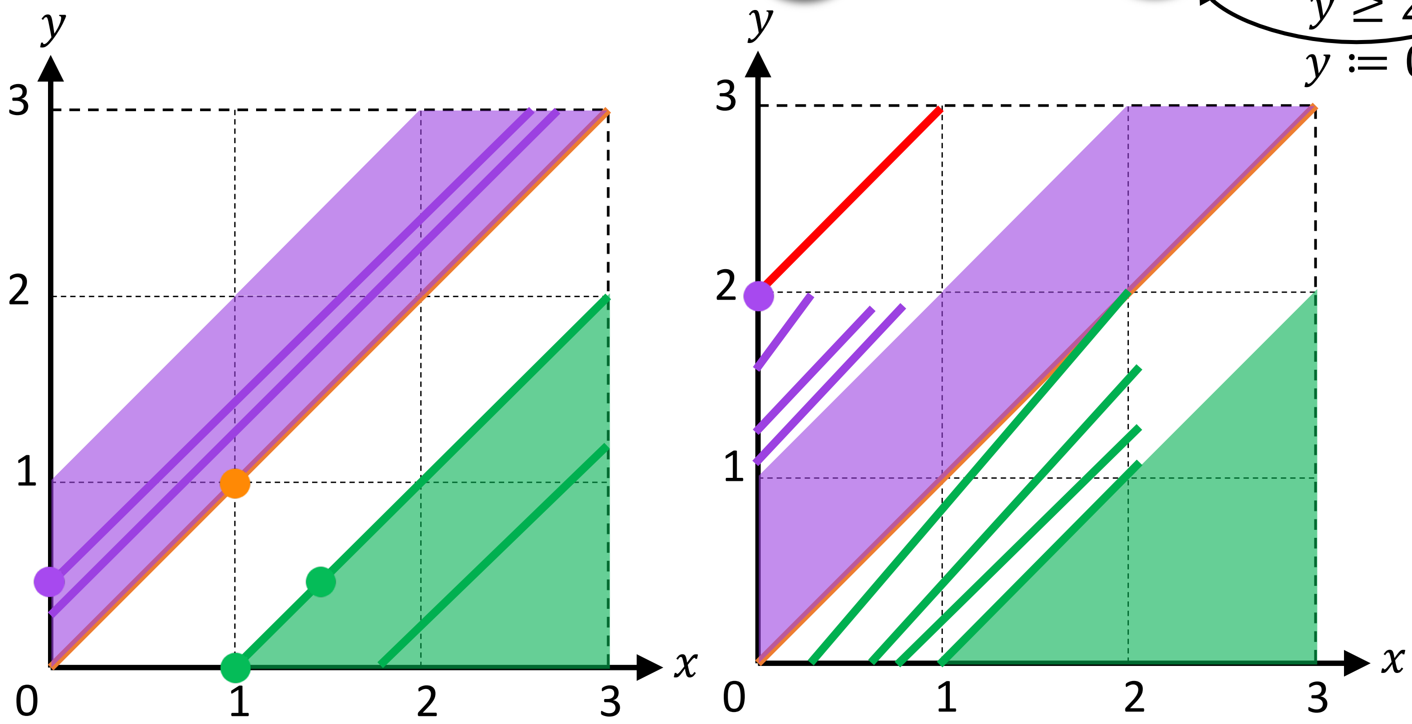

- Trajectory: an infinite sequence of continuous and discrete transition

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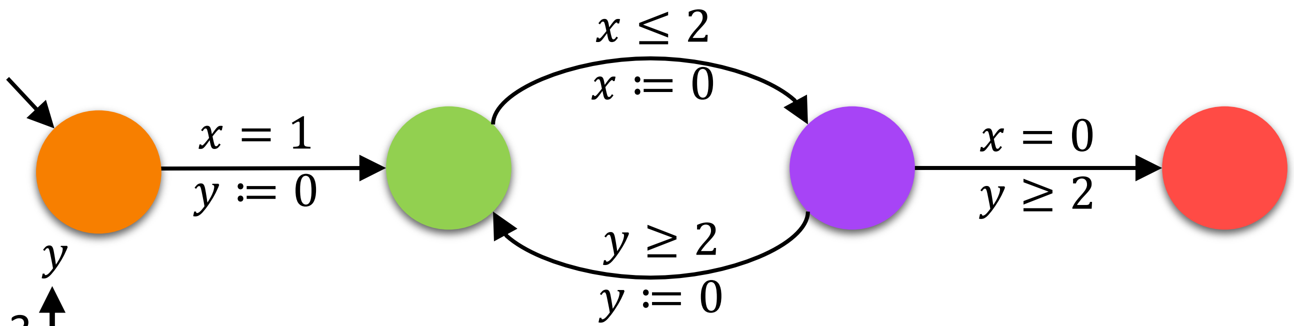
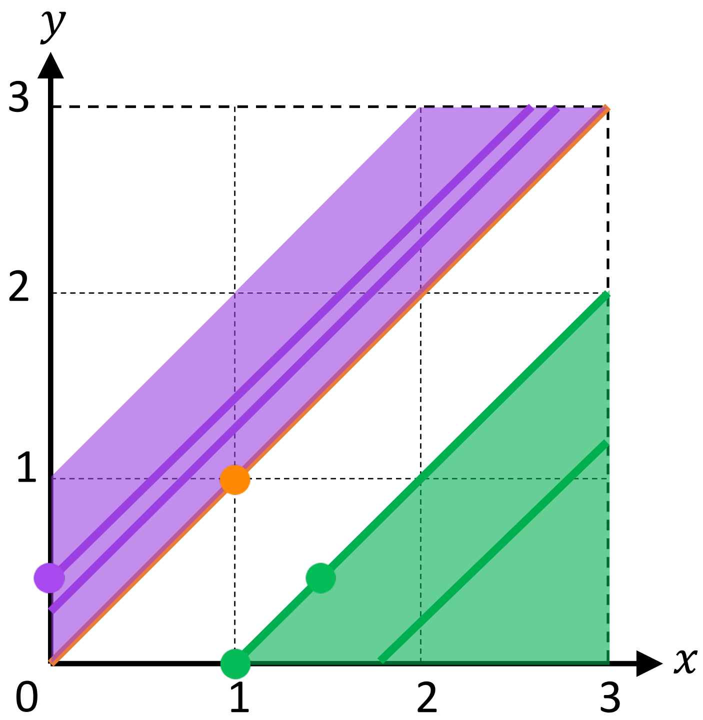
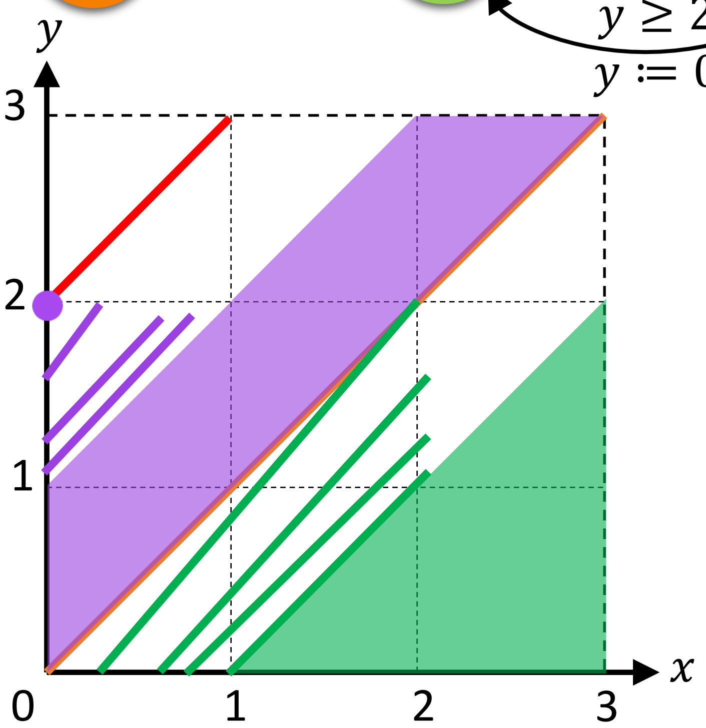


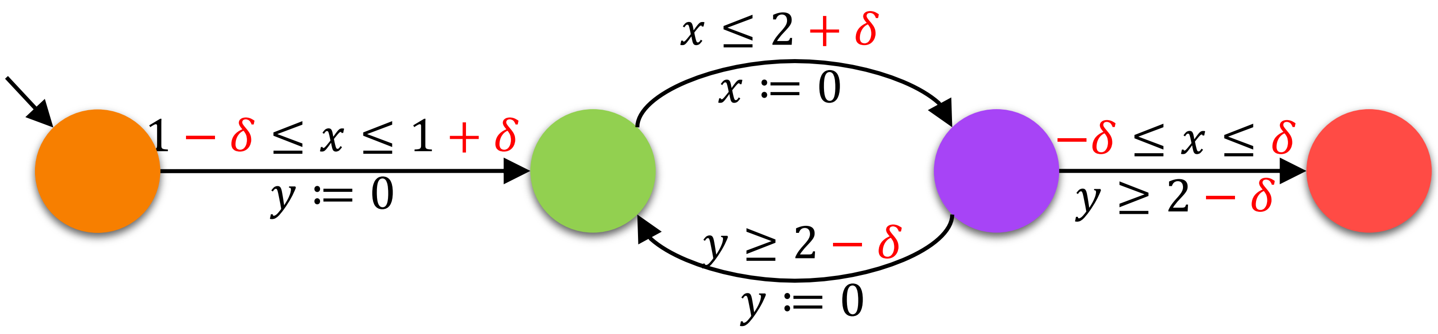




- Trajectory: an infinite sequence of continuous and discrete transition
- Execution: a trajectory that starts from the initial state
  - The set of executions $[\![\mathcal{T}]\!]$

Types of Perturbation

- Only guards are perturbed by δ
  - $[\![\mathcal{T}_\delta]\!]$



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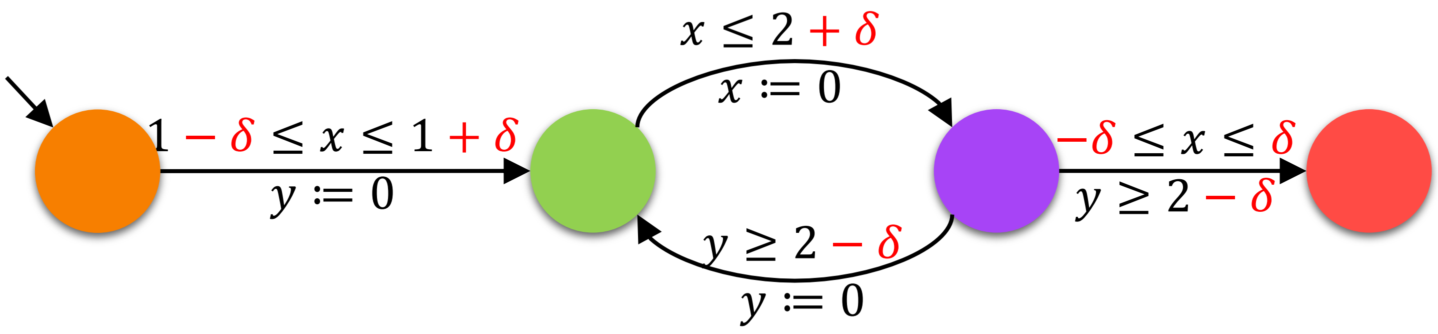

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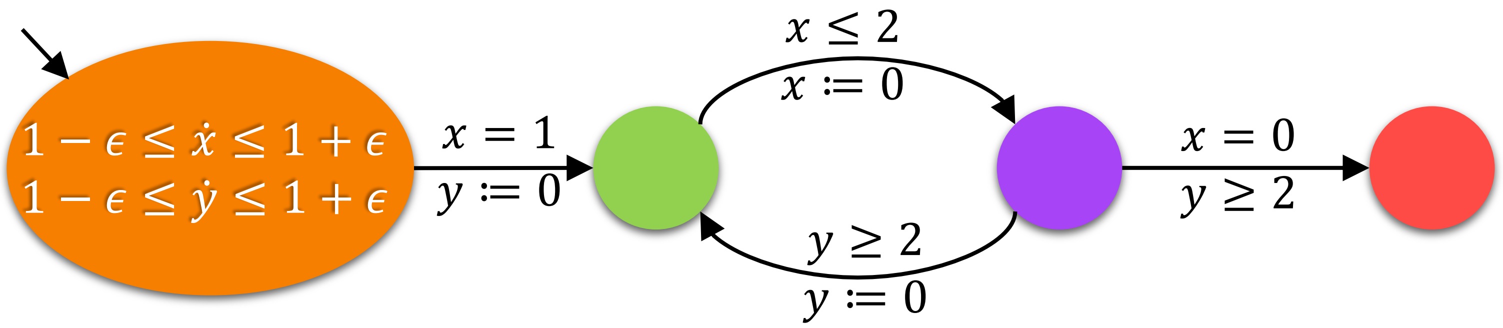

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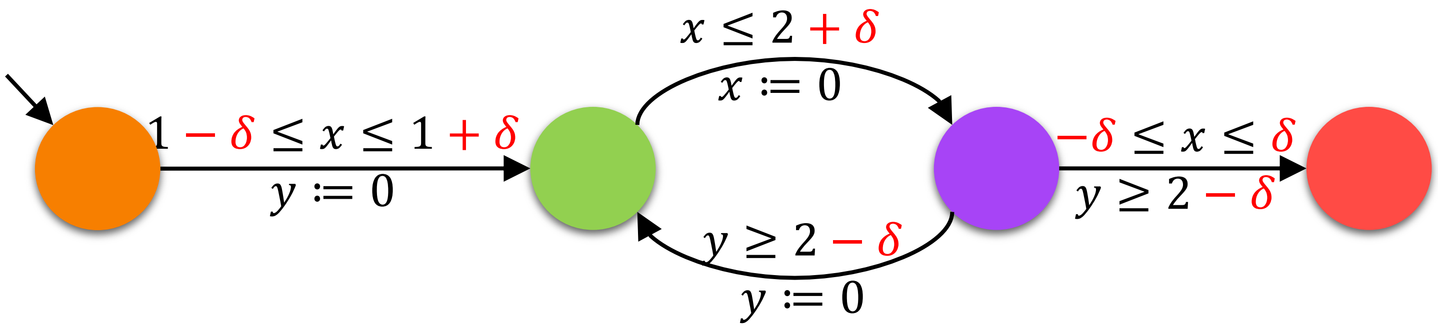

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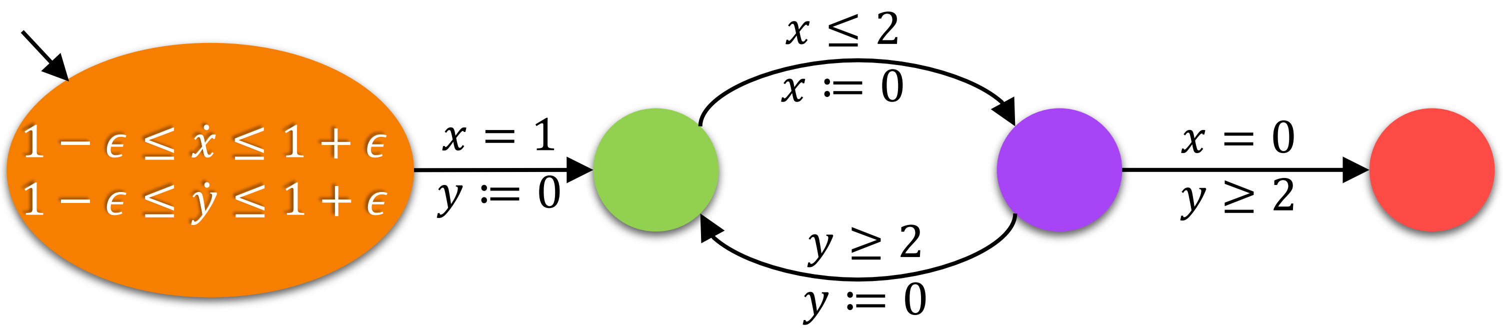

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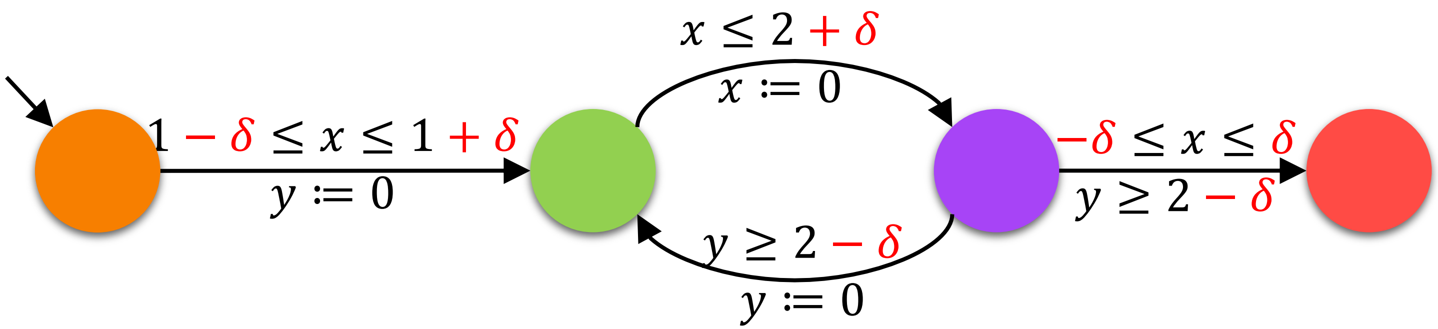

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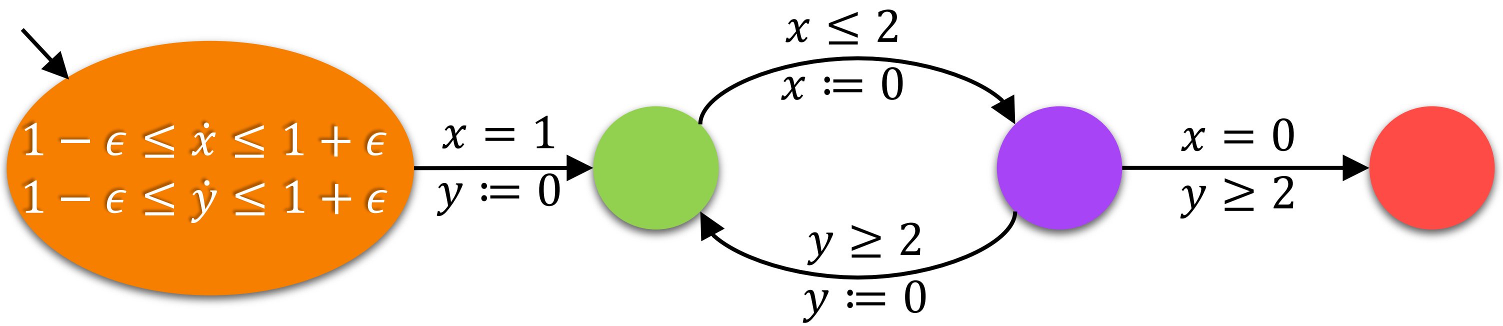

- Guards are perturbed by δ
  Clocks are perturbed by ϵ
  - $[\![\mathcal{T}_\delta^\epsilon]\!]$
- Only positive guards are perturbed by δ
  - $[\![\mathcal{T}_{+\delta}]\!]$

ω-Regular Properties

- We only consider Repeated Reachability $\Box\Diamond E$
  - Only to simplify presentation

$$\exists \epsilon : \mathbb{R}_+ \bullet \forall \tau : [\![\mathcal{T}^{\epsilon}]\!] \bullet \tau \vDash \Box\Diamond E$$

$$\exists \delta : \mathbb{R}_+ \bullet \forall \tau : [\![\mathcal{T}_{\delta}]\!] \bullet \tau \vDash \Box\Diamond E$$

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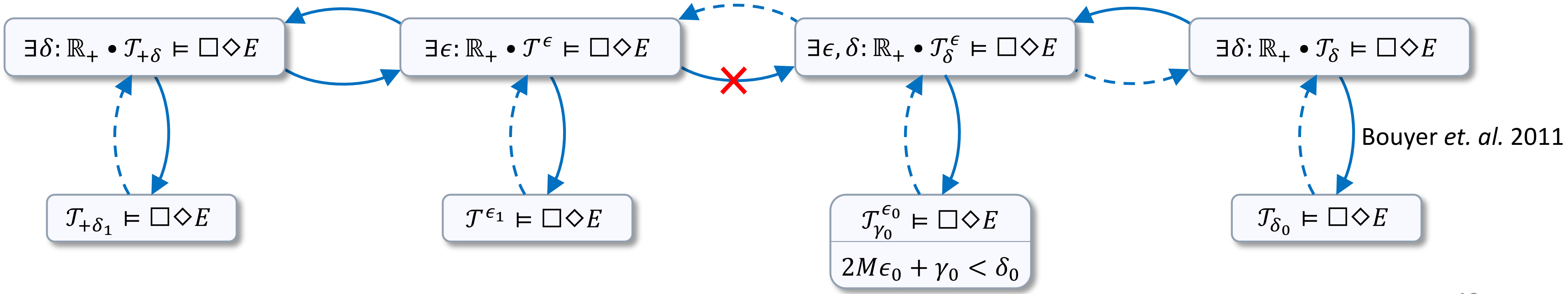
  - Proofs directly apply to Büchi Condition

ω-Regular Model Checking Results

- $\delta_0 \coloneqq \frac{1}{2}\left(5(W+1)|X|^3\left(2|Q|(|X|!)4^{|X|}+4\right)^2\right)^{-1}$
  - Only Exponentially Small
  - Adding one location makes δ_0 at most 12 times smaller
  - Independent of Number of Edges
- M is the maximum constant in $\mathcal{T}$
- $\delta_1 \coloneqq \frac{\delta_0}{24}$
- $\epsilon_1 \coloneqq \frac{\delta_1}{2\mathrm{M}}$



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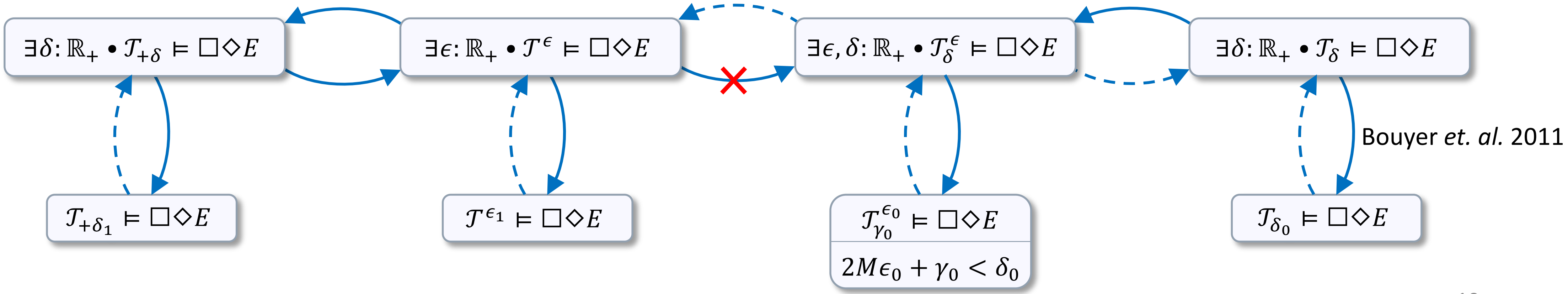
- All Problems are PSPACE-complete



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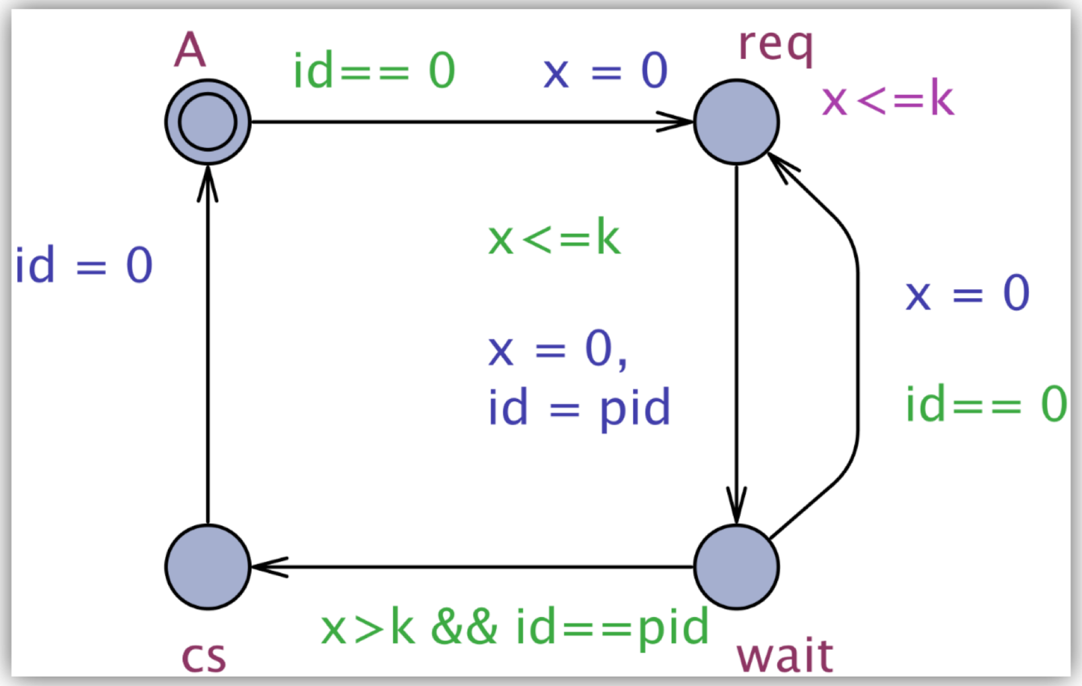

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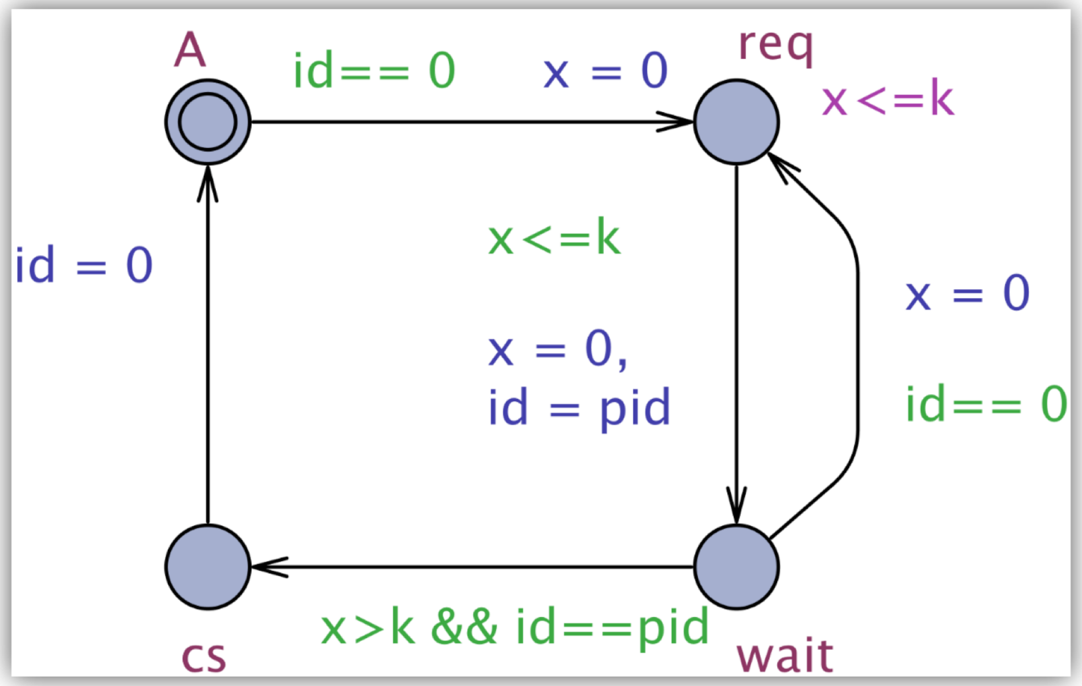

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- We tested it for 6 processes
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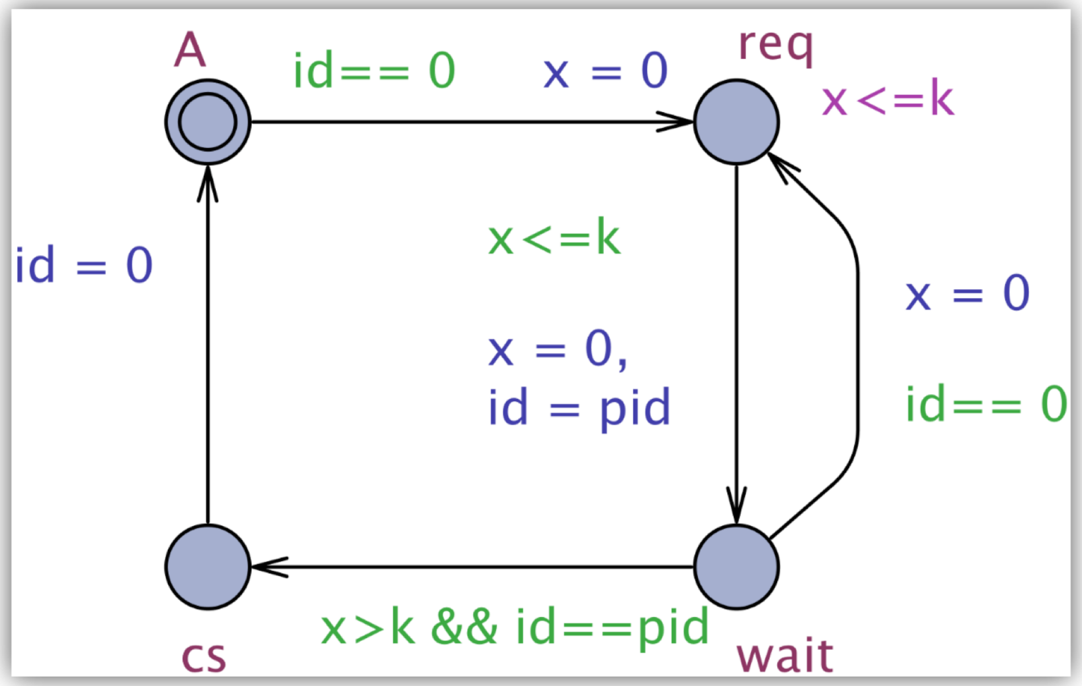
- $\mathcal{T}_{0.01}$ satisfies all these properties
  - Less than 2 seconds
- We conclude $\mathcal{T}_{\delta}^{\epsilon}$ does the same
  - For $\epsilon \coloneqq \frac{0.01}{12}$ and $\delta \coloneqq \frac{0.01}{2}$



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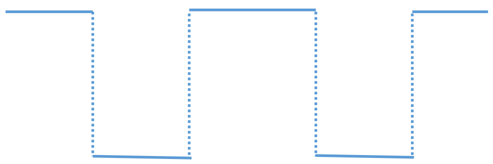

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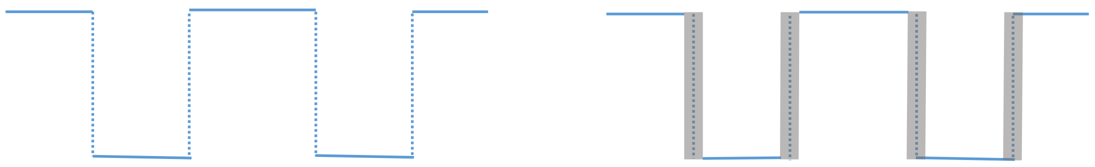
Similar to early robustness def.



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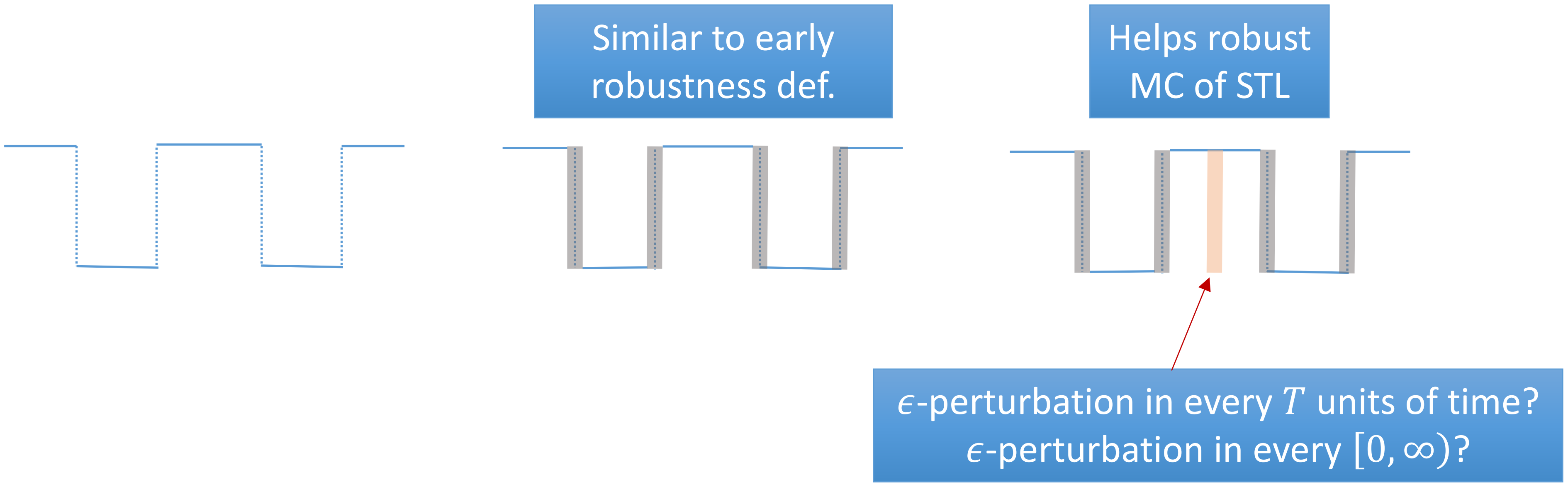

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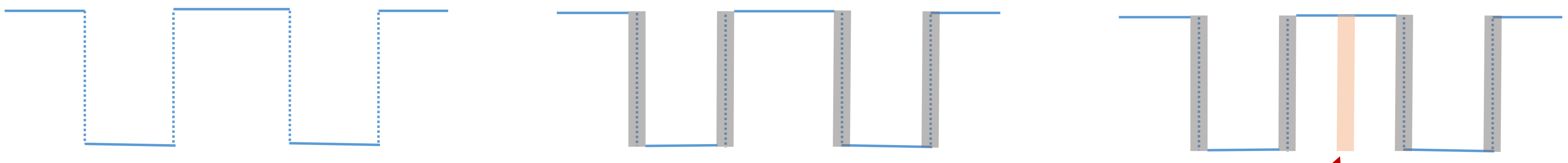

$\square_{\geq 0}(p \rightarrow \Diamond_{(0+\epsilon,3-\epsilon]} q)$

Robustness Helps

Statistical Verification of Hybrid Automata

HSCC 2015, 2017

ADHS 2015, 2018

CDC 2016

Temporal Properties about CTMC

- System is expressed using a Continuous Time Markov Chains
  - Rate matrix A is given
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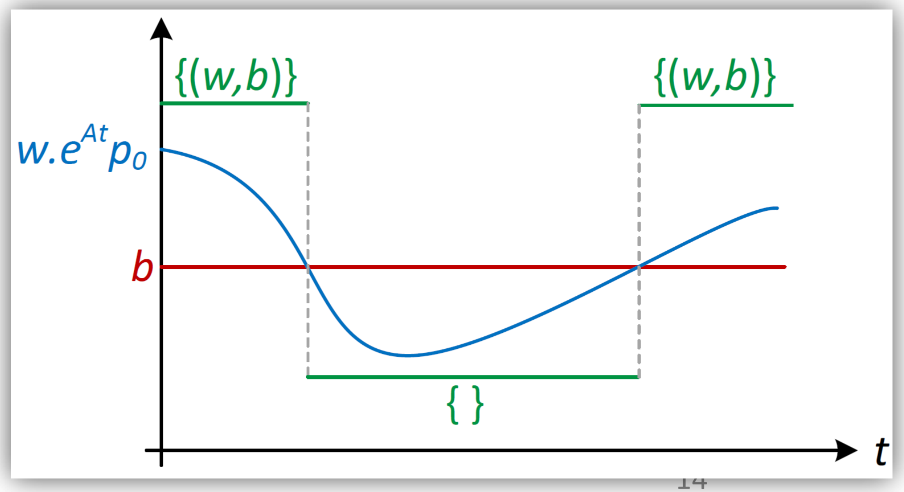

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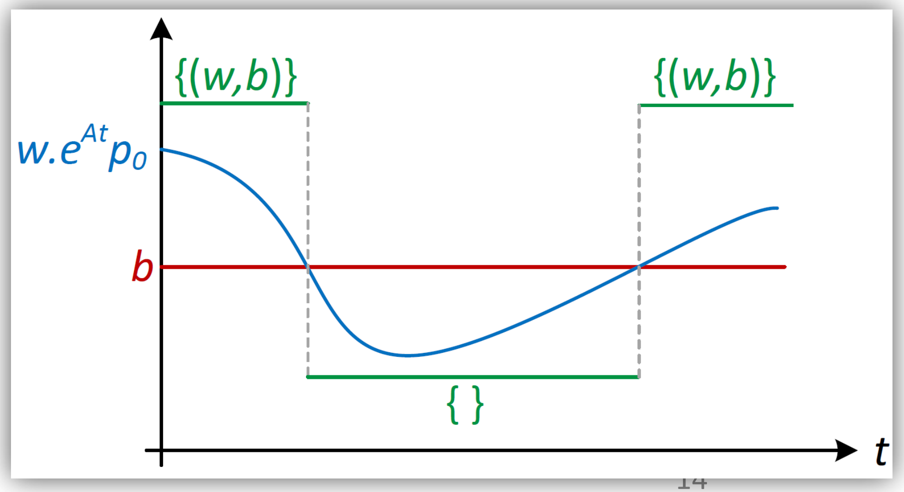
  - Atomic propositions are in the form of $w \cdot e^{At}p_0 \geq b$
- Deterministic behavior
  - Non-probabilistic
  - Unique signal



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- So problem is decidable
  - They use algebraic numbers
  - What is complexity of checking $\ln\frac{a}{b} \geq c$ when $a, b, c: \mathbb{N}_+$?
- To improve performance, we wanted to use statistical techniques
  - Simulate the system enough number of times
  - Provide some error guarantee

What can be guaranteed?

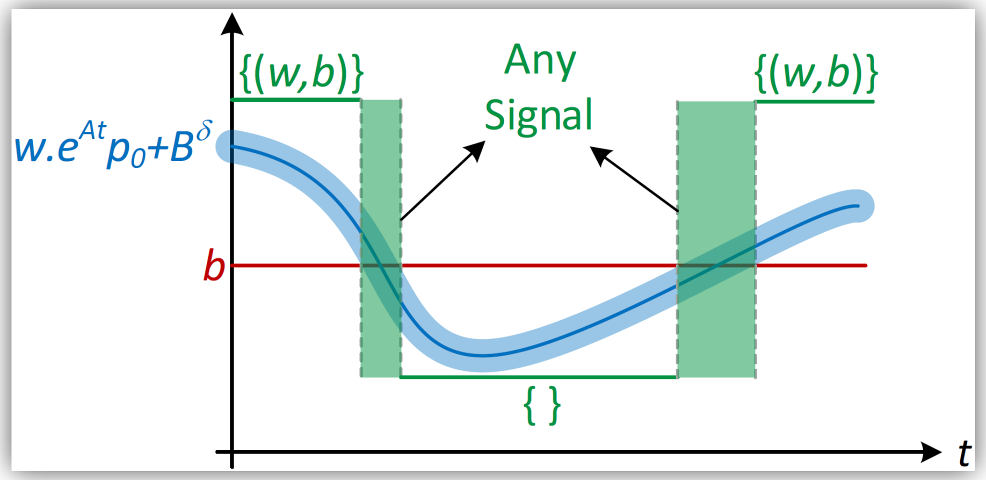
- Probability of returning wrong YES/NO is bounded

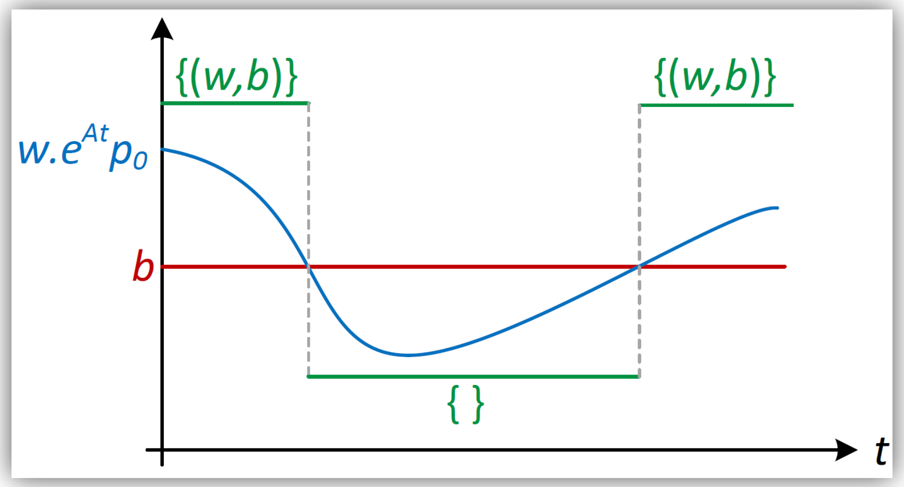
$$\mathbb{P}[res = \texttt{no} \mid C \models \phi] \leq \alpha$$

$$\mathbb{P}[res = \texttt{yes} \mid C \not\models \phi] \leq \alpha$$

- Probability of returning UNKNOWN is also bounded

$$\mathbb{P}[res = \texttt{unknown}] \leq \alpha + \beta$$





What is Next?

- When and how we can do this?
  - Verify deterministic (non-probabilistic) system using statistical techniques?
  - Much better performance
- What kind of robustness we need?

Abstraction Helps

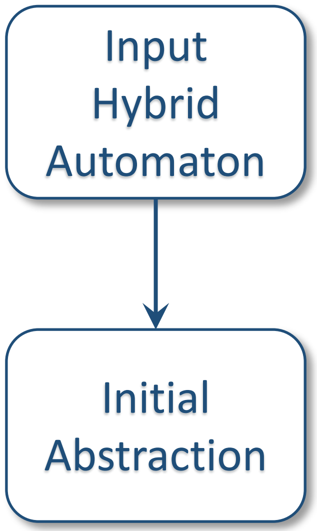
Reachability in Hybrid Automata

TACAS 2016-2017

CONCUR 2018

CEGAR Loop Edmund Clarke, 2000

- Simpler Differential Inclusions
- Abstraction
  - Finite vs. Infinite
  - Merging Locations Location
  - Removing Variables
  - Must over-approximate



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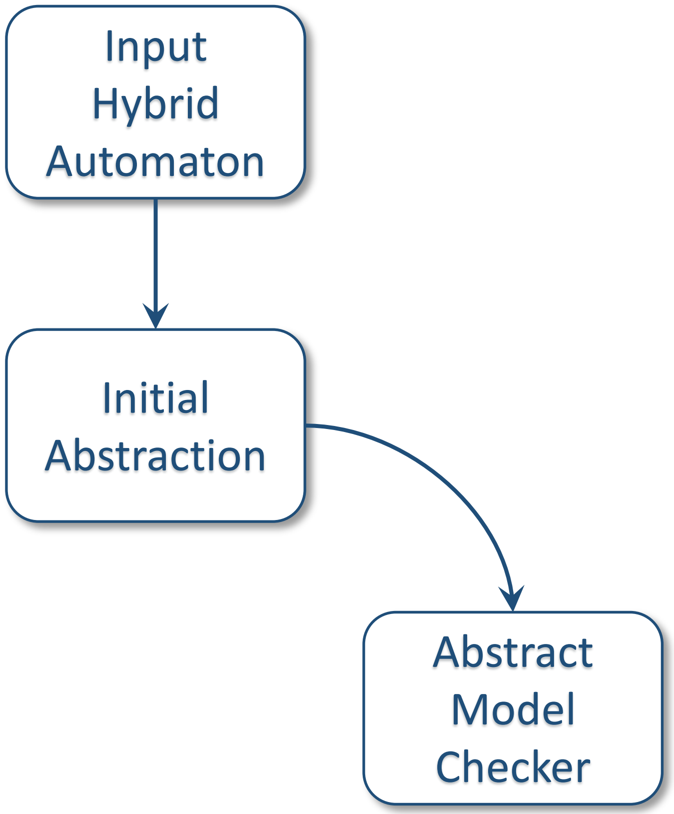

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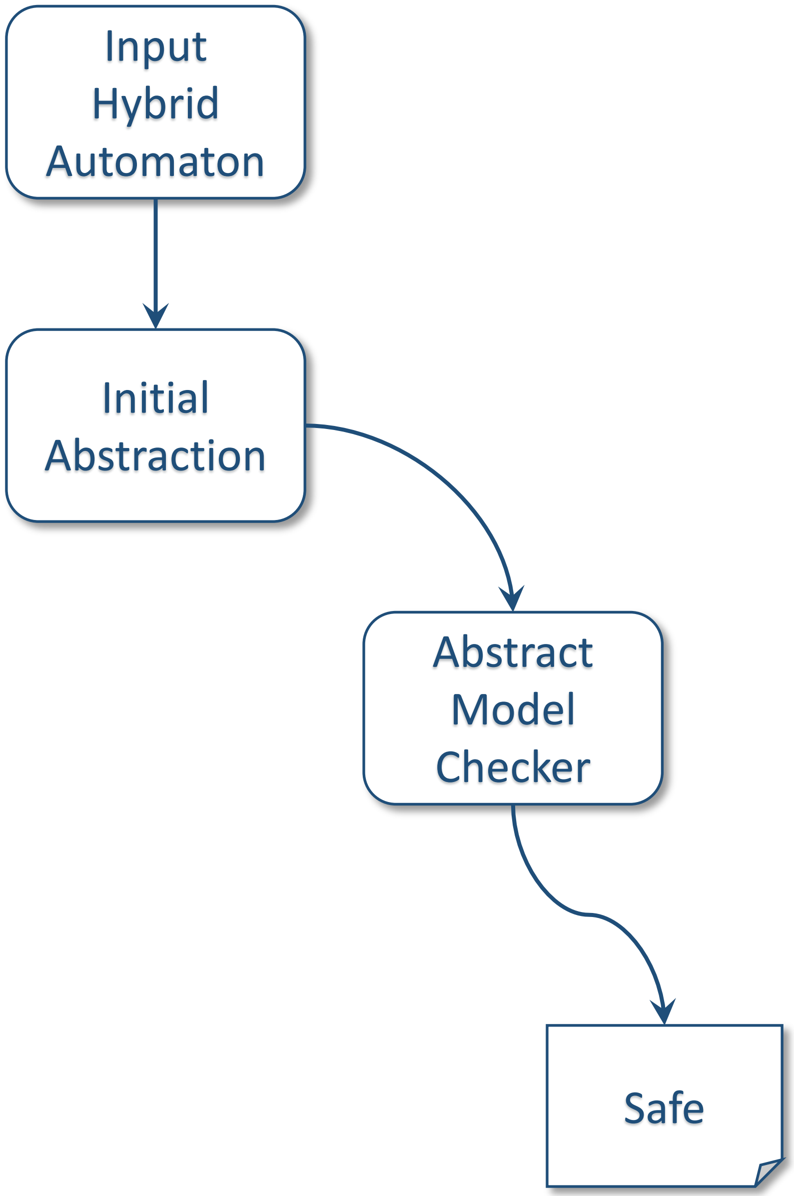

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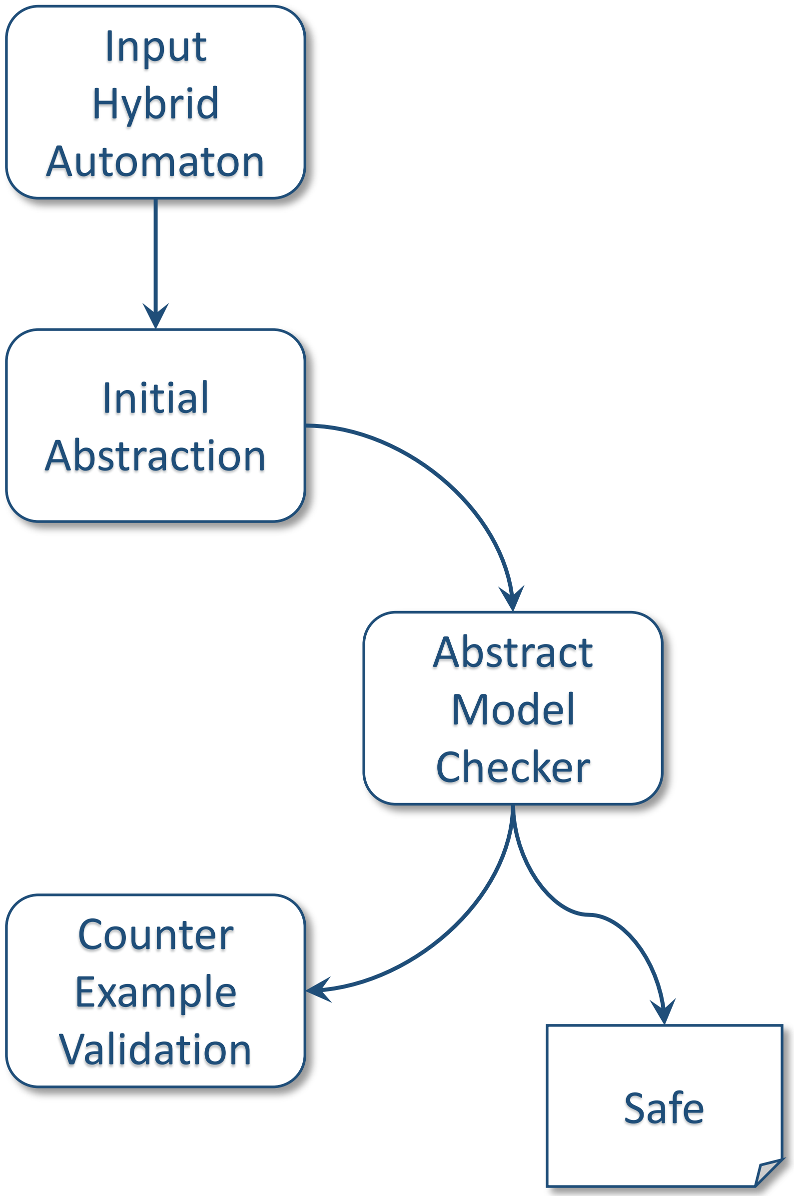

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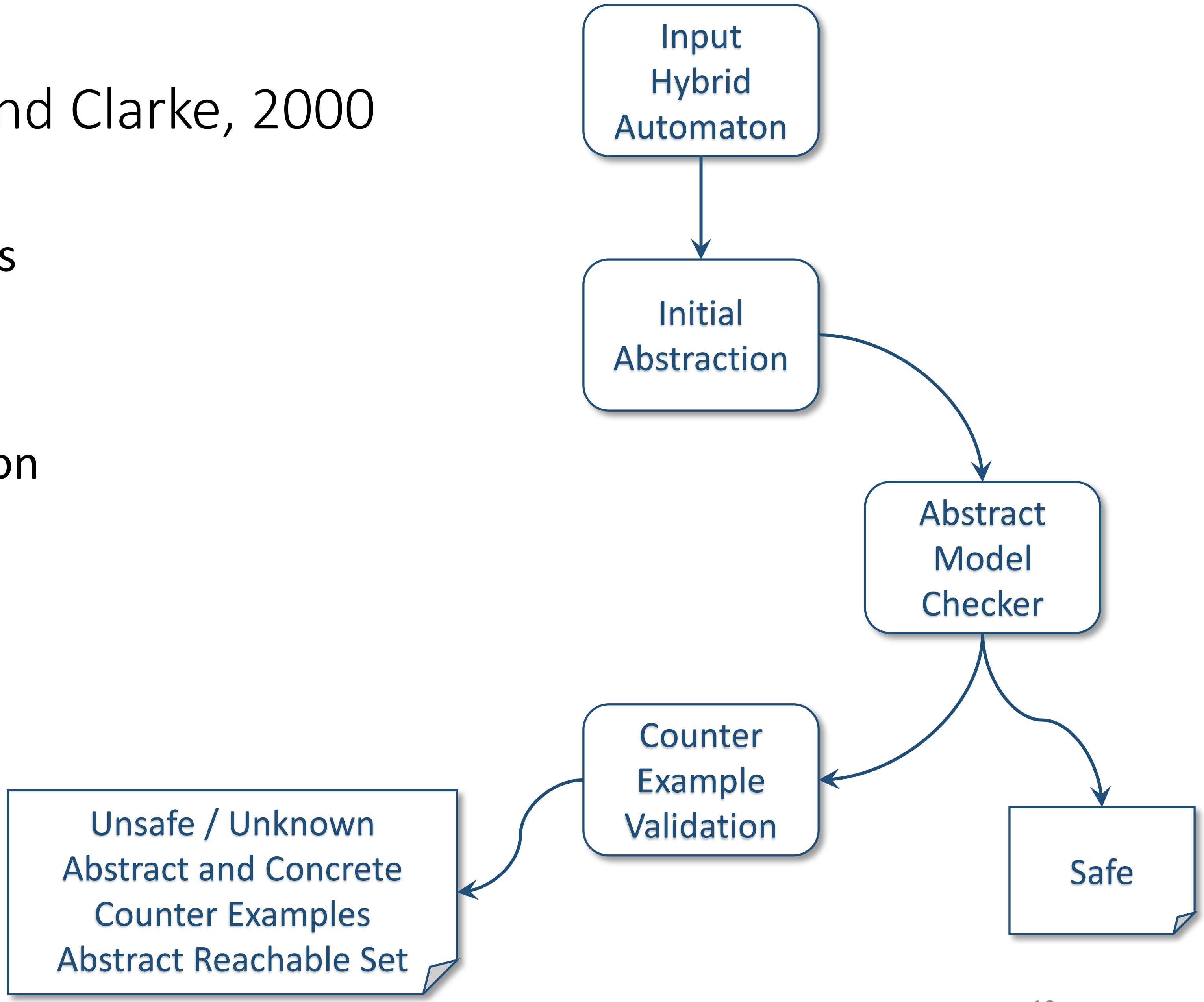

CEGAR Loop Edmund Clarke, 2000

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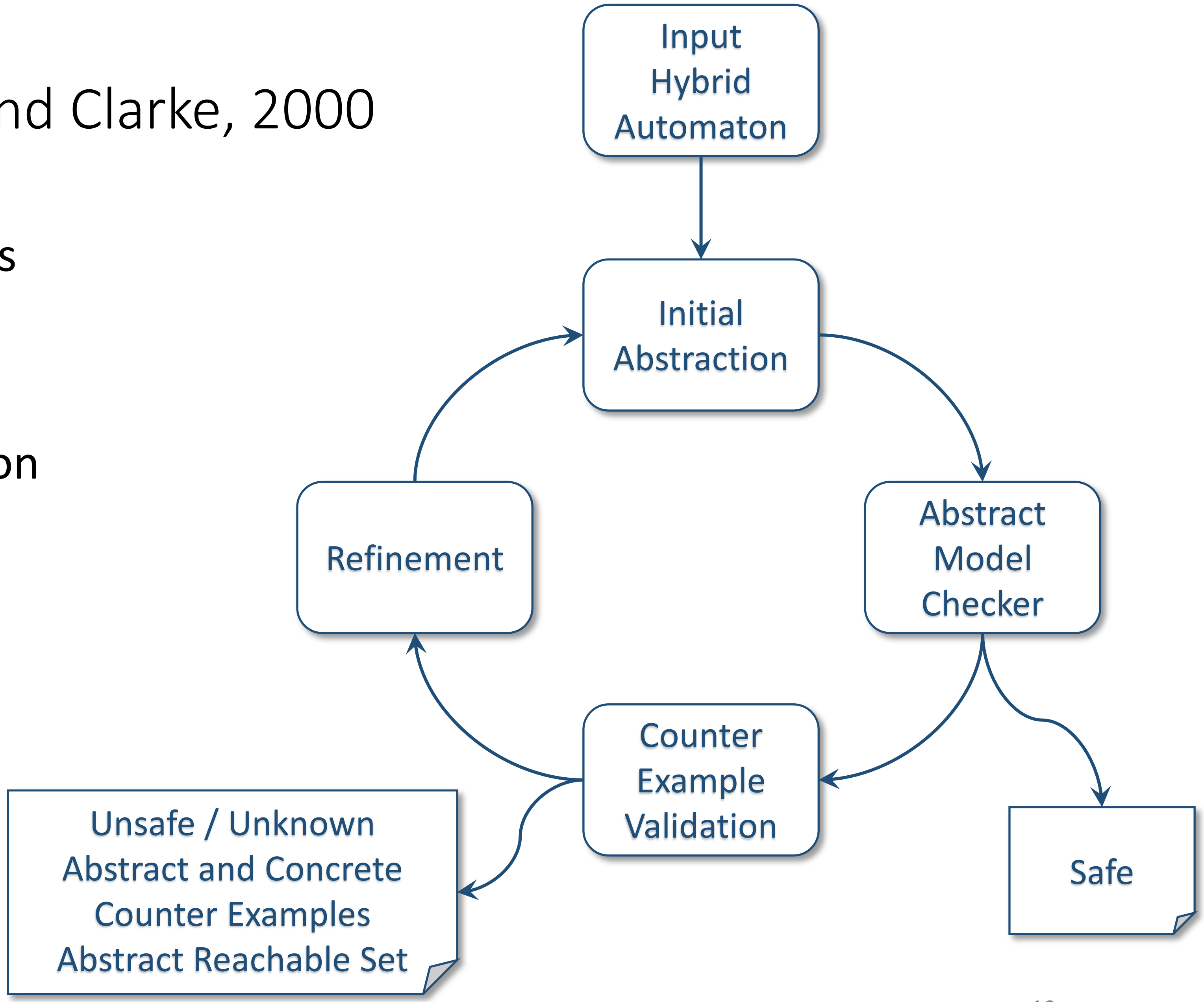
- What should be refined?



Experimental Results (affine dynamics)

- Constraints and continuous dynamics are specified using polyhedra

			HARE						SpaceEx			PHAVer			SpaceEx AGAR			
Model	Dim.	Size	Time		Iters.		Safe		Time	FP.	Safe	Time	FP.	Safe	Merged Locs.	Time	FP.	Safe
			old	new	old	new	old	new										
Tank 16	3	3 / 6	< 1	< 1	1	1	✓	✓	3	✗	✗	1414	✗	✓	2	1133	✗	✓
Tank 17	3	3 / 6	< 1	< 1	1	1	✓	✓	5	✗	✓	1309	✗	✓	2	1041	✗	✓
Satellite 03	4	64 / 198	91	< 1	1	1	✗	✓	< 1	✗	✗	1804	✗	✗	28	> 600	---	---
Satellite 04	4	100 / 307	< 1	< 1	1	1	✓	✓	< 1	✗	✓	< 1	✓	✓	91	49	✓	✓
Satellite 11	4	576 / 1735	1	< 1	1	1	✓	✓	< 1	✗	✓	< 1	✓	✓	449	> 600	---	---
Satellite 15	4	1296 / 3895	2	< 1	1	1	✓	✓	< 1	✗	✓	< 1	✓	✓	264	> 600	---	---
Heater 03	3	4 / 6	> 600	54	---	1	---	✓	84	✗	✓	< 1	✓	✗	---	---	---	---
Heater 05	3	4 / 6	< 1	58	1	38	✗	✓	61	✗	✓	< 1	✓	✗	---	---	---	---
Heater 09	3	4 / 6	< 1	80	1	15	✗	✓	42	✗	✗	< 1	✓	✗	---	---	---	---
Nav 01	4	25 / 80	9	18	11	11	✓	✓	< 1	✓	✓	< 1	✓	✓	21	5	✓	✓
Nav 08	4	16 / 48	7	< 1	13	1	✓	✓	685	✗	✓	< 1	✓	✓	10	< 1	✓	✓
Nav 09	4	16 / 48	7	< 1	10	1	✓	✓	< 1	✗	✗	< 1	✓	✗	4	< 1	✓	✗
Nav 13	4	9 / 18	8	< 1	15	1	✓	✓	< 1	✗	✓	< 1	✓	✓	4	< 1	✓	✓
Nav 19	4	33 / 97	29	< 1	17	1	✓	✓	2	✗	✓	< 1	✓	✓	11	< 1	✓	✓

Experimental Results (non-linear dynamics)

- Constraints are specified using polyhedra
- Continuous dynamics are specified using (non-linear) ODEs
  - Whatever can be supported by dReach

			HARE			C2E2	HSolver	FLOW*
Model	Dim.	Size	Reached Abst. Size	Time Bound	Time	Time	Time	Time
Van der Pol	2	1 / 0	26 / 194	∞	< 1	56	3*	> 600
Jet Engine	2	1 / 0	189 / 1330	∞	55	56	2*	> 600
Cardiac Cell	2	2 / 2	249 / 1783	∞	16	50	< 1*	25
Cardiac Control	3	2 / 2	270 / 3974	∞	153	> 600	> 600*	41
Clock	3	1 / 0	9 / 56	∞	< 1	---	< 1	< 1
Sinusoid	2	1 / 0	32 / 62	10	< 1	1	7	---

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- Assume every constraint uses non-strict inequality

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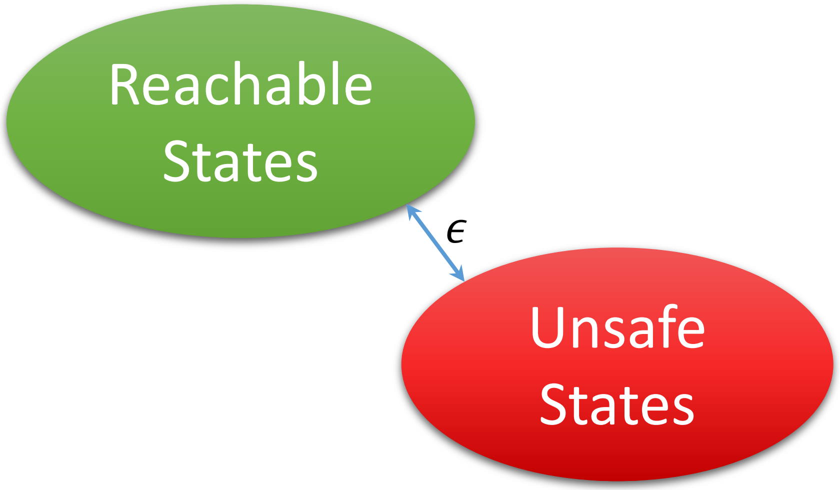

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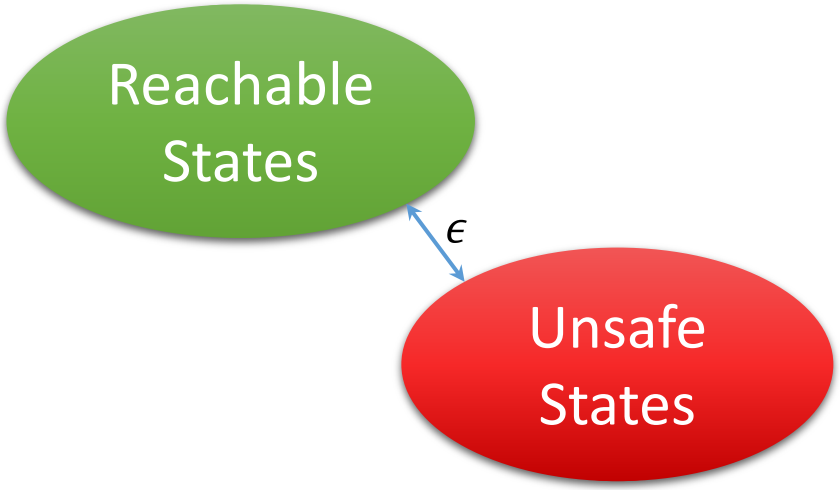

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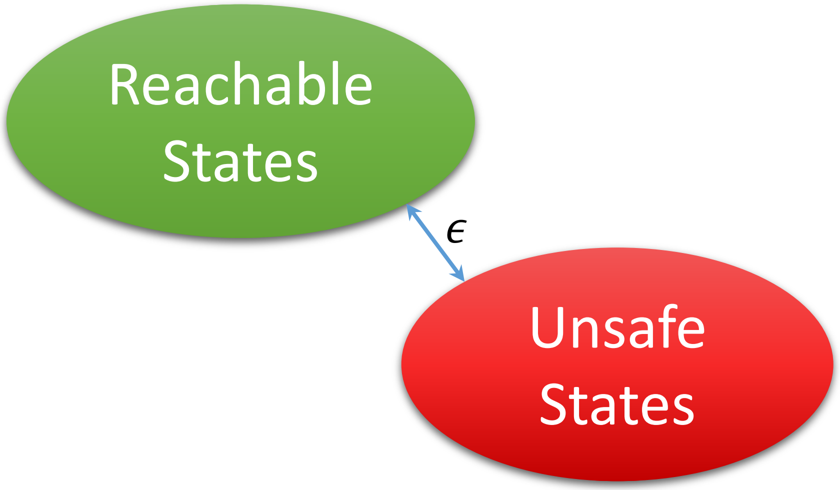

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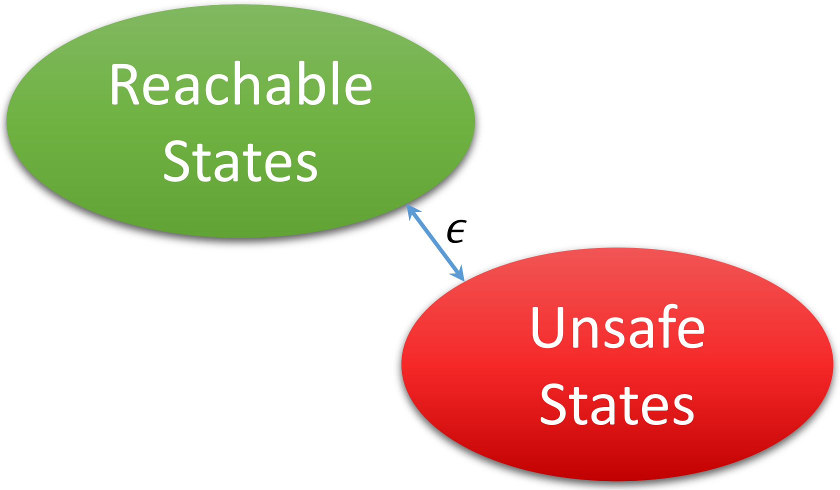

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- dReach uses dReal

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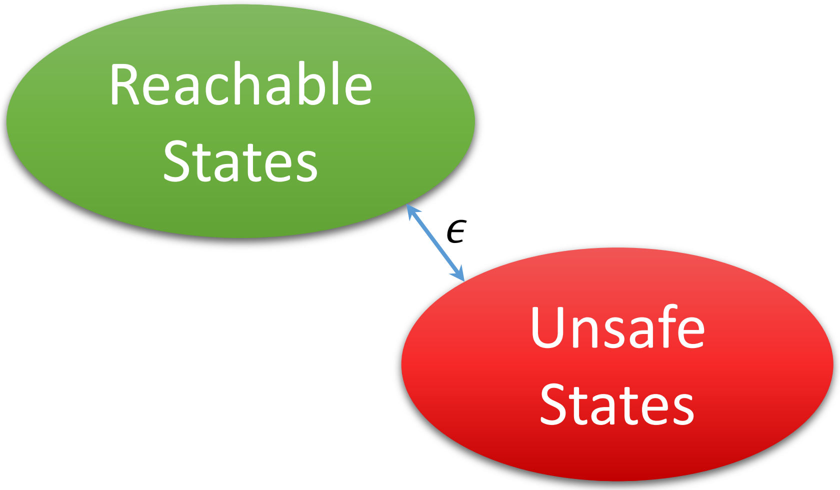

- We use dReach
- dReach uses dReal
- dReal perturbs syntax of formulas
  - UNSAT: the system is safe (spurious counter-example)
  - δ-SAT: the perturbed system is unsafe

What can be guaranteed?

- What is the relation between syntactic and semantic perturbations/robustness?
  - Can they become arbitrary close?
  - Syntactic perturbation is used to deal with computational complexity
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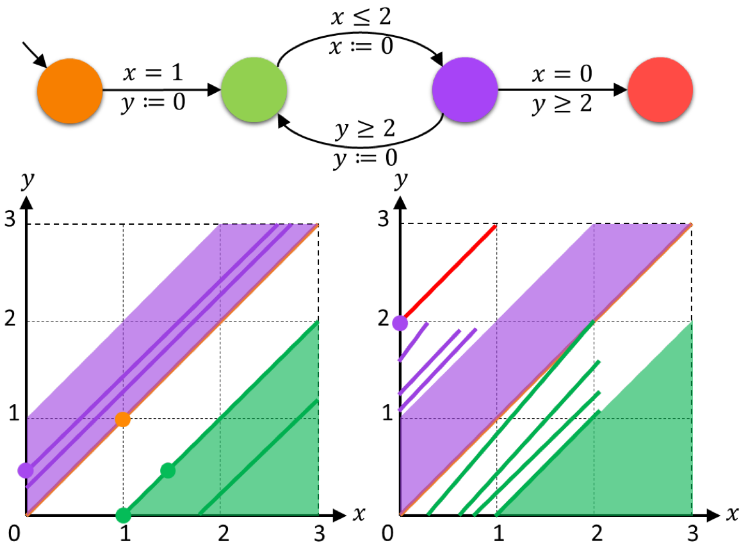

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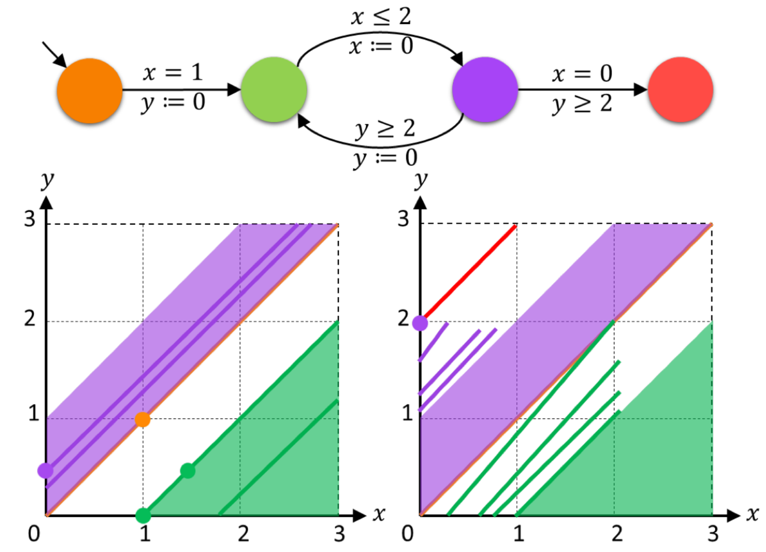
- We proved bounded ϵ-Simulation is possible

$$\forall \epsilon \in \mathbb{R}_+, k \in \mathbb{N} \bullet \exists \delta \in \mathbb{R}_+ \bullet \mathcal{H}^\delta \preceq_k^\epsilon \mathcal{H} \wedge \mathcal{H} \preceq_k^\epsilon \mathcal{H}^\delta$$

- Bisimulation is impossible



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- Find δ for the given ϵ
  - Anything more expressive than Timed Automata

Thank You