Information Design in Common Value Auction with Moral Hazard: Application to OCS Leasing Auctions*

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April 2023

Abstract

This paper explores the impact of information design on the auctioneer's revenue in the US offshore oil/gas lease auctions where, post-auction, the winner decides whether to explore the auctioned tract and must pay the government a royalty on its production value. I first document that there is a positive correlation between the exploration rate and publicly observed losing bids. This suggests that the winning bidder uses the rivals' bids to infer their private information about the tract's potential. I then characterize the equilibrium bidding strategy when the auctioneer designs and commits to how to reveal information on losing bids to the winning bidder. Counterfactual exercises reveal that alternative bid disclosure policies significantly improve auctioneer revenue.

Keywords: Information Design; Common-Value Auction
JEL Classification: C57, D44, D82.

*I thank David Childers, Chris Conlon, Hanming Fang, Gagan Gosh, Karam Kang, Phillip Haile, Brent Hickman, Akshaya Jha, Robert Miller, Robert Porter, Paulo Somaini, Matthew Shum, Teck Yong Tan, Mo Xiao, and seminar and conference participants at Harvard University, Johns Hopkins University, Toulouse School of Economics, University of Michigan-Ann Arbor, University of Nebraska-Lincoln, University of Pennsylvania, University of Texas-Austin, Washington University in St. Louis, AEA Winter Meeting 2021, NBER SI 2021, IIOC 2021, and Stanford SITE 2022 for helpful comments and discussions.

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1 Introduction

In many common value auction settings, the winning bidder has to make investment decisions that depend on the expected value of the auctioned object. Often, this value remains uncertain to the winning bidder even after the auction. To address this uncertainty, the winning bidder utilizes the information contained in the bids of rival bidders to form a belief about the value of the auctioned object in addition to using its own private signal. In such cases, whether and how the losing bids are revealed impact not only the equilibrium bids but also subsequent strategic decisions and social welfare.

The often-studied Outer Continental Shelf (OCS) leasing auctions are an example of such a setting. In an OCS leasing auction, the auctioneer (the government) auctions off the right to explore and produce oil and gas on federal offshore tracts via the first-price sealed-bid format. After winning the auction, the winning bidder remains uncertain about the resource potential of the tract and must undertake costly exploratory drilling to resolve this uncertainty. Here, the auctioneer's revenue consists of both the winning bid from the auction and a royalty on the production revenue of the tract. After the auction outcomes are determined, the losing bids are disclosed to the winning bidder. Given the common-value nature of the tract, lower losing bids provide a discouraging signal to the winning bidder against further exploratory drilling. When the tract is unexplored, the royalty payment to the government is zero.

This paper studies how strategically withholding information about the losing bids can increase the auctioneer's revenue in OCS auctions from 2000 to 2019.[1] Relative to a full disclosure benchmark in which all losing bids are disclosed to the winning bidder, the main economic tradeoff of an alternative disclosure policy is between lower bid revenue and potentially higher royalty revenue. The reduction in bid revenue is due to the increase in the uncertainty faced by the winning bidder when deciding whether to explore, resulting in a lower profit from winning the auction. However, by strategically pooling 'bad' information (i.e., low losing bids) with 'good' information (i.e., high losing bids), the auctioneer is able

[1]Bid disclosure policies are utilized in practice in other auction settings, but there has been no systematic study of their effectiveness. For example, in 2009, the FDIC stopped releasing the identity of losing bidders and the second-highest bid in failed bank acquisition auctions, which sparked outrage from industry insiders (Fajt, 2009). In the US spectrum auction, the FCC utilizes a 'limited disclosure' rule in which only the number of bidders and the provisional winning bid are announced after each round (Xiao and Yuan, Forthcoming). Similarly, bidders on Google Ads auctions were concerned about the lack of transparency regarding information on losing bids (Joseph, 2020). In some cases, losing bidders prefer to withhold information on their bids. For instance, several US states blocked the release of information on their offers to host Amazon's second headquarters.

to increase the expected probability of drilling and hence royalty revenue.

To motivate this exercise, I first provide suggestive evidence that shows the significance of the aforementioned economic tradeoff. First, in this setting, the royalty revenue accounts for approximately 85% of the total revenue of the auctioneer. However, the exploratory drilling rate is low—only 24.5% of leased tracts were explored. This low exploration rate is an ongoing concern for the federal government, and multiple policy adjustments designed to improve the exploratory drilling rate have been proposed.[2] Second, the reduced-form analysis highlights a strong and positive correlation between the winning bidder's exploration likelihood and the value of the losing bids. According to the regression results, a 1% increase in the second-highest bid is associated with a 3.9 percentage-point increase in the probability of exploratory drilling. This suggests that the winning bidder utilizes the rival firms' bids to infer their private information on the tract's production value. As a comparison, a 1% increase in the winning bid is associated with a 4.5 percentage-point increase in the probability of exploratory drilling. As a firm's bid is increasing in its expectation of the tract's production value, this implies that the winning firm's exploration decision is similarly affected by the (inferred) information of its rival firms as by its own private information.

I construct a model of first-price sealed-bid common value auction in which an auctioneer designs and commits to how the information on the losing bids is revealed. This disclosure policy is publicly announced to all bidders prior to bid submission. The winning bidder, upon receiving information about the losing bids, updates its belief about the profitability of a tract before deciding whether to conduct costly exploratory drilling. My first contribution is to characterize the equilibrium bidding strategy in such a setting. Under a policy of full disclosure of the losing bids, the equilibrium bidding strategy resembles that of a standard common value auction, exhibiting the *'winner's curse'* phenomenon: winning the auction indicates that the winning firm is overly optimistic on the production value of the tract; anticipating this effect, every firm depresses its bid. Under a nondisclosure policy in which the winning bidder does not receive any information about losing bids, bidders further reduce their bids due to two effects. First, the expected value from winning the auction now decreases because of the lack of information available to the winning bidder ex post. Second, by not observing the second-highest bid, the winning bidder perceives the auction as being less competitive. I show that this intuition generalizes to the case of a more general bid disclosure policy, and under a monotonic symmetric bidding equilibrium, the equilibrium

[2] For example, the 1995 Deepwater Royalty Relief Act provided large royalty suspension volumes to lessees of deepwater tracts to promote exploration, development, and production.

bid is always lower than that under a full-disclosure policy.

While the theoretical prediction on the reduction of bid revenue when switching away from a full-disclosure policy is clear, whether this reduction can be offset by an increase in royalty revenue is an empirical question. My empirical analysis relies on two datasets. The first is a publicly available dataset on the US offshore auction outcomes, which are under a full-disclosure policy, and the tracts' drilling and production activities post-auction. The second dataset is a proprietary dataset on all the contracts between the oil/gas firms and the offshore rig company employed by the oil/gas firms to engage in drilling. The objects of interests are (1) the joint distribution of the firm's private signals and the true latent oil/gas potential of the tract and (2) the distribution of the exploration cost. In the absence of any unobserved heterogeneity in the tracts' values, the identification of the joint distribution of private signals and the latent value of the tract is straightforward when this latent value is observed ex post. In my setting, additional difficulty arises because the ex post production value of a tract is available only if the tract is explored, but the exploration decision is endogenous to the winning firm's belief after the auction and the unobserved exploration cost.

Therefore, another of my contributions is to provide an alternative way to identify the parameters of a common value auction without relying on the ex post value. Identifying the joint distribution of the firms' private signals and the latent tract's value is equivalent to identifying (i) the marginal distribution of the signals and (ii) the conditional mean of the latent value as a function of all bidders' realized signals. The former object is identified from the empirical bid distribution. The identification argument of the remaining objects—i.e., the conditional mean function and the distribution of the exploration cost—utilizes the variation in exploration decisions across bid realizations. Since this variation comprises the variation in both the conditional mean across bids and the ex post exploration costs, which are not observed, I disentangle these two objects using a cost shifter that affects the distribution of the exploration costs but does not influence the firm's posterior belief about a tract's resource potential. I show that when such a cost shifter exists, the objects of the model are nonparametrically identified. In the estimation, the cost shifter is constructed using rig costs, which are a significant component of the exploration cost, of past contracts associated with the nearby tracts. Thus, this cost shifter represents the rig market condition of a tract prior to its auction date.

My estimated parameters show that the OCS auctions are a favorable setting for implementing an alternative bid disclosure policy to increase the auctioneer's expected revenue.

First, although bidders' private signals are positively correlated, the correlation is low (0.05 and 0.15 for deep and shallow tracts, respectively). This implies that there is substantial uncertainty about the private information of rival firms, and the winning bidder's posterior belief about the profitability of a tract is sensitive to the information on the losing bids. Second, I find that the relationship between the likelihood of exploratory drilling and the expected value of the tract under full disclosure is nonlinear. This nonlinearity arises due to both the option value of not drilling and the shape of the exploration cost distribution. As understood from the information design literature (Kamenica and Gentzkow, 2011), the nonlinearity suggests scope for revenue gains from strategically designing the bid disclosure policy.

In the first counterfactual exercise, I consider the effect of the nondisclosure policy. In this environment, the bidders decrease their bids by 2% for shallow tracts and by 3% for deep tracts. However, the post-auction exploration rate increases by 0.46 percentage points on average, which leads to a 1.52% and 2.96% increase in royalty revenue from shallow and deep tracts, respectively, resulting in a net increase in overall revenue of $0.6 million and $1.11 million for each type of tract. The differential effects between deep and shallow tracts reflect the differences in the characteristics of the tracts. For example, bidders have more precise information about shallow tracts (i.e., a more informed prior belief) than about deep tracts; therefore, the effect of a change in the bid disclosure policy on drilling outcomes tends to be smaller for shallow tracts. I further show that much of the royalty revenue gain comes from the increase in exploration for more productive tracts.

In the next counterfactual exercise, I explore the gain from using more general disclosure policies. A natural generalization that nests both the nondisclosure and full-disclosure policies is a policy in which the k highest (e.g., the highest and the second-highest) losing bids are not revealed. This policy is of particular empirical interest because it has been applied in other procurement settings.[3] I also consider a threshold disclosure policy in which the auctioneer releases information on all losing bids that are below a preset threshold and withholds information on the exact magnitude of bids that are above this threshold. The results suggest a nondisclosure policy is revenue maximizing among the class of disclosure policies being considered.

Although the auctioneer's revenue increases significantly when an alternative policy is implemented, it is important to examine whether such a policy induces welfare loss. In my model, the socially optimal outcome is achieved only when the royalty rate is zero

[3]See the discussion in Section 6.2.1.

and the winning bidder receives all available information before engaging in exploratory drilling. Given a positive royalty rate, which is a tax on the production revenue for the winning firm, the probability of exploratory drilling is always distorted downward under a full-disclosure policy. By removing some information on the losing bids ex post, we introduce another distortion to the firm's drilling incentive. However, I show that deviating from a full-disclosure policy might result in overall welfare *gains*. For example, at the current royalty rate, which is 12% on average in the sample, the nondisclosure policy eliminates 29.15% of the distortion for shallow tracts. This welfare gain arises because much of the positive impact from a nondisclosure policy on drilling is concentrated in the more productive tracts. Therefore, a nondisclosure policy, on average, restores the drilling incentives for productive tracts but does not cause excessive drilling for unproductive tracts.

In the last counterfactual exercise, I consider the effect of the nondisclosure policy on the bidders' incentive to participate in the auction via an entry model in which the entry cost is the cost of signal acquisition. This exercise reveals that the nondisclosure policy does not necessarily have a negative impact on the firm's expected profit from entering the auction. This is because, under the nondisclosure policy, despite the reduction in the expected revenue from winning the auction relative to that under the full-disclosure policy, bidders enjoy an even greater reduction in equilibrium bids. Therefore, on average, relative to the full-disclosure benchmark, the probability of entry under the nondisclosure policy remains the same for bidders on deep tracts, and increases by 2 percentage points for shallow tracts. The net revenue increase for the auctioneer when bidders' entry decision is taken into account is \$0.08 million for deep tracts and \$1.12 million for shallow tracts.

The remainder of this paper proceeds as follows. I first discuss the related literature in the next section. I then discuss the data and motivating evidence for information design on bids in Section 3. I present the model in Section 4 and provide the identification argument and estimation results in Section 5. The counterfactual exercises are in Section 6. Finally, I conclude in Section 8.

2 Literature Review

My paper adds to the empirical literature on auction design with ex post actions. Earlier works, such as Athey and Levin (2001), Lewis and Bajari (2011), and Bajari et al. (2014), study the equilibrium bidding strategy in scoring auctions in which each bid is an incentive contract. For example, Athey and Levin (2001) study timber auctions in which firms bid a per-unit price for each timber species, and bidders have private information about volumes of species. In these papers, however, there is no ex post uncertainty. More recently, Bhattacharya et al. (2022) studies auction design for onshore oil auctions in which they model the winning bidders observing the production value of the lease ex post and strategically delaying production in response to oil price uncertainty. In my OCS setting, the incentive to strategically delay production is not a first-order issue because the production duration of an offshore lease can be longer than 30 years. Instead, I focus on the effect of auction outcomes on the winning bidder's belief about the tract's latent production value and subsequent drilling decisions.

My paper also complements the literature on the role of information in altering an economic agent's behavior in general and in auctions in particular. Since the work of Kamenica and Gentzkow (2011), there is a rapidly growing theoretical literature on how to optimally design information revelation policy to persuade economic agents; however, there is still limited attention on quantifying the potential benefit of information design in empirical markets. In the context of auctions, theoretically, Milgrom and Weber (1982) and Ető and Szentes (2007) study the auctioneer's information disclosure policy when the auctioneer has access to exogenous signals that are affiliated with the bidders' valuations. Empirically, Takahashi (2018); Allen et al. (2022), and Krasnokutskaya et al. (2020) study environments in which the bidders are uncertain about the scoring rule of an auction with multidimensional bids. In these papers, the uncertainty in the scoring rule stems from the auctioneer having private information on the project, and whether this information is revealed to the bidder has implications for the bidding strategies. In contrast, in this paper, I study the strategic decision of an auctioneer to disclose the rival bidders' information that is privately revealed to the auctioneer through their sealed bids.

In addition, my paper contributes to the literature on the identification of auction models. Hendricks et al. (2003) and Athey and Haile (2007) establish the identification result for a standard pure common value when the ex post values are observed, utilizing the bid inversion method of Guerre et al. (2000). However, as previously mentioned, in my setting, the ex post values are not perfectly observed because only the explored tracts can be productive, and the exploration decision is endogenous to the latent production value. Similar to Somaini (2020), my identification argument requires a cost shifter that affects the distribution of the ex post drilling cost of the winning bidders; however, the role of the cost shifter is different here than in Somaini (2020). Whereas in Somaini (2020), bidders' costs are privately observed and interdependent, and the cost shifters are bidder-specific, in my model, bidders do not have private information about their drilling costs, and the cost shifter is needed only to recover the unobserved distribution of drilling costs.

Finally, my paper is related to the literature on the role of information on strategic exploration decisions in the oil and gas market. Porter (1995) and Hendricks and Porter (1996) document the determinants of exploratory drilling in US offshore oil and gas auctions from 1954 to 1979, focusing on the information spillover among neighboring leases and the information effect from auction outcomes. Lin (2009, 2013) and Hodgson (2018) study the strategic decision of oil and gas companies to delay exploration to free ride on the information from the exploration outcomes of neighboring leases. In this paper, I abstract from the timing of the exploration decision and focus on the impact of auction outcomes on whether exploration occurs and how it affects the optimal auction design.

3 Data and Motivating Evidence

3.1 Institutional Setting and Data

I use the publicly available data from the Bureau of Ocean Energy Management (BOEM) on OCS auctions from 2000 to 2019.[4] Each year, the BOEM holds sales in which the right to drill and extract the oil and gas from each available tract is auctioned. The BOEM data include the bid amounts of all bids submitted to each auction, the identity of the bidders, and whether the government accepts the highest bid as the winning bid. The data also include information on all well activities, such as when a well was drilled and when it was abandoned, and production activities of each well. If there is no drilling activity within a fixed lease term, which is between 5 and 10 years, the ownership of the tract reverts to the government. Otherwise, the lessee can continue exploration and production activities on the lease.

I supplement the BOEM data with the rig contract data from Rystad, which document all the rig contracts that were active at any point in the period from 2000 to 2019. Rigs are vital capital required for well drilling and are not owned by the oil and gas companies that are the tracts' leaseholders. Rigs are often contracted on a daily rate for a period between 3 months and a year. The data specify the tender date, the commencement date, the parties involved in the contracts, the daily rig rates, and the scope for each rig contract. Table 24 in the Appendix summarizes the average duration and costs of rig contracts.

[4] Hendricks and Porter (2014) provides an comprehensive overview of the OCS auctions and how they evolved over the period from 1954 to 2002.

During this period, all of the available unleased areas are available to be auctioned. The auctions' format is first-price sealed-bid with an announced minimum bid and a fixed royalty rate. The royalty rate determines the amount of postproduction revenue that the winning bidder must pay to the government should production occur. The government might choose to reject the highest bid if it deems that there is insufficient competition for the lease. During our data period, the government rejects the highest bid only when there is one bidder (0.84% of all auctions).

Prior to the auction, potential bidders first acquire seismic information from a geophysical company, which is then processed by geophysicists at the oil and gas firms and will be further updated as new data arrive. The methods used by the bidders to process the data are proprietary. After the auction's outcome is revealed, the winning bidder decides whether to conduct exploratory drilling to determine if the tract contains profitable natural resources. If the outcome of an exploratory well is successful, the firm may decide to drill development wells to start production, and the result of this development stage is more certain than that of the exploration stage.

Table 1: Summary statistics of 2000-2019 auctions

Statistic	N	Mean	St. Dev.	Max	Min
First Bid	2,510	5,089	11,732	157,111	10
Second Bid	2,510	2,066	6,180	84,391	4
Number of Bids	2,510	2.71	1.30	13	2
Fraction of Explored Tracts	2,510	0.25	0.43	1	0
Oil Production*	288	1,819	5,571	42,879	0
Gas Production*	288	11,498	27,151	253,421	0

Note: The bids are expressed in thousands of dollars. Oil production is measured in thousand barrels of crude oil, and gas production is measured in thousand MCF (1 MCF is equal to 1032 cubic feet). Some tracts produce only oil or gas, and some tracts produce both resources.

*In the empirical analysis, I utilize all available production data, which cover 1954 to 2019. Table 19 in the Appendix presents the summary statistics of this extended data period.

Table 1 presents the summary statistics of the auction outcomes and subsequent exploration and production outcomes from 2000 to 2019 with at least 2 bidders. In this setting, firms are allowed to bid jointly subject to restrictions.[5] For my analysis, I do not distinguish

[5]For example, firms whose production is above a threshold for either crude oil or natural gas are not allowed to form joint bids. They are also barred from bidding separately if they have an agreement with another restricted bidder. They are also prohibited from making any prebidding agreement to convey any potential lease interest to any person on the list of restricted joint bidders.

between joint bids and single bids. There is substantial bid heterogeneity both across and within the auctions. For example, the average highest bid is more than twice as high as the average second bid. The exploration rate during this period is low: only 24.5% of leases were explored, among which 46.8% were subsequently developed. The low exploration rate in this period is mainly due to the change in the composition of available tracts which are now heavily concentrated in the deepwater area. Deeper tracts require higher costs of exploration and development, and their resource potential is inherently more uncertain. Figure 16 in the Appendix shows the increase in the average tract depth and the decline in the percentage of successfully developed tracts among explored tracts over time. In the Appendix, I also provide evidence that the low exploration rates are not due to oil/gas firms' binding exploration capacity constraints (Figure 17 and Table 18).

In the OCS setting, due to the spatial correlation in oil and gas deposits, the observed characteristics of a tract are high-dimensional. For example, the presence of a productive tract makes it more likely for surrounding tracts to also be productive. Therefore, the observed characteristics of a tract can include the relative distance of a tract to nearby explored tracts, the relative distance to nearby productive tracts, and the production quantities of those tracts. These characteristics affect both the equilibrium bidding strategies and the drilling probabilities. Table 22 in the Appendix displays the spatial distribution of exploration outcomes (whether oil/gas was found) and the production quantities of productive tracts in my data.

To reduce the dimension of the observed characteristics of a tract, I construct a set of parameters to summarize the distribution of latent oil and gas quantities of each tract based only on the exploration and production outcomes of previously explored tracts. In this construction, oil and gas deposits are assumed to distribute in space according to a Gaussian random field. For each tract, the distribution of a deposit type (oil or gas) is assumed to be 0 with some random probability and follows a Gamma distribution otherwise. The scale parameters of the Gamma distributions are also random. These random components are assumed to be correlated between tracts with the degree of correlation being determined by the distance between tracts. The parameters of the Gaussian field are estimated using all available production data (Table 19 in the Appendix); however, the predicted oil/gas quantities of a tract are computed based only on the realized outcomes of previously explored tracts. Further details on this construction appear in Appendix E.

Under this construction, each tract is now characterized by four parameters ($\mu_{Bernoulli,oil}$, $\mu_{Bernoulli,gas}$, $\mu_{Gamma,oil}$, $\mu_{Gamma,gas}$) where $\mu_{Bernoulli,oil}(\mu_{Bernoulli,gas})$ is the probability that

Table 2: Distribution of ($\mu_{Bernoulli,oil}$, $\mu_{Bernoulli,gas}$, $\mu_{Gamma,oil}$, $\mu_{Gamma,gas}$) across tracts

Statistic	N	Mean	St. Dev.	Max	Min
Bernoulli Prior Mean (Oil)	2,510	0.102	0.108	0.553	0.001
Bernoulli Prior Mean (Gas)	2,510	0.236	0.274	0.884	0.001
Gamma Prior Mean (Oil)	2,510	24,198	24,648	113,176	49
Gamma Prior Mean (Gas)	2,510	57,962	27,224	321,959	9,152

Note: the mean column is the average of the estimated parameters across all tracts in the sample of tracts that were auctioned between 2000 and 2019. The standard deviation is also across tracts. The standard errors of these estimates are not reported. $\mu_{Gamma,oil}$ is measured in thousand of barrels of oil, and $\mu_{Gamma,gas}$ is measured in thousand of MCF.

the tract contains oil (gas), and $\mu_{Gamma,oil}(\mu_{Gamma,gas})$ is the predicted quantity of oil (gas) if oil (gas) is found. The constructed values of ($\mu_{Bernoulli,oil}$, $\mu_{Bernoulli,gas}$, $\mu_{Gamma,oil}$, $\mu_{Gamma,gas}$) across tracts are summarized in Table 2. It features substantial variation across tracts and a low ex ante probability of finding any oil or gas. For example, based on these estimates, a tract in the sample, on average, has a 10% probability of containing oil and a 24% probability of containing gas.

3.2 Motivating Evidence

In this paper, I focus on the extent to which the negative informational content from the auction outcomes influences the drilling decision of the winning bidder. Column (1) of Table 3 shows the correlation between the exploration decisions and the first and second bids. In this regression, I control for the neighboring tracts' characteristics such as whether a neighboring tract was explored and whether that happens before or after the auction.[6] I also control for the winning bidder's characteristics, including whether the winning bidder owns neighboring leases and whether these leases have been explored and productive. The results suggest that conditional on the winning bid, the firms are more likely to explore when the second bid is higher, which is consistent with a model in which the winning bidder uses the losing bids to update its beliefs about the tract's profitability before deciding whether to explore. Furthermore, the magnitude of the effect of the second-highest bid on the exploration rate is similar to that of the winning firm's own bid, suggesting that the winning firm's exploration decision is similarly affected by the inferred information of its rivals as by its own private

[6] For each tract, I define 'neighboring tracts' using a similar metric as in Hendricks et al. (2003), which are tracts within 0.11 degrees of latitude and 0.12 degrees of longitude.

information.

Table 3: Correlation between the exploratory drilling rate and the auction outcomes

	Exploration drilling = TRUE				
	(1)	(2)	(3)	(4)	(5)
Log first bid	0.045	0.084	0.045	0.048	0.081
	(0.010)	(0.009)	(0.010)	(0.010)	(0.010)
Log second bid	0.039		0.040	0.033	
	(0.013)		(0.013)	(0.013)	
Log (first bid) - Log (second bid)		−0.039			−0.033
		(0.013)			(0.013)
Number of bids	0.005	0.005	0.004	0.009	0.009
	(0.008)	(0.008)	(0.008)	(0.009)	(0.009)
Year FE	Yes	Yes	Yes	Yes	Yes
Area FE	Yes	Yes	Yes	Yes	Yes
Neighbor Characteristics	Yes	Yes	Yes	Yes	Yes
Winning Bidder Characteristics	Yes	Yes	Yes	Yes	Yes
Tract's predicted production	No	No	Yes	Yes	Yes
Exclude Shared Ownership	No	No	No	Yes	Yes
Observations	2,316	2,316	2,316	2,014	2,014
Adjusted R^2	0.318	0.318	0.320	0.356	0.356

Note: The variables that describe the characteristics of winning bidders include the number of tracts owned by lessees that have not been explored or under production, those under explored, and those under production. The variables that describe the characteristics of neighboring tracts include the average bids from neighboring tracts, the number of neighboring tracts that were explored, and the number of neighboring tracts that were under production. In Column (3)-(5), the constructed tract's potential ($\mu_{Gamma,oil}$, $\mu_{Gamma,gas}$, $\mu_{Bernoulli,oil}$, $\mu_{Bernoulli,gas}$) is included in the regression. In Column (4) and (5), leases with shared owners with nearby tracts are excluded. Standard errors are in parentheses and adjusted for heteroskedasticity.

The above regression result might suffer from omitted variable bias due to the unobserved quantity of oil and gas, which is affiliated with the losing bids. Therefore, I also consider an alternative specification that relies on the relative proportion between the highest bid and the second-highest bid instead of the magnitude of the losing bid (Column (2) of Table 3). Note that under the standard affiliation assumption on the joint distribution of firms' private signals, the log difference between bids is not necessarily affiliated with the latent value of the tract, which alleviates the concern about omitted variable bias. The correlation between the log difference between bids and the exploration outcome is similar in magnitude to that of the correlation between the highest losing bid and the auction outcome. In addition, Column (3) of Table 3 shows that the positive correlation between the losing bids and the exploration likelihood is still present when the parameters characterizing the oil and gas potential of each tract ($\mu_{Gamma,oil}$, $\mu_{Gamma,gas}$, $\mu_{Bernoulli,oil}$, $\mu_{Bernoulli,gas}$) are included.

In my analysis, I assume that each winning bidder considers its exploration decision independently between tracts. However, because the exploration outcomes on the adjacent tracts are usually positively correlated, the leaseholders of the adjacent tracts might choose to explore cooperatively to mitigate potential losses from overdrilling inefficiencies (Hendricks and Porter, 2014). To measure the extent to which firms cooperate on their exploration decisions among neighboring leases, I exclude explored (unexplored) tracts whose owners entered into ownership agreements with nearby unexplored (explored) tracts, which account for 13.2% of leases in the sample. The qualitative results of the regressions remain the same (Columns (4) and (5) of Table 3).[7]

4 Model

I first present a model of a common value auction with ex post moral hazard under full revelation of the bids ex post. This benchmark model is also the model used for identification and estimation. I then discuss an extension of the model in which the auctioneer does not disclose the losing bids. Last, I present a general version of this extension in which the auctioneer can partially disclose information on the losing bids. Throughout this section, I abstract from some empirically relevant details, which will be introduced in Section 5.

4.1 Full-Disclosure (FD) Model

The federal government (auctioneer) wants to auction off the lease of an offshore tract via a first-price sealed-bid auction. There are $N \geq 2$ firms/bidders. N is common knowledge. The expected production value during the lease term is a random variable $Q \in \mathbb{R}_+$, and the common prior belief about Q is represented by the distribution F^Q with density f^Q. Prior to the auction, each firm i receives a *private* signal $S_i \in \mathbb{R}_+$ about Q, which represents each firm's private information about the potential production value of the tract. Let $\boldsymbol{S} = (S_1, \ldots, S_N)$ and $f^{\boldsymbol{S}}(\boldsymbol{S}|q)$ be the joint density of the firms' signals conditional on $Q = q$. I assume that for all $q \in \mathbb{R}_+$, $f^{\boldsymbol{S}}(\boldsymbol{S}|q)$ has full support.[8] Let the expected production value

[7]When the exploration decision is not independent between tracts, the bidding behavior will also be interdependent. In my model, I account only for the effect of the realized production outcomes of neighboring tracts on bidding behavior. Kong (2021) studies a model in which both the affiliation between tracts and the potential synergies (for example, due to economies of scale in drilling activities) in a private value paradigm can arise.

[8]Since $f^{\boldsymbol{S}}(\cdot|q)$ has full support, by Bayes rule, $f^Q(q|\boldsymbol{S}=\boldsymbol{s}) = \frac{f^{\boldsymbol{S}}(\boldsymbol{S}|q)}{\int f^{\boldsymbol{S}}(\boldsymbol{s}|q') f^Q(q') dq'}$.

conditional on $\boldsymbol{S} = \boldsymbol{s}$ be denoted by

$$V(\boldsymbol{s}) := \int q \frac{f^{\boldsymbol{S}}(\boldsymbol{s}|q) f^{Q}(q)}{\int f^{\boldsymbol{S}}(\boldsymbol{s}|q') f^{Q}(q') dq'} dq$$

where I assume that $V(\cdot)$ is differentiable almost everywhere and is symmetric in its arguments. Without loss of generality, I also assume that $V(\boldsymbol{s})$ is increasing in s_1 to s_N and normalize the signals such that for $\boldsymbol{s} = (0, .., 0)$, $V(\boldsymbol{s})$ is 0. I make the standard assumption that $S_1, S_2, ..., S_N$ are affiliated.[9]

Besides the winning bid, the government also charges a royalty rate of $r < 1$ on the tract's production value, where r is announced before the auction. To extract the natural resources from the tract, the firm must incur an exploration cost. This cost is realized only *after* the auction—for example, such exploration costs depend on the geophysical and geological characteristics of the tract and can be fully assessed by a firm only after it has won the auction and gained access to the tract. We let $C > 0$ denote the random variable of the exploration cost and F^C be its distribution. F^C is also common knowledge.

To summarize, suppose that firm i wins the auction with a bid of b. Under the realizations $Q = q$ and $C = c$, if firm i explores the tract, then its overall profit is $(1 - r) q - c - b$, and the government's profit is $b + rq$. On the other hand, if firm i does not explore, its profit is $-b$, whereas the government's profit is b. The game proceeds as follows:

1. Each firm i *privately* observes the realization of S_i.

2. All the firms simultaneously submit a sealed bid to the auctioneer.

3. The government announces the winner, who is the highest bidder; the winner then pays its bid. In the event of a tie, the winner is randomly chosen from among the highest bidders.

4. The auctioneer reveals all the submitted bids.

5. The winner observes the realization of the exploration cost C.

[9] That is, for any $\boldsymbol{s} = (s_1, \ldots, s_N)$ and $\boldsymbol{s}' = (s'_1, \ldots, s'_N)$,

$$f^{\boldsymbol{S}}(\boldsymbol{s} \vee \boldsymbol{s}') f^{\boldsymbol{S}}(\boldsymbol{s} \wedge \boldsymbol{s}') \geq f^{\boldsymbol{S}}(\boldsymbol{s}) f^{\boldsymbol{S}}(\boldsymbol{s}'),$$

where $\boldsymbol{s} \vee \boldsymbol{s}'$ and $\boldsymbol{s} \wedge \boldsymbol{s}'$ are the component-wise maximum (i.e., join) and minimum (i.e., meet) between $\boldsymbol{s}$ and $\boldsymbol{s}'$, respectively.

6. The winner decides whether to explore the tract. If the winner decides to explore, it incurs C, and the value of Q is realized. Subsequently, the firm collects Q and pays rQ to the government. On the other hand, if the firm decides not to explore, the game ends.

Assumption 1 *C and Q are independent.*

Assumption 1 assumes that the realization of the exploration cost in Stage 5 does not change the winning bidder's posterior belief about the tract's production value. In the OCS setting, the exploration cost is mainly determined by the available rigs' locations and the tract's water depth. Therefore, the realization of the exploration cost is unlikely to carry any additional information about the tract's production value beyond the information observed by the bidders prior to the auction.

4.1.1 Equilibrium Analysis

Each firm's strategy consists of a bidding function $\beta_i : S_i \to \mathbb{R}_+$ and an exploration decision (conditional on winning the auction) that depends on the realization of C and all the information about the expected production value after the auction. The winner of the auction will explore if and only if its expected value of $(1-r)\,Q$ conditional on all the available information is higher than the realization of C. Given the simplicity of the drilling decision, it suffices to focus on only the equilibrium bidding functions in the equilibrium analysis. Henceforth, I abuse terminology and refer to an equilibrium as only the profile of the bidding functions of the equilibrium strategies. I restrict attention to symmetric Bayes Nash equilibria in which each firm i bids according to the same bidding function $\beta : S_i \to \mathbb{R}_+$, where $\beta(\cdot)$ is strictly increasing.

Let Y_i denote the first-order statistics of the $N-1$ random variables S_j for $j \neq i$ (i.e., $Y_i = \max_{j\neq i} S_j$). Let $G^Y(Y_i|s)$ denote the distribution of Y_i conditional on $S_i = s$, and let $g^Y(Y_i|s)$ be the corresponding density. Let $\boldsymbol{Z}_i$ denote the $(N-2)$-dimensional random vector $\left(Z_i^{(2)}, Z_i^{(3)}, \ldots, Z_i^{(N-1)}\right)$, where $Z_i^{(k)}$ denote the k-order statistics. Let $G^{\boldsymbol{Z}}(\boldsymbol{Z}_i|s, y)$ denote the distribution of $\boldsymbol{Z}_i$ conditional on $S_i = s$ and $Y_i = y$. Let $g^{\boldsymbol{Z}}(\boldsymbol{Z}_i|s, y)$ be the corresponding density. Finally, let $G^{Y,\boldsymbol{Z}}(Y_i, Z_i|s)$ denote the joint distribution of (Y_i, Z_i) conditional on $S_i = s$, and let $g^{Y,\boldsymbol{Z}}(Y_i, Z_i|s)$ be the corresponding density.

Next, let $\psi(V)$ denote the net revenue from the tract when its ex post expected production value is V, taking into account the firm's drilling decision.

$$\psi\left(V\right):=\int_{0}^{\infty}\max\left\{0,\left(1-r\right)V-c\right\}dF^{C}\left(c\right)\ =\int_{0}^{(1-r)V}\left(\left(1-r\right)V-c\right)dF^{C}(c) \qquad (1)$$

I make two observations. First, $\psi(V)$ is nonlinear in V because of the option value of not drilling. Because of this nonlinearity, changes in the observability of V affect the bidder's expected net revenue and hence actions, both at the drilling and the bidding stages, making bid disclosure policy a useful market design tool. Second, the curvature of $\psi(V)$ depends on the cost distribution F^C. To see this, note that $\psi'(V)=(1-r)F^{C}\left(\left(1-r\right)V\right)$.

Let $v(s,y)$ denote the expected net revenue conditional on $S=s$ and $Y=y$ when the winning bidder observes the realization of $\boldsymbol{S}$ before drilling.

$$v(s,y):=\int_{\boldsymbol{z}}\psi\left(V\left(s,y,\boldsymbol{z}\right)\right)dG^{\boldsymbol{Z}}(\boldsymbol{z}|s,y) \qquad (2)$$

Therefore, $v\left(s,y\right)$ is also firm i's expected profit from winning the auction conditional on $S_i=s$ and $Y_i=y$, where the expectation is taken at the start of Stage 2. In deriving $v(\cdot)$, note that the assumption that all firms are playing according to an increasing strategy $\beta(\cdot)$ is utilized; therefore, observing all losing bids is the same as observing all private signals of losing firms.

Consequently, at the start of Stage 2, if $S_i=s$ and firm i conjectures that all the other firms bid according to $\beta(\cdot)$, firm i's expected overall profit from bidding b is

$$\int_{0}^{\beta^{-1}(b)}\left[v\left(s,y\right)-b\right]g^{Y}\left(y|s\right)dy \qquad (3)$$

Let $L\left(s'|s\right):=\exp\left(-\int_{s'}^{s}\frac{g^{Y}(t|t)}{G^{Y}(t|t)}dt\right)$.

Proposition 1 *There exists a unique increasing and symmetric Bayes Nash equilibrium. The equilibrium strategy, denoted by $\beta^{FD}(\cdot)$, is as follows*

$$\beta^{FD}\left(s\right)=\int_{0}^{s}v\left(s',s'\right)dL\left(s'|s\right). \qquad (4)$$

The proof of Proposition 1 is found in Appendix A.3.1. The proof here is similar to the standard common value auction (Milgrom and Weber, 1982). In the proof, I also show that, for any s, $L\left(s'|s\right)$ is a proper distribution that is first-order stochastically dominated by the distribution $G^{Y}(s'|s'\leq s)$. Therefore, $\beta^{FD}(s)<\mathbb{E}[v(s',s')|s'\leq s]$ for all $s>0$. This bidding strategy thus reflects the firm's understanding of the "winner's curse" — winning the auction indicates that the firm is overly optimistic about the tract's production value; anticipating this effect, the firm depresses its bid.

4.2 Nondisclosure Model

I now consider the equilibrium bidding and drilling strategies when the auctioneer announces at the start of the auction that the losing bids will no longer be revealed, i.e., a nondisclosure (ND) policy. The modification to the baseline game is that Stage 4 is now removed, and this is common knowledge.

4.2.1 Drilling Decision: FD vs. ND

Before turning to the equilibrium analysis of the bidding strategy, I first discuss the difference in the drilling decision under FD and ND. Conditional on bidder i's signal being $S_i = s$, consider the winner's expected drilling probability at the end of Stage 3—–i.e., before the realization of the cost of drilling.

Under an FD policy, the winning bidder's expected drilling probability after winning the auction but before the realization of its cost of drilling is given by

$$\Delta^{FD}(s) = \mathbb{E}\left[F^C(V(\boldsymbol{S}))|S_i = s, S_{-i} \leq s\right] \tag{5}$$

Under an ND policy, winning bidder i does not observe S_{-i}. Therefore, the expected drilling probability is given by

$$\Delta^{ND}(s) = F^C\left(\mathbb{E}\left[V\left(\boldsymbol{S}\right)|S_i = s, S_{-i} \leq s\right]\right) \tag{6}$$

Although the bidders are assumed to be risk neutral, when $F^C(\cdot)$ is nonlinear, $\Delta^{FD}(s)$ and $\Delta^{ND}(s)$ are generally different. The difference between these two objects can be readily understood by treating $F^C(\dot{c})$ as a utility function. Under ND, conditional on S, the bidder receives deterministically the value $\mathbb{E}\left[V\left(\boldsymbol{S}\right)|S_i = s, S_{-i} \leq s\right]$. Under FD, conditional on S, the bidder receives a lottery with the payoff distribution being determined by the distribution of $S_{-i}|S_i = s$. Therefore, when $F^C(\cdot)$ is always concave (convex), the drilling rates under an ND policy are always greater (smaller) than those under an FD policy, using Jensen's inequality. When $F^C(\cdot)$ admits both concave and convex regions, the theoretical predictions on the relative magnitude of Δ^{ND} and Δ^{FD} are indeterminate. The effect of an ND policy on the expected drilling probability depends on two factors: (1) the shape of the cost distribution $F^C(\cdot)$, and (2) the distribution of the posterior mean $V(\boldsymbol{S}|S_i = s, S_{-i} \leq s)$ for different realizations s.

Figure 4: Illustration of the effect of an ND policy on the drilling probability

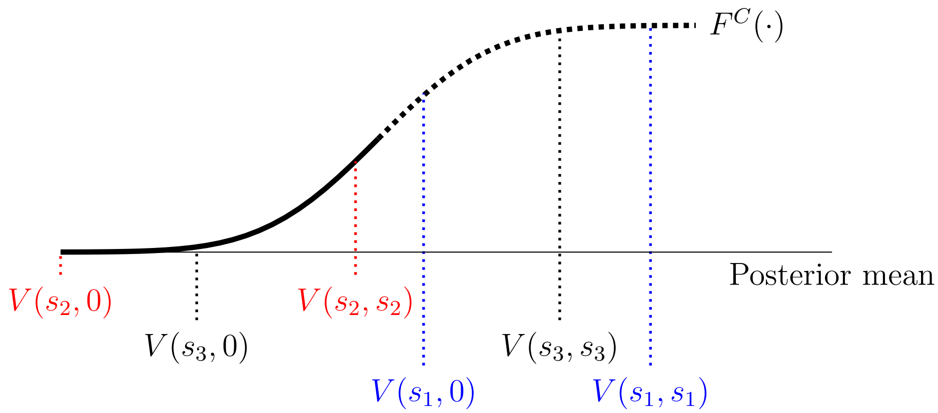


Figure 4 illustrates this intuition when there are two bidders. I consider three scenarios. In the first scenario, the winning signal is s_1, the support of the posterior mean is $\left[V(\boldsymbol{S}|S_i = s_1, S_{-i} = 0), V(\boldsymbol{S}|S_i = s_1, S_{-i} = s_1)\right]$, and $F^C(\cdot)$ is concave on this interval. Therefore, $\Delta^{ND}(s_1) > \Delta^{FD}(s_1)$. Conversely, in the second scenario, the winning signal is s_2, and $F^C(\cdot)$ is convex on the interval $\left[V(\boldsymbol{S}|S_i = s_2, S_{-i} = 0), V(\boldsymbol{S}|S_i = s_2, S_{-i} = s_2)\right]$. As a result, $\Delta^{FD}(s_2) < \Delta^{FD}(s_2)$. In the last scenario, the winning signal is s_3, and $F^C(\cdot)$ is neither concave nor convex over the support of the posterior mean. Therefore, the effect of ND relative to that of FD at the winning signal s_3 depends the distribution of the posterior mean $V(\cdot)$ over its support.

4.2.2 Equilibrium Analysis

In the following analysis, I explain the difference in the equilibrium bidding strategy between the FD and ND auctions.

As before, I focus on symmetric Bayes Nash equilibria in which each firm i bids according to a strictly increasing function $\beta^{ND} : S_i \to \mathbb{R}_+$. Let

$$\hat{V}_0(s, y) := E\left[V\left(S_i, Y_i, \boldsymbol{Z}_i\right) | S_i = s, Y_i \leq y\right] \tag{7}$$

$$\bar{V}_0(s, y) := E\left[V\left(S_i, Y_i, \boldsymbol{Z}_i\right) | S_i = s, Y_i = y\right] \tag{8}$$

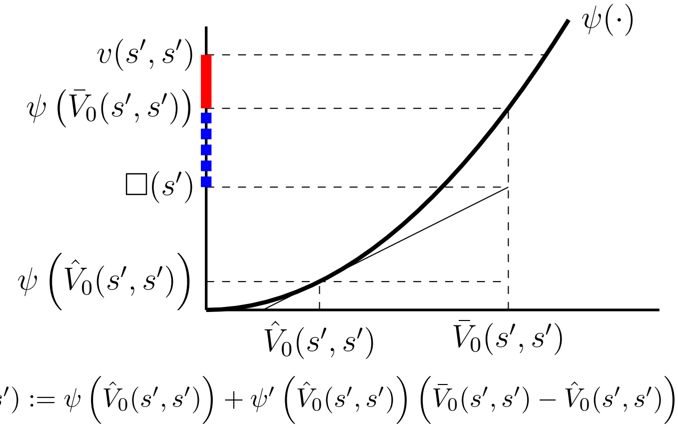
Therefore, $\hat{V}_0(s, y)$ represents firm i's expectation of Q conditional on $S_i = s$, and $S_{-i} \leq y$, and $\bar{V}_0(s,y)$ represents firm i's expectation conditional on more granular information that $\max_j S_j = y$. The affiliation property of the private signals implies that $\hat{V}_0(s,y) \leq \bar{V}_0(s,y)$. I adopt the same notations as in Section 4.1

Proposition 2 *In the ND auction, the increasing and symmetric Bayes Nash equilibrium is uniquely*

$$\beta^{ND}(s) = \int_0^s \left(\psi\left(\hat{V}_0(s',s')\right) + \left[\psi'\left(\hat{V}_0(s',s')\right)\left(\bar{V}_0(s',s') - \hat{V}_0(s',s')\right)\right]\right) dL(s'|s). \quad (9)$$

Moreover, $\beta^{ND}(s) < \beta^{FD}(s)$ for all s —i.e., the equilibrium bids in the ND auction are lower than those in the FD auction.

Figure 5: Illustration of the difference between $\beta^{FD}(\cdot)$ and $\beta^{ND}(\cdot)$



The proof of Proposition 2 is in Appendix A.3.2. The difference between $\beta^{ND}(\cdot)$ and $\beta^{FD}(\cdot)$ in Eq. (4) is illustrated in Figure 5. First, recall that $v(s',s') = \mathbb{E}[\psi(V(\boldsymbol{S}))|S_i = s', Y_i = s']$ is firm i's expected profit from winning the FD auction when conditioned on $S_i = Y_i = s'$. In the ND auction, firm i's expected profit from winning when conditioned on $S_i = Y_i = s'$ is $\psi\left(\bar{V}_0(s',s')\right)$. Note that $\psi(\cdot)$ is convex; therefore, $\psi\left(\bar{V}_0(s',s')\right) \leq v(s's')$. The difference between these two terms (the red difference in Figure 5) reflects the anticipated decrease in revenue resulting from less information being available to the winning bidder in the ND auction.

The difference between $\psi\left(\bar{V}_0(s',s')\right)$ and the term inside the integral of $\beta^{ND}(\cdot)$ in (9) represents the effect of the unobservability of the second-highest bid in the ND auction. This is represented by the blue difference in Figure 5. Intuitively, the winning bidder now places more weight on the possibility that its closest rival will place a lower bid; therefore, this effect resembles the effect of lessening competition in the auction.

In summary, the ND auction aggravates the winner's curse relative to the FD auction, and the equilibrium bids are always lower in the ND auction. Therefore, the auctioneer faces a tradeoff when concealing the losing bids. On the one hand, the ND auction yields strictly lower bid revenue than the FD auction. On the other hand, the expected drilling rate may be higher in the ND auction. Thus, it is unclear which of the two auctions yields the auctioneer a higher expected total profit.

4.3 General Disclosure Policy

In addition to ND, my model can also be extended to accommodate more general disclosure policies. Below, I briefly describe the setup for this extension. Further details are found in Appendix A.

Let $\mathcal{M} = \mathbb{R}_+^{N-1}$ denote the message space, and let $\Delta\mathcal{M}$ denote the set of distributions on $\mathcal{M}$, endowed with the weak topology. I define a *disclosure policy* by a measurable map $D : \mathbb{R}_+^{N-1} \to \Delta(\mathcal{M})$, where, fixing a set of N bids received by the auctioneer and ordering them in decreasing order, with $b^{(1)} \geq b^{(2)} \geq \cdots \geq b^{(N)}$, $D\left(\cdot|b^{(2)}, \ldots, b^{(N)}\right) \in \Delta(\mathcal{M})$ is the distribution of the message that is sent to the winner after the auction—i.e., the bidder who submitted $b^{(1)}$. Each message $m \in \mathcal{M}$ is an $(N-1) \times 1$ vector. Representing each message by $\boldsymbol{m} = (m_2, m_3, \ldots, m_N)$, the natural convention is that m_k is a "message" about $b^{(k)}$, although this formulation admits more general interpretations.

There are two implicit assumptions on the set of admissible disclosure policies. First, the message received is independent of the highest bid. This eliminates any strategic concerns of firms choosing their bids to influence the message that a firm receives upon winning. In terms of information provision, this restriction implies that the message does not carry any information on the winning bid, but this is without loss of generality because the message is meant for only the winning firm, which clearly knows its own bid. Second, the message carries information only about the values of the losing bids but not information about the identity of the firm that places each bid—this is natural because the firms are ex ante identical, and I focus solely on symmetric equilibria.

The modifications to the baseline model are as follows: at the start of Stage 1, the auctioneer announces the disclosure policy D. Subsequently, in Stage 4, instead of observing all the losing bids, the winner now receives only a message from the auctioneer according to D. Upon receiving the message, the winning firm then updates its belief about the values of the bids submitted by the other firms using Bayes' rule before making its drilling decision in the subsequent stage.

In Proposition 4 of Appendix A, I characterize the necessary and sufficient conditions for a monotonic function $\beta(\cdot)$ to be an equilibrium for a given disclosure policy D. Furthermore, when a monotonic bidding strategy equilibrium exists, the tradeoff highlighted in the discussion of Proposition 2 still holds. Therefore, such an equilibrium will yield a weakly lower expected bid revenue for the auctioneer than in the FD auction.

5 Identification and Estimation

In my data, the winning firm observes all losing bids, i.e., an FD auction. In this section, I discuss how the parameters of the model in Section 4.1 are identified given the available data. I first introduce three additional empirically relevant features that were not included in the theoretical model. I then discuss the identification and estimation strategies.

5.1 Additional Empirical Features

First, in my data, a lease can produce both oil and gas. Therefore, I decompose the oil and gas quantities in each tract into an oil quantity, Q^o (measured in barrels), and a gas quantity, Q_g (measured in MCF). The total value of each tract is given by

$$Q = Q^o P^o + Q^g P^g,$$

where P^o and P^g are the prices of oil and gas, respectively. I choose P^o and P^g to be the annual average offshore U.S. Gulf Coast crude oil first purchase price and the Henry Hub spot price in the year of the auction, respectively.

Second, I introduce two additional cost components to account for development costs, which are costs that must be incurred after exploration if oil/gas is found to extract these resources. These additional cost components are $0 \leq \delta, \kappa \leq 1$, which are known to all bidders prior to the auction and are objects to be identified. δ represents yearly development costs such as maintenance and operation costs, and κ is an upfront development cost. Therefore, if the firm engages in exploratory drilling after winning the auction with a bid of b and observing that the exploration cost is $C = c$ and the production value is $Q = q$, its profit is $(\delta(1-r) - \kappa)q - c - b$, where $\delta(1-r) - \kappa$ is assumed to be positive.

Third, I assume that there exists a cost shifter $\mathcal{I} > 0$ observed by bidders prior to the auction such that $C = C^0\zeta(\mathcal{I})$ where $C_0 > 0$ is a random variable with full support and with distribution $F^{C^0}(\cdot)$, and $\zeta : \mathbb{R}_+ \to \mathbb{R}_+$ is a differentiable and nonconstant function. Since it is not possible to distinguish the scale of $\zeta(\cdot)$ from the scale of C^0, I also normalize $\zeta(\mathbb{E}(\mathcal{I})) = 1$. Conditional on $\mathcal{I} = \iota$, the upfront development cost is now $\kappa(\iota)$ where $\kappa : \mathbb{R}_+ \to \mathbb{R}_+$ is differentiable.

Assumption 2 *C^0, Q, and $\mathcal{I}$ are pairwise independent.*

Therefore, $\mathcal{I}$ is a tract-level observed variable that affects the exploration and upfront development costs but does not carry any information about the tract's production value Q. C^0 represents the aspect of the exploration cost whose distribution F^{C^0} is common across all tracts. Henceforth, I refer to C^0 as the *base* exploration cost.

In my estimation, I construct an index that captures the lagged rig rental costs and use it as the cost shifter $\mathcal{I}$. Lagged rig rental costs are useful predictors of future rig rental costs, which are an important component of the exploration costs. To construct $\mathcal{I}$, I first examine the rig contracts that were signed within one year of and prior to a tract's auction date. Since a rig contract is signed between a lease operator (an oil/gas company) and a rig owner, for each rig contract, I identify all leases with the same operator and drilling activities during the contract's duration. The set of these leases are called matching leases. I then compute $\mathcal{I}$ as the average rig rental costs of these prior contracts, weighted by the distance between the tracts of matching leases to the tract being considered. Because this cost shifter reflects the rig market condition at the tract's location before the auction occurs, a higher $\mathcal{I}$ is intuitively associated with a lower exploration rate. Table 20 in the Appendix shows that this is indeed the case, providing evidence that this constructed cost shifter is relevant.

5.2 Identification

To facilitate exposition, I discuss identification while setting aside auction and tract heterogeneity. The objects of interest are $F^{\boldsymbol{S}}(\cdot), F^{C^0}(\cdot), V(\cdot), \kappa(\cdot), \zeta(\cdot)$ and δ. I observe the joint distribution of bids conditional on the cost shifter and royalty rate $F^{\boldsymbol{B}}(\cdot|\mathcal{I}, r)$ and the probability of drilling $\Pr(d = 1|\boldsymbol{B}, \mathcal{I}, r)$.

First, note that it is without loss of generality to normalize the marginal distribution of S_i to be $U[0, 1]$ for all i.[10] Let $F^B(\cdot)$ denote the marginal distribution of a firm's bid. Under this normalization, together with the symmetry assumption and that firms are playing $\beta^{FD}(\cdot)$,

[10]By Sklar's theorem, by normalizing the signals to be uniformly distributed, the joint distribution that we identify is the copula of the true joint distribution.

which is strictly increasing, firm i's signal can be computed as follows

$$S_i = F^B(B_i|\mathcal{I}, r)$$

Therefore, $F^{\boldsymbol{S}}(\cdot)$ is identified. Using the first-order inversion of Guerre et al. (2000), $\Omega(s, \mathcal{I}, r) := v(s, s|\mathcal{I}, r)$ is also identified.[11]
Next, conditional on $\boldsymbol{S} = \boldsymbol{s}$, let $\Delta(\boldsymbol{s}, \mathcal{I}, r) := \Pr(d = 1|\boldsymbol{s}, \mathcal{I}, r)$ and $\mathcal{Z}(\mathcal{I}, r) := \frac{\delta(1-r)-\kappa(\mathcal{I})}{\zeta(\mathcal{I})}$. Therefore:

$$\Delta(\boldsymbol{s}, \mathcal{I}, r) = F^{C^0}\left(\mathcal{Z}(\mathcal{I}, r)V(\boldsymbol{s})\right) \tag{10}$$

Conditional on $S = s$ and $Y = s$, using integration by parts on $v(s, s|\mathcal{I}, r)$, it can be shown that:

$$\Omega(s, \mathcal{I}, r) = \begin{cases} \mathcal{Z}(\mathcal{I}, r)\zeta(\mathcal{I}) \int_0^s \int_{\mathbf{0}}^{\tilde{\boldsymbol{z}}} \Delta\left((s, s, \tilde{\boldsymbol{z}}), \mathcal{I}, r\right) \frac{\partial}{\partial \tilde{\boldsymbol{z}}} V\left((s, s, \tilde{\boldsymbol{z}})\right) d\boldsymbol{z} dG^{\boldsymbol{Z}}(\boldsymbol{z}|s, s) \ , & \text{if } N > 2 \\ \mathcal{Z}(\mathcal{I}, r)\zeta(\mathcal{I}) \int_0^s \Delta\left((s', s'), \mathcal{I}, r\right) \frac{\partial}{\partial s'} V\left((s', s')\right) ds' \ , & \text{if } N = 2 \end{cases} \tag{11}$$

Proposition 3 *$F^{C^0}(\cdot)$ and $\zeta(\cdot)$ are identified. $V(\cdot)$, $\kappa(\cdot)$, and δ are identified up to a multiplicative constant.*

The proof of Proposition 3 is in Appendix B. Intuitively, $F^{C^0}(\cdot)$, $V(\cdot)$, $\zeta(\cdot)$, $\kappa(\cdot)$, and δ must satisfy both (10) and (11). The left-hand sides of both (10) and (11) are already identified from the data. Conditional on r, the slope of the level curves of $\Delta(\cdot, \cdot, r)$ is the same as the ratio of the derivative of the logs of $V(\cdot)$ and $\mathcal{Z}(\cdot, r)$. Because of the separability of $V(\cdot)$ and $\mathcal{Z}(\cdot, r)$, the logs of $V(\cdot)$ and $\mathcal{Z}(\cdot, r)$ are identified up to an affine transformation. They are further pinned down up to an additive constant using (11). Therefore, $V(\cdot)$ and $\mathcal{Z}(\cdot, r)$ are identified up to a multiplicative constant. From here, I identify F^{C^0} using the slope of $\Delta(\cdot, \mathcal{I}, r)$.

Assumption 3 *The mean of the prior belief $\int_{supp(Q)} q f^Q dq$ is observed by the econometrician.*

[11]Using the first-order inversion, conditional on $S = s$:

$$v(s, s|\mathcal{I}, r) = F^S(s|\mathcal{I}, r) + \frac{G(s|s, \mathcal{I}, r)}{g(s|s, \mathcal{I}, r)}$$

Corollary 1 *Under Assumption 3, $V(\cdot)$, $\kappa(\cdot)$, and δ are identified.*

Because the expected value of the posterior mean is always equal to the prior mean by Bayes' theorem, Assumption 3 provides one way to pin down the multiplicative constant in Proposition 3 by setting $\mathbb{E}(V(\cdot))$ to be equal to a known number. In my empirical application, I use Assumption 3 and set the prior mean to be $\mu_{Bernoulli,oil}\mu_{Gamma,oil}P^o + \mu_{Bernoulli,gas}\mu_{Gamma,gas}P^g$, using the variables constructed in Section 3. Further details are provided in the next section 5.3.

Much of my identification strategy relies on the variation in the drilling probability across bids and the presence of the cost shifter $\mathcal{I}$. This argument differs from the standard identification argument in the auction literature for two reasons. First, the standard method of identifying the joint distribution between the private signals and the latent value Q requires Q to be observed and utilizes the joint distribution of bids and Q.[12] However, in my setting, Q is not observed when exploration does not take place. Second, for my counterfactual purposes, the primary objects that need to be identified are $V(\cdot)$ and $F^C(\cdot)$ (see, for example, the equilibrium bidding strategy $\beta^{ND}(\cdot)$). Even when Q is always observed, it is not possible to identify $F^C(\cdot)$ based only on the bid information without utilizing observed drilling decisions.

5.3 Estimation

As discussed in Section 3, the main source of observed heterogeneity between tracts arises from the spatial correlation in oil and gas deposits. For example, a tract located near a productive tract is more likely to be productive. This heterogeneity affects the distribution of the prior belief $F^Q(\cdot)$, which translates into heterogeneity in $V(\cdot)$. As stated in Section 3, I assume that the heterogeneity in $F^Q(\cdot)$ can be summarized by the heterogeneity in $(\mu_{Bernoulli,oil},\ \mu_{Bernoulli,gas},\ \mu_{Gamma,oil},\ \mu_{Gamma,gas})$. Therefore, the object of interest is now $V(\cdot|\mu_{Bernoulli,oil},\ \mu_{Bernoulli,gas},\ \mu_{Gamma,oil},\ \mu_{Gamma,gas})$. However, due to the limit in sample size, it is not possible to nonparametrically incorporate $(\mu_{Bernoulli,oil},\ \mu_{Bernoulli,gas},\ \mu_{Gamma,oil},\ \mu_{Gamma,gas})$ into $V(\cdot)$. Thus, I parameterize $V(\cdot)$ for estimation purposes as follows.

Let F^{Q^o} (F^{Q^g}) denote the distribution characterized by $\mu_{Bernoulli,oil}$ ($\mu_{Bernoulli,gas}$) and $\mu_{Gamma,oil}$ ($\mu_{Gamma,gas}$) constructed in Section 3. The functional form of $V(\cdot)$ is microfounded as follows. For each tract, the joint distribution of $(S_1, S_2, ..., S_N, Q^o, Q^g)$ has a Gaussian

[12] For a review of the literature, see Athey and Haile (2007)

copula.

$$F^{\boldsymbol{S},Q}(S_1,S_2,...,S_N,Q) = \Phi_R\left(\Phi^{-1}\left(F^S(S_1)\right),...,\Phi^{-1}\left(F^S(S_N)\right),\Phi^{-1}\left(F^{Q^o}(Q^o)\right),\Phi^{-1}\left(F^{Q^g}(Q^g)\right)\right)$$

where Φ_R is the joint CDF of a multivariate normal distribution with mean 0 and correlation matrix R and Φ^{-1} is the inverse of the CDF of a standard normal distribution. Since the bidders are ex ante symmetric, R must satisfy the following constraints:

$$\begin{cases} R_{ij} = R_{ik} & \text{for } i,j,k \leq N \\ R_{i,N+1} = R_{j,N+1} & \text{for } i,j \leq N \\ R_{i,N+2} = R_{j,N+2} & \text{for } i,j \leq N \end{cases}$$

R_{12} is thus the correlation between two bidders' private signals. To satisfy the assumption that the signals are affiliated, R_{12} must be weakly positive. $R_{1,N+1}$ and $R_{1,N+2}$ indicate the degree of correlation between the bidders' signals and the latent value of oil and gas, respectively.

The conditional distribution of $\left(\Phi^{-1}\left(F^{Q^o}\left(Q^o\right)\right),\Phi^{-1}\left(F^{Q^g}\left(Q^g\right)\right)\right)|\boldsymbol{S}$ is thus

$$\mathcal{N}\left(\begin{bmatrix} M^o\sum_i S_i \\ M^g\sum_i S_i \end{bmatrix},\Sigma_Q\right) \tag{12}$$

where

$$M^o = \frac{R_{1,N+1}}{R_{11}+R_{12}}, \quad M^g = \frac{R_{1,N+2}}{R_{11}+R_{12}}$$
$$\Sigma_Q = \begin{bmatrix} R_{N+1,N+1} & 0 \\ 0 & R_{N+2,N+2} \end{bmatrix} - \frac{N}{R_{11}+R_{12}}\begin{bmatrix} R^2_{1,N+1} & R_{1,N+1}R_{1,N+2} \\ R_{1,N+1}R_{1,N+2} & R^2_{1,N+2} \end{bmatrix}$$

The distribution specified in (12) then fully determines $V(\cdot)$. M^o and M^g quantify the marginal effects of the signals on the posterior belief about oil and gas quantities, respectively. Conditional on the prior belief $F^Q(\cdot)$, the larger M^o and M^g are, the more informative the signals. The precision of the distribution in (12) increases as the number of bidders increases. This represents the informational advantage of a winning bidder in an auction with more bidders because it receives more information ex post, thus having a more precise posterior belief.

My estimation procedure is conducted in two steps. In the first step, I use kernel estimation to estimate the CDF of the bids, which then yields the estimated private signals of the bidders. Because $V(\cdot)$ varies with N due to the informational advantage of having more bidders, in this step, the kernel estimation is done on 6 separate samples, which are categorized based on the number of bidders (2 bidders, 3 bidders, greater than 3 bidders) and the tract's depth (shallow tracts (less than 400m deep) and deep tracts (at least 400m deep)).[13] For each subsample, I follow the method of Haile et al. (2003) to 'homogenize' the bids to account for the following auction-level observed heterogeneity variables: year fixed effects, royalty rate, and tract's water depth. To implement Haile et al. (2003)'s approach, I first regress the log of the observed bids on year fixed effects and a fifth-order spline of water depth for each subsample with the same royalty rate. The residuals are then treated as the log of the bids of auctions with homogeneous observed characteristics, conditional on $\mathcal{I}$ and $F^Q(\cdot)$. The homogenized bids preserve the optimality of the original bids when the observed heterogeneity commonly affects both the exploration cost and the tract's expected production value $V(\cdot)$ in a multiplicatively separable manner. I also make a simplification assumption that the prior mean is a sufficient statistic for $F^Q(\cdot)$. I then estimate the marginal distribution of the a firm's bid $F^B(B|\mathcal{I}, \mathbb{E}(Q))$ using the Gaussian kernel and least-squares cross-validation method to select bandwidths (Li et al., 2013). The estimated $F^B(\cdot)$ yields estimates of $\boldsymbol{S}$.

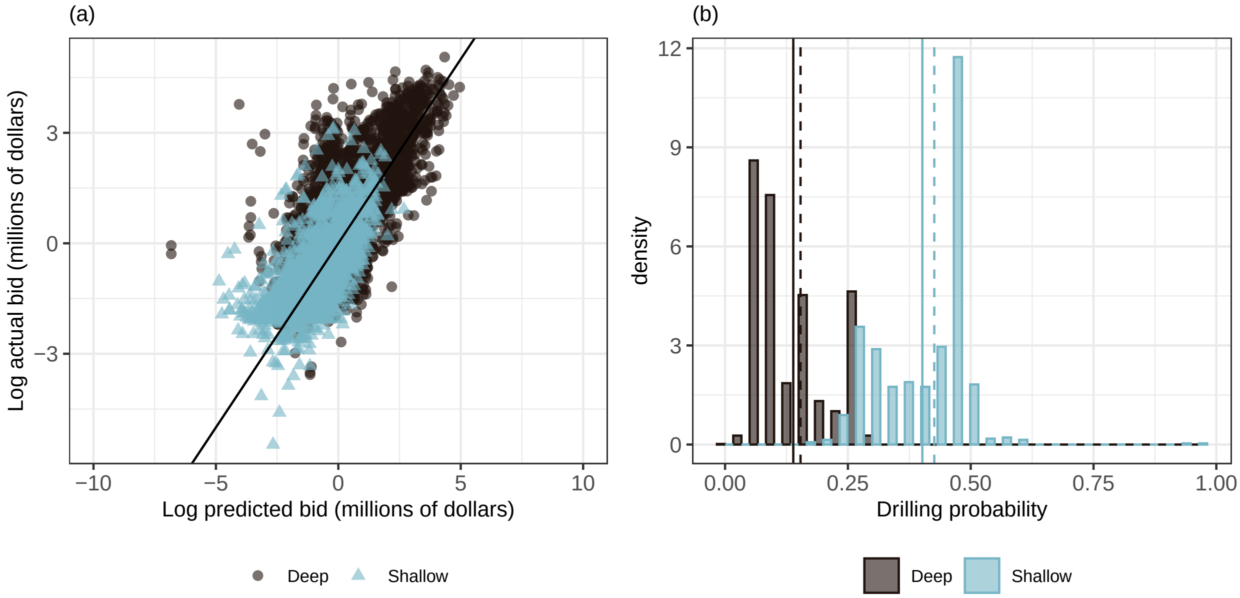
In the second step, the equilibrium bidding strategies $\beta^{FD}(\cdot)$ and the expected drilling probability before C is realized are computed for each draw of the model's parameters. I then estimate the parameters of the model by minimizing the sum of the squared difference between the predicted bids and the actual bids and the squared difference between the predicted drilling probabilities and the actual drilling outcomes. The standard errors are bootstrapped based on 100 iterations.

5.4 Estimation results

My model fit is summarized in Figure 6. In Panel (a), I plot the predicted bids, computed using the equilibrium bidding strategy under FD (Proposition 1), against the actual bids. The predicted median bids are \$339K and \$698K for shallow and deep tracts, which are close to the observed medians in the data of \$335K and \$850K. In Panel (b), I plot the distribution of the predicted drilling probability for deep and shallow tracts, which also fits well with the data. The average predicted drilling probabilities are 0.4 and 0.14 for shallow and deep tracts, similar to the average drilling rates of 0.43 and 0.15 in the data.

[13] The sample sizes of each sub-samples are listed in Table 23

Figure 6: Model fit



Note: The solid black line in panel (a) is the 45-degree line that goes through the origin. In panel (b), the solid lines are the mean of the fitted drilling probabilities, and the dashed lines are the average drilling rates in the data.

Table 7 summarizes the parameters that measure the informativeness of the bidders' private signals (M^o and M^g in (12)) and the correlation between these private signals (R_{12} in (12)). The correlation between bidders' signals is significantly nonnegative, thus implying that the bidders' signals are indeed affiliated. The estimated correlations are 0.05 for deep tracts and 0.15 for shallow tracts, which are significantly smaller than 1. Therefore, there is substantial uncertainty about other bidders' private information, which suggests that bid disclosure policies may be an effective means of influencing a bidder's posterior belief.

The interpretations of M^o and M^g are as follows. Conditional on $S_i = 0.5\ \forall i$, the posterior mean at the realized signal $V(\boldsymbol{S})$ is the same as the median of the prior belief. If one firm's signal increases to $S_i = 1$, the posterior mean conditional on the realized signals increases to the 76% percentile (51% percentile) of the prior distribution for oil (gas) for deep tracts. For shallow tracts, the increases are to the 85%-percentile (56%-percentile) of the prior distribution for oil (gas). My estimates imply that firms' private signals carry less information about gas than oil because most of the tracts explored during the data period are primarily for oil production.[14]

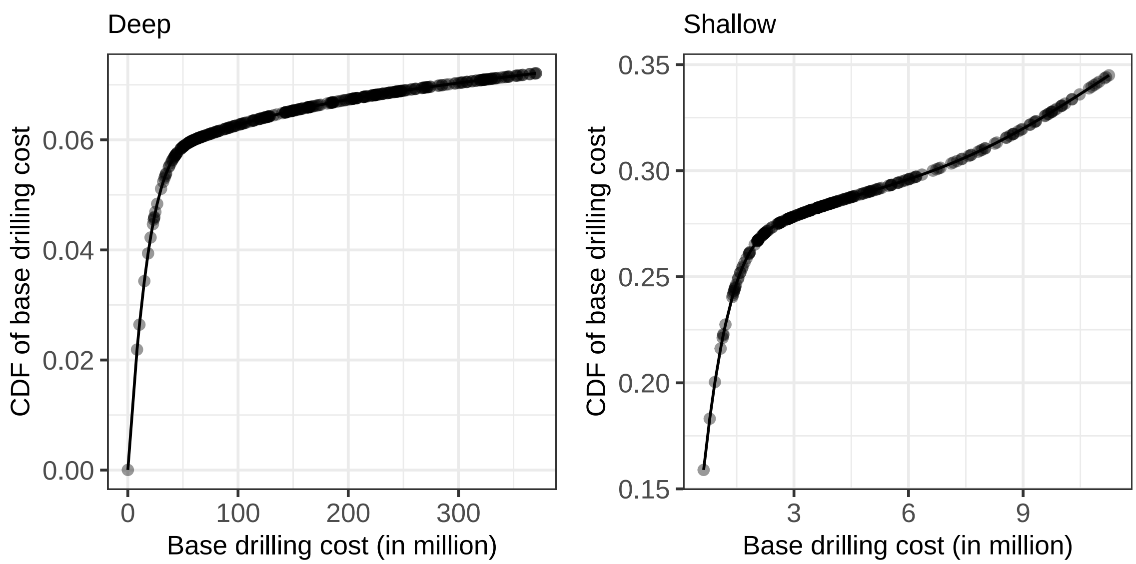
[14] Much of the natural gas production in the Gulf of Mexico is from shallow tracts that were explored before my data period (Bureau of Ocean Energy Management, 2019).

Table 7: Estimated parameters

Tract's depth	$Corr(S_i, S_j)$	Marginal effect of S on conditional mean of Q^o (M^o)	Marginal effect of S on conditional mean of Q^g (M^g)
Deep	0.05	1.41	0.03
	(0.02)	(0.02)	(0)
Shallow	0.15	2.09	0.32
	(0.04)	(0.45)	(0.07)

Note: $Corr(S_i, S_j)$ is the correlation between signals S_i in (12). The values in the second and third columns (M^o and M^g) are the increase in the expected value of oil and gas of the tract as S_i increases, measured as a percentile of the distribution of the posterior mean. Standard errors are bootstrapped.

Figure 8: Illustration of the shape of the CDF of the base drilling cost F^{C^0}



Note: Each dot depicts the value of a tract's drilling probability at the estimated realized signals on the CDF of the base drilling cost F^{C^0}, computed at a randomly chosen draw. The range of the cost estimates is similar to industry estimates. For example, the cost of a deep exploration well in the Gulf of Mexico in 2018 is said to be between 30 to 60 million on average (Sandrea and Stark, 2020), and prior to 2013 it was twice as expensive. For shallow exploration wells drilled after 1998, the average per-meter depth cost is about $25,000/meter (Kaiser, 2021), which translates into less than $10 million for shallow (less than 400m, as defined by the BOEM) wells.

Figure 8 illustrates the estimated shape of the CDF of the base drilling cost C^0 across deep and shallow tracts, evaluated at a random draw of the bootstrapped estimates. I make two observations. First, the estimated CDFs are significantly nonlinear. The p values of the Kolmogorov-Smirnov test against a linear function for F^{C^0} are less than 0.01 for both deep and shallow tracts. As explained in Section 4.2, a bid disclosure policy can affect the auctioneer's revenue only if the CDF of the exploration cost is nonlinear in the support of the posterior mean. Second, the CDF of the base exploration costs is weakly concave for deep tracts, but for shallow tracts, the cost distribution is neither concave nor convex.[15] Therefore, an ND policy is expected to weakly increase the drilling probability for deep tracts, but it is not theoretically clear whether an ND policy will improve the expected drilling probability for shallow tracts.

6 Counterfactual

In the counterfactual exercises, I study the effect of alternative bid disclosure policies on the bid revenue, drilling rate, and auctioneer revenue. I also consider the interaction between the royalty rate and disclosure policy. In these exercises, I fix the realizations of the bidders' private signals as estimated from the data, and the royalty revenue is computed based on the expected value of the tract conditional on the realized signals.

I first discuss the effect of the ND policy in Section 6.1, followed by two partial disclosure policies in Section 6.2.2 and Section 6.2.1. I then consider the potential changes in firms' entry decisions in Section 6.3 when the entry cost is modeled as a signal acquisition cost.

6.1 Nondisclosure Policy

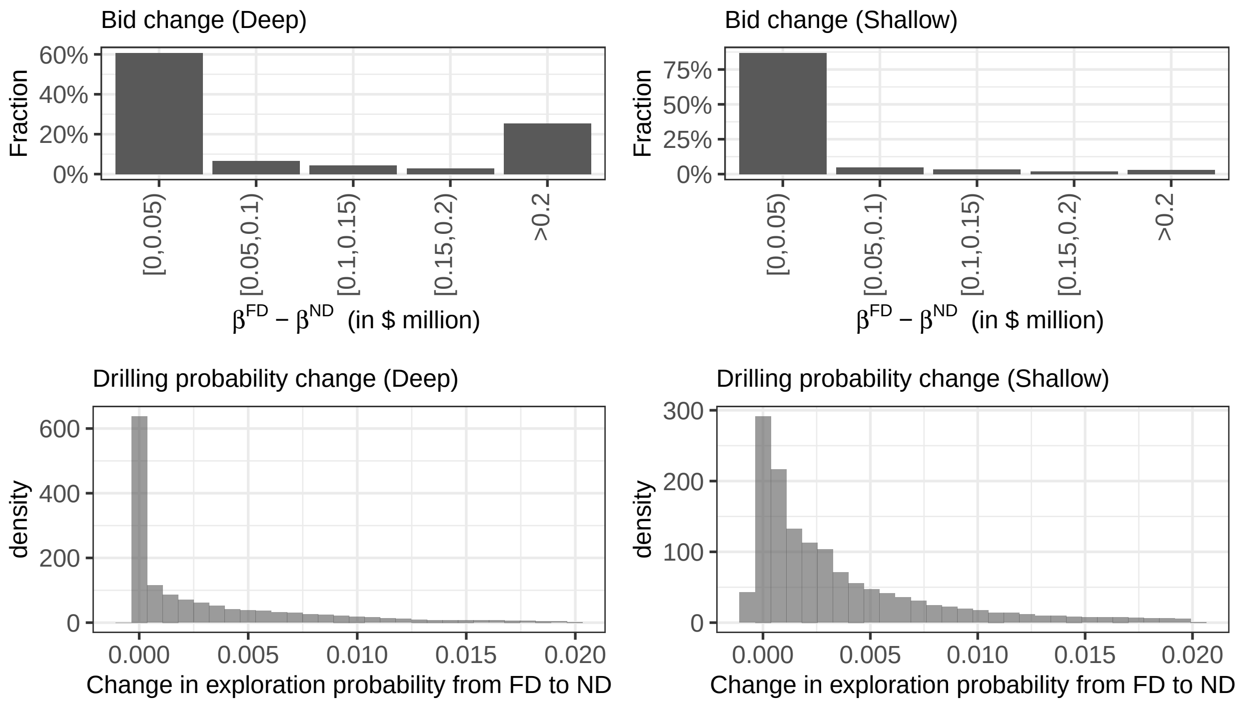
For each auction, I compare the drilling probability between the ND auction and the FD auction at the end of Stage 3, i.e., before the drilling cost is realized. Under ND, this drilling probability is determined when a bidder receives its own private signal S_i because it does not receive any further information ex post. Under FD, I compute the expected drilling probability before the losing bids are transmitted to the winning bidder.

Figure 9 shows the distribution of the changes in the drilling probability (upper panels) and the bid revenue (lower panels) across tracts. As shown in Proposition 2, the ND policy results in lower equilibrium bids. For shallow tracts, the decrease is small for most tracts. For deep tracts, the distribution of the bid decrease is bimodal because of the large right tail of the bid distribution.[16] Since $F^{C^0}(\cdot)$ and thus $F^C(\cdot)$ is concave for deep tracts, the ND

[15]To test these hypotheses, I compute the 95% bootstrapped confidence interval of the minimum of the second derivative of F^{C^0} for each type of tract.

[16]The average bid for the tracts with a more than \$0.2M drop in bids has an average bid under FD of \$16.63M. As a comparison, the average bid under FD of tracts with between a \$0.15M to \$0.2M drop in bids is \$4.7M.

Figure 9: Distribution of the changes (per tract) in the drilling probability and the winning bids from FD to ND



Note: The changes in the drilling probability and the bid revenue are computed using the mean of the estimated parameters. The standard errors are not reported here. The bid revenue is computed at the realized signal of the highest bidder in the data and is in millions of dollars. The probability of drilling is between 0 and 1, and a positive change implies that an ND auction yields a higher expected drilling probability.

policy increases the drilling probabilities for all tracts. On the other hand, for shallow tracts, there is a small portion (6%) of tracts with a decrease in the drilling probability because $F^{C^0}(\cdot)$ (and thus $F^C(\cdot)$) is not everywhere concave for shallow tracts. Overall, the higher exploration rate under an ND policy results in an average royalty increase of \$1.41M per tract for deep tracts and \$0.63M for shallow tracts, significantly larger than the drop in bid revenue (Table 10). The large impact of the ND policy on the royalty revenue arises from the heterogeneity in the effects of the ND policy across tracts. Specifically, the increase in the drilling probability is higher for more productive tracts (Figure 25 in the Appendix).

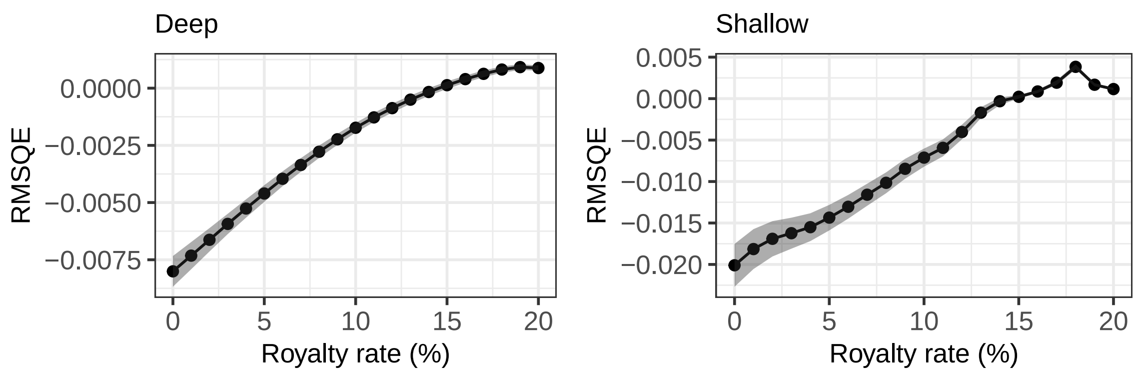
Welfare discussion In addition to increasing the auctioneer's revenue, the counterfactual estimates also indicate that the ND policy does not worsen social welfare (Figure 11). In the scope of the model, under the FD policy, the incentive misalignment between the auctioneer and the winning bidder arises due to the presence of the positive royalty rate. This is because

Table 10: Change in the bid and royalty revenues (per tract) from FD to ND (millions of dollars)

	Δ Bid	%Δ Bid	Δ Royalty (undiscounted)	Δ Royalty (discounted)*	% Δ Royalty
Deep	-0.3	-0.03	3.16	1.41	2.96
	(0.05)	(0.04)	(0.64)	(0.28)	(1.73)
Shallow	-0.03	-0.02	1.41	0.63	1.52
	(0.01)	(0.00)	(0.34)	(0.15)	(0.35)

Note*: To compute the discounted value, I assume a discount factor of 0.9 and that the royalty revenues are received in equal payments over 20 years. The numbers in the parentheses are bootstrapped standard errors.

Figure 11: Difference in welfare between a full-disclosure and a non-disclosure policy



Note: I measure welfare as the squared root of the mean-squared error between the first-best drilling rate and the drilling rate under a disclosure policy (FD or ND) at a given royalty rate. The y-axis in this figure is the difference in welfare between an ND and an FD policy where a positive value signifies welfare gain. The shaded area is the 95% bootstrapped confidence interval.

the royalty rate is a tax on production *revenue*, which induces the winning bidder to drill less than the socially optimal drilling level. Thus, the first-best drilling rate is defined as the drilling probability under FD but without any royalty. In this exercise, I measure welfare as the mean squared error between the first-best drilling probability of each tract and the drilling probability with a positive royalty rate under a disclosure policy (FD or ND). Figure 11 shows the difference in welfare between FD and ND at different levels of royalty rate r. These estimates show that at a high level of royalty rates, the ND policy is able to offset some of the distortions from the positive royalty rate, resulting in an overall welfare gain. At the current royalty rates in the data (approximately 12% across all tracts), the ND policy does not have a large impact on welfare for deep tracts because it causes only a 0.94% increase in the distortion. However, for shallow tracts, the ND policy reduces the distortion from the positive royalty rate by 29.15%.

6.2 Partial-disclosure Policy

Next, instead of considering the winner of the auction either observing all the submitted bids or not observing anything, I allow the auctioneer to commit to more general types of bid disclosure policies.

As explained in the ND analysis, switching away from the FD policy presents a tradeoff between the bid revenue and the royalty revenue. On the one hand, the ND policy decreases the bid revenue. On the other hand, the ND policy increases the expected royalty revenue. It is then natural to consider whether some intermediate policy, i.e., a partial disclosure policy, can perform better than the ND policy. A partial disclosure policy might outperform the ND policy for two reasons. First, the distribution of the drilling cost $F^C(\cdot)$ for shallow tracts is not concave; therefore, for shallow tracts, a partial disclosure policy might yield the highest expected drilling probability. Second, a partial disclosure policy might have a less adverse impact on the bid revenue than the ND policy.

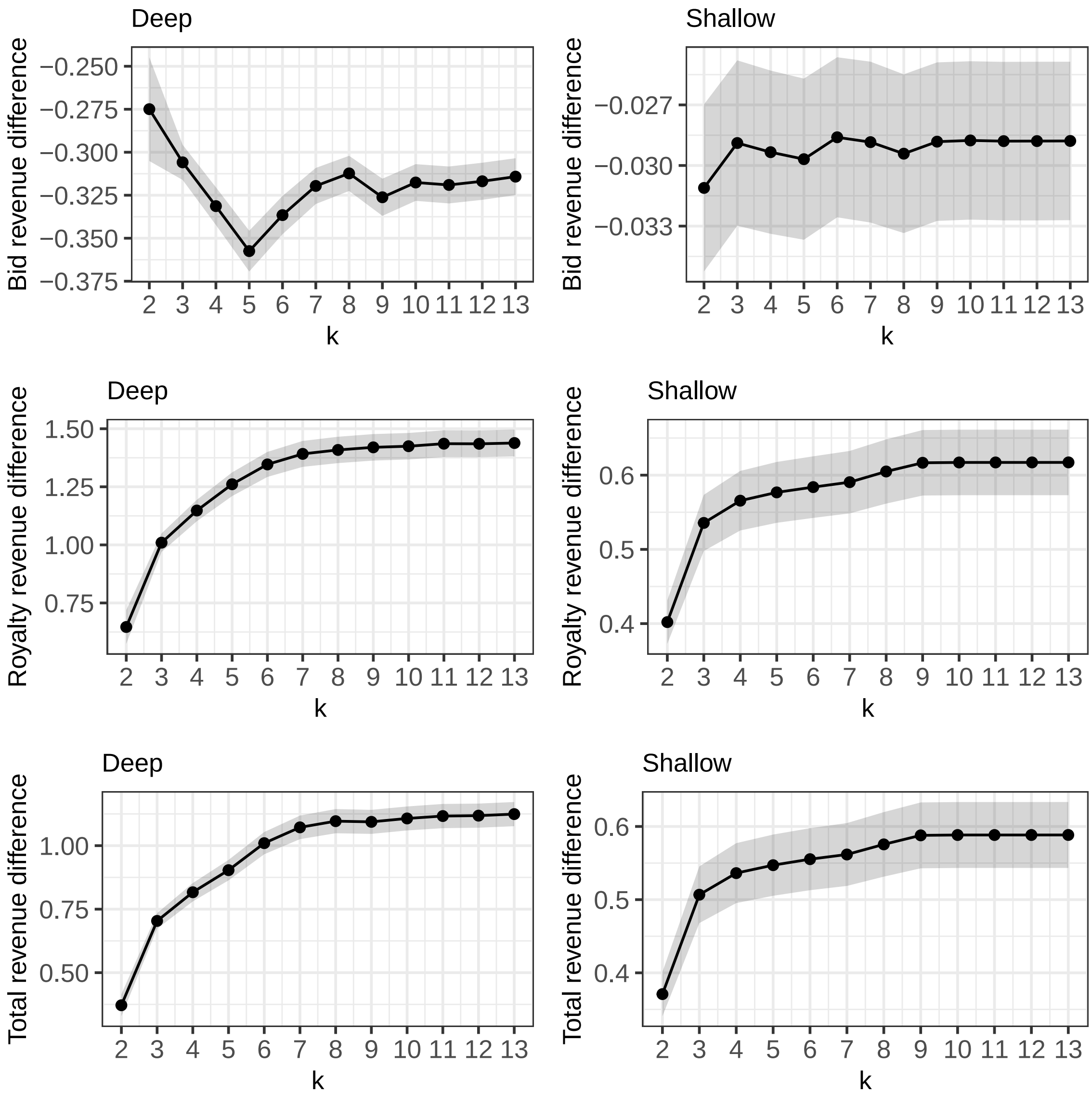
6.2.1 Partial disclosure: Withhold-k policy

In this exercise, I consider a partial disclosure policy in which the auctioneer announces that the k highest bids, not including the winning bid, are not revealed. For example, when $k = 4$, the second, third, and fourth highest bids are not revealed. Henceforth, I refer to this disclosure as the *withhold-k* policy. Such policy has been implemented in other US auction settings such as in failed financial institution auctions in which the information on the second-highest bid is withheld ($k = 2$).[17] The ND policy is a special case in which $k = N$. Proposition 5 in Appendix A shows that the withhold-k policy has a symmetric and strictly increasing bidding strategy.

The revenue estimates of withhold-k policies are depicted in Figure 12. I make two observations. First, the relationship between the amount of information withheld, which is represented by the magnitude of k, and the bid revenue is nonmonotonic. For example, for deep tracts, the policy that yields the lowest bid revenue is when $k = 5$. Second, on average, the withhold-k policy that yields the highest revenue for the auctioneer is still the ND policy. As a comparison, withholding only the second-highest bid achieves 33.07% (63.04%) of the revenue increase from the ND policy for deep (shallow) tracts. When the optimal withhold-k

[17] For further details on the bid disclosure policy of failed financial institution acquisition auctions, see `https://www.fdic.gov/resources/resolutions/bank-failures/failed-bank-list/biddocs.html`

Figure 12: Effect on the per-tract revenue of a withhold-k policy relative to the FD benchmark (in millions of dollars)



Note: When $k = 2$, only the second highest bid is not disclosed. When $k = 3$, the second and the third highest bid are not disclosed. The revenue on the y-axis is the revenue relative to the FD revenue. As $k \to N$, the bid revenue, the royalty revenue, and the total revenue all converge to those under the ND policy. In the computation of the royalty revenue, the government's discount factor is assumed to be 0.9, and the royalty revenue for each tract is paid in equal payments over 20 years. The gray band is the 95% bootstrapped confidence interval.

policy for each tract is considered (Figure 26 in the Appendix), the ND policy is optimal for

93% of deep tracts and 97% of shallow tracts.

6.2.2 Partial disclosure: Threshold disclosure

Another interesting disclosure policy that is often studied in the context of information design is a threshold disclosure policy.[18] In this exercise, I consider the auctioneer's revenue when the bid disclosure policy reveals the bids only if they are below a threshold α. When $\alpha = 0$, this is equivalent to the ND policy. When $\alpha = \infty$, this is the same as the FD policy. Proposition 6 in Appendix A establishes that a symmetric and increasing bidding strategy exists under a threshold disclosure policy.

To simplify the comparison between tracts, the threshold for each tract is normalized as follows. For a given value of $\alpha \in [0, 1]$, the threshold value of each tract is the α quantile value of the bid distribution of the same tract under the FD policy. For example, at $\alpha = 0.5$, the threshold of each tract is the median value of the equilibrium FD bid distribution of the tract. At $\alpha = 0$, this is now equivalent to the ND policy. At $\alpha = 1$, this is now equivalent to the FD policy.

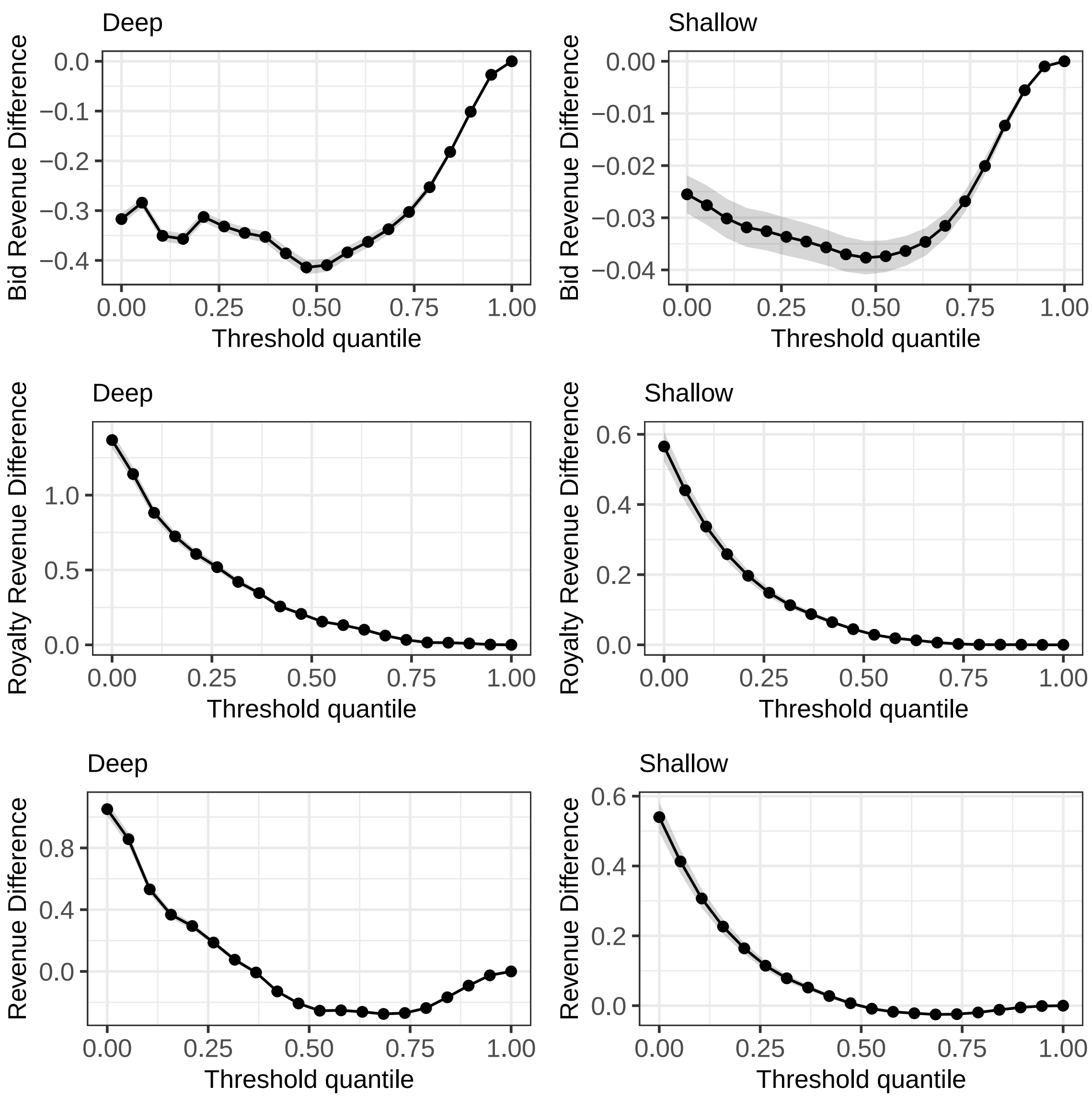
Figure 13 shows the revenue composition per auction for deep and shallow tracts under threshold disclosure policies for different values of α. Similar to the analysis in Section 6.2.1, here, the ND policy still yields the highest total revenue for both deep and shallow tracts among the threshold disclosure policies being considered, and choosing a larger threshold (i.e., when more information is revealed) does not always result in an increase in bid revenue. Furthermore, an intermediate value of α has a detrimental effect on revenue for the auctioneer. For example, choosing $\alpha = 0.5$ decreases total revenue per tract by \$0.25M for deep tracts and \$0.01M for shallow tracts relative to the FD benchmark.

6.3 Entry and Information Acquisition

In this section, I consider an extension where the number of bidders in the auction is endogenously determined in an entry game. To simplify the analysis, I focus only on the comparison between the FD policy and the ND policy. Because the ND policy reduces the value from winning the auction for the winning bidder, one might expect that bidders might have less incentive to participate in the auction, thereby reducing the number of bidders and potentially the auctioneer's revenue.

[18] A threshold disclosure policy is interesting because it belongs to the class of monotone partitional disclosure policies. Such disclosure policies have been shown to be optimal in many contexts in the theoretical literature (Dworczak and Martini, 2019). However, these theoretical results do not apply here.

Figure 13: Effect on the per-tract revenue of threshold disclosure policies relative to the FD benchmark (in millions of dollars)



Note: the x-axis of this graph is the α quantile of the equilibrium bid distribution under FD. For example, for $\alpha = 0.5$, the threshold being considered is the same as the median of the bid distribution for a particular tract under FD. The government's discount factor is assumed to be 0.9, and the royalty revenue for each tract is paid in equal payments over 20 years. The gray band is the 95% bootstrapped confidence interval.

I consider an entry game similar to that of Hendricks et al. (2003). The timeline is as follows. Before the auction, each potential bidder, among the set of $\bar{N}$ bidders,[19] draws a private signal acquisition cost η, where η is iid across bidders and η is drawn from a log-normal distribution. If a bidder pays the signal acquisition cost, its private signal S is then realized, and I assume that S and η are independent. Only bidders that obtain the signals can participate in the auction. Before bidding, the number of active bidders N becomes public information.[20] The rest of the game follows as in Section 4.1 (for the FD policy) and Section 4.2 (for the ND policy).

To solve this entry game, it is important to consider the effect of the disclosure policy on a potential bidder's *profit*, which will later determine the equilibrium entry decision. I first note that the ND policy does not necessarily lower a bidder's expected profit. A bidder's profit comprises two components. The first component is the revenue from the tract (taking into account the royalty payment), conditional on winning. Given a winning signal s, the difference in this revenue component between the ND and the FD policies is negative and equal to

$$s\psi\left(\mathbb{E}\left(V\left(s,s'\right)|s'\leq s\right)\right)-s\mathbb{E}\left(\psi\left(V\left(s,s'\right)\right)|s'\leq s\right)$$

The second component is the equilibrium bid. As explained in Section 4.2, the reduction in the bid $(\beta^{ND}-\beta^{FD})$ arises not only from the reduction in the value from winning the auction but also from the winning bidder's inability to observe the second-highest bid. Therefore, it is possible that the reduction in the bids more than offsets the reduction in the expected revenue from the tract, resulting in an *increase* in a bidder's expected profit. In Appendix C, I provide a parametric example to illustrate this point.

In this entry game, a potential bidder will acquire a signal if its realized acquisition cost is below a threshold, and this threshold is determined by (1) its conjecture about other bidders' bidding strategies and (2) the expected profit from bidding in the auction. The main complication of the entry game here is that the threshold strategy might not be unique. This is because the expected profit from bidding in the auction is not necessarily a decreasing function of the realized number of bidders N. Specifically, fixing a disclosure policy, a bidder who wins against a greater number of bidders receives more information after the auction,

[19] I set $\bar{N}$ to be the maximum number of bidders on neighboring tracts—i.e., tracts within 0.11 degrees of latitude and 0.12 degrees of longitude—of the same depth type (shallow or deep). On average, shallow tracts have 9.8 potential bidders, and deep tracts have 11 potential bidders.

[20] The assumption that N becomes public is important in this setting for two reasons. First, when N is unknown, it is possible that a strictly increasing bidding strategy does not exist, and there might be bunching (Lauermann and Speit, 2022). Second, the winning bidder's drilling decision now depends on both its belief about the losing bidders' signals *and* its belief about the number of actual bidders based on the message it receives.

and this informational advantage increases its expected payoff. The equilibrium threshold strategy is unique only if the competition effect, which is a negative effect on profit due to the higher equilibrium bids, dominates the information effect. In the estimation, I verify that the estimates of the auction stage yield a monotonic relationship between a bidder's profit and the number of bidders; therefore, the entry threshold is unique.

Table 14: Estimated entry cost, entry probabilities, and auctioneer's revenue under FD and ND policies

	Deep tracts	Shallow tracts
Distribution of log entry cost		
Expected value (across tracts)	2.57	0.84
	(0.11)	(0.34)
Standard deviation (across tracts)	5.06	4.75
	(0.15)	(0.44)
Average probability of entry under FD	0.25	0.23
	(0.01)	(0)
Average probability of entry under ND	0.25	0.25
	(0.01)	(0.01)
Average auctioneer's revenue under FD	6.51	5.18
	(0.21)	(1.18)
Average auctioneer's revenue under ND	6.59	6.3
	(0.21)	(0.36)

Note: The numbers in parentheses are the bootstrapped standard errors. The revenues under FD and ND are measured in millions of dollars. The distribution of the log entry cost is normalized and computed relative to the homogenized bids.

The estimates and implications of the entry model are reported in Table 14. The parameters of the entry game are the expected value of the distribution of the log entry cost, which is allowed to vary across tracts based on tracts' observed characteristics (as discussed in Section 5.3), and the standard deviation of this distribution, which is assumed to be fixed across tracts. On average, the cost of entry is higher for deep tracts than for shallow tracts, which is consistent with the interpretation that the entry cost is the signal acquisition cost.

The results in Table 14 show that the ND policy does not reduce the average number of (active) bidders in equilibrium. Under ND, the average probability of entry for deep tracts is similar to that under FD and is equal to 25% on average. For shallow tracts, the ND policy increases the probability of entry by approximately 2 percentage points. However, for deep tracts, the ND policy has heterogeneous impacts across tracts. The ND policy tends to increase (decrease) the entry probability for tracts that generate lower (higher) revenue for the auctioneer (Figure 27 in the Appendix). Therefore, the positive impact of the ND policy on deep tracts is dampened when the entry decision is taken into account, and the increase in revenue is approximately $80K per tract. On the other hand, because the probability of entry increases for most shallow tracts under ND, it results in a significant increase in revenue for the auctioneer, approximately $1.12 million per tract on average.

7 Conclusion

In this paper, I empirically study how information on the losing bids should be strategically revealed to the winning bidder in the context of US OCS auctions and quantify the potential revenue gain from such an information mechanism. In an OCS auction, the auctioneer (the government) auctions off the right to extract oil and gas on federal offshore tracts via a first-price sealed-bid auction. In addition to receiving the winning cash bid from the auction, the government also charges a royalty on the tract's production value. After the auction, the winning bidder decides whether to explore the tract, which is a costly decision. Unexplored tracts do not produce oil and gas; therefore, the winning bidder's post-auction action also affects the auctioneer's payoff.

I first construct and estimate a model of a first-price sealed-bid pure common value auction in which the winner also chooses whether to explore the tract after the auction at a cost after observing all the losing bids. I show that by combining the bid data, the variation in the exploration rate across auctions, and an exploration cost shifter, the firms' posterior beliefs on the production value of the tract conditional on all the bidders' private signals are nonparametrically identified.

I then extend the model to the case in which the auctioneer can use an alternative bid disclosure policy. I provide a characterization of the equilibrium bidding strategy for a large class of disclosure policies of the losing bids. Under an alternative bid disclosure policy, the equilibrium bids decrease for two reasons. First, when less than full information is transmitted to the winning bidder, the winning bidder makes more ex post mistakes, which lowers the revenue from winning the auction, resulting in an incentive to lower its bid. Second, when the winning bidder is unable to observe the second-highest bid, the effect on the equilibrium bidding strategy is akin to a decrease in the competitiveness of the auction. On the other hand, an alternative bid disclosure policy might be able to improve the likelihood of exploratory drilling, which can ultimately result in higher royalty revenue. The counterfactual analysis reveals that an alternative bid disclosure policy can significantly improve the government's revenue and offset some of the negative consequences of a positive royalty rate on welfare. Furthermore, revealing no information to the winning bidder does not result in a lower expected profit for the bidders; therefore, the positive findings are robust to when an endogenous entry decision is considered.

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A Equilibrium under General Disclosure Policy

In this section, I provide the theoretical analysis of my model under a general disclosure policy for the bids. This general disclosure policy nests full disclosure (Section 4.1), nondisclosure (considered in Section 4.2 and the partial disclosure policies considered in Section 6. I proceed as follows: I first describe the model in Section A.1. In Section A.2, I analyze the bidding equilibrium and provide necessary and sufficient conditions for an equilibrium in Proposition 4. Subsequently, in Section A.3, I use Proposition 4 to prove Proposition 1 (i.e., equilibrium under full disclosure) and Proposition 2 (i.e., equilibrium under nondisclosure) in the main text. Finally, in Section A.4, I use Proposition 4 to show that the auctions under the partial disclosure policies considered in Section 6 in the main text admit a symmetric and increasing bidding equilibrium.

A.1 General Disclosure Policy

Consider the setup described in Section 4.3. Abusing notations slightly, I also let D denote the Borel probability measure on $\mathcal{M}$. The following are the disclosure policies considered in the main text formulated in the current setup:

(Section 4.1) Full disclosure: for all $\mathbf{b}$,

$$D\left(\left(b^{(2)},\ldots,b^{(N)}\right) \mid \left(b^{(2)},\ldots,b^{(N)}\right)\right) = 1.$$

(Section 4.2) Nondisclosure: for all $\mathbf{b}$,

$$D\left(\mathbf{0} \middle| \left(b^{(2)},\ldots,b^{(N)}\right)\right) = 1,$$

where $\mathbf{0}$ is the zero vector.

(Section 6.2.1) Withholding the k highest bids: for all $\mathbf{b}$,

$$D\left(\left(0,\ldots,0,b^{(k+1)},\ldots,b^{(N)}\right) \mid \left(b^{(2)},\ldots,b^{(N)}\right)\right) = 1 \tag{13}$$

i.e., the auctioneer withholds the k highest bids (not including the winning bid) and discloses the rest.

(Section 6.2.2) Threshold Disclosure: for all **b**,

$$D\left(\left(\hat{b}^{(2)},\ldots,\hat{b}^{(N)}\right)\mid\left(b^{(2)},\ldots,b^{(N)}\right)\right)=1, \tag{14}$$

where $\hat{b}^{(j)}=\min\{\alpha,b^{(j)}\}$ — i.e., the auctioneer only discloses bids that are lower than α.

A.2 Equilibrium Bids

Fix a disclosure rule D. Note that besides information from the message, the knowledge that a firm has won the auction with a bid of b also provides information that all the other bids are lower than b, and this is potentially a piece of information that is not revealed by the message (e.g., in the non-disclosure case). Therefore, the winning firm essentially updates its belief twice—first, upon knowing that its bid is the highest, and second, upon receiving the message. Because the firms are Bayes-rational, the order of belief-updating is inconsequential, and it is more instructive to consider the winning firm updating its belief using the message first, before using the knowledge that it is the winner.

Let $B^{(k)}$ denote the kth order statistic of the realized bids. Accordingly, let $P^{Y,\boldsymbol{Z}}_{D,\beta}\left(Y_i,\boldsymbol{Z}_i|S_i;\boldsymbol{m}\right)$ be bidder i's *posterior* joint distribution of $(Y_i,\boldsymbol{Z}_i)$, conditioned on $S_i=s$, firm i observing message $\boldsymbol{m}$ that is generated from distribution $D\left(\cdot|B_i^{(2)},\ldots,B_i^{(N-1)},B_i^{(N)}\right)$, and firm i conjecturing that every firm bids according to an increasing function $\beta\left(\cdot\right)$.[21] In addition, let $P^{Y}_{D,\beta}\left(Y_i|S_i,\boldsymbol{m}\right)$ be the associated posterior distribution of Y_i. When firm i further updates its belief based on the knowledge that its bid b_i is the winning bid, its posterior belief is

$$P^{Y,\boldsymbol{Z}}_{D,\beta}\left(Y_i,\boldsymbol{Z}_i|S_i;\boldsymbol{m},b_i\right)=\begin{cases}\frac{P^{Y,\boldsymbol{Z}}_{D,\beta}\left(Y_i,\boldsymbol{Z}_i|S_i,\boldsymbol{m}\right)}{P^{Y}_{D,\beta}\left(\beta^{-1}(b_i)|S_i,\boldsymbol{m}\right)} & \text{if } Y\le\beta^{-1}\left(b_i\right)\\ 1 & \text{if } Y>\beta^{-1}\left(b_i\right)\end{cases} \tag{15}$$

Note that if $B_i^{(2)}$ is always perfectly revealed, then $P^{Y,\boldsymbol{Z}}_{D,\beta}\left(Y_i,\boldsymbol{Z}_i|S_i;\boldsymbol{m},b_i\right)=P^{Y,\boldsymbol{Z}}_{D,\beta}\left(Y_i,\boldsymbol{Z}_i|S_i;\boldsymbol{m}\right)$ for all b_i—i.e., firm i's posterior is independent of its own bid.

Therefore, conditional on $S_i=s$, firm i winning the auction with a bid of b and receiving

[21] i.e., given D, β and $S_i=s$, for any $\left(Y_i,\hat{\boldsymbol{Z}}_i\right)=(\hat{y},\hat{\boldsymbol{z}})$ and $\hat{\boldsymbol{m}}\in\mathcal{M}$,

$$\int_{y_i\in[0,\bar{s}]}\int_{\boldsymbol{z}_i\in[0,\bar{s}]^{N-2}}\left[\int_{\boldsymbol{m}\le\hat{\boldsymbol{m}}}P^{Y,\boldsymbol{Z}}_{D,\beta}\left(\hat{y},\hat{\boldsymbol{z}}|s,\boldsymbol{m}\right)dD\left(\boldsymbol{m}|\beta\left(y_i\right),\beta\left(z_i^{(2)}\right),\ldots,\beta\left(z_i^{(N)}\right)\right)\right]h\left(y_i,\boldsymbol{z}_i|s\right)d\boldsymbol{z}_i dy_i$$
$$=\int_{y_i\le\hat{y}}\int_{\boldsymbol{z}_i\le\hat{\boldsymbol{z}}}D\left(\hat{\boldsymbol{m}}|\beta\left(y_i\right),\beta\left(z_i^{(2)}\right),\ldots,\beta\left(z_i^{(N)}\right)\right)h\left(y_i,\boldsymbol{z}_i|s\right)d\boldsymbol{z}_i dy_i.$$

message $\boldsymbol{m}$, firm i's expected value of the oil/gas reserve at the end of Stage 4 is

$$\tilde{V}_{D,\beta}\left(s;\boldsymbol{m},b\right) := \frac{1}{P^{Y}_{D,\beta}\left(\beta^{-1}\left(b\right)|s,\boldsymbol{m}\right)}\int_{\boldsymbol{z}}\int_{0}^{\beta^{-1}(b)} V\left(s,y,\boldsymbol{z}\right)P^{Y,\boldsymbol{Z}}_{D,\beta}\left(dy,d\boldsymbol{z}|s;\boldsymbol{m}\right).$$

In turn, conditional on $S_i = s$, $Y_i = y$ and firm i winning the auction with a bid of b, firm i's expected profit from the perspective at the start of Stage 2 is

$$\tilde{v}_{D,\beta}\left(s,y;b\right) := \int \psi\left(\tilde{V}_{D,\beta}\left(s;\boldsymbol{m},b\right)\right)dR_{D,\beta}\left(\boldsymbol{m}|s,y\right),$$

where, $R_{D,\beta}\left(\cdot|s,y\right) \in \Delta\left(\mathcal{M}\right)$ denotes the ex ante distribution of the message that firm i receives after winning the auction when conditioned on $S_i = s$, $Y_i = y$, and every firm $j \neq i$ using the bidding strategy β.[22] Therefore, at the start of Stage 2, under disclosure policy is D, if $S_i = s$ and firm i conjectures that every other firm $j \neq i$ bids according to β, firm i's expected profit from bidding b is

$$\int_{0}^{\beta^{-1}(b)}\left[\tilde{v}_{D,\beta}\left(s,y;b\right)-b\right]g^{Y}\left(y|s\right)dy. \tag{16}$$

Let

$$\begin{aligned}\hat{V}^{*}_{D,\beta}\left(s,y;\boldsymbol{m}\right) :=& E_{P^{Y,\boldsymbol{Z}}_{D,\beta}(\cdot|s,\boldsymbol{m})}\left[V\left(s,Y_i,\boldsymbol{Z}_i\right)|Y_i \leq y\right]\\ \bar{V}^{*}_{D,\beta}\left(s,y;\boldsymbol{m}\right) :=& E_{P^{Y,\boldsymbol{Z}}_{D,\beta}(\cdot|s,\boldsymbol{m})}\left[V\left(s,Y_i,\boldsymbol{Z}_i\right)|Y_i = y\right],\end{aligned}$$

where $E_{P^{Y,Z}_{D,\beta}(\cdot|s,\boldsymbol{m})}$ denote the expectation operator with respect to the distribution $P^{Y,Z}_{D,\beta}\left(Y_i,Z_i|s,\boldsymbol{m}\right)$.[23] Let

$$\Delta^{*}_{D,\beta}\left(s,y;\boldsymbol{m}\right) := \bar{V}^{*}_{D,\beta}\left(s,y;\boldsymbol{m}\right) - \hat{V}^{*}_{D,\beta}\left(s,y;\boldsymbol{m}\right),$$

and let $H^{Y}_{D,\beta}\left(Y_i|S_i,\boldsymbol{m}\right)$ denote the reverse hazard rate associated with the distribution $P^{Y}_{D,\beta}\left(Y_i|S_i,\boldsymbol{m}\right)$.[24]

Proposition 4 *Fix a disclosure policy D. If $\beta\left(\cdot\right)$ is an increasing and symmetric Bayes*

[22] i.e., for any $\boldsymbol{m} \in \mathcal{M}$, $R_{D,\beta}\left(\boldsymbol{m}|s,y\right) = \int_{\boldsymbol{z}} D\left(\boldsymbol{m}|\beta\left(y\right),\beta\left(z^{(2)}\right),\ldots\beta\left(z^{(N)}\right)\right)g^{\boldsymbol{z}}\left(z^{(2)},\ldots,z^{(N)}|s,y\right)d\boldsymbol{z}$

[23] It is useful to note that $\tilde{V}_{D,\beta}\left(s;\boldsymbol{m},b\right) = \hat{V}^{*}_{D,\beta}\left(s,\beta^{-1}\left(b\right);\boldsymbol{m}\right)$.

[24] The reversed hazard rate is the ratio of the probability density/mass function and the cumulative distribution function. We take the convention that if $P^{Y}_{D,\beta}\left(y|s,\boldsymbol{m}\right) = 0$, then $H^{Y}_{D,\beta}\left(y|s,\boldsymbol{m}\right) = 0$.

Nash equilibrium of the auction, then $\beta(\cdot)$ *satisfies*

$$\beta(s) = \int_0^s \left[v^*_{D,\beta}(s', s') + w^*_{D,\beta}(s', s')\right] dL(s'|s) \quad \forall s \in [0, \bar{s}], \tag{17}$$

where

$$v^*_{D,\beta}(s, y) := \int_{\boldsymbol{m}} \psi\left(\hat{V}^*_{D,\beta}(s, y; \boldsymbol{m})\right) dR_{D,\beta}(\boldsymbol{m}|s, y),$$
$$w^*_{D,\beta}(s, y) := \left(\int_{\boldsymbol{m}} \left[\psi'\left(\hat{V}^*_{D,\beta}(s, y; \boldsymbol{m})\right) \Delta^*_{D,\beta}(s, y; \boldsymbol{m})\right] dR_{D,\beta}(\boldsymbol{m}|s, y)\right)$$

Moreover, if a function $\beta(\cdot)$ *satisfies Eq. (17) and* $v^*_{D,\beta}(s, y) + w^*_{D,\beta}(s, y)$ *is increasing in both* s *and* y*, then* $\beta(\cdot)$ *is an increasing and symmetric Bayes Nash equilibrium of the auction.*

Proposition 4 provides necessary and sufficient conditions for a bidding strategy to be an increasing and symmetric Bayes Nash equilibrium.

Lemma 1 *If* $\beta^D(\cdot)$ *is the symmetric and increasing Bayes Nash equilibrium of an auction with disclosure policy* D*, then:*

$$\beta^D(s) \leq \beta^{FD}(s)$$

Lemma 1 establishes that the full-disclosure auction yields the highest expected bid revenue. This is intuitive—relative to a full-disclosure policy, *any* partial disclosure policy provides less information to the winning bidder at the drilling stage, hence decreasing the winner's expected profit. This implies that the ex ante expected value of winning the auction is lower; thus, the equilibrium bids are also lower.

I provide the proofs of Proposition 4 and Lemma 1 next.

A.2.1 Proof of Proposition 4

Proof:
Fix any strictly increasing $\beta(\cdot)$ and suppose that all firms $j \neq i$ play according to β. Firm i's expected profit from bidding b when $S_i = s$ is in Eq. (16). Differentiating Eq. (16) with respect to b, the first-order necessary condition (FOC) is:

$$\begin{aligned} 0 = &- G^Y\left(\beta^{-1}(b)|s\right) + \frac{1}{\beta'\left(\beta^{-1}(b)\right)}\left[\tilde{v}_{D,\beta}\left(s, \beta^{-1}(b); b\right) - b\right] g^Y\left(\beta^{-1}(b)|s\right) \\ &+ \int_0^{\beta^{-1}(b)} \frac{\partial \tilde{v}_{D,\beta}(s, y; b)}{\partial b} g^Y(y|s)\, dy. \end{aligned} \tag{18}$$

Note that

$$\tilde{v}_{D,\beta}\left(s,\beta^{-1}(b);b\right)=\int_{\boldsymbol{m}}\psi\left(\hat{V}^{*}_{D,\beta}\left(s,\beta^{-1}(b);\boldsymbol{m}\right)\right)dR_{D,\beta}\left(\boldsymbol{m}|s,\beta^{-1}(b)\right)=v^{*}_{D,\beta}\left(s,\beta^{-1}(b)\right)$$

and

$$\frac{\partial\tilde{v}_{D,\beta}(s,y;b)}{\partial b}=\int_{\boldsymbol{m}}\psi'\left(\hat{V}^{*}_{D,\beta}\left(s,\beta^{-1}(b);\boldsymbol{m}\right)\right)\left[\frac{\partial\tilde{V}_{D,\beta}(s;\boldsymbol{m},b)}{\partial b}\right]dR_{D,\beta}(\boldsymbol{m}|s,y)$$

where

$$\begin{aligned}
&\frac{\partial\tilde{V}_{D,\beta}(s;\boldsymbol{m},b)}{\partial b}\\
&=\left(\frac{1}{\beta'(\beta^{-1}(b))}\right)\left[\frac{\int_{\boldsymbol{z}}V\left(s,\beta^{-1}(b),\boldsymbol{z}\right)P^{Y,\boldsymbol{Z}}_{D,\beta}\left(\beta^{-1}(b),d\boldsymbol{z}|s;\boldsymbol{m}\right)}{P^{Y}_{D,\beta}\left(\beta^{-1}(b)|s,\boldsymbol{m}\right)}-H^{Y}_{D,\beta}\left(\beta^{-1}(b_i)|s,\boldsymbol{m}\right)\tilde{V}_{D,\beta}(s;\boldsymbol{m},b)\right]\\
&=\left(\frac{1}{\beta'(\beta^{-1}(b))}\right)H^{Y}_{D,\beta}\left(\beta^{-1}(b_i)|s,\boldsymbol{m}\right)\left[\bar{V}^{*}_{D,\beta}\left(s,\beta^{-1}(b);\boldsymbol{m}\right)-\hat{V}^{*}_{D,\beta}\left(s,\beta^{-1}(b);\boldsymbol{m}\right)\right]
\end{aligned}$$

Therefore, $\int_0^{\beta^{-1}(b)}\frac{\partial\tilde{v}_{D,\beta}(s,y;b)}{\partial b}g^{Y}(y|s)\,dy=\frac{g^{Y}\left(\beta^{-1}(b)|s\right)}{\beta'(\beta^{-1}(b))}w^{*}_{D,\beta}\left(s,\beta^{-1}(b)\right)$. In turn, FOC (18) becomes

$$0=-G^{Y}\left(\beta^{-1}(b)|s\right)+\frac{g^{Y}\left(\beta^{-1}(b)|s\right)}{\beta'(\beta^{-1}(b))}\left[v^{*}_{D,\beta}\left(s,\beta^{-1}(b)\right)+w^{*}_{D,\beta}(s,y)-b\right]\tag{19}$$

At a symmetric equilibrium in which every firm plays β, FOC (19) must be satisfied at $b=\beta(s)$, which implies that

$$\begin{aligned}
&0=-G^{Y}(s|s)+\frac{g^{Y}(s|s)}{\beta'(s)}\left[v^{*}_{D,\beta}(s,s)+w^{*}_{D,\beta}(s,s)-\beta(s)\right]\\
\iff\quad&\beta'(s)+\beta(s)\frac{g^{Y}(s|s)}{G^{Y}(s|s)}=\left[v^{*}_{D,\beta}(s,s)+w^{*}_{D,\beta}(s,s)\right]\frac{g^{Y}(s|s)}{G^{Y}(s|s)}
\end{aligned}\tag{20}$$

Eq. (20) is a linear ordinary differential equation (ODE). It is well-known that the solution to a linear ODE of the form $\beta'(s)+\beta(s)\xi(s)=\gamma(s)\xi(s)$ is

$$\theta(s)\beta(s)=\theta(0)\beta(0)+\int_0^s\gamma(s')\xi(s')\theta(s')\,ds',$$

where $\theta(s) = \exp\left(\int_0^s \xi(s')\,ds'\right)$. Using the boundary condition of $\beta(0) = 0$, we have

$$\begin{aligned}
\beta(s) &= \int_0^s \left[v^*_{D,\beta}(s', s') + w^*_{D,\beta}(s', s')\right] \left(\frac{g^Y(s'|s')}{G^Y(s'|s')} \frac{\exp\left(\int_0^{s'} \frac{g^Y(t|t)}{G^Y(t|t)}dt\right)}{\exp\left(\int_0^{s} \frac{g^Y(t|t)}{G^Y(t|t)}dt\right)}\right) ds' \\
&= \int_0^s \left[v^*_{D,\beta}(s', s') + w^*_{D,\beta}(s', s')\right] \left(\frac{g^Y(s'|s')}{G^Y(s'|s')} \exp\left(-\int_{s'}^{s} \frac{g^Y(t|t)}{G^Y(t|t)}dt\right)\right) ds' \\
&= \int_0^s \left[v^*_{D,\beta}(s', s') + w^*_{D,\beta}(s', s')\right] dL(s'|s). \qquad (21)
\end{aligned}$$

This proves the first (necessity) part of the proposition.

Next, to prove the sufficiency part, suppose that $v^*_{D,\beta}(s', s') + w^*_{D,\beta}(s', s')$ is also increasing. Note that for any s, $L(0|s) = 0$, $L(s|s) = 1$, and $L(\cdot|s)$ is increasing; therefore, $L(\cdot|s)$ is a CDF with support on $[0, s]$. Moreover, if $\hat{s} > s$, then $L(y|\hat{s}) < L(y|s)$ for all y — i.e., first-order stochastic dominance. In turn, because the integrand term in Eq. (21) is increasing in s', $\beta(s)$ is (indeed) an increasing function.

To show that bidding $b = \beta(s)$ is indeed firm i's best response against all other firms playing β, first note that bidding above $\beta(\bar{s})$ is worse than bidding $\beta(s)$; therefore, firm i's best response (if it exists) is a bid in $[\beta(0), \beta(\bar{s})]$. Suppose that firm i bids $\beta(\tilde{s})$, where $\tilde{s} \neq s$. Its expected profit (using Eq. (16)) is

$$U(\tilde{s}|s) := \int_0^{\tilde{s}} \left[\tilde{v}_{D,\beta}(s, y; \beta(\tilde{s})) - \beta(\tilde{s})\right] g^Y(y|s)\,dy.$$

Let

$$\begin{aligned}
W(\tilde{s}|s) &= \frac{\partial U(\tilde{s}|s)}{\partial \tilde{s}} \\
&= -G^Y(\tilde{s}|s)\,\beta'(\tilde{s}) + \left[\tilde{v}_{D,\beta}(s, \tilde{s}; \beta(\tilde{s})) - \beta(\tilde{s})\right] g^Y(\tilde{s}|s) + \beta'(\tilde{s}) \int_0^{\tilde{s}} \frac{\partial \tilde{v}_{D,\beta}(s, y; \beta(\tilde{s}))}{\partial b} g^Y(y|s)\,dy \\
&= G^Y(\tilde{s}|s) \left(\left[v^*_{D,\beta}(s, \tilde{s}) + w_{D,\beta}(s, \tilde{s}) - \beta(\tilde{s})\right] \frac{g^Y(\tilde{s}|s)}{G^Y(\tilde{s}|s)} - \beta'(\tilde{s})\right).
\end{aligned}$$

From FOC (20), $W(\tilde{s}|\tilde{s}) = 0$. If $\tilde{s} > s$, then $v^*_{D,\beta}(s, \tilde{s}) + w_{D,\beta}(s, \tilde{s}) < v^*_{D,\beta}(\tilde{s}, \tilde{s}) + w_{D,\beta}(\tilde{s}, \tilde{s})$; moreover, because the signals are affiliated, $\frac{g^Y(\tilde{s}|s)}{G^Y(\tilde{s}|s)} < \frac{g^Y(\tilde{s}|\tilde{s})}{G^Y(\tilde{s}|\tilde{s})}$. Together with $W(\tilde{s}|\tilde{s}) = 0$, we have $W(\tilde{s}|s) < 0$. On the other hand, if $\tilde{s} < s$, then $v^*_{D,\beta}(s, \tilde{s}) + w_{D,\beta}(s, \tilde{s}) > v^*_{D,\beta}(\tilde{s}, \tilde{s}) + w_{D,\beta}(\tilde{s}, \tilde{s})$ and $\frac{g^Y(\tilde{s}|s)}{G^Y(\tilde{s}|s)} > \frac{g^Y(\tilde{s}|\tilde{s})}{G^Y(\tilde{s}|\tilde{s})}$, which implies that $W(\tilde{s}|s) > 0$. Therefore, bidding $\beta(s)$ is firm i's best response. ■

A.2.2 Proof of Lemma 1

Proof:

To show that $\beta^{D}(s) < \beta^{FD}(s)$, note that

$$
\begin{aligned}
&\psi'\left(\hat{V}^{*}_{D,\beta}(s',s';\boldsymbol{m})\right)\left[\bar{V}^{*}_{D,\beta}(s',s';\boldsymbol{m})-\hat{V}^{*}_{D,\beta}(s',s';\boldsymbol{m})\right] \\
=&\int_{0}^{(1-r)\hat{V}^{*}_{D,\beta}(s',s';\boldsymbol{m})}(1-r)\left[\bar{V}^{*}_{D,\beta}(s',s';\boldsymbol{m})-\hat{V}^{*}_{D,\beta}(s',s';\boldsymbol{m})\right]dF^{C}(c) \\
=&\int_{0}^{(1-r)\hat{V}^{*}_{D,\beta}(s',s';\boldsymbol{m})}\left((1-r)\bar{V}^{*}_{D,\beta}(s',s';\boldsymbol{m})-c\right)dF^{C}(c)- \\
&\quad\int_{0}^{(1-r)\hat{V}^{*}_{D,\beta}(s',s';\boldsymbol{m})}\left((1-r)\hat{V}^{*}_{D,\beta}(s',s';\boldsymbol{m})-c\right)dF^{C}(c) \\
\leq&\psi\left(\bar{V}^{*}_{D,\beta}(s',s';\boldsymbol{m})\right)-\psi\left(\hat{V}^{*}_{D,\beta}(s',s';\boldsymbol{m})\right)
\end{aligned}
$$

Therefore,

$$
\begin{aligned}
&\psi\left(\hat{V}^{*}_{D,\beta}(s',s';\boldsymbol{m})\right)+\psi'\left(\hat{V}^{*}_{D,\beta}(s',s';\boldsymbol{m})\right)\left[\bar{V}^{*}_{D,\beta}(s',s';\boldsymbol{m})-\hat{V}^{*}_{D,\beta}(s',s';\boldsymbol{m})\right] \\
\leq&\psi\left(\bar{V}^{*}_{D,\beta}(s',s';\boldsymbol{m})\right) \\
=&\psi\left(E\left[V\left(S_i,Y_i,\boldsymbol{Z}_i\right)|S_i=s',Y_i=s',\boldsymbol{m}\right]\right) \\
\leq&E\left[\psi\left(V\left(S_i,Y_i,\boldsymbol{Z}_i\right)\right)|S_i=s',Y_i=s'\right] \\
=&v\left(s',s'\right),
\end{aligned}
$$

where the inequality in the third line follows from ψ being convex. Therefore, $\beta^{D}(s) < \beta^{FD}(s)$. ■

A.3 Proof for Propositions 1 and 2

A.3.1 Proof of Proposition 1

Proof:

Suppose first that there exists a symmetric and increasing bidding equilibrium β. By Proposition 4, β must satisfy satisfies Eq. (17). Under full disclosure, for any $\boldsymbol{m}=\left(b^{(2)},\ldots,b^{(N)}\right)$, because β is increasing, the posterior belief $P^{Y,\boldsymbol{Z}}_{D,\beta}\left(Y_i,\boldsymbol{Z}_i|S_i,\boldsymbol{m}\right)$ is always a Dirac measure on

$\left(Y_i, Z_i^{(2)}, \ldots, Z_i^{(N-1)}\right) = \left(\beta^{-1}\left(b^{(2)}\right), \beta^{-1}\left(b^{(3)}\right), \ldots, \beta^{-1}\left(b^{(N-1)}\right)\right)$. This implies that

$$\hat{V}_{D,\beta}^{*}\left(s, y; \left(b^{(2)}, \ldots, b^{(N)}\right)\right) = V\left(s, y, \beta^{-1}\left(b^{(3)}\right), \ldots, \beta^{-1}\left(b^{(N-1)}\right)\right)$$

and $\bar{V}_{D,\beta}^{*}(s, y; \boldsymbol{m}) = \hat{V}_{D,\beta}^{*}(s, y; \boldsymbol{m})$. Therefore, $w_{D,\beta}^{*}$ is the zero function. Next, for any m in which $b^{(2)} = \beta(y)$,

$$R_{D,\beta}\left(\left(b^{(2)}, b^{(3)}, \ldots, b^{(N)}\right)|s, y\right) = G^{\boldsymbol{Z}}\left(Z_i^{(2)} = \beta^{-1}\left(b^{(3)}\right), \ldots, Z_i^{(N-1)} = \beta^{-1}\left(b^{(N)}\right)|s, y\right).$$

Therefore,

$$\begin{aligned} & v_{D,\beta}^{*}(s, y) \\ =& \int_{\beta^{-1}\left(b^{(3)}\right), \ldots, \beta^{-1}\left(b^{(N-1)}\right)} \psi\left(V\left(s, y, \beta^{-1}\left(b^{(3)}\right), \ldots, \beta^{-1}\left(b^{(N-1)}\right)\right)\right) dG^{\boldsymbol{Z}}\left(\beta^{-1}\left(b^{(3)}\right), \ldots, \beta^{-1}\left(b^{(N)}\right)|s, y\right) \\ =& v(s, y), \end{aligned}$$

as defined in Eq. (2). Therefore, if there exists an increasing β that satisfies satisfies Eq. (17), it must be uniquely β^{FD} in the proposition. By Proposition 4, if there exists an increasing and symmetric equilibrium, then it must be β^{FD}. Moreover, if $v(s, y)$ is strictly increasing in both its arguments, then β^{FD} is indeed an equilibrium, and this holds because V is strictly increasing and the signals are affiliated. ■

A.3.2 Proof of Proposition 2

Proof:

Under nondisclosure, for any β, $R_{D,\beta}(\cdot|s, y)$ is a Dirac measure on $\boldsymbol{m} = \mathbf{0}$, and $P_{D,\beta}^{Y,\boldsymbol{Z}}(Y_i, \boldsymbol{Z}_i|S_i, \boldsymbol{m} = \mathbf{0}) = G^{Y,\boldsymbol{Z}}(Y_i, \boldsymbol{Z}_i|S_i)$. Therefore, for all β, $\hat{V}_{D,\beta}^{*}(s, y; \mathbf{0}) = \hat{V}_0(s, y)$ (as defined in Eq. (7)), and $\bar{V}_{D,\beta}^{*}(s, y; \phi) = \bar{V}_0(s, y)$ (as defined in Eq. (8)). In turn,

$$v_{D,\beta}^{*}(s, y) := \psi\left(\hat{V}_0(s, y)\right),$$

and, because $H^Y_{D,\beta}(y|s,\phi) = \frac{g^Y(y|s)}{G^Y(y|s)}$,

$$
\begin{aligned}
w^*_{D,\beta}(s,y) =& \int_0^y \psi'\left(\hat{V}_0(s,y)\right) \frac{g^Y(y|s)}{G^Y(y|s)} \left[\bar{V}_0(s,y) - \hat{V}_0(s,y)\right] \frac{g^Y(y'|s)}{g^Y(y|s)} dy'. \\
=& \psi'\left(\hat{V}_0(s,y)\right) \frac{1}{G^Y(y|s)} \left[\bar{V}_0(s,y) - \hat{V}_0(s,y)\right] \int_0^y g^Y(y'|s)\, dy' \\
=& \psi'\left(\hat{V}_0(s,y)\right) \left[\bar{V}_0(s,y) - \hat{V}_0(s,y)\right]
\end{aligned}
$$

Therefore, if there exists an increasing β that satisfies Eq. (17), it must be β^{ND} in the proposition. By Proposition 4, if there exists an increasing and symmetric equilibrium, then it must be β^{ND}. Next, because the signals are affiliated, $\bar{V}_0(s,y)$ and $\hat{V}_0(s,y)$ are both strictly increasing with s and y. By Lemma 2 (below) , $v^*_{D,\beta}(s,y)+w^*_{D,\beta}(s,y)$ is strictly increasing in s and y. Therefore, by Proposition 4, β^{ND} is a symmetric and increasing equilibrium. Finally, it is readily observed that in the proof of Lemma 1, if the disclosure policy is nondisclosure, all the inequalities are strict. This implies that $\beta^{ND}(s) < \beta^{FD}(s)$. ■

A.4 Equilibrium under the Partial Disclosure Policies in Section 6

I first prove an intermediate lemma.

Lemma 2 *If $\bar{\phi}$ and $\hat{\phi}$ are two nonnegative real functions that are strictly increasing and $\bar{\phi}(x) \geq \hat{\phi}(x)$ for all x, then $\xi(x) := \psi\left(\hat{\phi}(x)\right) + \psi'\left(\hat{\phi}(x)\right)\left[\bar{\phi}(x) - \hat{\phi}(x)\right]$ is strictly increasing in x.*

Proof:
Consider any $x' > x$.

$$
\begin{aligned}
\xi(x') - \xi(x) =& \psi\left(\hat{\phi}(x')\right) - \psi\left(\hat{\phi}(x)\right) \\
&+ \psi'\left(\hat{\phi}(x')\right)\left[\bar{\phi}(x') - \hat{\phi}(x')\right] - \psi'\left(\hat{\phi}(x)\right)\left[\bar{\phi}(x) - \hat{\phi}(x)\right]
\end{aligned}
$$

Because ψ is increasing and convex, we have, for any $z' > z$,

$$
\psi(z') - \psi(z) > \psi'(z)(z' - z).
$$

This implies that

$$
\begin{aligned}
\psi\left(\hat{\phi}\left(x'\right)\right)-\psi\left(\hat{\phi}\left(x\right)\right) >&\psi'\left(\hat{\phi}\left(x\right)\right)\left(\hat{\phi}\left(x'\right)-\hat{\phi}\left(x\right)\right)\\
=&\psi'\left(\hat{\phi}\left(x\right)\right)\left(\bar{\phi}\left(x'\right)-\hat{\phi}\left(x\right)\right)-\psi'\left(\hat{\phi}\left(x\right)\right)\left(\bar{\phi}\left(x'\right)-\hat{\phi}\left(x'\right)\right)\\
>&\psi'\left(\hat{\phi}\left(x\right)\right)\left(\bar{\phi}\left(x\right)-\hat{\phi}\left(x\right)\right)-\psi'\left(\hat{\phi}\left(x'\right)\right)\left(\bar{\phi}\left(x'\right)-\hat{\phi}\left(x'\right)\right),
\end{aligned}
$$

where the last inequality follows from, first, $\bar{\phi}(x') > \bar{\phi}(x)$ because $\bar{\phi}$ is increasing, and, second, $\psi'\left(\hat{\phi}(x)\right) < \psi'\left(\hat{\phi}(x')\right)$ because ψ is convex and $\hat{\phi}$ is increasing. This means that $\xi(x') > \xi(x)$. ■

Proposition 5 *Under a disclosure policy in which the auctioneer withholds the k highest losing bid—i.e., D defined in (13)—there exists a symmetric and increasing Bayes Nash equilibrium for the auction.*

Proof:
If $k = N-1$, this is the nondisclosure policy, which has already been considered. Thus, assume that $k < N-1$. Let $\bar{\boldsymbol{Z}}_i = \left(Z_i^{(2)}, \ldots, Z_i^{(k)}\right)$ and $\underline{\boldsymbol{Z}}_i = \left(Z_i^{(k+1)}, \ldots, Z_i^{(N)}\right)$. (If $k=1$, ignore $\bar{\boldsymbol{Z}}_i$.) Suppose that every player plays according to some strictly increasing bidding strategy β. Conditional on $\underline{\boldsymbol{Z}}_i = \underline{\boldsymbol{z}} = \left(z^{(k+1)}, \ldots, z^{(N)}\right)$ and bidder i winning the auction, bidder i always receives message $m_\beta(\underline{\boldsymbol{z}}) = \left(0, \ldots, 0, \beta\left(z^{(k+1)}\right), \ldots, \beta\left(z^{(N)}\right)\right)$. Conversely, the support of the messages that i receives is $m_\beta(\underline{\boldsymbol{z}})$, with $\underline{\boldsymbol{z}}$ in the support of $\underline{\boldsymbol{Z}}_i$. Thus, I can represent each possible message by $m_\beta(\underline{\boldsymbol{z}})$. Upon receiving $m_\beta(\underline{\boldsymbol{z}})$, conditional on $S_i = s$ and $Y_i \leq y$, the expected value of the tract is

$$
\begin{aligned}
\hat{V}^*_{D,\beta}\left(s, y; m_\beta(\underline{\boldsymbol{z}})\right) &= E_{Y_i, \bar{\boldsymbol{Z}}_i}\left[V\left(S_i, Y_i, \bar{\boldsymbol{Z}}_i, \underline{\boldsymbol{Z}}_i\right) | S_i = s, Y_i \leq y, \underline{\boldsymbol{Z}}_i = \underline{\boldsymbol{z}}\right]\\
&= \hat{e}\left(s, y, \underline{\boldsymbol{z}}\right).
\end{aligned}
$$

Upon receiving $m_\beta(\underline{\boldsymbol{z}})$, conditional on $S_i = s$ and $Y_i = y$ (instead), the expected value of the tract is

$$
\begin{aligned}
\bar{V}^*_{D,\beta}\left(s, y; m_\beta(\underline{\boldsymbol{z}})\right) &= E_{Y_i, \bar{\boldsymbol{Z}}_i}\left[V\left(S_i, Y_i, \bar{\boldsymbol{Z}}_i, \underline{\boldsymbol{Z}}_i\right) | S_i = s, Y_i = y, \underline{\boldsymbol{Z}}_i = \underline{\boldsymbol{z}}\right]\\
&= \bar{e}\left(s, y, \underline{\boldsymbol{z}}\right).
\end{aligned}
$$

Because the winner receives $m_\beta(\underline{\boldsymbol{z}})$ only if $\underline{\boldsymbol{Z}}_i = \underline{\boldsymbol{z}}$, this implies that $R_{D,\beta}\left(m_\beta(\underline{\boldsymbol{z}}) | s, y\right) = G^{\underline{\boldsymbol{Z}}}\left(\underline{\boldsymbol{z}} | S_i = s, Y_i = y\right)$, the distribution of $\underline{\boldsymbol{Z}}_i$ conditional on $S_i = s$ and $Y_i = y$. Therefore, we have

$$
\begin{aligned}
v^*_{D,\beta}(s,y) &= \int_{\underline{\boldsymbol{z}}} \psi\left(\hat{e}\left(s,y,\underline{\boldsymbol{z}}\right)\right) dG^{\underline{\boldsymbol{Z}}}\left(\underline{\boldsymbol{z}}|s,y\right) \\
&=: v^*_D(s,y) \\
w^*_{D,\beta}(s,y) &= \int_{\underline{\boldsymbol{z}}} \psi'\left(\hat{e}\left(s,y,\underline{\boldsymbol{z}}\right)\right)\left[\bar{e}\left(s,y,\underline{\boldsymbol{z}}\right) - \hat{e}\left(s,y,\underline{\boldsymbol{z}}\right)\right] dG^{\underline{\boldsymbol{Z}}}\left(\underline{\boldsymbol{z}}|s,y\right) \\
&=: w^*_D(s,y)
\end{aligned}
$$

Therefore, the bidding function $\beta^k(s) := \int_0^{s'} \left[v^*_D(s',s') + w^*_D(s',s')\right] dL(s'|s)$ satisfies Eq. (17).

Next, because the signals are affiliated, both $\bar{e}$ and $\hat{e}$ are strictly increasing in each of its arguments and $\bar{e}(s,y,\underline{\boldsymbol{z}}) > \hat{e}(s,y,\underline{\boldsymbol{z}})$. By Lemma 2, $\psi(\hat{e}(\cdot)) + \psi'(\hat{e}(\cdot))[\bar{e}(\cdot) - \hat{e}(\cdot)]$ is strictly increasing in each of its arguments. In turn, because $\underline{\boldsymbol{Z}}_i$ is affiliated with S_i and Y_i, $v^*_{D,\beta}(s,y) + w^*_{D,\beta}(s,y)$ must also be increasing in s and Y.[25] By Proposition 4, β^k is a symmetric and increasing equilibrium. ■

Proposition 6 *Under a disclosure policy in which the auctioneer discloses only bids that are lower than α — i.e., D defined in (14) — there exists a symmetric and increasing Bayes Nash equilibrium for the auction.*

Proof:

I first construct a β function that satisfies Eq. (17). Fix some $a \in [0,\bar{s}]$ and let $\hat{Z}^{(j)}_i = \min\left\{a, \hat{Z}^{(j)}_i\right\}$ and $\hat{\boldsymbol{Z}}_i = \left(\hat{Z}^{(2)}_i, \ldots, \hat{Z}^{(n)}_i\right)$. Let

$$
\begin{aligned}
\hat{e}(s,y,\hat{\boldsymbol{z}}) =& E_{Y_i,\boldsymbol{Z}_i}\left[V\left(S_i,Y_i,\boldsymbol{Z}_i\right)|S_i = s, Y_i \leq y, \hat{\boldsymbol{Z}}_i = \hat{\boldsymbol{z}}\right] \\
\bar{e}(s,y,\hat{\boldsymbol{z}}) =& E_{Y_i,\boldsymbol{Z}_i}\left[V\left(S_i,Y_i,\boldsymbol{Z}_i\right)|S_i = s, Y_i \leq y, \hat{\boldsymbol{Z}}_i = \hat{\boldsymbol{z}}\right]
\end{aligned}
$$

Let $\tilde{R}(\cdot|s,y)$ denote the distribution of $\hat{\boldsymbol{Z}}_i$ conditional on $S_i = s$ and $Y_i = y$—i.e.,

$$
\tilde{R}\left(\hat{z}^{(2)},\ldots,\hat{z}^{(N-1)}|s,y\right) = \int_{z^{(N-1)}>a} \cdots \int_{z^{(a)}>a} G^{\boldsymbol{Z}}\left(z^{(2)},\ldots,z^{(N-1)}|s,y\right) dz^{(2)} \ldots dz^{(N-1)}.
$$

[25]See Theorem 5 in Milgrom and Weber (1982).

Let

$$\tilde{v}^*(s,y) = \int_{\hat{\boldsymbol{z}}} \psi\left(\hat{e}\left(s,y,\hat{\boldsymbol{z}}\right)\right) d\tilde{R}\left(\hat{\boldsymbol{z}}|s,y\right)$$

$$\tilde{w}^*(s,y) = \int_{\hat{\boldsymbol{z}}} \psi'\left(\hat{e}\left(s,y,\hat{\boldsymbol{z}}\right)\right)\left[\bar{e}\left(s,y,\hat{\boldsymbol{z}}\right) - \hat{e}\left(s,y,\hat{\boldsymbol{z}}\right)\right] d\tilde{R}\left(\hat{\boldsymbol{z}}|s,y\right).$$

Let

$$\tilde{\beta}(s) = \int_0^s \left[\tilde{v}^*(s',s') + \tilde{w}^*(s',s')\right] dL(s'|s), \tag{22}$$

Using a similar argument as that used in Proposition 5, $\tilde{v}^* + \tilde{w}^*$ is strictly increasing in both its argument, thereby implying that $\tilde{\beta}$ is strictly increasing. Given that $\tilde{\beta}$ is increasing, under a disclosure policy with threshold $\alpha = \tilde{\beta}(a)$, $\tilde{\beta}$ satisfies Eq. (17) by construction.[26] By Proposition 4, $\tilde{\beta}$ is a symmetric and increasing equilibrium. ■

B Proof of Proposition 3

Proof:

As stated in the main text, $F^{C^0}(\cdot)$, $V(\cdot)$, $\zeta(\cdot)$, $\kappa(\cdot)$, and δ are objects to be identified, and they must satisfy both (10) and (11). The left-hand side (LHS) of (10) and (11) are identified from the data.

Consider (10). The slopes of the level curves of $\Delta(\cdot,\cdot,r)$ are also identified and equal to $-\frac{\frac{\partial \log \Delta(\boldsymbol{s},\mathcal{I},r)}{\partial \boldsymbol{s}}}{\frac{\partial \log \Delta(\boldsymbol{s},\mathcal{I},r)}{\partial \mathcal{I}}}$. Therefore:

$$\frac{\frac{\partial \log \Delta(\boldsymbol{s},\mathcal{I},r)}{\partial \boldsymbol{s}}}{\frac{\partial \log \Delta(\boldsymbol{s},\mathcal{I},r)}{\partial \mathcal{I}}} = \frac{\frac{\partial \log V(\boldsymbol{s})}{\partial \boldsymbol{s}}}{\frac{\partial \log \mathcal{Z}(\mathcal{I},r)}{\partial \mathcal{I}}} \tag{23}$$

Suppose that there exists $\left\{\tilde{V}(\cdot), \tilde{\zeta}(\cdot), \tilde{F}^{C^0}, \tilde{\zeta}(\mathcal{I}), \tilde{\delta}, \tilde{\kappa}\right\}$ and $\left\{V(\cdot), \zeta(\cdot), F^{C^0}, \zeta(\mathcal{I}), \delta, \kappa\right\}$ that both satisfy (10), (11), and thus (23). From (23), for any $\boldsymbol{s}$,$\mathcal{I}$, and r. Similar to the main

[26]If every player plays according to $\tilde{\beta}$, then the disclosure policy is equivalent to revealing the realization of $\hat{\boldsymbol{Z}}_i$.

text, I define

$$\mathcal{Z}(\mathcal{I},r) := \frac{\delta(1-r)-\kappa(\mathcal{I})}{\zeta(\mathcal{I})}$$
$$\tilde{\mathcal{Z}}(\mathcal{I},r) := \frac{\tilde{\delta}(1-r)-\tilde{\kappa}(\mathcal{I})}{\tilde{\zeta}(\mathcal{I})}$$

Thus,

$$\frac{\frac{\partial}{\partial \boldsymbol{s}}\log \tilde{V}(\boldsymbol{s})}{\frac{\partial}{\partial \boldsymbol{s}}\log V(\boldsymbol{s})} = \frac{\frac{\partial}{\partial \mathcal{I}}\log \tilde{\mathcal{Z}}(\mathcal{I},r)}{\frac{\partial}{\partial \mathcal{I}}\log \mathcal{Z}(\mathcal{I},r)}$$

Since the LHS of the above equation is a function of only $\boldsymbol{s}$ whereas the right-hand side (RHS) is a function of only $(\mathcal{I},r)$, there exists two constants $k_0, k_1, k_2 > 0$ such that

$$\tilde{\mathcal{Z}}(\mathcal{I},r) = k_2 \mathcal{Z}(\mathcal{I},r)^{k_0}$$
$$\text{and} \quad \tilde{V}(\boldsymbol{s}) = k_1 V(\boldsymbol{s})^{k_0}$$

WLOG, let $k_0 \geq 1$. I now prove that $k_0 = 1$. Suppose, for a contradiction, that $k_0 > 1$. Consider two cases:

Case 1: $N > 2$**.** From Eq (11), conditional on $S_i = s$

$$\begin{aligned}
0 &= \int^s \int^z \Delta(s,s,\tilde{\boldsymbol{z}},I)\left[\mathcal{Z}(\mathcal{I},r)\zeta(\mathcal{I})\frac{\partial}{\partial \tilde{\boldsymbol{z}}}V(s,s,\tilde{\boldsymbol{z}}) - \tilde{\mathcal{Z}}(\mathcal{I},r)\tilde{\zeta}(\mathcal{I})k_1\frac{\partial}{\partial \tilde{\boldsymbol{z}}}V^{k_0}(s,s,\tilde{\boldsymbol{z}})\right]\, d\tilde{\boldsymbol{z}}\; dG^{\boldsymbol{Z}}(\boldsymbol{z}|s,s)\\
&= \int^s \int^z \Delta(s,s,\tilde{\boldsymbol{z}},I)\left[\frac{\partial}{\partial \tilde{\boldsymbol{z}}}V(s,s,\tilde{\boldsymbol{z}})\left(\mathcal{Z}(\mathcal{I},r)\zeta(\mathcal{I}) - \tilde{\mathcal{Z}}(\mathcal{I},r)\tilde{\zeta}(\mathcal{I})k_1k_0V^{k_0-1}(s,s,\tilde{\boldsymbol{z}})\right)\right]\, d\tilde{\boldsymbol{z}}\; dG^{\boldsymbol{Z}}(\boldsymbol{z}|s,s)\\
&\geq \int^s \int^z \Delta(s,s,\tilde{\boldsymbol{z}},I)\left[\frac{\partial}{\partial \tilde{\boldsymbol{z}}}V(s,s,\tilde{\boldsymbol{z}})\left(\mathcal{Z}(\mathcal{I},r)\zeta(\mathcal{I}) - k_1k_0V^{k_0-1}(s,s,\tilde{\boldsymbol{z}})\right)\right]\, d\tilde{\boldsymbol{z}}\; dG^{\boldsymbol{z}}(\boldsymbol{z}|s,s)
\end{aligned} \tag{24}$$

where the last inequality is due to $\tilde{\mathcal{Z}}(\mathcal{I},r)\tilde{\zeta}(\mathcal{I}) = \tilde{\delta}(1-r)-\tilde{\kappa}(\mathcal{I}) \leq 1$. Since $V(\cdot)$ is strictly increasing, $\frac{\partial}{\partial \tilde{\boldsymbol{z}}}V(s,s,\tilde{\boldsymbol{z}}) > 0$. Since $V(\mathbf{0}) = 0$, for $S_i = s$ sufficiently small, the right hand side of Eq. (24) is positive, which is a contradiction.

Case 2: $N = 2$**.** From (11), conditional on $S_i = s$

$$0 = \int^s \Delta(s',s',I)\left(\mathcal{Z}(\mathcal{I},r)\zeta(\mathcal{I})\frac{\partial}{\partial s'}V(s',s') - \tilde{\mathcal{Z}}(\mathcal{I},r)\tilde{\zeta}(\mathcal{I})k_1\frac{\partial}{\partial s'}V^{k_0}(s',s')\right)\, ds' \tag{25}$$

Similar to the $N > 2$ case, for $S_i = s$ sufficiently small, the right hand side of Eq. (25) is positive, which is a contradiction.
Therefore, $k_0 = 1$. Therefore, $V(\cdot)$ and $\mathcal{Z}(\cdot,\cdot)$ are identified up to two multiplicative constant k_1 and k_2, respectively.
I now show that $\zeta(\cdot), \kappa(\cdot)$, and δ are also identified up to a multiplicative constant. Note that if $\mathcal{Z}(\mathcal{I}, r)$ is identified, $\delta/\zeta(\mathcal{I})$ and $\frac{\kappa(\mathcal{I})}{\zeta(\mathcal{I})}$ is identified due to the variations in r in the data. Using the normalization $\zeta(\mathbb{E}(\mathcal{I})) = 1$, δ is identified. Therefore, $\zeta(\mathcal{I})$ and $\kappa(\mathcal{I})$ are both identified. In other words

$$\begin{aligned}
\tilde{V}(\boldsymbol{s}) &= k_1 V(\boldsymbol{s}) \\
\tilde{\zeta}(\mathcal{I}) &= \zeta(\mathcal{I}) \\
\tilde{\kappa}(\mathcal{I}) &= k_2 \kappa(\mathcal{I}) \\
\tilde{\delta}(\mathcal{I}) &= k_2 \delta
\end{aligned} \tag{26}$$

Since (11) must be satisfied, $k_2 = \frac{1}{k_1}$. Therefore, $V(\cdot)$, $\kappa(\cdot)$, and δ are identified up to the same multiplicative constant, and $\zeta(\cdot)$ is identified.
I now show that F^{C^0} is also identified as long as $V(\cdot)$,$\kappa(\cdot)$, and δ is identified. Consider (10) and take derivative of $\Delta(\cdot, \mathcal{I}, r)$ at $\boldsymbol{s}$ and $\boldsymbol{s'} \neq \boldsymbol{s}$

$$\frac{\partial/\partial \boldsymbol{s} \Delta(\boldsymbol{s}, \mathcal{I}, r)}{\partial/\partial \boldsymbol{s} \Delta(\boldsymbol{s'}, \mathcal{I}, r)} = \frac{f^{C^0}\left(V\left(\boldsymbol{s}\right)\mathcal{Z}\left(\mathcal{I}, r\right)\right)V'\left(\boldsymbol{s}\right)}{f^{C^0}\left(V\left(\boldsymbol{s'}\right)\mathcal{Z}\left(\mathcal{I}, r\right)\right)V'\left(\boldsymbol{s'}\right)}$$

and

$$\frac{\partial/\partial \boldsymbol{s} \Delta(\boldsymbol{s}, \mathcal{I}, r)}{\partial/\partial \boldsymbol{s} \Delta(\boldsymbol{s'}, \mathcal{I}, r)} = \frac{\tilde{f}^{C^0}\left(\tilde{V}\left(\boldsymbol{s}\right)\left(\mathcal{I}, r\right)\right)\tilde{V}'\left(\boldsymbol{s}\right)}{\tilde{f}^{C^0}\left(\tilde{V}\left(\boldsymbol{s'}\right)\left(\mathcal{I}, r\right)\right)\tilde{V}'\left(\boldsymbol{s'}\right)}$$

Note that the above equations are well defined because $V(\cdot)$ is strictly increasing. Using (26) and that $k_2 = 1/k_1$, the above two equations imply

$$\frac{f^{C^0}\left(V\left(\boldsymbol{s}\right)\mathcal{Z}\left(\mathcal{I}, r\right)\right)}{\tilde{f}^{C^0}\left(V\left(\boldsymbol{s}\right)\mathcal{Z}\left(\mathcal{I}, r\right)\right)} = \frac{f^{C^0}\left(V\left(\boldsymbol{s'}\right)\mathcal{Z}\left(\mathcal{I}, r\right)\right)}{\tilde{f}^{C^0}\left(V\left(\boldsymbol{s'}\right)\mathcal{Z}\left(\mathcal{I}, r\right)\right)} \tag{27}$$

This implies that there exists a constant k_3 such that

$$\tilde{f}^{C^0}(\cdot) = k_3 f^{C^0}(\cdot) \tag{28}$$

Since f^{C^0} and $\tilde{f}^{C^0}$ are both density functions, $k_3 = 1$. Therefore, f^{C^0} and thus F^{C^0} are identified. ■

C Parametric Example

In this parametric example, I show that the bidders' expected profit may be higher under an ND policy than under an FD policy.

There are two bidders ($N = 2$). S_i, S_j are independent and are uniformly distributed in $[0, 1]$. Conditional on $S_i = s$, and $S_j = s'$, the tract's value is given by

$$V(s, s') = \frac{s + s'}{2}$$

The cumulative distribution of the drilling cost is $F^C(x) = 1 - (1 - x)^2$. Using the same notations as in the main text:

$$\psi(x) = \int^{(1-r)x} (1 - r)x - c dF^C(c)$$

and

$$L(s'|s) = \frac{s'}{s}$$

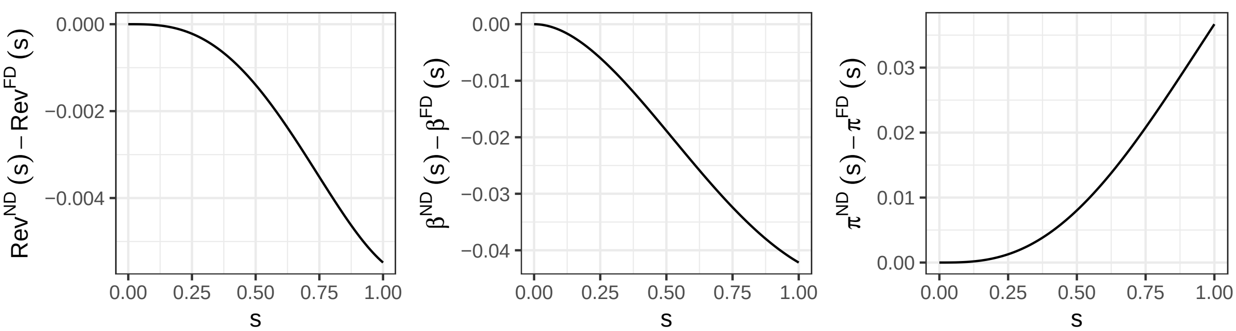
Therefore, using Proposition 1 and 2, the equilibrium bidding strategies under FD and ND are given by:

$$\beta^{FD}(s) = \frac{1}{s} \int_0^s \psi\left(s'\right) ds'$$
$$\beta^{ND}(s) = \frac{1}{s} \int_0^s \psi\left(\frac{3}{4}s'\right) + (1 - r)\frac{3}{4}s'\left(\psi\left(s'\right) - \psi\left(\frac{3}{4}s'\right)\right) ds'$$

The firms' profits under FD and ND are thus:

$$\pi^{FD}(s) = \int_0^s \psi(\frac{s + s'}{2})ds' - s\beta^{FD}(s)$$
$$\pi^{ND}(s) = \psi\left(\frac{3}{4}s\right) - s\beta^{ND}(s)$$

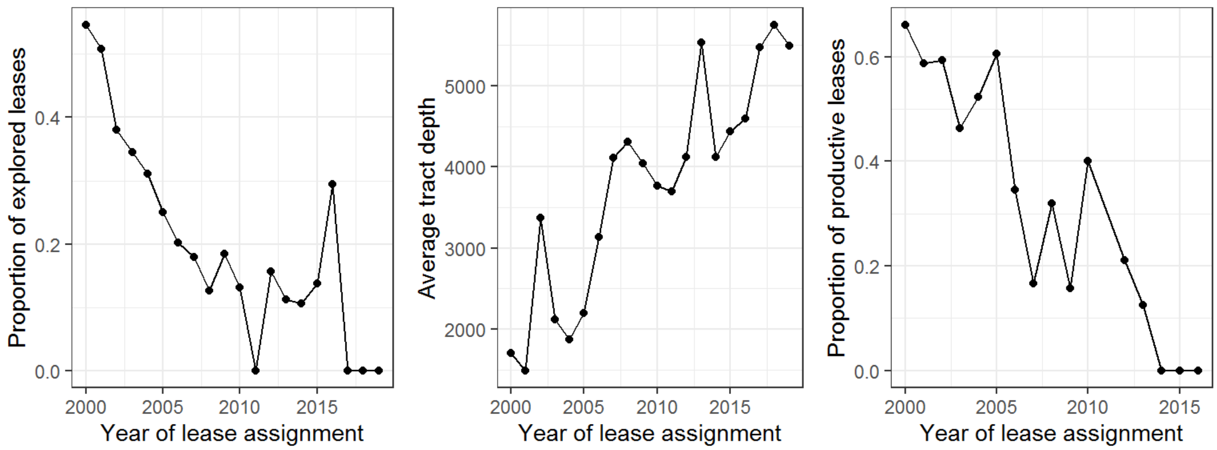
Figure 15: Difference between bidder's profits under ND and FD when $r = 0.1$



The first figure shows the difference in expected revenue from the tract, conditional on winning with signal s, between the ND policy and the FD policy. Since the ND policy creates more ex-post mistakes, the difference in revenue is always negative. However, the difference in the bids in the second figure is greater in magnitude than the difference in the expected revenue. Therefore, a bidder's expected profit is higher under ND than under FD.

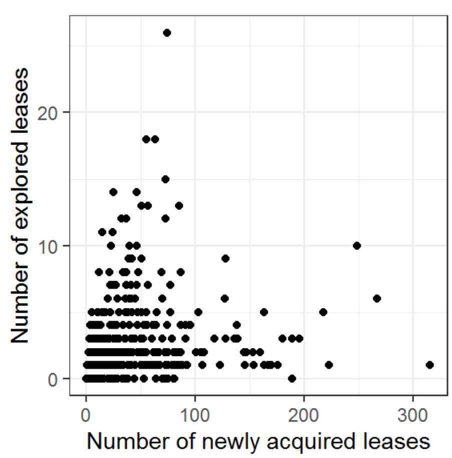
D Tables and Figures

Figure 16: Average exploration rate, tract depth, and hit rate over time



Note: The left panel and middle panel show the average exploration rate and average water depth among tracts that were assigned within a given year. The right panel shows the proportion of tracts that were successfully developed among tracts that were explored.

Figure 17: Number of explored leases among newly acquired leases for each oil/gas company in 3-year rolling windows



Note: Each dot in the graph represents the number of newly acquired leases and explored leases within a 3-year window for a particular year for each oil/gas company. The number of leases includes only leases with at least 50% ownership. The three-year benchmark was chosen because, during my data period, most explored leases (68.59%) were explored within the first three years. Most firms acquire many more leases than the number of tracts that will eventually be explored, and some firms do not explore any leases. For example, on average, firms acquire 28 leases but explore only 2 in a three-year window.

Table 18: Correlation between the number of explored leases and the number of all newly acquired leases for each owner

	Number of explored leases
Number of newly added leases (at least 50% ownership)	0.029
	(0.008)
Year FE	Yes
Lease owner FE	Yes
Observations	633
Adjusted R^2	0.543

Note: In this table, I regress the number of explored leases among newly acquired tracts in a given year for owners with majority ownership percentage ($\geq$ 50%) on the total number of tracts acquired in the same year, year fixed effects, and owner fixed effects. Standard errors are in parentheses and adjusted for heteroskedasticity. The positive correlation suggests that gas and oil companies are more likely to explore new tracts if they have more newly acquired tracts. Therefore, exploration capacity is unlikely to be the main reason for low exploration rates.

Table 19: Summary statistics of all available auctions and production data

Statistic	N	Mean	St. Dev.	Max	Min
First Bid	4,006	3,773	9,583	157,111	10
Second Bid	4,006	1,555	5,015	84,391	4
Number of Bids	4,006	2.73	1.30	13	2
Fraction of Explored Tracts	30,757	0.28	0.45	1	0
Oil Production	4,119	4,702	19,445	581,873	0
Gas Production	4,117	38,546	80,892	1,100,616	0

Note: The bids are expressed in hundreds of thousands of dollars. Oil production is measured in thousand barrels of crude oil, and gas production is measured in million cubic feet.

Table 20: Relevance of Cost Instrument

	Explored	
	(1)	(2)
Log Average Rig Daily Costs	−0.741	−0.896
	(0.063)	(0.068)
Number of Bids		0.012
		(0.007)
Log First Bid		0.056
		(0.010)
Log Second Bid		0.043
		(0.012)
Area FE	No	Yes
Observations	2,510	2,510
Adjusted R^2	0.053	0.216

Note: The rig rental costs are in thousands of dollars. The average rig renting cost is the constructed cost shifter $\mathcal{I}$, which is a tract-level variable, and is a distance-weighted daily rig renting costs of nearby tracts prior to a tract's auction. The regression estimates suggest that this constructed average daily rig renting cost is a useful predictor of the realized cost of exploration as it is negatively correlated with the exploration probability.

Figure 21: Current composition of source of revenue

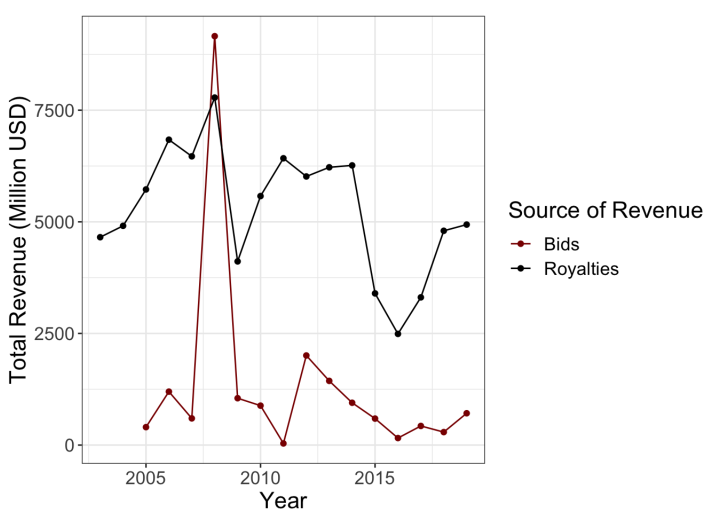
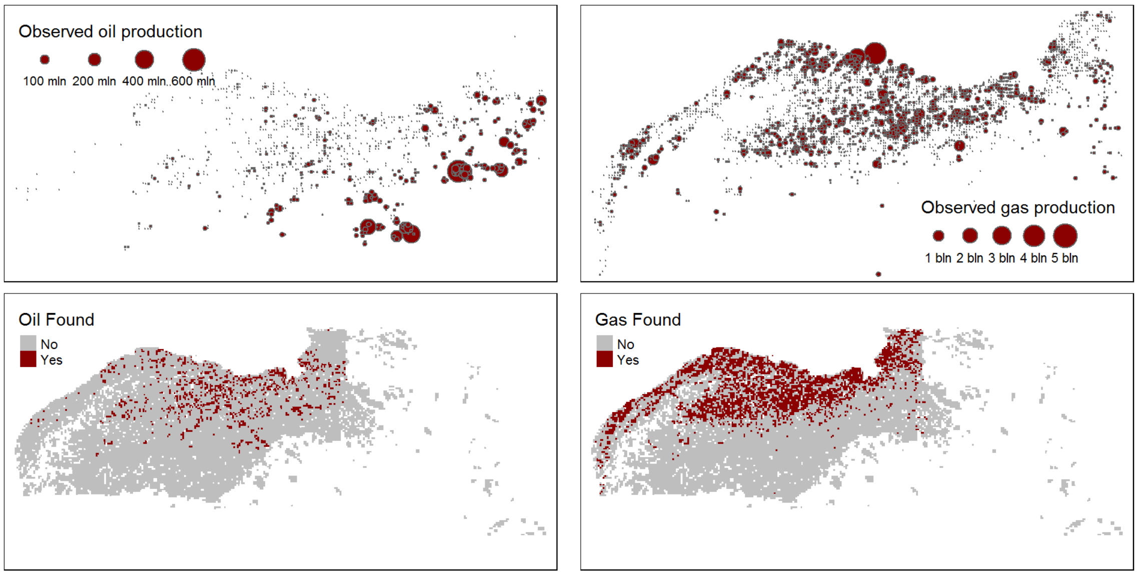


Figure 22: Observed oil and gas production



Note: The upper graphs show the locations and production quantities of tracts that have produced oil or gas or both. The units of production are barrels for oil and MCF for gas. The lower graphs show the locations of tracts that were explored. The gray tracts are tracts that were explored but were subsequently abandoned, and the red tracts are tracts that were explored and subsequently developed. Note that the lower graphs cover a larger area than the upper graphs.

Table 23: Subsample statistics

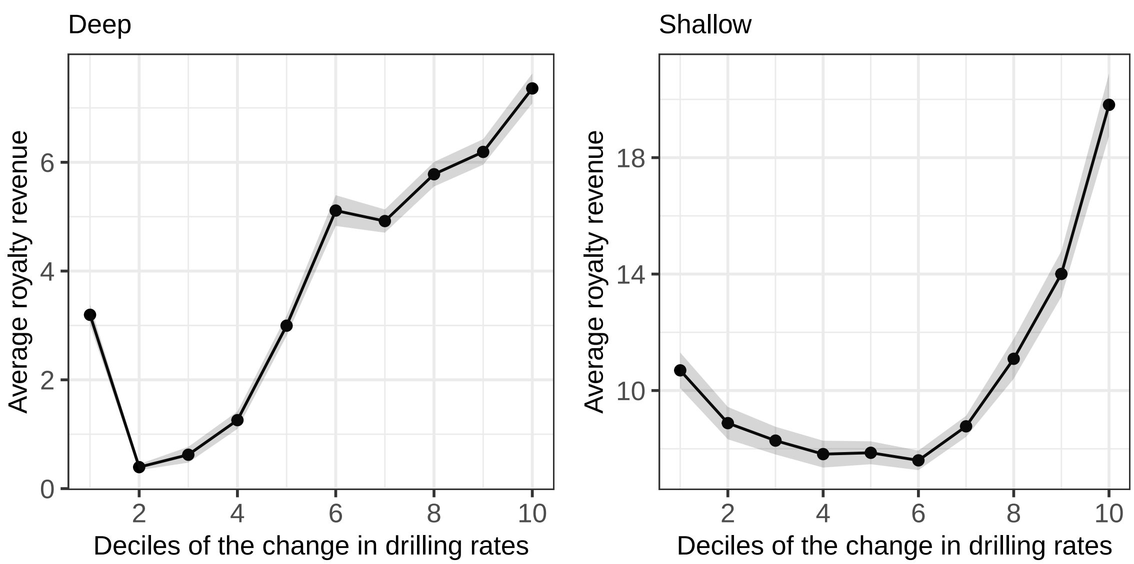
Number of bidders	Tract type	Percentage	Number of observations
2	Deep	41%	1072
2	Shallow	14%	358
3	Deep	12%	302
3	Shallow	22%	578
> 3	Deep	7%	194
> 3	Shallow	4%	105

Table 24: Summary statistics of rig contracts

Statistic	Mean	N	St. Dev.
Contract Duration (Days)	143.53	4,649	275.47
Daily Rate	90.82	4.65	113.67
Total Rig Cost	27,551.33	4,649	121,521.98

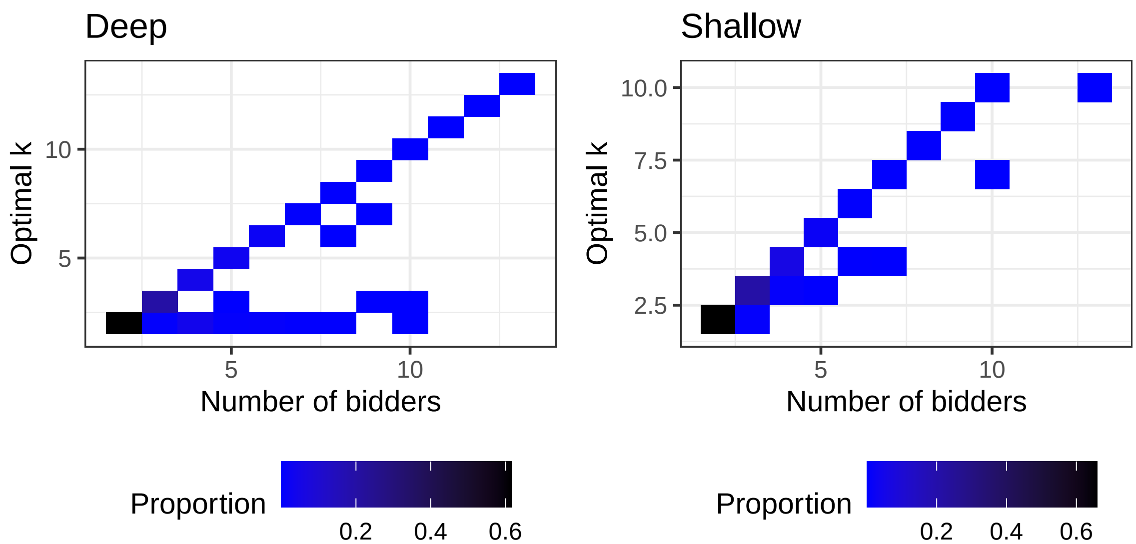
Note: The rig contracts are only available between 2000 and 2019. The data is recorded at the contract level. The units of the daily rate and the total rig rental cost are thousands of dollars.

Figure 25: Average royalty revenue (in millions of dollars) across deciles of drilling rate change



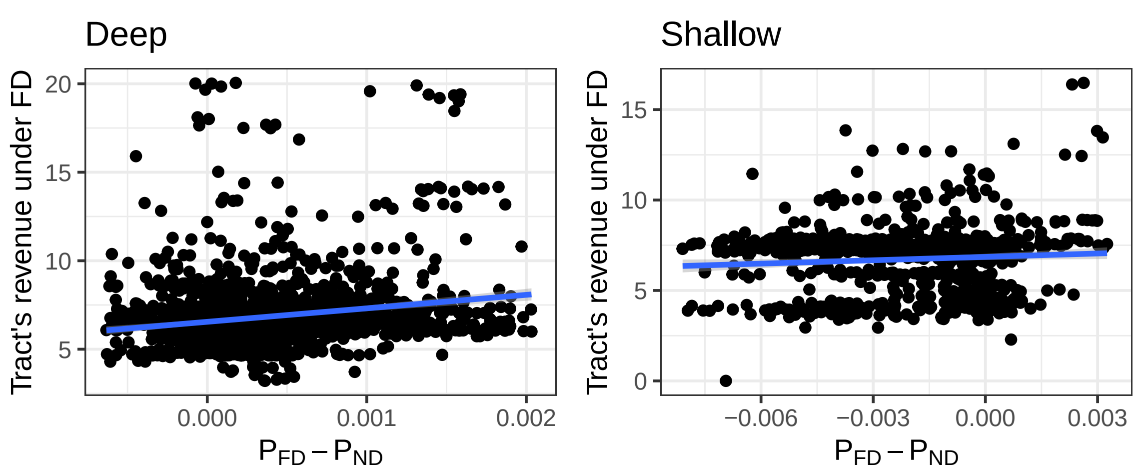
Note: In this figure, I categorized tracts into 10 groups according to the impact of a non-disclosure policy on the tracts' drilling probabilities. For example, a tract in the 5th decile has a median increase in the drilling probability relative to other tracts in the sample. On the vertical axis, I compute the average royalty revenue of those tracts. This figure shows that the most productive deep tracts tend to be associated with the largest increase in the drilling probability when a non-disclosure policy is implemented. This explains the significant increase in the royalty revenue for deep tracts despite the modest average increase in the drilling probability.

Figure 26: Distribution of optimal k values across tracts



Note: In this exercise, I find the optimal withhold-k policy for each tract. When $N = k$, the optimal policy is the same as the ND policy.

Figure 27: Correlation between the difference in probability of entry and tract's revenue



Note: The slope of the blue line represents the correlation between the tract's revenue under FD and the difference in the probability of entry between the FD policy and the ND policy

E Construction of tract's potential ($\mu_{gamma,oil}$, $\mu_{gamma,gas}$, $\mu_{bernoulli,oil}$, $\mu_{bernoulli,gas}$)

I assume that once production begins at a tract, all potential bidders observe that tract's production value. For leases that have expired—which I term 'completed' leases—the oil and gas quantities of a tract are equal to the total amount of oil and gas the tract has produced. For tracts that are still in production at the end of my data period, I construct a measure of the total amount of oil and gas that will be produced using the production data of the previous 2 years.

Construction of production values For leases that have not expired by the end date of my data set, I use the following regression on completed leases to predict the total amount of oil and lease in the tract:

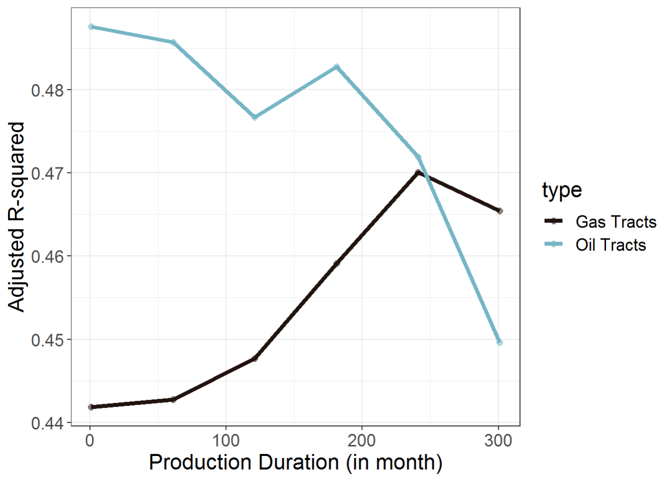
$$\log\left(\sum_{\tau=T}^{\infty} Q_t^{i,T}\right) = \beta_0 + \sum_{j=1}^{\underline{T}} \beta_j Q_t^{i,T-j} + \alpha X_t + \gamma_T + \epsilon_t^i \tag{29}$$

where $i \in \{o, g\}$, and $Q_t^{i,\tau}$ is the total amount of reserve type i produced by tract t after τ months of production, and X_t are tract characteristics. I allow the regression coefficients $\{\beta_0, \beta_{j,j=1,2,...,\underline{T}}, \alpha, \gamma\}$ to be dependent on the range of T. In my benchmark estimates, $\underline{T} = 2$ years, and I estimate regressions separately for $T < 5$ years, $T < 10$ years, ..., up to 30 years. Table 28 summarizes the number of completed and uncompleted tracts in my data, and figure 29 shows the model fit of (29).

Table 28: Summary of tract types

Tract type	Number of observations	Average production duration (in month)
Uncompleted oil tracts with sufficient data	770	335.4
Completed oil tracts	691	157.6
Uncompleted oil tracts without sufficient data	56	9
Uncompleted gas tracts with sufficient data	724	335.4
Completed gas tracts	2786	147.9
Uncompleted gas tracts without sufficient data	40	10.7

Figure 29: Oil and gas production model fit



For tracts that were explored but not subsequently developed, their production quantities are assumed to be 0 ($Q^o = Q^g = 0$).

Constructing $(\mu_{Gamma,oil}, \mu_{Gamma,gas}, \mu_{Bernoulli,oil}, \mu_{Bernoulli,gas})$ I decompose the prior F^Q into two priors F^{Q^o} and F^{Q^g} for oil and gas quantities, respectively. For each deposit type $i \in \{o, g\}$, the unconditional (with respect to any ex post outcomes of nearby tracts) prior $F_t^{Q^i}$ of a tract t is assumed to be a mixture distribution as follows:

$$\begin{cases} Q_t^i = 0 \text{ with probability } \frac{\exp(\beta_0^i + \nu_{0t}^i)}{1+\exp(\beta_0^i + \nu_{0t}^i)} \\ Q_t^i \sim Gamma(\cdot, \cdot) \text{ with probability } \frac{1}{1+\exp(\beta_0^i + \nu_{0t}^i)} \end{cases} \tag{30}$$

where β_0^i is a parameter to be estimated. ν_{0t}^i is assumed to be a Gaussian random field with a Matérn correlation function, where, for two tracts t and t', the covariance between ν_{0t}^i and $\nu_{0t'}^i$ is

$$\sigma_0^{i\,2} \left(\gamma_0^i ||s_t - s_{t'}||\right) K_1 \left(\gamma_0^i ||s_t - s_{t'}||\right), \tag{31}$$

where σ_0^i and γ_0^i are parameters to be estimated, $||s_t - s_{t'}||$ denotes the distance between the centroid of tract t and tract t', and $K_1(.)$ is the modified Bessel function of the second kind with a smoothness parameter of one (Krainski et al., 2018). Next, the Gamma distribution in Eq. (30) is assumed to have mean μ_t^i and standard deviation $\frac{(\mu_t^i)^2}{\phi^i}$, where

$$\log \mu_t^i = \beta_1^i + \nu_{1t}^i,$$

in which ϕ^i and β_1^i are parameters to be estimated, and ν_{1t}^i is also a Gaussian random field similarly defined as in (31) with parameters σ_1^i and γ_1^i. To estimate γ_0^i, I estimate the range value $\rho_0^i = \frac{\sqrt{8}}{\gamma_0^i}$. I follow a similar procedure for γ_1^i—i.e., we estimate $\rho_1^i = \frac{\sqrt{8}}{\gamma_1^i}$.

Thus, in summary, the parameters that need to be estimated are $\{\beta_0^i, \sigma_0^i, \rho_0^i, \phi^i, \beta_1^i, \sigma_1^i, \rho_1^i\}$ for $i \in \{o, g\}$. These parameters represent the spatial distribution of oil and gas deposits. I estimate these parameters using R-INLA (Rue et al., 2009). The estimated parameters of the priors are summarized in Table 30.

Table 30: Estimated parameters of the priors

	β_0^o	β_1^o	ϕ_1^o	ρ_1^o	$\sigma_1^{2^o}$	ρ_0^o	$\sigma_0^{2^o}$
Estimate	-6.560	14.799	0.385	2.412	1.923	2.080	3.108
SE	1.124	0.873	0.016	0.874	0.370	0.432	0.505

	β_0^g	β_1^g	ϕ_1^g	ρ_1^g	$\sigma_1^{2^g}$	ρ_0^g	$\sigma_0^{2^g}$
Estimate	-5.437	17.555	0.659	0.440	0.891	2.284	3.337
SE	1.074	0.113	0.016	0.084	0.060	0.391	0.468

The prior distribution of a particular tract t being auctioned in year τ is then the distribution in (30) conditional on ex-post production and exploration outcomes of all tracts that were explored prior to year τ. This naturally implies that tracts that were auctioned in earlier years have less informative priors than those that were auctioned in later years. In addition, the same tract can have different priors based on the year of the auction.