Good Data and Bad Data: The Welfare Effects of Price Discrimination

Maryam Farboodi Nima Haghpanah Ali Shourideh

MIT, Yale, CMU

ESSET July 2025

Effects of Price Discrimination

- Classic question: how does third-degree price discrimination affect payoffs of buyers and sellers?
    - Much more relevant with the rise of personalized pricing and availability of data

Effects of Price Discrimination

- Classic question: how does third-degree price discrimination affect payoffs of buyers and sellers?
  - Much more relevant with the rise of personalized pricing and availability of data
- Common folk wisdom: personalized pricing hurts consumers
  - Also: Pigou (1920), Joan Robinson (1969), etc.

Effects of Price Discrimination

- Classic question: how does third-degree price discrimination affect payoffs of buyers and sellers?
  - Much more relevant with the rise of personalized pricing and availability of data
- Common folk wisdom: personalized pricing hurts consumers
  - Also: Pigou (1920), Joan Robinson (1969), etc.
- Bergemann, Brooks and Morris (2015) or BBM:
  - It can go either way
  - Every rationalizable CS-PS pair is feasible!

A Modified Version of Pigou's Logic

- Two types of consumers

$$D(p, \theta_1) = \theta_1 - p < \theta_2 - p = D(p, \theta_2)$$

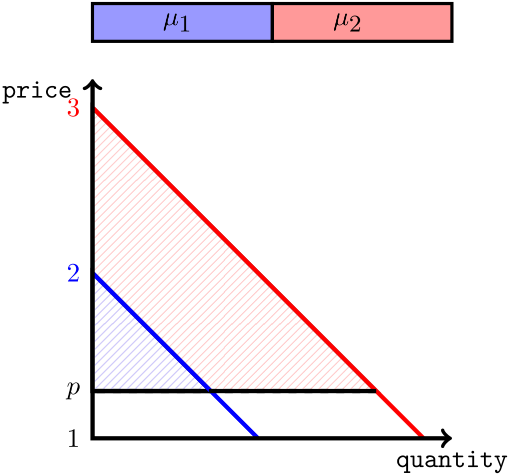
- Any market segment with distribution: $\mu_1 + \mu_2 = 1$
- Optimal price:

$$p = \frac{\mu_1 \theta_1 + \mu_2 \theta_2}{2}$$

- Total quantity

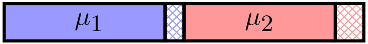
$$Q = \frac{\mathbb{E}\theta}{2}$$

Pigou's Logic: Example

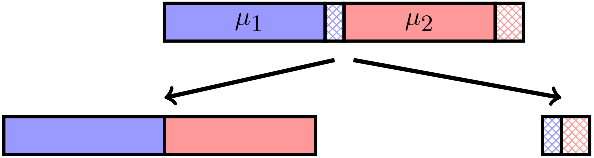


$$\texttt{CS} = \frac{3}{4}\texttt{var}[\theta] + \frac{1}{4}\mathbb{E}[\theta^2]$$

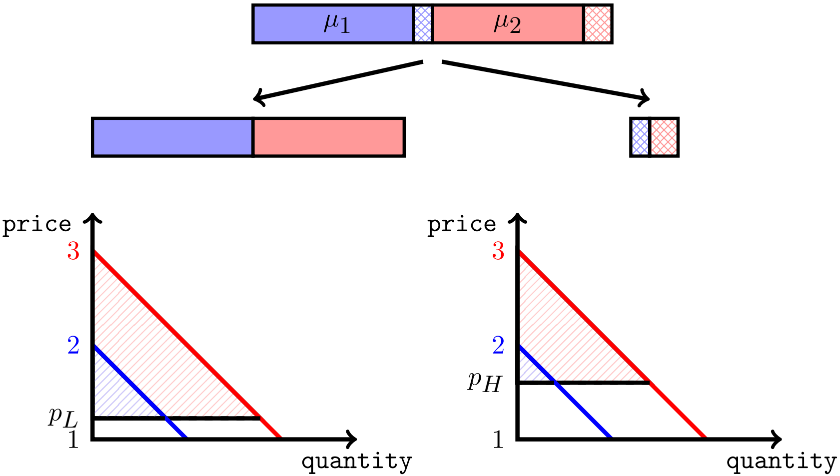
Pigou's Logic: Example



Pigou's Logic: Example



Pigou's Logic: Example



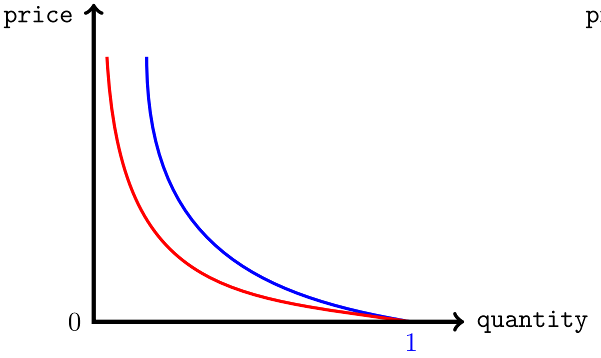
$\texttt{CS}_L = \frac{3}{4}\texttt{var}_L[\theta] + \frac{1}{4}\mathbb{E}_L[\theta^2]$ $\qquad$ $\texttt{CS}_H = \frac{3}{4}\texttt{var}_H[\theta] + \frac{1}{4}\mathbb{E}_H[\theta^2]$

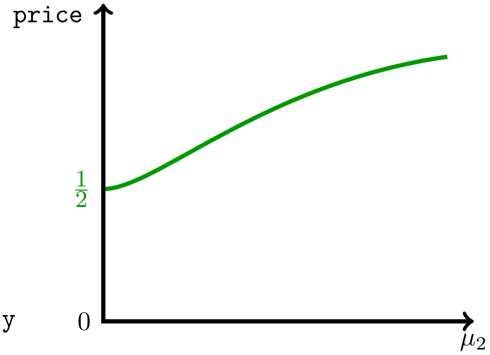
Pigou's Logic

- Finer segmentation: two effects
    1. Market size effect: depending on how the seller reacts, total quantity can go up or down
        1.1 For linear demand family, it does not change
    2. Misallocation effect: finer segmentation decreases CS – inefficiency of delivering fixed quantity via multiple markets
- Linear demand: only misallocation effect

Piguo's Logic is more General

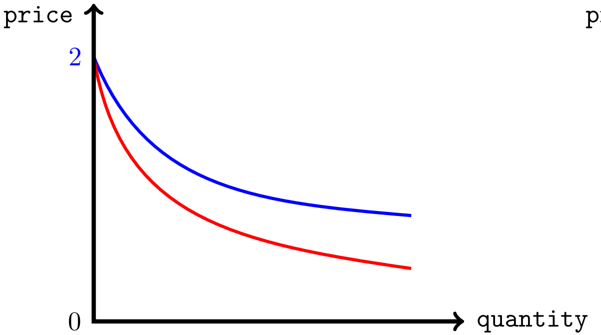
- Another example: $D\left(p,\theta_1\right)=\left(1+p\right)^{-3}, D\left(p,\theta_2\right)=\left(1+p\right)^{-2}$
  - Price is almost linear $\Rightarrow$ Misallocation effect dominates

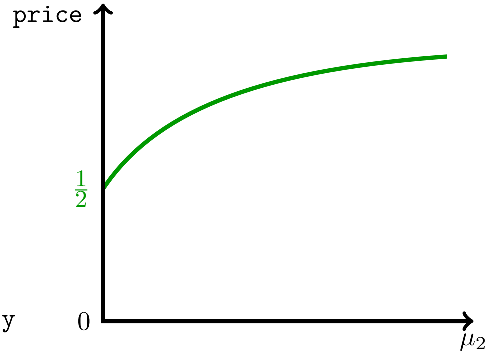




Good Data!

- Another example: $D(p,\theta_1) = 1 - p + \frac{2}{p}, D(p,\theta_2) = 3 - 2p + \frac{2}{p}$
  - Price is concave in prior $\Rightarrow$ Quantity effect dominates





Our Paper

- Question: when is it that any coarsening is good? In other words, how general is Pigou's logic?
- Useful determinant for outright bans of personalized pricing
  - cannot really verify who knows what
- Alternative: banning use of specific type of data
  - standard information design
  - not this paper: but related

Two Main Results

- Key concept: IMB (or IMG): $\alpha \cdot CS + (1-\alpha) \cdot PS$

`Information is Monotonically Bad!  (or Good!)`

Two Main Results

- Key concept: IMB (or IMG): $\alpha \cdot CS + (1-\alpha) \cdot PS$

`Information is Monotonically Bad!  (or Good!)`

- `Theorem 1:` Full characterization of demand systems (class of demand functions) where more data is always bad (or always good):
  - No exclusion, i.e., all types are active
  - demand functions are generated by linear combinations of two demands

Two Main Results

- Key concept: IMB (or IMG): $\alpha \cdot CS + (1 - \alpha) \cdot PS$

`Information is Monotonically Bad!  (or Good!)`

- `Theorem 1:` Full characterization of demand systems (class of demand functions) where more data is always bad (or always good):
  - No exclusion, i.e., all types are active
  - demand functions are generated by linear combinations of two demands
- `Theorem 2:` Provide bounds for any demand system for loss/gain from more data.
  - Key idea:
    - rank of Hessian of the (KG) value function ≤ 2
    - eigenvalues!

Related Literature

- 2–demand models: Pigou (1920), Robinson(1933), Varian(1985), Aguirre, Cowan, Vickers(2010)
- All segmentations: Bergemann, Brooks, Morris (2014), ..., Strack and Yang (2025)
- Bayesian Persuasion:
  - Kamenica and Gentzkow (2011), ...
  - First order approach and duality: Kolotilin (2018); Dworczak, Martini (2019); Kolotilin, Corrao, Wolitzky (2023); Smolin, Yamashita (2023); Dworczak, Kolotilin (2023)

The Model

- A family of demand curves $\mathcal{D} = \{D(p,\theta)\}_{\theta\in\Theta}$, a distribution $\mu_0 \in \Delta(\Theta)$.
  - Each $D(p,\theta)$ is decreasing, C^1, and $R(p,\theta) = pD(p,\theta)$ is st. concave
  - $D(p,\theta) : [0, \overline{p}(\theta)] \to \mathbb{R}_+$
- An interpretation:
  - unit demand but θ is the finest information
    - 1D-PD is not possible

Market Segmentation

- Segmentation: $\sigma \in \Delta\Delta\Theta$, $\mathbb{E}\mu = \mu_0$; each μ is a "market"
- Seller chooses a price for every market $\mu \in \mathrm{Supp}\sigma, p^*(\mu) \in \arg\max_{p \in \mathbb{R}_+} \mathbb{E}[R(p,\theta)]$.
- (Regulator) objective

$$V^\alpha(\sigma) = \alpha \int\int CS\left(p^*(\mu), \theta\right) d\mu d\sigma + (1-\alpha) \int\int R\left(p^*(\mu), \theta\right) d\mu d\sigma$$

Market Segmentation

- Segmentation: $\sigma \in \Delta\Delta\Theta$, $\mathbb{E}\mu = \mu_0$; each μ is a "market"
- Seller chooses a price for every market
  $\mu \in \text{Supp}\sigma, p^*(\mu) \in \arg\max_{p \in \mathbb{R}_+} \mathbb{E}[R(p,\theta)]$.
- (Regulator) objective

$$V^\alpha(\sigma) = \alpha \int\int CS\left(p^*(\mu),\theta\right) d\mu d\sigma + (1-\alpha)\int\int R\left(p^*(\mu),\theta\right) d\mu d\sigma$$

Definition. Demand system $\mathcal{D}$ is:

1. *IMB* if for all σ, σ' with $\sigma \succcurlyeq_{\text{MPS}} \sigma'$, $V^\alpha(\sigma) \leq V^\alpha(\sigma')$,
2. *IMG* if for all σ, σ' with $\sigma \succcurlyeq_{\text{MPS}} \sigma'$, $V^\alpha(\sigma) \geq V^\alpha(\sigma')$,

IMB: Information is Monotonically Bad!

Theorem 1

Theorem 1

Theorem. For demand system $\mathcal{D}$, let D_1, D_2 be demands in $\mathcal{D}$ with lowest and highest monopoly price p_1^*, p_2^*. $\mathcal{D}$ is IMB (IMG) if and only if:

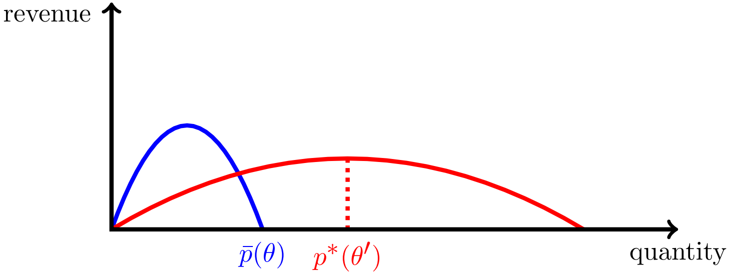
1. there is no exclusion: $p^*(\theta) \leq \bar{p}(\theta')$ for all $\theta, \theta' \in \Theta$,
2. D_1, D_2 is a basis for $\mathcal{D}$, i.e.,

$$D(p,\theta) = f_1(\theta) D_1(p) + f_2(\theta) D_2(p), \forall \theta, p \in \left(p_1^*, p_2^*\right)$$

3. $\{D_1(p), D_2(p)\}$ is IMB (IMG).

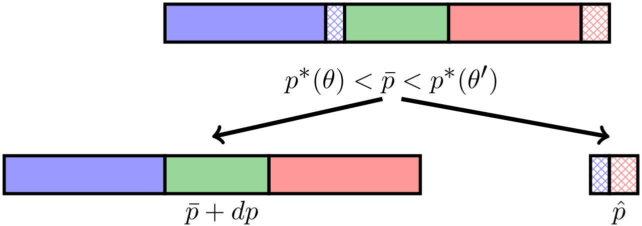
No Exclusion

- Similar to BBM



- $\Pr(\theta') \gg 0$:A segmentation that separates some θ from the rest does better.
- $\Pr(\theta) \gg 0$:A segmentation that separates some θ from the rest does worse.

Proof: Reduction to Two Demands



- Effects:
  - Composition effect: $\varepsilon\left(V\left(\hat{p}, \theta\right) - V\left(\overline{p}, \theta\right)\right) + \varepsilon'\left(V\left(\hat{p}, \theta'\right) - V\left(\overline{p}, \theta'\right)\right)$
  - Behavioral response of the seller:

$$dp = \frac{\varepsilon R_p\left(\overline{p}, \theta\right) + \varepsilon' R_p\left(\overline{p}, \theta'\right)}{\mathbb{E}\left[R_{pp}\left(\overline{p}, \theta\right)\right]}, \qquad 0 = \varepsilon R_p\left(\hat{p}, \theta\right) + \varepsilon' R_p\left(\hat{p}, \theta'\right)$$

Reduction to Two Demands

- Total Effect:

$$dV \approx \frac{\frac{\mathbb{E}V_p(\overline{p},\theta)}{\mathbb{E}R_{pp}(\overline{p},\theta)} R_p\left(\overline{p},\theta'\right) + V\left(\hat{p},\theta'\right) - V\left(\overline{p},\theta'\right)}{R_p\left(\hat{p},\theta'\right)} - \frac{\frac{\mathbb{E}V_p(\overline{p},\theta)}{\mathbb{E}R_{pp}(\overline{p},\theta)} R_p\left(\overline{p},\theta\right) + V\left(\hat{p},\theta\right) - V\left(\overline{p},\theta\right)}{R_p\left(\hat{p},\theta\right)}$$

- Should always have the same sign under IMB/IMG for any two types
- At $\overline{p} = \hat{p}$, $dV = 0$, dV same sign for all $\hat{p}$

$$\left.\frac{d}{d\hat{p}} dV\right|_{\hat{p}=\overline{p}} = 0 \Rightarrow \frac{V_p\left(\overline{p},\theta'\right)}{R_p\left(\overline{p},\theta'\right)} - \frac{\mathbb{E}V_p\left(\overline{p}\right)}{\mathbb{E}R_{pp}\left(\overline{p}\right)} \frac{R_{pp}\left(\overline{p},\theta'\right)}{R_p\left(\overline{p},\theta'\right)} = \frac{V_p\left(\overline{p},\theta\right)}{R_p\left(\overline{p},\theta\right)} - \frac{\mathbb{E}V_p\left(\overline{p}\right)}{\mathbb{E}R_{pp}\left(\overline{p}\right)} \frac{R_{pp}\left(\overline{p},\theta\right)}{R_p\left(\overline{p},\theta\right)}$$

- Has to hold for all pairs at all beliefs

$$\Rightarrow D\left(p,\theta\right) \in \textbf{span}\left\{D\left(p,\theta_1\right), D\left(p,\theta_2\right)\right\}$$

Proof: Reduction to Two Demands

- In the paper, use duality approach of Kolotilin, Corrao, Wolitzky (2025)

Two Types

- Curvature of KG value function

$$
\begin{aligned}
v''(\mu) = {} & (p^*(\mu))^2 \; \mathbb{E}\Big[V''(p^*(\mu))\Big] \\
& + 2\frac{d}{d\mu}p^*(\mu) \quad \Big[V_2'(p^*(\mu)) - V_1'(p^*(\mu))\Big] \\
& + \frac{d^2}{d\mu^2}p^*(\mu) \; \mathbb{E}\Big[V'(p^*(\mu)\Big]
\end{aligned}
$$

- Sufficient conditions for IMB (with $V = CS$): p^* convex enough, $D_2 > D_1$
- Sufficient conditions for IMG (with $V = CS$): p^* concave, $D_2 < D_1$

CES Demand

Example. Consider two demand curves $(c+p)^{-\theta_1}, (c+p)^{-\theta_2}$ for $\theta_1 > \theta_2 > 1$ and some constant $c > 0$. Then α-IMB holds if and only if $\theta_1 \leq \theta_2 + \frac{1}{2}$ for all $\alpha \geq 1/2$.

Theorem 2

General Demand Systems

- Theorem 1: silent on arbitrary demand systems

General Demand Systems

- Theorem 1: silent on arbitrary demand systems
- Assume finite number of types $|\Theta| = N$ – possible to make it more general
- Value of a segmentation

$$W(\sigma) = \int \overbrace{\int V\left(p^*(\mu), \theta\right) d\mu}^{\text{KG's } v(\mu)} d\sigma, v : \mathbb{R}^{N-1} \to \mathbb{R}$$

General Demand Systems

- Theorem 1: silent on arbitrary demand systems
- Assume finite number of types $|\Theta| = N$ – possible to make it more general
- Value of a segmentation

$$W(\sigma) = \int \overbrace{\int V\left(p^*(\mu), \theta\right) d\mu}^{\text{KG's } v(\mu)} d\sigma, v : \mathbb{R}^{N-1} \to \mathbb{R}$$

Proposition. Hessian of v, $\nabla^2 v(\mu)$ is of rank at most 2 and satisfies

$$\nabla^2 v(\mu) = -\frac{1}{\mathbb{E}R_{pp}} \left[\left(\Delta \mathbf{V}_p - \frac{d}{dp} \frac{\mathbb{E}V_p}{\mathbb{E}R_{pp}} \Delta \mathbf{R}_p \right) \Delta \mathbf{R}_p^T + \Delta \mathbf{R}_p \left(\Delta \mathbf{V}_p - \frac{d}{dp} \frac{\mathbb{E}V_p}{\mathbb{E}R_{pp}} \Delta \mathbf{R}_p \right)^T \right]$$

$\Delta \mathbf{V}, \Delta \mathbf{R}$ is the stacked version of difference between all types (but 1) and type 1.

Theorem 2

- $\underline{\lambda}(\mu)$ and $\overline{\lambda}(\mu)$ are the lowest and the highest eigenvalues of $\nabla^2 v(\mu)$

Theorem 2. Suppose $\mathcal{D}$ is finite and there is no exclusion. For any two $\sigma \succ_{MPS} \sigma'$,

$$\frac{1}{2}\min_{\mu\in\Delta\Theta}\underline{\lambda}(\mu) \leq \frac{V^{\alpha}(\sigma)-V^{\alpha}(\sigma')}{\mathrm{var}_{\sigma}(\|\mu\|_2)-\mathrm{var}_{\sigma'}(\|\mu\|_2)} \leq \frac{1}{2}\max_{\mu\in\Delta\Theta}\overline{\lambda}(\mu).$$

The bounds are tight.

A CES example

Implications of the first result:

1. $\frac{1}{2}$-IMB holds for $\{(c+p)^{-1.5}, (c+p)^{-2}\}$.
2. $\frac{1}{2}$-IMB does not hold for $\{(c+p)^{-1.5}, (c+p)^{-2}, (c+p)^{-\theta}\}$, $1.5 < \theta < 2$.

A CES example

Implications of the first result:

1. $\frac{1}{2}$-IMB holds for $\{(c+p)^{-1.5}, (c+p)^{-2}\}$.
2. $\frac{1}{2}$-IMB does not hold for $\{(c+p)^{-1.5}, (c+p)^{-2}, (c+p)^{-\theta}\}$, $1.5 < \theta < 2$.

Implication of the second result for total surplus:

θ	Lower bound	Upper bound
1.9	-0.460	4.6×10^{-5}
1.8	-0.395	1.47×10^{-4}
1.7	-0.332	2.19×10^{-4}
1.6	-0.282	1.48×10^{-4}

Implications

Example: The class $aD + b$

Proposition. Consider $\mathcal{D} = \left\{a(\theta)D(p) + b(\theta)\right\}_{\theta}$. α-IMB (α-IMG) holds if and only if

$$(2\alpha - 1)\,p + \alpha\frac{pD'(p)}{R''(p)}$$

is increasing (decreasing) over $[\min_{\theta} p^*(\theta), \max_{\theta} p^*(\theta)]$.

Example: The class $aD+b$

Proposition. Consider $\mathcal{D} = \left\{a(\theta)D(p)+b(\theta)\right\}_{\theta}$. α-IMB (α-IMG) holds if and only if

$$(2\alpha-1)\,p+\alpha\frac{pD'(p)}{R''(p)}$$

is increasing (decreasing) over $[\min_{\theta} p^*(\theta), \max_{\theta} p^*(\theta)]$.

- $\alpha = 1/2$, $V^{\alpha} = TS$:
  - 1/2-IMB holds if and only if $-p^2 D'(p)$ is log-concave.
  - 1/2-IMG holds if and only if $-p^2 D'(p)$ is log-concave.

Example: The class $aD + b$

Proposition. Consider $\mathcal{D} = \left\{ a(\theta) D(p) + b(\theta) \right\}_\theta$. α-IMB (α-IMG) holds if and only if

$$(2\alpha - 1)\, p + \alpha \frac{pD'(p)}{R''(p)}$$

is increasing (decreasing) over $[\min_\theta p^*(\theta), \max_\theta p^*(\theta)]$.

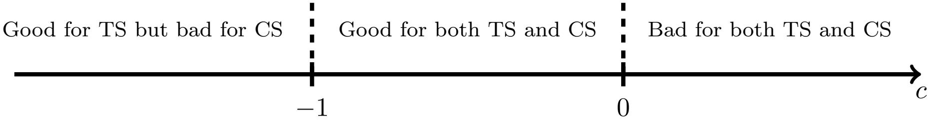
Example: The class $aD + b$

Proposition. Consider $\mathcal{D} = \left\{a(\theta)D(p) + b(\theta)\right\}_{\theta}$. α-IMB (α-IMG) holds if and only if

$$(2\alpha - 1)\,p + \alpha\frac{pD'(p)}{R''(p)}$$

is increasing (decreasing) over $[\min_{\theta} p^*(\theta), \max_{\theta} p^*(\theta)]$.

- Three cases for how information affects CS and TS. $D'(p) = -p^{-2}(1 + cp)^{c}$

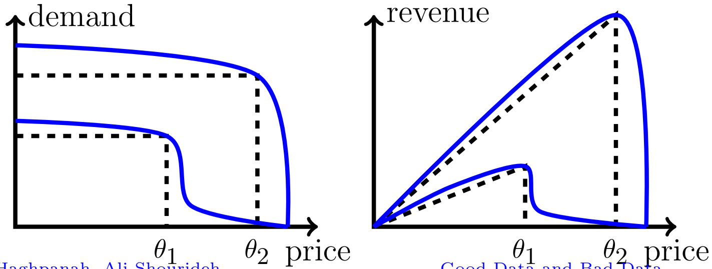


CES Demand

Example. Consider two demand curves $(c+p)^{-\theta_1}, (c+p)^{-\theta_2}$ for $\theta_1 > \theta_2 > 1$ and some constant $c > 0$. Then $\frac{1}{2}$-IMB holds if and only if $\theta_1 \leq \theta_2 + \frac{1}{2}$.

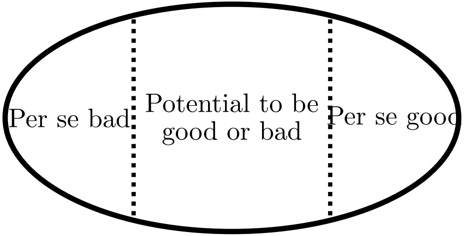
What if we approach unit-demand curves without violating no exclusion?

Corollary. Consider $\mathcal{D}^\epsilon$ that uniformly converges to a family of unit-demand curves as $\epsilon \to 0$ and revenue is concave for every $\epsilon > 0$. For small enough ϵ, the partial-inclusion condition is violated and therefore information is neither monotonically good nor bad.



Conclusion

A characterization of:



- More examples in the paper
- Methodologically:
  - We apply modern tools (concavification and duality) to study a classical problem
  - Bounds

Thank You!