Optimal Communication Design

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Motivation

- Should we allow experts to collude before providing advice?
- Do we want people to observe other's opinions before providing their own?
- Can we improve the informativeness of communication online by only providing summary statistics of opinions?

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The Question We Answer

What transfer-free mechanism maximizes the payoff of a DM receiving advice from two identically biased experts with independent information?

The Model

Players: 2 senders, 1 receiver

Each sender $i \in \{1, 2\}$ has private type $s_i \in [-1, 1]$

- $s_1 \perp\!\!\!\perp s_2$, and $s_i \sim F$.

Payoff Relevant State variable: $\omega = \frac{s_1 + s_2}{2}$

Receiver Chooses: $a \in \{0, 1\}$

Sender payoff: $a(\omega + b)$, with $b > 0$

Receiver payoff: $a\omega$

The Model: Timing and Communication

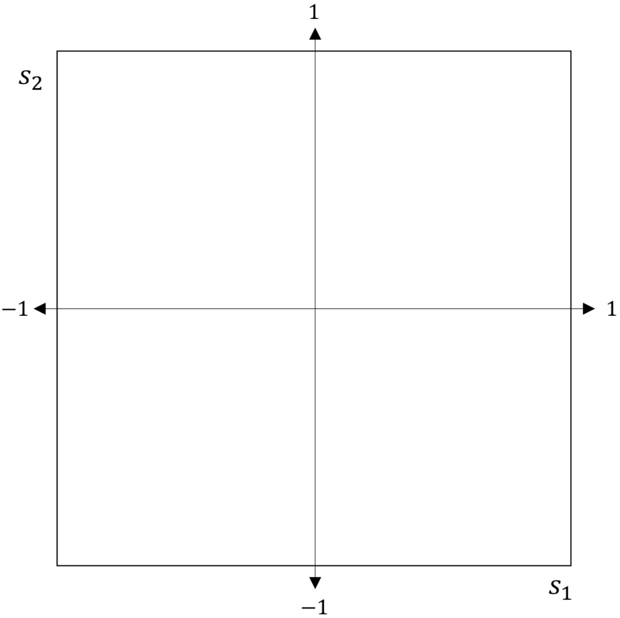
1. Mechanism designer commits to $\sigma(\tilde{s}_1, \tilde{s}_2) \in [0, 1]$
   - The probability of recommending $a = 1$ given reports $(\tilde{s}_1, \tilde{s}_2)$.

2. Senders simultaneously report their types to the mechanism.

3. The mechanism makes a recommendation to the receiver.

4. Receiver chooses action $a \in \{0, 1\}$.

Receiver and Sender First-Best Allocations



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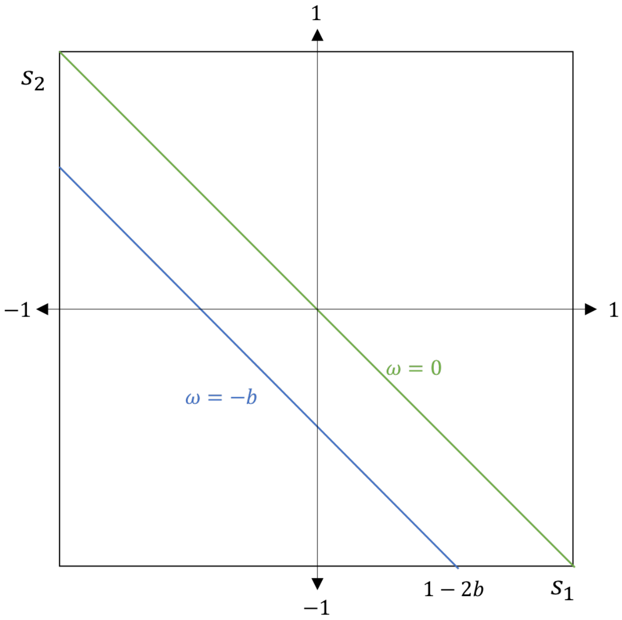

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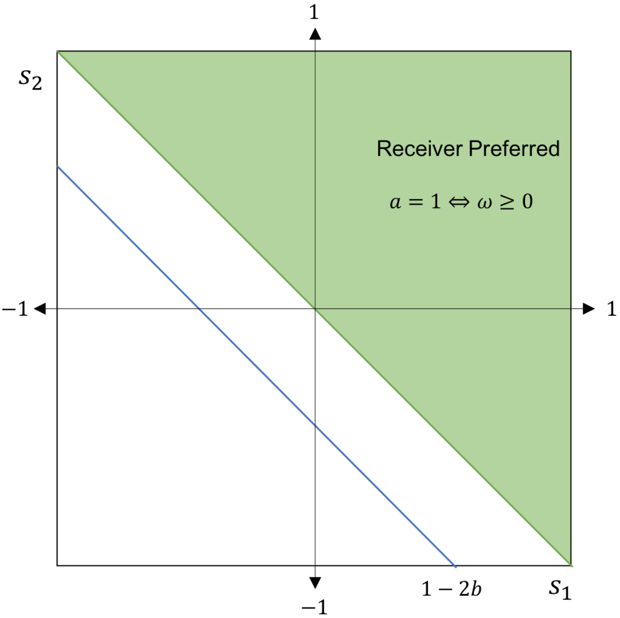

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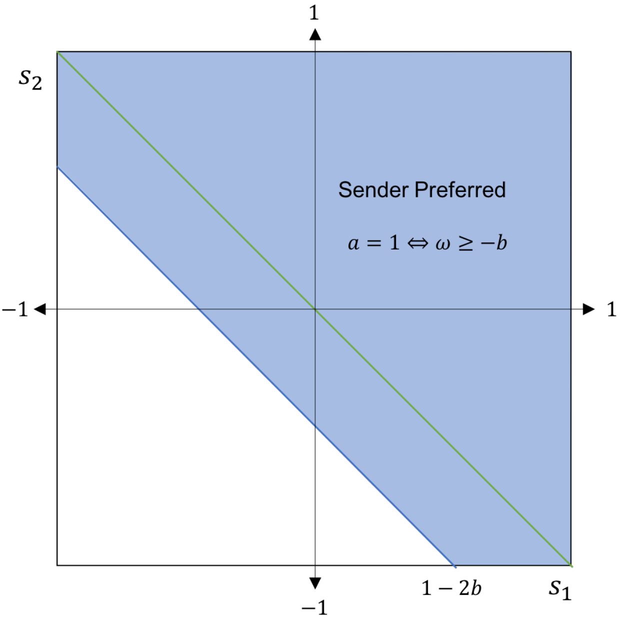

Communication Design Problem

Maximize Expected Receiver Payoff via a Transfer Free Direct Mechanism

$\sigma(s_1, s_2) \in [0, 1]$ is the probability of $a = 1$ given reports (s_1, s_2).

$$\max_{\sigma} E[a\omega] \quad \text{s.t.}$$

1. Receiver Obedience; and
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1. Receiver Obedience; and
2. Sender Incentive Compatibility for each i:

$$\Pr(1|\sigma, s_i)\left[E[s_{-i} \mid 1, s_i] + s_i + 2b\right] \geq \Pr(1|\sigma, \hat{s}_i)\left[E[s_{-i} \mid 1, \hat{s}_i] + s_i + 2b\right], \forall s_i, \hat{s}_i$$

Main Result

We make two assumptions:

(i.) Receiver obeys collusive advice: $E[\omega | s_1 + s_2 \geq -2b] \geq 0$.

(ii.) $F(s) + 2bf(s)$ is weakly increasing in s for all $s \in [0, 1]$

Theorem 1

Under (i.) and (ii.), a symmetric mechanism is optimal for the receiver if and only if it achieves the senders' first best allocation.

Proof (Sketch of Method):

- Problem with randomized recommendation is a linear programming problem in $L_\infty([-1, 1]^2)$.
- Show sender's first best solves a relaxed version of this problem by constructing appropriate Lagrange multipliers.

Monotonic Mechanisms

We only provide the intuition for monotonic mechanisms in this talk.

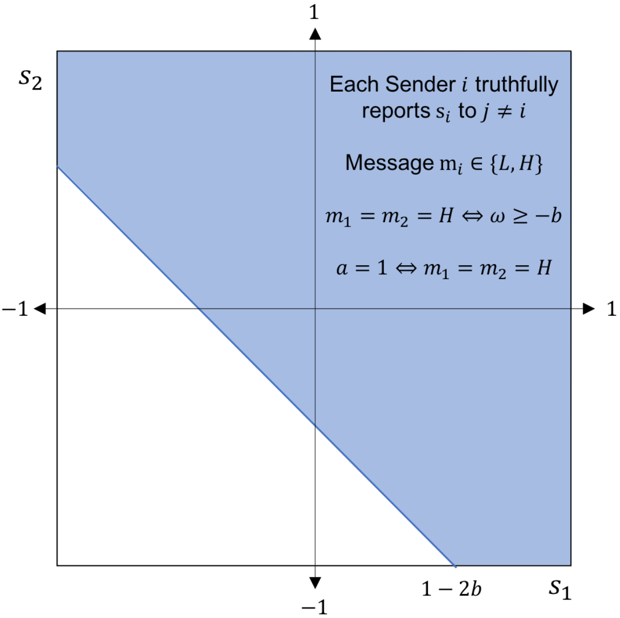
A mechanism is monotonic *iff*

$$s_i > s_i' \Rightarrow \sigma(s_i, s_{-i}) \geq \sigma(s_i', s_{-i})$$

If a monotonic mechanism satisfies IC and obedience then either it is:

1. The equilibrium of a collusive game; or
   - Sender's share information before sending messages to the receiver.
2. An equilibrium of a simultaneous move cheap-talk game
   - Each sender sends a message simultaneously to the receiver.

Collusive Equilibrium



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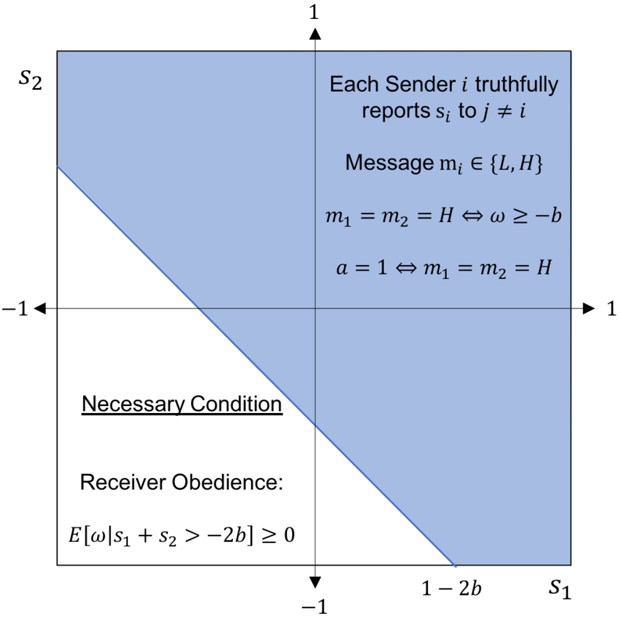

Independent Equilibrium: 2 Messages, $m_i \in \{L, H\}$

Sender Strategy:

$$m_i = H \Leftrightarrow s_i \geq \hat{s}$$

$\hat{s}$ is the type who is indifferent between recommending L and H:

$$E[s|s \geq \hat{s}] + \hat{s} + 2b = 0$$

Receiver:

$$a = 1 \Leftrightarrow m_1 = m_2 = H$$

If this is an equilibrium, then:

- $E[s|s > \hat{s}] > 0$
- $-1 < \hat{s} < -2b$

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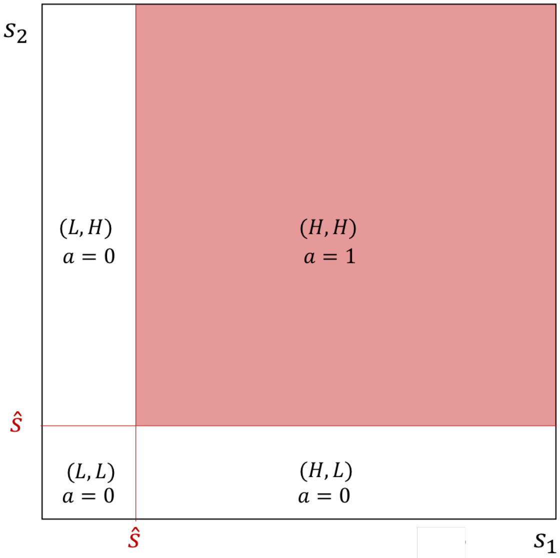
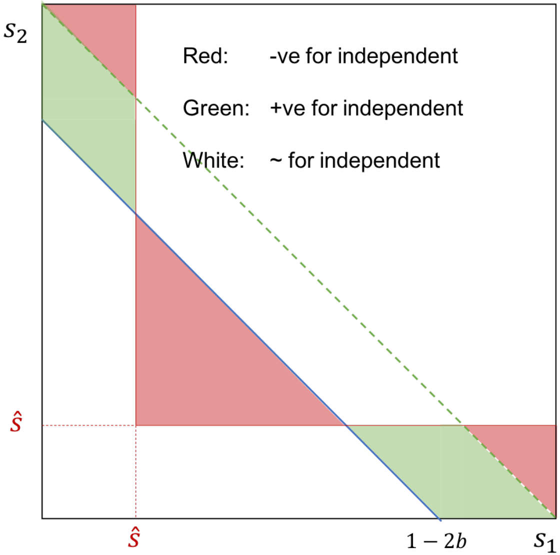


Collusive vs. Independent



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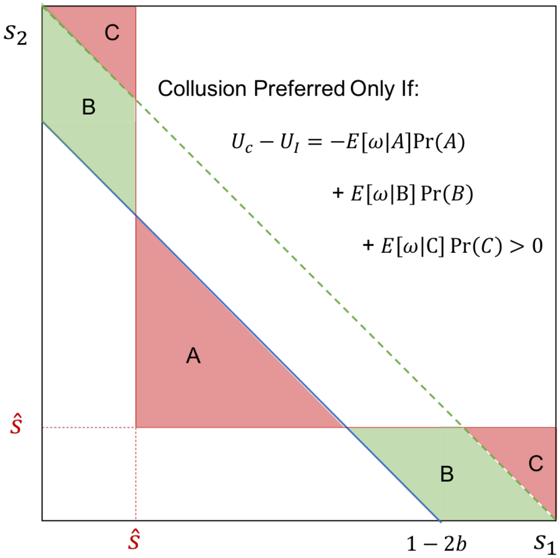
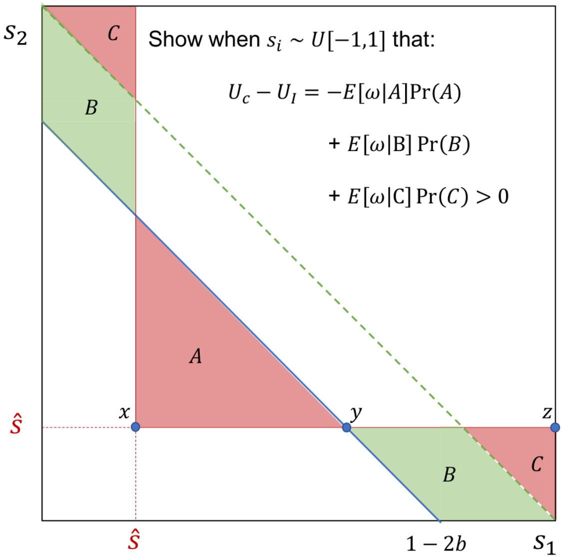


Collusive vs. Independent: Uniform $s_i \sim U[-1,1]$



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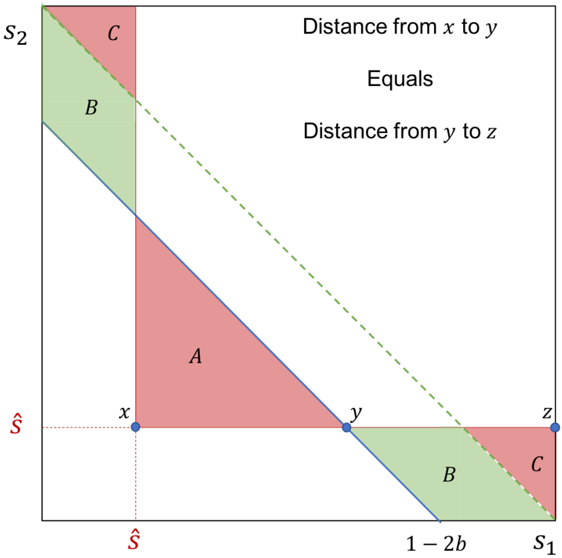

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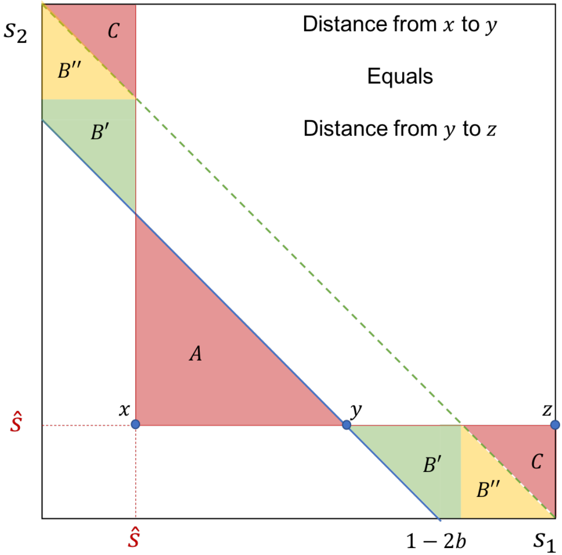

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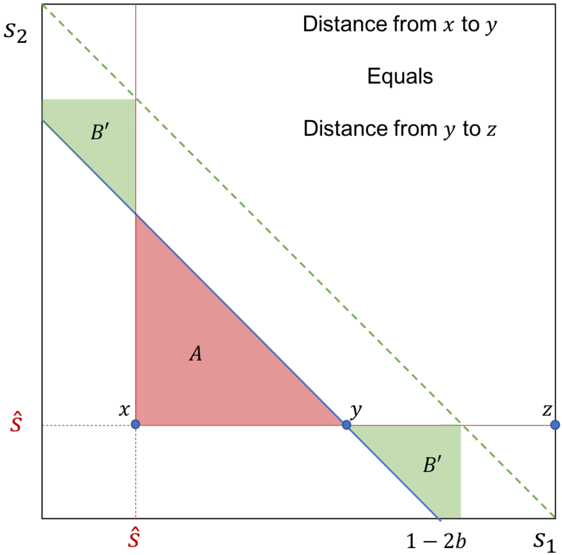

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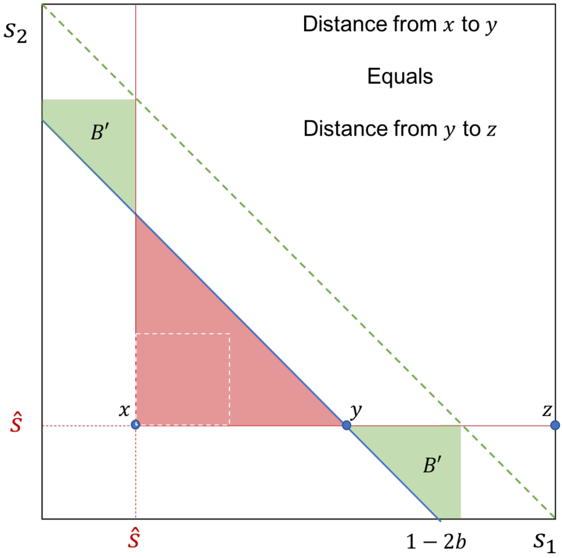

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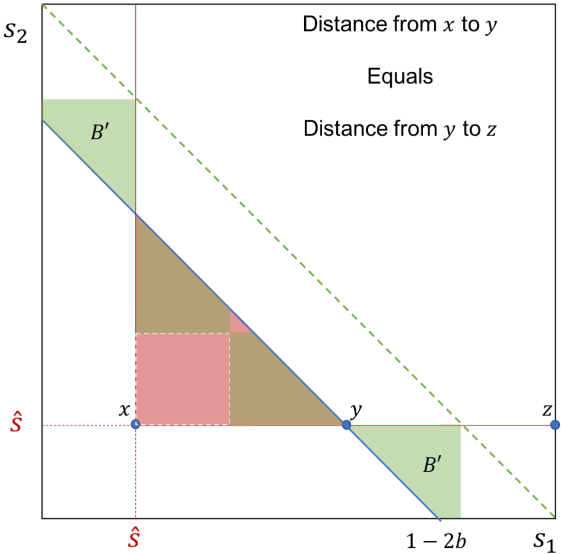

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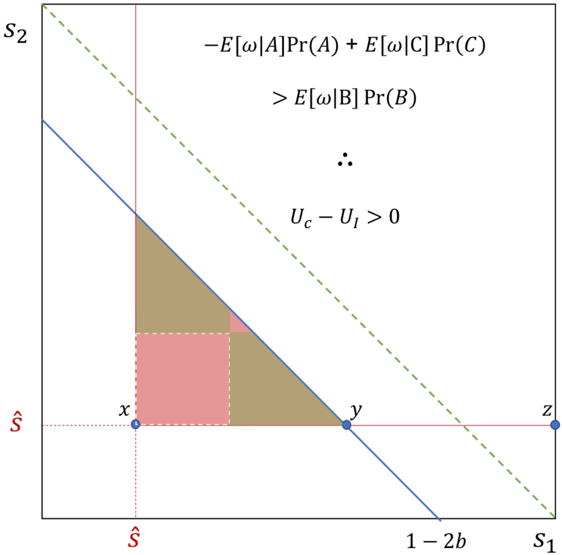


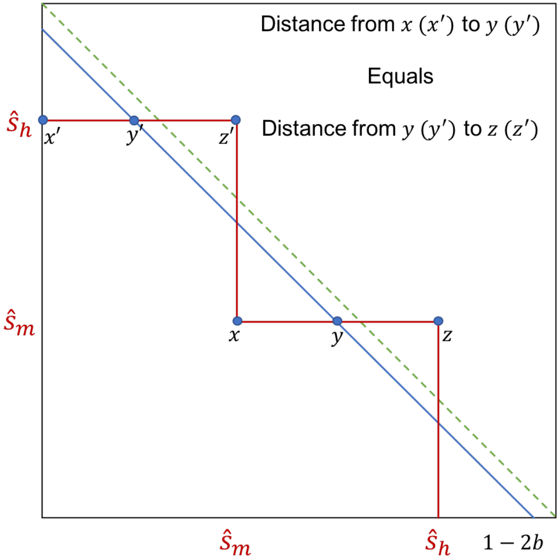
Collusive vs Independent: $|M| > 2$

As $b \to 0$:

- The number of messages (receiver's payoff) increases in the independent equilibrium.
  - Receiver preferred cheap-talk equilibrium approaches first best.
- The senders' first best (collusive) approaches the receiver's first best.

The uniform case can illustrate why the collusive outcome wins the race.

Collusive vs. Independent: Uniform with $|M| = 3$



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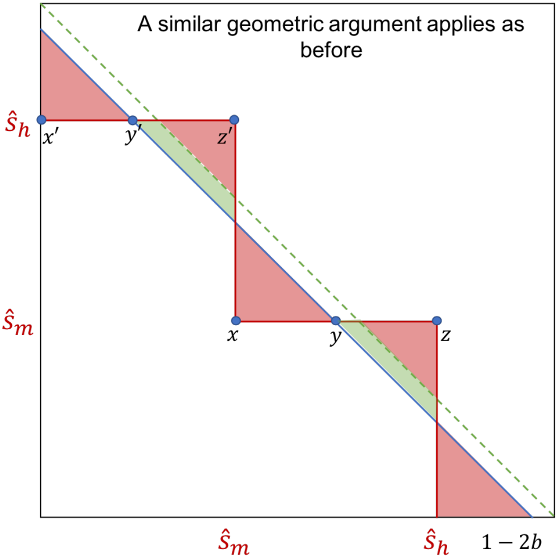
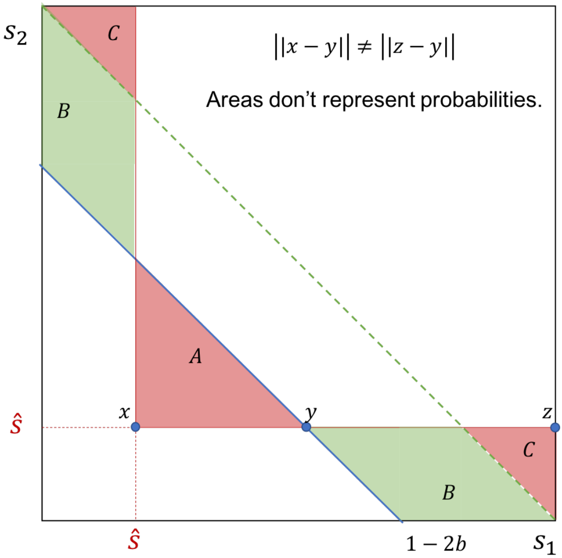


Collusive vs. Independent: General F



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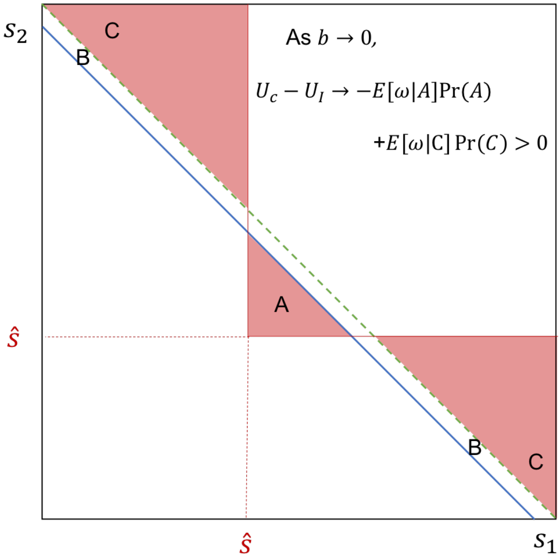

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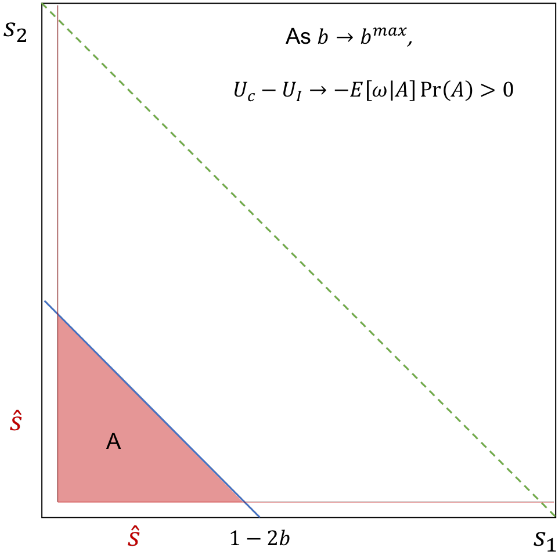

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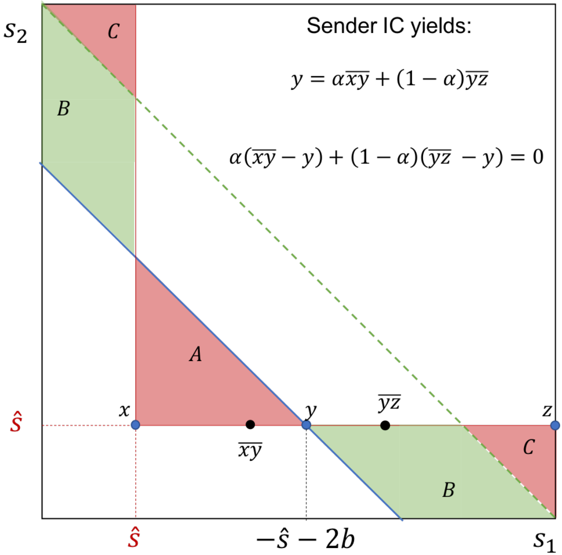

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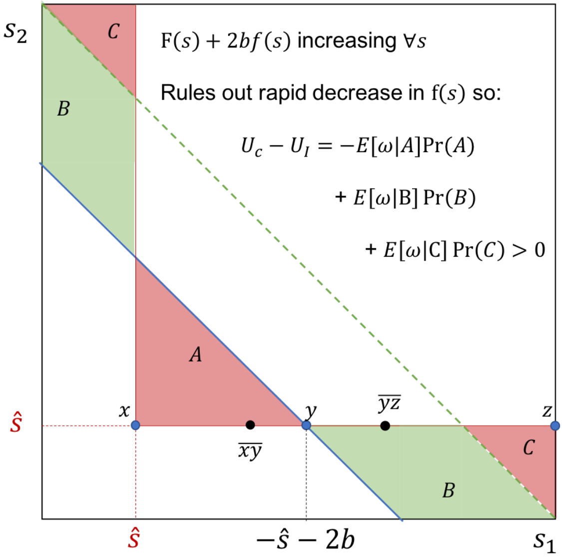

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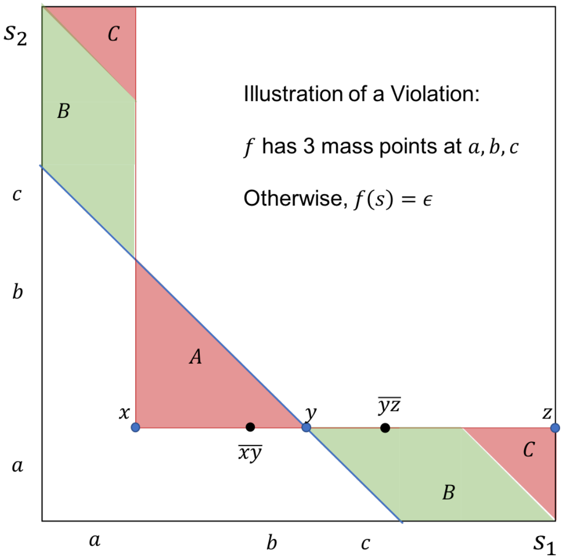

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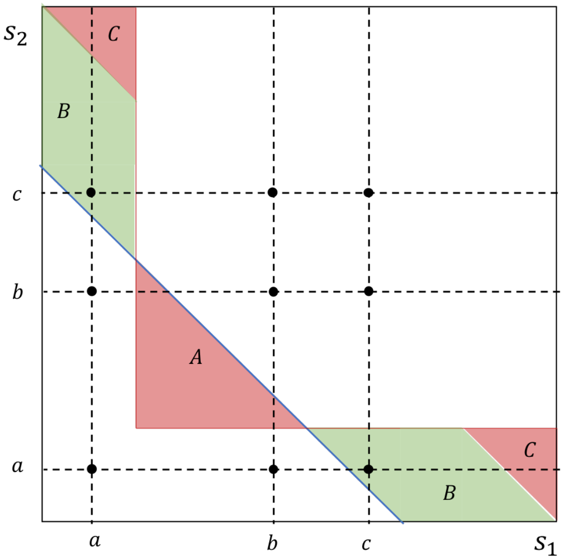

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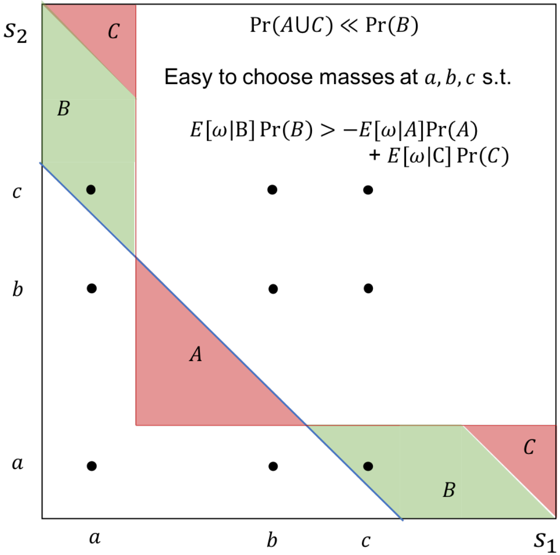

Summary and Extensions

In this setting a laissez-faire approach is optimal:

- Let your biased experts talk with each other before proffering advice.
- Allow the free sharing of opinions between agents online.

Further work, extend results to cases of:

- Correlated types.
- Differing biases.
- Larger number of senders.
- Richer action spaces.