Supplemental Appendix for "Information Design in Common Value Auction with Moral Hazard: Application to OCS Leasing Auctions"

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1 Equilibrium under General Disclosure Policy

In this section, I provide the theoretical analysis of my model under a general disclosure policy for the bids. This general disclosure policy nests full disclosure (Section 4.1 in the main text), nondisclosure (considered in Section 4.2 in the main text) and the partial disclosure policies considered in Section 6 in the main text. I proceed as follows: I first describe the model in Section 1.1. In Section 1.2, I analyze the bidding equilibrium and provide necessary and sufficient conditions for an equilibrium in Proposition 1. Subsequently, in Section 1.3, I use Proposition 1 to show that the auctions under the partial disclosure policies considered in Section 6 in the main text admit a symmetric and increasing bidding equilibrium.

1.1 General Disclosure Policy

Let $\mathcal{M} = \mathbb{R}_+^{N-1}$ denote the message space, and let $\Delta\mathcal{M}$ denote the set of distributions on $\mathcal{M}$, endowed with the weak topology. I define a *disclosure policy* by a measurable map $D : \mathbb{R}_+^{N-1} \to \Delta(\mathcal{M})$, where, fixing a set of N bids received by the auctioneer and ordering them in decreasing order, with $b^{(1)} \geq b^{(2)} \geq \cdots \geq b^{(N)}$, $D\left(\cdot|b^{(2)}, \ldots, b^{(N)}\right) \in \Delta(\mathcal{M})$ is the distribution of the message that is sent to the winner after the auction—i.e., the bidder who submitted $b^{(1)}$. Each message $m \in \mathcal{M}$ is an $(N-1) \times 1$ vector. Representing each message

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by $\boldsymbol{m} = (m_2, m_3, \ldots, m_N)$, the natural convention is that m_k is a "message" about $b^{(k)}$, although this formulation admits more general interpretations.

There are two implicit assumptions on the set of admissible disclosure policies. First, the message received is independent of the highest bid. This eliminates any strategic concerns of firms choosing their bids to influence the message that a firm receives upon winning. In terms of information provision, this restriction implies that the message does not carry any information on the winning bid, but this is without loss of generality because the message is meant for only the winning firm, which clearly knows its own bid. Second, the message carries information only about the values of the losing bids but not information about the identity of the firm that places each bid—this is natural because the firms are ex ante identical, and I focus solely on symmetric equilibria.

The modifications to the baseline model are as follows: at the start of Stage 1, the auctioneer announces the disclosure policy D. Subsequently, in Stage 4, instead of observing all the losing bids, the winner now receives only a message from the auctioneer according to D. Upon receiving the message, the winning firm then updates its belief about the values of the bids submitted by the other firms using Bayes' rule before making its drilling decision in the subsequent stage.

Abusing notations slightly, I also let D denote the Borel probability measure on $\mathcal{M}$. The following are the disclosure policies considered in the main text formulated in the current setup:

(Section 4.1) Full disclosure: for all $\mathbf{b}$,

$$D\left(\left(b^{(2)}, \ldots, b^{(N)}\right) \mid \left(b^{(2)}, \ldots, b^{(N)}\right)\right) = 1.$$

(Section 4.2) Nondisclosure: for all $\mathbf{b}$,

$$D\left(\mathbf{0} \mid \left(b^{(2)}, \ldots, b^{(N)}\right)\right) = 1,$$

where $\mathbf{0}$ is the zero vector.

(Section 6.2.2) Withholding the k highest bids: for all $\mathbf{b}$,

$$D\left(\left(0, \ldots, 0, b^{(k+1)}, \ldots, b^{(N)}\right) \mid \left(b^{(2)}, \ldots, b^{(N)}\right)\right) = 1 \tag{1}$$

i.e., the auctioneer withholds the k highest bids (not including the winning bid) and discloses the rest.

(Section 6.2.1) Threshold Disclosure: for all $\mathbf{b}$,

$$D\left(\left(\hat{b}^{(2)},\ldots,\hat{b}^{(N)}\right)\mid\left(b^{(2)},\ldots,b^{(N)}\right)\right)=1, \tag{2}$$

where $\hat{b}^{(j)}=\min\{\alpha,b^{(j)}\}$ — i.e., the auctioneer only discloses bids that are lower than α.

1.2 Equilibrium Bids

Fix a disclosure rule D. Note that besides information from the message, the knowledge that a firm has won the auction with a bid of b also provides information that all the other bids are lower than b, and this is potentially a piece of information that is not revealed by the message (e.g., in the non-disclosure case). Therefore, the winning firm essentially updates its belief twice—first, upon knowing that its bid is the highest, and second, upon receiving the message. Because the firms are Bayes-rational, the order of belief-updating is inconsequential, and it is more instructive to consider the winning firm updating its belief using the message first, before using the knowledge that it is the winner.

Let $B^{(k)}$ denote the kth order statistic of the realized bids. Accordingly, let $P_{D,\beta}^{Y,\boldsymbol{Z}}\left(Y_i,\boldsymbol{Z}_i|S_i;\boldsymbol{m}\right)$ be bidder i's *posterior* joint distribution of $(Y_i,\boldsymbol{Z}_i)$, conditioned on $S_i=s$, firm i observing message $\boldsymbol{m}$ that is generated from distribution $D\left(\cdot|B_i^{(2)},\ldots,B_i^{(N-1)},B_i^{(N)}\right)$, and firm i conjecturing that every firm bids according to an increasing function $\beta(\cdot)$.[1] In addition, let $P_{D,\beta}^{Y}\left(Y_i|S_i,\boldsymbol{m}\right)$ be the associated posterior distribution of Y_i. When firm i further updates its belief based on the knowledge that its bid b_i is the winning bid, its posterior belief is

$$P_{D,\beta}^{Y,\boldsymbol{Z}}\left(Y_i,\boldsymbol{Z}_i|S_i;\boldsymbol{m},b_i\right)=\begin{cases}\frac{P_{D,\beta}^{Y,\boldsymbol{Z}}(Y_i,\boldsymbol{Z}_i|S_i,\boldsymbol{m})}{P_{D,\beta}^{Y}(\beta^{-1}(b_i)|S_i,\boldsymbol{m})} & \text{if } Y\leq\beta^{-1}(b_i)\\ 1 & \text{if } Y>\beta^{-1}(b_i)\end{cases} \tag{3}$$

[1] i.e., given D, β and $S_i=s$, for any $\left(Y_i,\hat{\boldsymbol{Z}}_i\right)=(\hat{y},\hat{\boldsymbol{z}})$ and $\hat{\boldsymbol{m}}\in\mathcal{M}$,

$$\int_{y_i\in[0,\bar{s}]}\int_{\boldsymbol{z}_i\in[0,\bar{s}]^{N-2}}\left[\int_{\boldsymbol{m}\leq\hat{\boldsymbol{m}}}P_{D,\beta}^{Y,\boldsymbol{Z}}\left(\hat{y},\hat{\boldsymbol{z}}|s,\boldsymbol{m}\right)dD\left(\boldsymbol{m}|\beta\left(y_i\right),\beta\left(z_i^{(2)}\right),\ldots,\beta\left(z_i^{(N)}\right)\right)\right]h\left(y_i,\boldsymbol{z}_i|s\right)d\boldsymbol{z}_i dy_i$$
$$=\int_{y_i\leq\hat{y}}\int_{\boldsymbol{z}_i\leq\hat{\boldsymbol{z}}}D\left(\hat{\boldsymbol{m}}|\beta\left(y_i\right),\beta\left(z_i^{(2)}\right),\ldots,\beta\left(z_i^{(N)}\right)\right)h\left(y_i,\boldsymbol{z}_i|s\right)d\boldsymbol{z}_i dy_i.$$

Note that if $B_i^{(2)}$ is always perfectly revealed, then $P_{D,\beta}^{Y,\boldsymbol{Z}}(Y_i, \boldsymbol{Z}_i | S_i; \boldsymbol{m}, b_i) = P_{D,\beta}^{Y,\boldsymbol{Z}}(Y_i, \boldsymbol{Z}_i | S_i; \boldsymbol{m})$ for all b_i—i.e., firm i's posterior is independent of its own bid.

Therefore, conditional on $S_i = s$, firm i winning the auction with a bid of b and receiving message $\boldsymbol{m}$, firm i's expected value of the oil/gas reserve at the end of Stage 4 is

$$\tilde{V}_{D,\beta}(s; \boldsymbol{m}, b) := \frac{1}{P_{D,\beta}^{Y}(\beta^{-1}(b) | s, \boldsymbol{m})} \int_{\boldsymbol{z}} \int_0^{\beta^{-1}(b)} V(s, y, \boldsymbol{z}) \, P_{D,\beta}^{Y,\boldsymbol{Z}}(dy, d\boldsymbol{z} | s; \boldsymbol{m}).$$

In turn, conditional on $S_i = s$, $Y_i = y$ and firm i winning the auction with a bid of b, firm i's expected profit from the perspective at the start of Stage 2 is

$$\tilde{v}_{D,\beta}(s, y; b) := \int \psi\left(\tilde{V}_{D,\beta}(s; \boldsymbol{m}, b)\right) dR_{D,\beta}(\boldsymbol{m} | s, y),$$

where, $R_{D,\beta}(\cdot | s, y) \in \Delta(\mathcal{M})$ denotes the ex ante distribution of the message that firm i receives after winning the auction when conditioned on $S_i = s$, $Y_i = y$, and every firm $j \neq i$ using the bidding strategy β.[2] Therefore, at the start of Stage 2, under disclosure policy is D, if $S_i = s$ and firm i conjectures that every other firm $j \neq i$ bids according to β, firm i's expected profit from bidding b is

$$\int_0^{\beta^{-1}(b)} \left[\tilde{v}_{D,\beta}(s, y; b) - b\right] g^Y(y | s) \, dy. \tag{4}$$

Let

$$\begin{aligned}
\hat{V}_{D,\beta}^{*}(s, y; \boldsymbol{m}) &:= E_{P_{D,\beta}^{Y,\boldsymbol{Z}}(\cdot | s, \boldsymbol{m})}\left[V(s, Y_i, \boldsymbol{Z}_i) | Y_i \leq y\right] \\
\bar{V}_{D,\beta}^{*}(s, y; \boldsymbol{m}) &:= E_{P_{D,\beta}^{Y,\boldsymbol{Z}}(\cdot | s, \boldsymbol{m})}\left[V(s, Y_i, \boldsymbol{Z}_i) | Y_i = y\right],
\end{aligned}$$

where $E_{P_{D,\beta}^{Y,Z}(\cdot | s, \boldsymbol{m})}$ denote the expectation operator with respect to the distribution $P_{D,\beta}^{Y,Z}(Y_i, Z_i | s, \boldsymbol{m})$.[3] Let

$$\Delta_{D,\beta}^{*}(s, y; \boldsymbol{m}) := \bar{V}_{D,\beta}^{*}(s, y; \boldsymbol{m}) - \hat{V}_{D,\beta}^{*}(s, y; \boldsymbol{m}),$$

and let $H_{D,\beta}^{Y}(Y_i | S_i, \boldsymbol{m})$ denote the reverse hazard rate associated with the distribution $P_{D,\beta}^{Y}(Y_i | S_i, \boldsymbol{m})$.[4]

[2] i.e., for any $\boldsymbol{m} \in \mathcal{M}$, $R_{D,\beta}(\boldsymbol{m} | s, y) = \int_{\boldsymbol{z}} D\left(\boldsymbol{m} | \beta(y), \beta\left(z^{(2)}\right), \ldots \beta\left(z^{(N)}\right)\right) g^{\boldsymbol{z}}\left(z^{(2)}, \ldots, z^{(N)} | s, y\right) d\boldsymbol{z}$

[3] It is useful to note that $\tilde{V}_{D,\beta}(s; \boldsymbol{m}, b) = \hat{V}_{D,\beta}^{*}\left(s, \beta^{-1}(b); \boldsymbol{m}\right)$.

[4] The reversed hazard rate is the ratio of the probability density/mass function and the cumulative distribution function. We take the convention that if $P_{D,\beta}^{Y}(y | s, \boldsymbol{m}) = 0$, then $H_{D,\beta}^{Y}(y | s, \boldsymbol{m}) = 0$.

Proposition 1 *Fix a disclosure policy D. If $\beta(\cdot)$ is an increasing and symmetric Bayes Nash equilibrium of the auction, then $\beta(\cdot)$ satisfies*

$$\beta(s) = \int_0^s \left[v^*_{D,\beta}(s', s') + w^*_{D,\beta}(s', s')\right] dL(s'|s) \quad \forall s \in [0, \bar{s}], \tag{5}$$

where

$$v^*_{D,\beta}(s, y) := \int_{\boldsymbol{m}} \psi\left(\hat{V}^*_{D,\beta}(s, y; \boldsymbol{m})\right) dR_{D,\beta}(\boldsymbol{m}|s, y),$$
$$w^*_{D,\beta}(s, y) := \left(\int_{\boldsymbol{m}} \left[\psi'\left(\hat{V}^*_{D,\beta}(s, y; \boldsymbol{m})\right) \Delta^*_{D,\beta}(s, y; \boldsymbol{m})\right] dR_{D,\beta}(\boldsymbol{m}|s, y)\right)$$

Moreover, if a function $\beta(\cdot)$ satisfies Eq. (5) and $v^*_{D,\beta}(s, y) + w^*_{D,\beta}(s, y)$ is increasing in both s and y, then $\beta(\cdot)$ is an increasing and symmetric Bayes Nash equilibrium of the auction.

Proposition 1 provides necessary and sufficient conditions for a bidding strategy to be an increasing and symmetric Bayes Nash equilibrium.

Lemma 1 *If $\beta^D(\cdot)$ is the symmetric and increasing Bayes Nash equilibrium of an auction with disclosure policy D, then:*

$$\beta^D(s) \leq \beta^{FD}(s)$$

Lemma 1 establishes that the full-disclosure auction yields the highest expected bid revenue. This is intuitive—relative to a full-disclosure policy, *any* partial disclosure policy provides less information to the winning bidder at the drilling stage, hence decreasing the winner's expected profit. This implies that the ex ante expected value of winning the auction is lower; thus, the equilibrium bids are also lower.

I provide the proofs of Proposition 1 and Lemma 1 next.

1.2.1 Proof of Proposition 1

Proof:
Fix any strictly increasing $\beta(\cdot)$ and suppose that all firms $j \neq i$ play according to β. Firm i's expected profit from bidding b when $S_i = s$ is in Eq. (4). Differentiating Eq. (4) with

respect to b, the first-order necessary condition (FOC) is:

$$
\begin{aligned}
0 = & - G^{Y}\left(\beta^{-1}(b)|s\right) + \frac{1}{\beta'\left(\beta^{-1}(b)\right)}\left[\tilde{v}_{D,\beta}\left(s,\beta^{-1}(b);b\right) - b\right] g^{Y}\left(\beta^{-1}(b)|s\right) \\
& + \int_0^{\beta^{-1}(b)} \frac{\partial \tilde{v}_{D,\beta}(s,y;b)}{\partial b} g^{Y}(y|s)\,dy. \qquad (6)
\end{aligned}
$$

Note that

$$
\tilde{v}_{D,\beta}\left(s,\beta^{-1}(b);b\right) = \int_{\boldsymbol{m}} \psi\left(\hat{V}^{*}_{D,\beta}\left(s,\beta^{-1}(b);\boldsymbol{m}\right)\right) dR_{D,\beta}\left(\boldsymbol{m}|s,\beta^{-1}(b)\right) = v^{*}_{D,\beta}\left(s,\beta^{-1}(b)\right)
$$

and

$$
\frac{\partial \tilde{v}_{D,\beta}(s,y;b)}{\partial b} = \int_{\boldsymbol{m}} \psi'\left(\hat{V}^{*}_{D,\beta}\left(s,\beta^{-1}(b);\boldsymbol{m}\right)\right)\left[\frac{\partial \tilde{V}_{D,\beta}(s;\boldsymbol{m},b)}{\partial b}\right] dR_{D,\beta}(\boldsymbol{m}|s,y)
$$

where

$$
\begin{aligned}
& \frac{\partial \tilde{V}_{D,\beta}(s;\boldsymbol{m},b)}{\partial b} \\
& = \left(\frac{1}{\beta'\left(\beta^{-1}(b)\right)}\right)\left[\frac{\int_{\boldsymbol{z}} V\left(s,\beta^{-1}(b),\boldsymbol{z}\right) P^{Y,\boldsymbol{Z}}_{D,\beta}\left(\beta^{-1}(b),d\boldsymbol{z}|s;\boldsymbol{m}\right)}{P^{Y}_{D,\beta}\left(\beta^{-1}(b)|s,\boldsymbol{m}\right)} - H^{Y}_{D,\beta}\left(\beta^{-1}(b_i)|s,\boldsymbol{m}\right)\tilde{V}_{D,\beta}(s;\boldsymbol{m},b)\right] \\
& = \left(\frac{1}{\beta'\left(\beta^{-1}(b)\right)}\right) H^{Y}_{D,\beta}\left(\beta^{-1}(b_i)|s,\boldsymbol{m}\right)\left[\bar{V}^{*}_{D,\beta}\left(s,\beta^{-1}(b);\boldsymbol{m}\right) - \hat{V}^{*}_{D,\beta}\left(s,\beta^{-1}(b);\boldsymbol{m}\right)\right]
\end{aligned}
$$

Therefore, $\int_0^{\beta^{-1}(b)} \frac{\partial \tilde{v}_{D,\beta}(s,y;b)}{\partial b} g^{Y}(y|s)\,dy = \frac{g^{Y}\left(\beta^{-1}(b)|s\right)}{\beta'(\beta^{-1}(b))} w^{*}_{D,\beta}\left(s,\beta^{-1}(b)\right)$. In turn, FOC (6) becomes

$$
0 = -G^{Y}\left(\beta^{-1}(b)|s\right) + \frac{g^{Y}\left(\beta^{-1}(b)|s\right)}{\beta'(\beta^{-1}(b))}\left[v^{*}_{D,\beta}\left(s,\beta^{-1}(b)\right) + w^{*}_{D,\beta}(s,y) - b\right] \qquad (7)
$$

At a symmetric equilibrium in which every firm plays β, FOC (7) must be satisfied at $b = \beta(s)$, which implies that

$$
\begin{aligned}
& 0 = -G^{Y}(s|s) + \frac{g^{Y}(s|s)}{\beta'(s)}\left[v^{*}_{D,\beta}(s,s) + w^{*}_{D,\beta}(s,s) - \beta(s)\right] \\
\iff \quad & \beta'(s) + \beta(s)\frac{g^{Y}(s|s)}{G^{Y}(s|s)} = \left[v^{*}_{D,\beta}(s,s) + w^{*}_{D,\beta}(s,s)\right]\frac{g^{Y}(s|s)}{G^{Y}(s|s)} \qquad (8)
\end{aligned}
$$

Eq. (8) is a linear ordinary differential equation (ODE). It is well-known that the solution to a linear ODE of the form $\beta'(s) + \beta(s)\xi(s) = \gamma(s)\xi(s)$ is

$$\theta(s)\beta(s) = \theta(0)\beta(0) + \int_0^s \gamma(s')\xi(s')\theta(s')\,ds',$$

where $\theta(s) = \exp\left(\int_0^s \xi(s')\,ds'\right)$. Using the boundary condition of $\beta(0) = 0$, we have

$$\begin{aligned}
\beta(s) &= \int_0^s \left[v^*_{D,\beta}(s', s') + w^*_{D,\beta}(s', s')\right] \left(\frac{g^Y(s'|s')}{G^Y(s'|s')} \frac{\exp\left(\int_0^{s'} \frac{g^Y(t|t)}{G^Y(t|t)} dt\right)}{\exp\left(\int_0^{s} \frac{g^Y(t|t)}{G^Y(t|t)} dt\right)} \right) ds' \\
&= \int_0^s \left[v^*_{D,\beta}(s', s') + w^*_{D,\beta}(s', s')\right] \left(\frac{g^Y(s'|s')}{G^Y(s'|s')} \exp\left(-\int_{s'}^{s} \frac{g^Y(t|t)}{G^Y(t|t)} dt\right)\right) ds' \\
&= \int_0^s \left[v^*_{D,\beta}(s', s') + w^*_{D,\beta}(s', s')\right] dL(s'|s). \qquad (9)
\end{aligned}$$

This proves the first (necessity) part of the proposition.

Next, to prove the sufficiency part, suppose that $v^*_{D,\beta}(s', s') + w^*_{D,\beta}(s', s')$ is also increasing. Note that for any s, $L(0|s) = 0$, $L(s|s) = 1$, and $L(\cdot|s)$ is increasing; therefore, $L(\cdot|s)$ is a CDF with support on $[0, s]$. Moreover, if $\hat{s} > s$, then $L(y|\hat{s}) < L(y|s)$ for all y — i.e., first-order stochastic dominance. In turn, because the integrand term in Eq. (9) is increasing in s', $\beta(s)$ is (indeed) an increasing function.

To show that bidding $b = \beta(s)$ is indeed firm i's best response against all other firms playing β, first note that bidding above $\beta(\bar{s})$ is worse than bidding $\beta(s)$; therefore, firm i's best response (if it exists) is a bid in $[\beta(0), \beta(\bar{s})]$. Suppose that firm i bids $\beta(\tilde{s})$, where $\tilde{s} \neq s$. Its expected profit (using Eq. (4)) is

$$U(\tilde{s}|s) := \int_0^{\tilde{s}} \left[\tilde{v}_{D,\beta}(s, y; \beta(\tilde{s})) - \beta(\tilde{s})\right] g^Y(y|s)\,dy.$$

Let

$$\begin{aligned}
W(\tilde{s}|s) &= \frac{\partial U(\tilde{s}|s)}{\partial \tilde{s}} \\
&= -G^Y(\tilde{s}|s)\beta'(\tilde{s}) + \left[\tilde{v}_{D,\beta}(s, \tilde{s}; \beta(\tilde{s})) - \beta(\tilde{s})\right] g^Y(\tilde{s}|s) + \beta'(\tilde{s}) \int_0^{\tilde{s}} \frac{\partial \tilde{v}_{D,\beta}(s, y; \beta(\tilde{s}))}{\partial b} g^Y(y|s)\,dy \\
&= G^Y(\tilde{s}|s) \left(\left[v^*_{D,\beta}(s, \tilde{s}) + w_{D,\beta}(s, \tilde{s}) - \beta(\tilde{s})\right] \frac{g^Y(\tilde{s}|s)}{G^Y(\tilde{s}|s)} - \beta'(\tilde{s}) \right).
\end{aligned}$$

From FOC (8), $W(\tilde{s}|\tilde{s}) = 0$. If $\tilde{s} > s$, then $v^*_{D,\beta}(s,\tilde{s}) + w_{D,\beta}(s,\tilde{s}) < v^*_{D,\beta}(\tilde{s},\tilde{s}) + w_{D,\beta}(\tilde{s},\tilde{s})$; moreover, because the signals are affiliated, $\frac{g^Y(\tilde{s}|s)}{G^Y(\tilde{s}|s)} < \frac{g^Y(\tilde{s}|\tilde{s})}{G^Y(\tilde{s}|\tilde{s})}$. Together with $W(\tilde{s}|\tilde{s}) = 0$, we have $W(\tilde{s}|s) < 0$. On the other hand, if $\tilde{s} < s$, then $v^*_{D,\beta}(s,\tilde{s}) + w_{D,\beta}(s,\tilde{s}) > v^*_{D,\beta}(\tilde{s},\tilde{s}) + w_{D,\beta}(\tilde{s},\tilde{s})$ and $\frac{g^Y(\tilde{s}|s)}{G^Y(\tilde{s}|s)} > \frac{g^Y(\tilde{s}|\tilde{s})}{G^Y(\tilde{s}|\tilde{s})}$, which implies that $W(\tilde{s}|s) > 0$. Therefore, bidding $\beta(s)$ is firm i's best response. ■

1.2.2 Proof of Lemma 1

Proof:

To show that $\beta^D(s) < \beta^{FD}(s)$, note that

$$\begin{aligned}
&\psi'\left(\hat{V}^*_{D,\beta}(s',s';\boldsymbol{m})\right)\left[\bar{V}^*_{D,\beta}(s',s';\boldsymbol{m}) - \hat{V}^*_{D,\beta}(s',s';\boldsymbol{m})\right] \\
=&\int_0^{(1-r)\hat{V}^*_{D,\beta}(s',s';\boldsymbol{m})} (1-r)\left[\bar{V}^*_{D,\beta}(s',s';\boldsymbol{m}) - \hat{V}^*_{D,\beta}(s',s';\boldsymbol{m})\right] dF^C(c) \\
=&\int_0^{(1-r)\hat{V}^*_{D,\beta}(s',s';\boldsymbol{m})} \left((1-r)\bar{V}^*_{D,\beta}(s',s';\boldsymbol{m}) - c\right) dF^C(c) - \\
&\quad\int_0^{(1-r)\hat{V}^*_{D,\beta}(s',s';\boldsymbol{m})} \left((1-r)\hat{V}^*_{D,\beta}(s',s';\boldsymbol{m}) - c\right) dF^C(c) \\
\leq&\psi\left(\bar{V}^*_{D,\beta}(s',s';\boldsymbol{m})\right) - \psi\left(\hat{V}^*_{D,\beta}(s',s';\boldsymbol{m})\right)
\end{aligned}$$

Therefore,

$$\begin{aligned}
&\psi\left(\hat{V}^*_{D,\beta}(s',s';\boldsymbol{m})\right) + \psi'\left(\hat{V}^*_{D,\beta}(s',s';\boldsymbol{m})\right)\left[\bar{V}^*_{D,\beta}(s',s';\boldsymbol{m}) - \hat{V}^*_{D,\beta}(s',s';\boldsymbol{m})\right] \\
\leq&\psi\left(\bar{V}^*_{D,\beta}(s',s';\boldsymbol{m})\right) \\
=&\psi\left(E\left[V(S_i,Y_i,\boldsymbol{Z}_i)\,|S_i = s', Y_i = s', \boldsymbol{m}\right]\right) \\
\leq&E\left[\psi\left(V(S_i,Y_i,\boldsymbol{Z}_i)\right)|S_i = s', Y_i = s'\right] \\
=&v(s',s'),
\end{aligned}$$

where the inequality in the third line follows from ψ being convex. Therefore, $\beta^D(s) < \beta^{FD}(s)$. ■

1.3 Equilibrium under the Partial Disclosure Policies in Section 6

I first prove an intermediate lemma.

Lemma 2 *If $\bar{\phi}$ and $\hat{\phi}$ are two nonnegative real functions that are strictly increasing and $\bar{\phi}(x) \geq \hat{\phi}(x)$ for all x, then $\xi(x) := \psi\left(\hat{\phi}(x)\right) + \psi'\left(\hat{\phi}(x)\right)\left[\bar{\phi}(x) - \hat{\phi}(x)\right]$ is strictly increasing in x.*

Proof:
Consider any $x' > x$.

$$\begin{aligned}\xi(x') - \xi(x) =& \psi\left(\hat{\phi}(x')\right) - \psi\left(\hat{\phi}(x)\right) \\ &+ \psi'\left(\hat{\phi}(x')\right)\left[\bar{\phi}(x') - \hat{\phi}(x')\right] - \psi'\left(\hat{\phi}(x)\right)\left[\bar{\phi}(x) - \hat{\phi}(x)\right]\end{aligned}$$

Because ψ is increasing and convex, we have, for any $z' > z$,

$$\psi(z') - \psi(z) > \psi'(z)(z' - z).$$

This implies that

$$\begin{aligned}\psi\left(\hat{\phi}(x')\right) - \psi\left(\hat{\phi}(x)\right) >& \psi'\left(\hat{\phi}(x)\right)\left(\hat{\phi}(x') - \hat{\phi}(x)\right) \\ =& \psi'\left(\hat{\phi}(x)\right)\left(\bar{\phi}(x') - \hat{\phi}(x)\right) - \psi'\left(\hat{\phi}(x)\right)\left(\bar{\phi}(x') - \hat{\phi}(x')\right) \\ >& \psi'\left(\hat{\phi}(x)\right)\left(\bar{\phi}(x) - \hat{\phi}(x)\right) - \psi'\left(\hat{\phi}(x')\right)\left(\bar{\phi}(x') - \hat{\phi}(x')\right),\end{aligned}$$

where the last inequality follows from, first, $\bar{\phi}(x') > \bar{\phi}(x)$ because $\bar{\phi}$ is increasing, and, second, $\psi'\left(\hat{\phi}(x)\right) < \psi'\left(\hat{\phi}(x')\right)$ because ψ is convex and $\hat{\phi}$ is increasing. This means that $\xi(x') > \xi(x)$. ■

Proposition 2 *Under a disclosure policy in which the auctioneer withholds the k highest losing bid—i.e., D defined in (1)—there exists a symmetric and increasing Bayes Nash equilibrium for the auction.*

Proof:
If $k = N - 1$, this is the nondisclosure policy, which has already been considered. Thus, assume that $k < N - 1$. Let $\bar{\boldsymbol{Z}}_i = \left(Z_i^{(2)}, \ldots, Z_i^{(k)}\right)$ and $\underline{\boldsymbol{Z}}_i = \left(Z_i^{(k+1)}, \ldots, Z_i^{(N)}\right)$. (If $k = 1$, ignore $\bar{\boldsymbol{Z}}_i$.) Suppose that every player plays according to some strictly increasing bidding strategy β. Conditional on $\underline{\boldsymbol{Z}}_i = \underline{\boldsymbol{z}} = \left(z^{(k+1)}, \ldots, z^{(N)}\right)$ and bidder i winning the auction, bidder i always receives message $m_\beta(\underline{\boldsymbol{z}}) = \left(0, \ldots, 0, \beta\left(z^{(k+1)}\right), \ldots, \beta\left(z^{(N)}\right)\right)$. Conversely, the support of the messages that i receives is $m_\beta(\underline{\boldsymbol{z}})$, with $\underline{\boldsymbol{z}}$ in the support of $\underline{\boldsymbol{Z}}_i$. Thus, I

can represent each possible message by $m_\beta(\underline{\boldsymbol{z}})$. Upon receiving $m_\beta(\underline{\boldsymbol{z}})$, conditional on $S_i = s$ and $Y_i \leq y$, the expected value of the tract is

$$\begin{aligned}\hat{V}^*_{D,\beta}(s, y; m_\beta(\underline{\boldsymbol{z}})) &= E_{Y_i, \bar{\boldsymbol{Z}}_i}\left[V\left(S_i, Y_i, \bar{\boldsymbol{Z}}_i, \underline{\boldsymbol{Z}}_i\right) | S_i = s, Y_i \leq y, \underline{\boldsymbol{Z}}_i = \underline{\boldsymbol{z}}\right] \\ &= \hat{e}(s, y, \underline{\boldsymbol{z}}).\end{aligned}$$

Upon receiving $m_\beta(\underline{\boldsymbol{z}})$, conditional on $S_i = s$ and $Y_i = y$ (instead), the expected value of the tract is

$$\begin{aligned}\bar{V}^*_{D,\beta}(s, y; m_\beta(\underline{\boldsymbol{z}})) &= E_{Y_i, \bar{\boldsymbol{Z}}_i}\left[V\left(S_i, Y_i, \bar{\boldsymbol{Z}}_i, \underline{\boldsymbol{Z}}_i\right) | S_i = s, Y_i = y, \underline{\boldsymbol{Z}}_i = \underline{\boldsymbol{z}}\right] \\ &= \bar{e}(s, y, \underline{\boldsymbol{z}}).\end{aligned}$$

Because the winner receives $m_\beta(\underline{\boldsymbol{z}})$ only if $\underline{\boldsymbol{Z}}_i = \underline{\boldsymbol{z}}$, this implies that $R_{D,\beta}(m_\beta(\underline{\boldsymbol{z}}) | s, y) = G^{\underline{\boldsymbol{Z}}}(\underline{\boldsymbol{z}} | S_i = s, Y_i = y)$, the distribution of $\underline{\boldsymbol{Z}}_i$ conditional on $S_i = s$ and $Y_i = y$. Therefore, we have

$$\begin{aligned}v^*_{D,\beta}(s, y) &= \int_{\underline{\boldsymbol{z}}} \psi(\hat{e}(s, y, \underline{\boldsymbol{z}}))\, dG^{\underline{\boldsymbol{Z}}}(\underline{\boldsymbol{z}} | s, y) \\ &=: v^*_D(s, y) \\ w^*_{D,\beta}(s, y) &= \int_{\underline{\boldsymbol{z}}} \psi'(\hat{e}(s, y, \underline{\boldsymbol{z}}))\left[\bar{e}(s, y, \underline{\boldsymbol{z}}) - \hat{e}(s, y, \underline{\boldsymbol{z}})\right] dG^{\underline{\boldsymbol{Z}}}(\underline{\boldsymbol{z}} | s, y) \\ &=: w^*_D(s, y)\end{aligned}$$

Therefore, the bidding function $\beta^k(s) := \int_0^{s'}\left[v^*_D(s', s') + w^*_D(s', s')\right] dL(s' | s)$ satisfies Eq. (5).

Next, because the signals are affiliated, both $\bar{e}$ and $\hat{e}$ are strictly increasing in each of its arguments and $\bar{e}(s, y, \underline{\boldsymbol{z}}) > \hat{e}(s, y, \underline{\boldsymbol{z}})$. By Lemma 2, $\psi(\hat{e}(\cdot)) + \psi'(\hat{e}(\cdot))\left[\bar{e}(\cdot) - \hat{e}(\cdot)\right]$ is strictly increasing in each of its arguments. In turn, because $\underline{\boldsymbol{Z}}_i$ is affiliated with S_i and Y_i, $v^*_{D,\beta}(s, y) + w^*_{D,\beta}(s, y)$ must also be increasing in s and Y.[5] By Proposition 1, β^k is a symmetric and increasing equilibrium. ■

Proposition 3 *Under a disclosure policy in which the auctioneer discloses only bids that are lower than α — i.e., D defined in (2) — there exists a symmetric and increasing Bayes Nash equilibrium for the auction.*

[5] See Theorem 5 in Milgrom and Weber (1982).

Proof:

I first construct a β function that satisfies Eq. (5). Fix some $a \in [0, \bar{s}]$ and let $\hat{Z}_i^{(j)} = \min\left\{a, \hat{Z}_i^{(j)}\right\}$ and $\hat{\boldsymbol{Z}}_i = \left(\hat{Z}_i^{(2)}, \ldots, \hat{Z}_i^{(n)}\right)$. Let

$$\begin{aligned} \hat{e}\left(s, y, \hat{\boldsymbol{z}}\right) =& E_{Y_i, \boldsymbol{Z}_i}\left[V\left(S_i, Y_i, \boldsymbol{Z}_i\right)|S_i = s, Y_i \leq y, \hat{\boldsymbol{Z}}_i = \hat{\boldsymbol{z}}\right] \\ \bar{e}\left(s, y, \hat{\boldsymbol{z}}\right) =& E_{Y_i, \boldsymbol{Z}_i}\left[V\left(S_i, Y_i, \boldsymbol{Z}_i\right)|S_i = s, Y_i \leq y, \hat{\boldsymbol{Z}}_i = \hat{\boldsymbol{z}}\right] \end{aligned}$$

Let $\tilde{R}\left(\cdot|s, y\right)$ denote the distribution of $\hat{\boldsymbol{Z}}_i$ conditional on $S_i = s$ and $Y_i = y$—i.e.,

$$\tilde{R}\left(\hat{z}^{(2)}, \ldots, \hat{z}^{(N-1)}|s, y\right) = \int_{z^{(N-1)} > a} \cdots \int_{z^{(a)} > a} G^{\boldsymbol{Z}}\left(z^{(2)}, \ldots, z^{(N-1)}|s, y\right) dz^{(2)} \ldots dz^{(N-1)}.$$

Let

$$\begin{aligned} \tilde{v}^*\left(s, y\right) &= \int_{\hat{\boldsymbol{z}}} \psi\left(\hat{e}\left(s, y, \hat{\boldsymbol{z}}\right)\right) d\tilde{R}\left(\hat{\boldsymbol{z}}|s, y\right) \\ \tilde{w}^*\left(s, y\right) &= \int_{\hat{\boldsymbol{z}}} \psi'\left(\hat{e}\left(s, y, \hat{\boldsymbol{z}}\right)\right)\left[\bar{e}\left(s, y, \hat{\boldsymbol{z}}\right) - \hat{e}\left(s, y, \hat{\boldsymbol{z}}\right)\right] d\tilde{R}\left(\hat{\boldsymbol{z}}|s, y\right). \end{aligned}$$

Let

$$\tilde{\beta}\left(s\right) = \int_0^s \left[\tilde{v}^*\left(s', s'\right) + \tilde{w}^*\left(s', s'\right)\right] dL\left(s'|s\right), \tag{10}$$

Using a similar argument as that used in Proposition 2, $\tilde{v}^* + \tilde{w}^*$ is strictly increasing in both its argument, thereby implying that $\tilde{\beta}$ is strictly increasing. Given that $\tilde{\beta}$ is increasing, under a disclosure policy with threshold $\alpha = \tilde{\beta}\left(a\right)$, $\tilde{\beta}$ satisfies Eq. (5) by construction.[6] By Proposition 1, $\tilde{\beta}$ is a symmetric and increasing equilibrium. ■

2 Entry and Information Acquisition

In this section, I consider an extension where the number of bidders in the auction is endogenously determined in an entry game. To simplify the analysis, I focus only on the comparison between the FD policy and the ND policy. Because the ND policy reduces the value from winning the auction for the winning bidder, one might expect that bidders might have less incentive to participate in the auction, thereby reducing the number of bidders and potentially the auctioneer's revenue.

[6] If every player plays according to $\tilde{\beta}$, then the disclosure policy is equivalent to revealing the realization of $\hat{\boldsymbol{Z}}_i$.

I consider an entry game similar to that of **?**. The timeline is as follows. Before the auction, each potential bidder, among the set of $\bar{N}$ bidders,[7] draws a private signal acquisition cost η, where η is iid across bidders and η is drawn from a log-normal distribution. If a bidder pays the signal acquisition cost, its private signal S is then realized, and I assume that S and η are independent. Only bidders that obtain the signals can participate in the auction. Before bidding, the number of active bidders N becomes public information.[8] The rest of the game follows as in Section 4.1 (for the FD policy) and Section 4.2 (for the ND policy).

To solve this entry game, it is important to consider the effect of the disclosure policy on a potential bidder's *profit*, which will later determine the equilibrium entry decision. I first note that the ND policy does not necessarily lower a bidder's expected profit. A bidder's profit comprises two components. The first component is the revenue from the tract (taking into account the royalty payment), conditional on winning. Given a winning signal s, the difference in this revenue component between the ND and the FD policies is negative and equal to

$$s\psi\left(\mathbb{E}\left(V\left(s,s'\right)|s'\leq s\right)\right)-s\mathbb{E}\left(\psi\left(V\left(s,s'\right)\right)|s'\leq s\right)$$

The second component is the equilibrium bid. As explained in Section 4.2, the reduction in the bid ($\beta^{ND}-\beta^{FD}$) arises not only from the reduction in the value from winning the auction but also from the winning bidder's inability to observe the second-highest bid. Therefore, it is possible that the reduction in the bids more than offsets the reduction in the expected revenue from the tract, resulting in an *increase* in a bidder's expected profit. In Section 2.1, I provide a parametric example to illustrate this point.

In this entry game, a potential bidder will acquire a signal if its realized acquisition cost is below a threshold, and this threshold is determined by (1) its conjecture about other bidders' bidding strategies and (2) the expected profit from bidding in the auction. The main complication of the entry game here is that the threshold strategy might not be unique. This is because the expected profit from bidding in the auction is not necessarily a decreasing function of the realized number of bidders N. Specifically, fixing a disclosure policy, a bidder who wins against a greater number of bidders receives more information after the auction, and this informational advantage increases its expected payoff. The equilibrium threshold

[7] I set $\bar{N}$ to be the maximum number of bidders on neighboring tracts—i.e., tracts within 0.11 degrees of latitude and 0.12 degrees of longitude—of the same depth type (shallow or deep). On average, shallow tracts have potential bidders, and deep tracts have potential bidders.

[8] The assumption that N becomes public is important in this setting for two reasons. First, when N is unknown, it is possible that a strictly increasing bidding strategy does not exist, and there might be bunching (**?**). Second, the winning bidder's drilling decision now depends on both its belief about the losing bidders' signals *and* its belief about the number of actual bidders based on the message it receives.

strategy is unique only if the competition effect, which is a negative effect on profit due to the higher equilibrium bids, dominates the information effect. In the estimation, I verify that the estimates of the auction stage yield a monotonic relationship between a bidder's profit and the number of bidders; therefore, the entry threshold is unique.

Table 1: Estimated entry cost, entry probabilities, and auctioneer's revenue under FD and ND policies

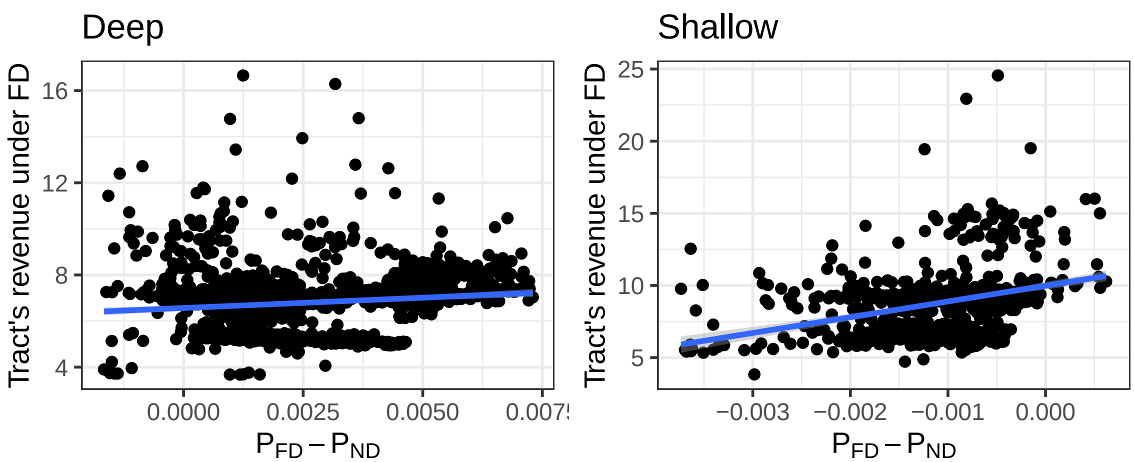
	Deep tracts	Shallow tracts
Distribution of log entry cost		
Expected value (across tracts)	1.36	0.95
	(0.07)	(0.34)
Standard deviation (across tracts)	3.31	5.26
	(0.15)	(0.48)
Average probability of entry under FD	0.24	0.25
	(2e-04)	(4e-04)
Average probability of entry under ND	0.24	0.25
	(1e-04)	(5e-04)
Average auctioneer's revenue under FD	6.81	9.27
	(0.08)	(0.44)
Average auctioneer's revenue under ND	6.93	9.4
	(0.08)	(0.46)

Note: The numbers in parentheses are the bootstrapped standard errors. The revenues under FD and ND are measured in millions of dollars. The distribution of the log entry cost is normalized and computed relative to the homogenized bids.

The estimates and implications of the entry model are reported in Table 1. The parameters of the entry game are the expected value of the distribution of the log entry cost, which is allowed to vary across tracts based on tracts' observed characteristics (as discussed in Section 5.3) in the main text, and the standard deviation of this distribution, which is assumed to be fixed across tracts. On average, the cost of entry is higher for deep tracts than for shallow tracts, which is consistent with the interpretation that the entry cost is the signal acquisition cost.

The results in Table 1 show that the ND policy does not reduce the average number of (active) bidders in equilibrium. Under ND, the average probabilities of entry for deep and shallow tracts are similar to that under FD and is equal to 24% and 25% on average. However, the ND policy has heterogeneous impacts across tracts. The ND policy tends to increase (decrease) the entry probability for tracts that generate lower (higher) revenue for the auctioneer (Figure 2). Therefore, the positive impact of the ND policy is dampened when the entry decision is taken into account, and the increase in revenue is approximately \$120K per tract for deep tracts and \$130K for shallow tracts on average.

Figure 2: Correlation between the difference in probability of entry and tract's revenue



Note: The slope of the blue line represents the correlation between the tract's revenue under FD and the difference in the probability of entry between the FD policy and the ND policy

2.1 Parametric Example

In this parametric example, I show that the bidders' expected profit may be higher under an ND policy than under an FD policy.

There are two bidders ($N = 2$). S_i, S_j are independent and are uniformly distributed in $[0, 1]$. Conditional on $S_i = s$, and $S_j = s'$, the tract's value is given by

$$V(s, s') = \frac{s + s'}{2}$$

The cumulative distribution of the drilling cost is $F^C(x) = 1 - (1 - x)^2$. Using the same notations as in the main text:

$$\psi(x) = \int^{(1-r)x} (1 - r)x - c dF^C(c)$$

and

$$L(s'|s) = \frac{s'}{s}$$

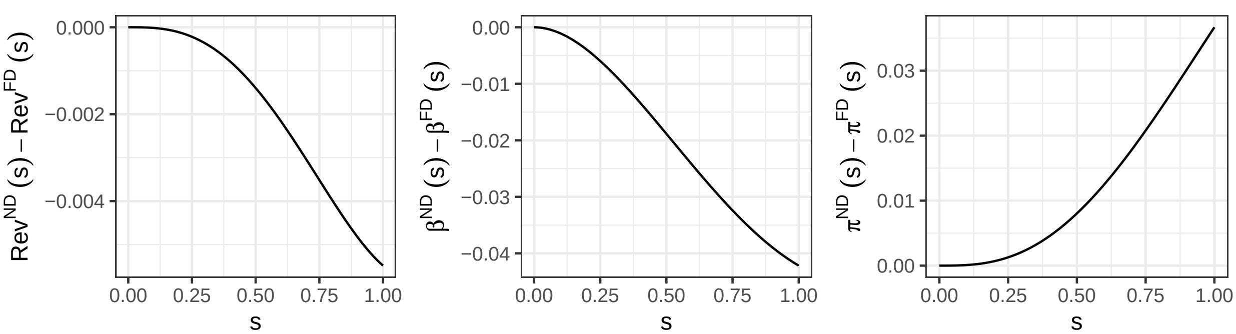
Therefore, using Proposition 1 and 2, the equilibrium bidding strategies under FD and ND are given by:

$$\beta^{FD}(s) = \frac{1}{s}\int_0^s \psi\left(s'\right) ds'$$
$$\beta^{ND}(s) = \frac{1}{s}\int_0^s \psi\left(\frac{3}{4}s'\right) + (1-r)\frac{3}{4}s'\left(\psi\left(s'\right) - \psi\left(\frac{3}{4}s'\right)\right) ds'$$

The firms' profits under FD and ND are thus:

$$\pi^{FD}(s) = \int_0^s \psi(\frac{s+s'}{2})ds' - s\beta^{FD}(s)$$
$$\pi^{ND}(s) = \psi\left(\frac{3}{4}s\right) - s\beta^{ND}(s)$$

Figure 3: Difference between bidder's profits under ND and FD when $r = 0.1$



The first figure shows the difference in expected revenue from the tract, conditional on winning with signal s, between the ND policy and the FD policy. Since the ND policy creates more ex-post mistakes, the difference in revenue is always negative. However, the difference in the bids in the second figure is greater in magnitude than the difference in the expected revenue. Therefore, a bidder's expected profit is higher under ND than under FD.