

Definition MunkTop.13.1: \mathcal{B} is a *basis* for a topology on X if and only if $\mathcal{B} \subseteq \wp(X)$ and for every $x \in X$, there exists $B \in \mathcal{B}$ such that $x \in B$ and for every $x \in X$, for every $B_1, B_2 \in \mathcal{B}$, if $x \in B_1 \cap B_2$ then there exists $B_3 \in \mathcal{B}$ such that $x \in B_3$ and $B_3 \subseteq B_1 \cap B_2$.

Definition MunkTop.13.2: If \mathcal{B} is a basis for a topology on X then *the topology on X generated by \mathcal{B}* is the unique $\mathcal{T} \subseteq \wp(X)$ such that for every $U \subseteq X$, $U \in \mathcal{T}$ if and only if for every $x \in U$, there exists $B \in \mathcal{B}$ such that $x \in B$ and $B \subseteq U$.

Definition MunkTop.13.3.a.basis: *The standard basis for a topology on \mathbb{R}* is the set of $U \subseteq \mathbb{R}$ such that there exist $a, b \in \mathbb{R}$ such that $U = \{x \in \mathbb{R} : a < x < b\}$.

Definition MunkTop.13.3.a: *The standard topology on \mathbb{R}* is the topology on \mathbb{R} generated by the standard basis for a topology on \mathbb{R} .

Definition MunkTop.13.3.b: *The lower-limit topology on \mathbb{R}* is the topology on \mathbb{R} generated by the set of $U \subseteq \mathbb{R}$ such that there exist $a, b \in \mathbb{R}$ such that $U = \{x \in \mathbb{R} : a \leq x < b\}$.

Definition MunkTop.13.3.c: *The K -topology on \mathbb{R}* is the topology on \mathbb{R} generated by the standard basis for a topology on \mathbb{R} union the set of $V \subseteq \mathbb{R}$ such that there exists W in the standard basis for a topology on \mathbb{R} such that $V = W \setminus \{1/n : n \in \mathbb{N}\}$.

Definition MunkTop.13.4.a: \mathcal{S} is a *subbasis* for a topology on X if and only if $\mathcal{S} \subseteq \wp(X)$ and $\cup \mathcal{S} = X$.

Definition MunkTop.13.4.b: *The topology on X generated by the subbasis \mathcal{S}* is the set of $U \subseteq X$ such that there exists $\mathcal{A} \subseteq \wp(\mathcal{S})$ such that for every $A \in \mathcal{A}$, A is finite and $U = \cup \{\cap S : S \in A\}$.