How It's Made: A General Theory of the Labor Implications of Technology Change*

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Abstract

We develop a general theory relating technology change and skill demand. Performers (human or machine) face stochastic issues that must be solved in order to complete tasks. Firms choose how production tasks are divided into steps, the rate at which steps need to be completed, and the skill of the performer assigned to a step. Longer steps are more complex. Performers face a tradeoff between the complexity of their step and the rate at which they can perform. Human performers tend to have an advantage in complex steps while machine performers have an advantage in high rates. The cost of fragmenting tasks into steps and the cost of allocating performers to multiple steps are both central to the theory. We derive the optimal division of tasks, the level of automation, and the demand for workers of different skill levels. The theory predicts that automation generates skill polarization at lower production volumes and is upskilling at higher volumes; in addition, the theory implies that a reduction in fragmentation costs (such as interchangeable parts) increases the demand for low skill; and that technology change that raises the cost of fragmenting tasks (such as parts consolidation) reduces the dispersion of skill demand. We find counterparts to the theory across a range of contexts and time periods, including the Hand-Machine Labor Study covering mechanization and process improvement at the end of the 19th century and in contemporary automotive body assembly and optoelectronic semiconductor manufacturing.

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1 Introduction

The goal of this paper is to understand why different technological advancements have had varying effects on workers throughout history. For instance, the introduction of the factory system and machinery in the 19th century resulted in a reduction in skill requirements (Hounshell, 1985; Goldin and Katz, 1998). Conversely, the automation of routine tasks from the 1970s to the 1990s led to an increase in skill demand (Autor, Levy, and Murnane, 2003).[1] While the literature has provided compelling explanations of these patterns in terms of substitutability between the capital that embodies a technology and worker skill, it does not explain why these differences in substitutability exist. This paper provides an explanation: we identify three pivotal factors that together can explain the impact of technology on the demand for skills. The first factor, recognized since Adam Smith's time, pertains to technology's role in shaping the division of labor. The second factor looks at the trade-offs between the complexity of tasks and the speed of execution, and ways in which the tradeoff differs for machines and workers. The final factor looks at the cost of redeploying workers or machines at different tasks.

The starting point of the model is the set of tasks that must be completed to make a product or a service. To minimize the cost of producing at a given volume, a firm chooses how to divide this set of tasks into production steps. The firm selects the performer type for each step (humans or machines of different ability level) and the rate of production for each step. The difficulty of a step is increasing in the number of tasks (the length of a step) and in the rate at which the step needs to be completed. How the difficulty of a step is impacted by the number of tasks or the rate of completion is specific to the type of performer: humans are less sensitive to the number of tasks than machines (they are more general than machines) but more sensitive to rate. In deciding the division of production, the firm faces a trade-off. More difficult steps require a more able and thus more costly performer. This mechanism provides an incentive for smaller steps. On the other hand, division of two sequential tasks incurs fragmentation costs, providing an incentive for longer steps. The firm must also take into account excess performer capacity, either by allowing a performer to be idle or by reallocating the performer to a different step. Reallocation incurs a performer-specific divisibility cost. This cost is higher for machines than humans. In the model, technological change can be described in terms of how it alters five dimensions: 1) the overall complexity of a process, 2) the cost of dividing tasks in a process, 3) the sensitivity of performers to the rate of production, 4) the sensitivity of performers to the number of tasks in a step, 5) the cost of dividing performers among multiple steps.

We characterize the impact of key technological changes on production and workers with three main results. First, we identify conditions under which it is optimal for firms to divide production into smaller steps. We show that heterogeneous costs of dividing different tasks are necessary for heterogeneity in ability demand within a firm. For division to occur, performer

[1] The literature on the impact of technology on workers and specifically the way in which different technologies differ is vast. As a starting point refer to: Caselli (1999), Bresnahan, Brynjolfsson, and Hitt (2002), Acemoglu and Autor (2011) Autor and Dorn (2013), Dinlersoz and Wolf (2018), Eden and Gaggl (2019), Acemoglu and Restrepo (2020), Jaimovich et al. (2021).

costs must be convex in the length of steps. This convexity occurs when wages are sufficiently convex with increasing skill or with a sufficiently high production volume. From a historical perspective, the optimality of division of labor under high volume explains the adoption of the factory system and of the assembly line.

Second, we provide conditions under which it is optimal to automate a step. We find that two dimensions determine the choices of automation: the volume of production and the step length. Within these two dimensions, our theoretical results identify a region we call a *cone of automation*. Specifically, we find that at sufficiently low production volumes no automation is optimal because the higher divisibility costs of machines lead firms to leave them idle, thus raising overall costs. At middle production volumes, it is optimal to automate middle-length steps. This causes machines to substitute for middle skill workers, generating skill polarization. Short steps are not automated because they have high rates of work and hence low machine utilization, leading to high idling costs. Long steps are also not automated due to their complexity. As steps increase in length, the cost of a machine performer increases faster than a human performer, because machine performers are less general than human performers. At high volumes, machine utilization is high even at high rates of work, and so only the longest steps are not automated (substituting for low and middle skill workers). The cone of automation is a useful result for understanding the root causes of historical variation in the effects of automation (Goldin and Katz, 1998) and the more recent polarization of occupational demand (Goos, Manning, and Salomons, 2009; Acemoglu and Autor, 2011). The result on the cone of automation is significant as it implies that the phenomenon of skill polarization is not only a relatively recent aggregate information-technology related phenomenon but is connected to any type of automation and is potentially observable at firm level.

For our third main result we explore how changes in the division of tasks can affect skill demand and hence wages. We show that declining costs of dividing tasks (occurring during the initial phases of the industrial revolution) decreases the lower bound of skill demand. We also consider technologies that reduce fragmentation costs but increase process complexity (such as modularization). We find that such technologies increase inequality between the highest and lowest wages by polarizing the upper and lower bounds of skill demand. This result also shows that technologies that reduce process complexity by eliminating opportunities to divide tasks (such as parts consolidation) can reduce inequality between the highest and lowest wages.

We take our model to the data and provide empirical counterparts to key results of the theory. The model presented in the paper is rich enough to provide a tight linkage with production operations data. This type of data has rarely been used in the economic analysis of technology change. We use three sources of detailed operations data. The first dataset is the *Hand and Machine Labor Study* (HML) (Wright, 1898), covering mechanization and process innovations at the time of the Second Industrial Revolution (1870s to 1910s). This dataset breaks down the production of products spanning mining, agricultural, manufacturing and transportation services. The other two data sets are contemporary, capturing in detail the optoelectronic semiconductor component production and assembly (Combemale, Whitefoot, Ales, and Fuchs, 2020) and the automotive body assembly (Fuchs, Field, Roth, and Kirchain, 2008). The optoelectronic semiconductor data involves hand-collected shop-floor-level production data for a single data communications product. The automotive body assembly data contains detailed data on process flow from multiple major U.S. vehicle manufacturers.

In taking the model to the data, we first look at some of the underlying assumptions of the model. We find evidence (using the automotive and optoelectronic semiconductor data) of the trade-offs between the number of tasks in steps and the rate of operations, consistent with the developed model. We also find evidence (using the optoelectronic semiconductor data) that the level of ability demand is indeed increasing with the number of tasks that make up a production step. We show that our theory can explain historical and contemporary changes in the distribution of worker ability demand when technology impacts the cost of dividing tasks. We show in the HML context that an increase in the division of tasks leads to polarization toward the highest and lowest wages. This finding is consistent with what the theory would predict for technology changes occurring at the time, such as the adoption of interchangeable parts. The theory is also consistent with our observations in optoelectronics that technologies that reduce the divisibility of tasks but also reduce process complexity (such as parts consolidation) lead to a convergence of ability demand, with less demand for the highest and lowest ability and higher demand for middle-level ability. Finally, our main result, we find that the theory can rationalize patterns of substitution of machines for human workers. We recover an empirical analog to the *cone of automation* directly from production data during the second industrial revolution, in the HML study. The empirical analogue cone of automation is built by comparing steps across products with different levels of utilization (a measure that can be related to production volumes in our theory). In modern production data on optoelectronics, we also show polarization of ability demand following automation, consistent with the implications of our theory at middle production volumes.

Literature Building on Adam Smith's insights and the pin-factory example, a small literature examines task division factors. Smith and Stigler (1951) suggest market size limits specialization due to low demand for high-output firms in small markets. An alternative view (Becker and Murphy (1992); Yang and Ng (1998)) considers coordination costs among team members as the limit. Our approach covers both aspects: volume is directly considered, and coordination costs are represented as fragmentation costs. Our findings expand on this literature, revealing how task division, technology change, and skill demand interact, showing that finely divided tasks are more susceptible to automation in high-volume settings.

We connect to a literature focusing on the task content of production (Autor, 2013; Acemoglu and Restrepo, 2018a,b) and featuring task-assignment models.[2] The former examines the long-run effects of displacement of workers by capital using a framework where jobs are bundles of

[2]See for example, Rosen (1978); Costinot and Vogel (2010); Ales et al. (2015); Lindenlaub (2017); Ocampo (2018); Haanwinckel (2020).

tasks that can be performed by humans or machines. The latter literature studies the optimal assignment of heterogeneous workers to jobs of varying complexity.[3] Similarly to Autor (2013) and Acemoglu and Restrepo (2018a,b), we consider a job as a bundle of steps and study the assignment of a step to either a human or a machine. Similarly to other task-assignment models, we consider heterogeneous workers with different abilities that can be assigned to jobs of varying complexity. We extend both approaches by considering technology change more broadly (going beyond automation) and by studying the endogenous bundling and assignment of work activities. In our model the assignment is endogenous and so is the complexity of the job, which is determined by the set of tasks and the rate of production. This approach is closer to the original motivation of Rosen (1978). We also study the effects of performer indivisibility on differential returns to scale, a feature whose importance Rosen emphasized but did not include in his model. Our focus is also distinct from most of this literature. This paper is well suited at analyzing changes that occur within firms adopting new technologies. On the other hand, papers such as Acemoglu and Restrepo (2018b), considering creation of new tasks, speak on the equilibrium of the entire economy.

The paper relates to the literature on polarization of occupational demand (Goos, Manning, and Salomons, 2009; Acemoglu and Autor, 2011; Goos, Rademakers, Salomons, and Vandeweyer, 2019; Jaimovich and Siu, 2020). This literature has identified aggregate changes in the occupational structure of advanced economies in the last few decades. Polarization refers to the fact that middle-wage occupations exhibit slower growth relative to low and high paying occupations. Relative to this literature, we provide a micro-founded mechanism for these occupational changes. We show the condition in which automation is more likely to occur for mid-level skills, and we also examine when automation occurs for low-level skills. In addition, the data presented in this paper provides additional plant-level evidence of the polarization phenomenon.

The paper also connects to the literature on the labor consequences of different forms of automation, from traditional mechanization (Goldin and Katz, 1998) to robotics (Graetz and Michaels, 2018; Acemoglu and Restrepo, 2020) to machine learning (Brynjolfsson et al., 2018). We do so by explaining how these and other technological changes affect task divisibility and may generate differential labor outcomes. For example, in our theory robotics offers more general performers than traditional mechanization, leading to more automation of high skill steps. Meanwhile machine learning offers both greater generality and greater divisibility, which leads to more automation of high and low skill steps.

Our modeling approach, like Garicano and Rossi-Hansberg (2006), focuses on hierarchical organization within firms, dividing tasks by complexity and assigning them to workers, creating an endogenous earnings-talent link. Unlike their work, our theory accommodates flexible task division, involving human and machine performers, and endogenous production rates.

The paper proceeds as follows: Section 2 motivates the key ingredients in the model which is formalized in Section 3. Section 4 analyzes the implications of the model: optimality of division

[3] Most of the task-assignment literature is fairly general in the set of jobs and skills analyzed. This is expected as the scope of the analysis encompasses the entirety of the labor market.

of tasks and provides results leading to the cone of automation. Section 5 provides empirical counterparts on the main findings of this paper. Section 6 concludes.

2 Empirical Motivation

This section introduces and provides support for our modeling ingredients using evidence from the economic, historic, and engineering literature.

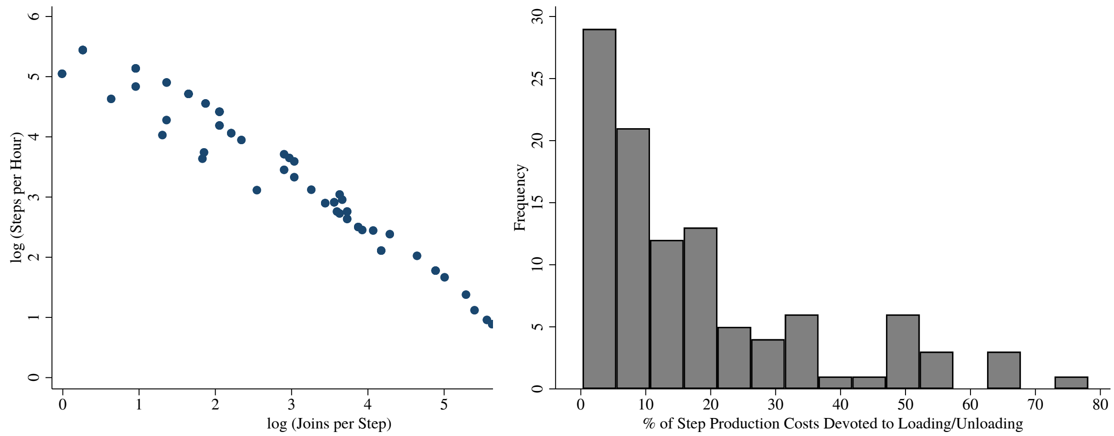
A key feature of the theory is the ability of a firm to divide production in multiple steps and assign these steps to either a human or machine performer. The historical literature provides extensive examples of the importance of the division of tasks for early US manufacturing. For example, Hounshell (1985) and Womak, Jones, and Roos (1990) provide measurement for Ford automotive assembly plants. They report that with the introduction of the moving assembly line around 1913, the average cycle time of a worker decreased from 2.3 to 1.2 minutes (the cycle time was 514 minutes before a fine division of tasks was introduced). Hounshell (1985) and Womak et al. (1990) also report that the demand for the skill of workers also changed during the move from craft production to factories to the adoption of the assembly line. In the time of craft production, a worker was trained via lengthy apprenticeships on many aspects of automobile fabrication and assembly; however, by the time the assembly line was in full usage, the average training time for a worker was measured in minutes. This is an important ingredient of our theory: the fewer the tasks to perform, the easier the job for a worker.[4]

The difficulty of completing a job is also driven by the overall time a worker or a machine has available to complete a task. The trade-off between measures of complexity and speed of execution has been extensively documented for both humans and machines.[5] Our own measurements confirm these regularities. In Figure 1a we display machine-level data from the automotive industry taken from Fuchs et al. (2008). In this case, it can be clearly seen how more complex part production (involving multiple welding joins per each step) is associated with an overall decrease in the number of completed steps per unit of time.[6] Dividing production into ever smaller steps is not costless. When production is divided, one task in a sequence is handled by a different performer from the next task. Transferring a work-in-progress from one performer to another takes time for both parties and creates errors. This phenomenon has been extensively studied, see for example Becker and Murphy (1992) and Baldwin (2008). Our own measurements illustrate the importance of these costs. In Figure 1b, we look at machine-level data from the optoelectronic semiconductor manufacturing industry taken from Combemale et al. (2020). A lower bound on

[4] For an example of this pattern for services refer to Autor et al. (2002) that looks at the division of tasks in a check-processing department before and after the introduction of computerized equipment.

[5] See the work of Fitts (1954), Welford (1981) and MacKay (1982) for the case of human motor movements; For application to robotic systems refer to Lin and Lee (2013). The common denominator of these empirical regularities resides in the fact that any task requires information to be completed, and any operator has a limited bandwidth for such information (Shannon, 1948).

[6] The time-per-join varies across steps (steps with more joins tend to require less time per-join), so that the relationship between complexity and the rate of steps completed is not merely a linear function of the number of joins.



(a) Speed Complexity Tradeoffs (b) Fragmentation Costs

Figure 1: (a) Relationship between rate and step complexity (Source: Fuchs et al. 2008), and (b) costs of fragmenting steps into less-complex bundles of tasks (Source: Combemale et al. 2020).

the step fragmentation costs is the time devoted by the operator to load and unload a machine. For a large number of steps, this time-cost alone amounts to more than ten percent of all step-wise production costs. Introducing fragmentation cost is also essential to model several technological developments. For example, a key development behind the growth of mass production is the introduction of exchangeable parts, which lowered the cost of splitting production across multiple workers (Hounshell, 1985). Technological progress does not always lead to decreases in costs of splitting production. For example, parts integration in electronics reduces divisibility due to monolithic part integration (Combemale et al., 2020).

The previous costs are embodied in the technology used in production. An additional source of costs in dividing production, and a final ingredient of the model, is incorporated in the cost of splitting performers across steps. Very short steps do not demand the full capacity of a performer, which introduces the possibility of a worker or a machine being under-utilized in production. Reallocating underutilized performers to other steps is not costless, for instance incurring time to reconfigure machines or for workers to change tooling or position. Differently from the previous costs, these opportunity costs now depend on the total level of production (see Hopp and Spearman (2011) and Laureijs, Fuchs, and Whitefoot (2019) for an extensive analysis).

The ingredients described in this section give intuitive dimensions to the problem of the firm in dividing production tasks: the firm must trade-off between the cost of complex steps and the cost of dividing tasks and performers. We formalize these dimensions in the following section.

3 Model

We first describe the nature of production introducing tasks and steps. Then we introduce the difficulty associated with each step. The section concludes with the problem of the firm.

3.1 Tasks and Steps

A good or service is produced by executing a set of tasks in the interval $\mathcal{V} = [0, \overline{v}]$ with $\overline{v}$ finite.[7] Tasks are indexed by $v \in \mathcal{V}$. A task can be performed by a human or a machine. A consecutive set of tasks $\mathcal{S}_i \subseteq \mathcal{V}$ performed by either a single human or a machine is referred to as a step.[8] To define a step, we introduce a series of $T \geq 1$ thresholds $\{s_i\}_{i=1}^{T}$ that split the set of tasks. For all i we have $s_i \in \mathcal{V}$ and $s_T = \overline{v}$. T thresholds define T steps as follows: $\mathcal{S}_i = (s_{i-1}, s_i]$ for $i = 2, \ldots, T$ and $\mathcal{S}_1 = [0, s_1]$. The type of performer in step t is defined with the indicator $o_i \in \{m, h\}$. When $o_i = h$, a human (or when $o_i = m$, a machine) is performing step i. For every step, we associate a length $l_i = s_i - s_{i-1}$ for all $i = 2, \ldots, T$ and $l_1 = s_1$.

Whenever production is split into multiple steps, a fragmentation cost is generated. Fragmentation costs are characterized by the point at which a step ends, and by the type of performer executing the step. The costs are described by the function $f(\cdot, \cdot) : \mathcal{V} \times \{h, m\} \to \mathbb{R}_+$, with $f(\overline{v}, \cdot) = 0$. For a given production process split over T steps and executed by performers according to $\{o_i\}_{i=1}^{T}$, total fragmentation costs are then by: $\sum_{i=1}^{T} f(s_i, o_i)$.

3.2 Steps and Difficulty

Firms assign the performer (either human or machine) to a step of a particular length and determine the rate at which the step needs to be completed. Length and rate drive the overall difficulty of a step for a performer and define the different margins on which humans and machines have an advantage.

Length & Complexity To complete a step, a performer needs to solve a number of issues that may arise. Issues arise according to a Poisson process so that the probability of n issues arising in a step of length l is given by: $P_n(l) = \frac{(\lambda l)^n}{n!} e^{-\lambda l}$. Parameter $\lambda > 0$ governs the relationship between step length (l) and the expected number of issues denoted by $N(l) = \lambda l$.

Just as the number of issues that need to be solved to complete the step is uncertain, so is the magnitude of issues. To capture both uncertainties, we model issues as a compound Poisson

[7] The interval of tasks does not indicate that tasks must be carried out sequentially over time. The model can represent a variety of production processes including the case of tasks being executed simultaneously as occurs with parallel production of subsystems that are later assembled together.

[8] In practice, multiple performers may be involved in the completion of a step through parallelization or coordination. It is also common for performers of different types to work simultaneously on a specific unit (e.g., a human and a collaborative robot). In this case, each performer is generally performing a different task: a human might be responsible for visual and cognitive tasks, while a robot may be responsible for strength-based tasks Vicentini (2021). In our model these cases are described as separate steps. Since steps are defined by a performer, they do not distinguish between a human performer and a tool used (following Frohm et al. 2008, a tool is an object that makes a task easier for a human, while a machine performs a task).

process. The complexity of each issue is modeled according to an *i.i.d.* random variable $X \in \mathcal{X} \subseteq \mathbb{R}_+$. We assume that all moments of X exist and are bounded. A key difference between a human and machine performer is the ability to solve closely related issues. For a human performer, the ability to solve an issue likely implies an ability to solve easier issues. Instead, for a typical machine, the ability to solve an issue is less informative on the ability to solve other issues. We formalize this distinction as follows. Given n issues X_i with $i = 1, \ldots, n$, step complexity is given by:

$$\mathbf{X}(n|\rho) = E\left[\left(\sum_{j=1}^{n}(X_j)^{\rho}\right)^{\frac{1}{\rho}}\right], \qquad n \geq 1.$$

Parameter $\rho \in [1, \infty)$ represents an important property of a performer; we will refer to ρ as the *degree of generality*.[9] To understand the role of ρ, it is convenient to consider two extreme examples:

1. **Perfect Generalist:** A perfect generalist is a performer with $\rho = \infty$. In this case: $\mathbf{X}^g(n) \equiv \lim_{\rho\to\infty} \mathbf{X}(n|\rho) = E[\max_{i=1,\ldots,n} X_i]$. In this scenario, only the most complex issue drives step-complexity for the performer. As a consequence, solving an issue of a given level implies the performer can solve all easier issues.

2. **Perfect Specialist:** At the opposite end, a perfect specialist is a performer with $\rho = 1$. In this scenario, each issue affects the step complexity separately. In this case $\mathbf{X}^s(n) \equiv \sum_{i=1}^{n} E[X_i]$, so that the realization of all issues contributes equally to the overall step complexity.

Differences between humans and machines can now be summarized as follows:

Assumption 1. *Let ρ_h (ρ_m) be the degree of generality of a human (machine) performer. Then $\rho_h > \rho_m$.*

To formally show the relationship between ρ and complexity, it is helpful to relate the definition of $\mathbf{X}(n|\rho)$ to an L^p norm. The result below follows from using Hermite-Hadamard inequalities for convex functions.

Lemma 1. *Suppose Assumption 1 holds. Then: (i) for all $n > 1$, $\mathbf{X}(n|\rho_m) > \mathbf{X}(n|\rho_h)$. (ii) $\lim_{n\to\infty} \mathbf{X}(n|\rho_m) - \mathbf{X}(n|\rho_h) = \infty$.*

Proof. In Appendix A.1. □

To gain intuition on how the complexity of a step for human and machine performers increases at a different rate, consider the case for large n. Let $\mathbf{S}_n(\{X_i\}_{i=1}^n) = \sum_{i=1}^{n}(X_i)^{\rho}$ and $\overline{\mathbf{S}}_n = E[S_n] = nE[X^{\rho}]$. We then have from Proposition 2 in Biau and Mason (2015) that:

$$\lim_{n\to\infty} \mathbf{X}(n|\rho) \approx E\left[\overline{\mathbf{S}}_n^{1/\rho} + \frac{1}{2}\frac{1-\rho}{\rho^2}\overline{\mathbf{S}}_n^{1/\rho-2}\left(\mathbf{S}_n - \overline{\mathbf{S}}_n\right)^2 + \cdots\right] \approx n^{1/\rho}(E[X^{\rho}])^{1/\rho}. \tag{1}$$

[9] The definition of complexity is reminiscent of a CES production function with degree of substitutability ρ (and elasticity of substitution equal to $1/(1-\rho)$).The model with $\rho = 1$ becomes a version of the Cramer-Lundberg model. See also, Cai (2014).

From (1) we see that $\mathbf{X}(n|\rho)$ increases more quickly with n for lower values of ρ.[10] Finally, $\mathbf{X}(n+1|\rho) - \mathbf{X}(n|\rho)$ is decreasing in n. This last observation is the basis for a concave relationship between step length and step complexity.

We can now define complexity of solving a step by a performer by:

$$c(l|\rho) = \sum_{n=0}^{\infty} P_n(l)\mathbf{X}(n|\rho). \tag{2}$$

Where we normalize $\mathbf{X}(0|\rho) = 0$. Complexity of a step inherits properties of $\mathbf{X}(n|\rho)$. The following lemma summarizes key properties of complexity used later in this paper.

Lemma 2. *The function $c(l|\rho)$ is: (i) strictly increasing, and (ii) strictly concave in step length l; (iii) If Assumption 1 holds, then $\lim_{l\to\infty}[c(l|\rho_m) - c(l|\rho_h)] = \infty$.*

Proof. In Appendix A.1. □

To fix intuition, it is helpful to go back to the case of performers being either perfect generalists or perfect specialists and see how different performer characteristics impact step complexity.

Example 1 (A Solved Case). *Let $X_{k:n}$ the k-th order statistic out of a sample of n draws of X, in this case the step complexity for the perfect generalist can be written as $\mathbf{X}^g(n) = E[X_{n:n}]$. Assume that each X_i is uniformly distributed in $[0,1]$. We then have that: $E[X_{n:n}] = \frac{n}{n+1}$. Since the number of issues and their complexity are assumed independent of each other, we have that the expected total difficulty to complete steps of length l by a perfect generalist is given by:*

$$c(l|\infty) = \sum_{n=0}^{\infty} \frac{n}{n+1} \frac{(\lambda l)^n}{n!} e^{-\lambda l} = \frac{1}{e^{\lambda l}\lambda l} + \frac{\lambda l - 1}{\lambda l}.$$

From the above, it is easy to see directly that $c(l|\infty)$ is increasing and strictly concave in l for all $\lambda > 0$. Similarly, for a perfect specialist, we have:

$$c(l|1) = \sum_{n=0}^{\infty} \frac{n}{2} \frac{(\lambda l)^n}{n!} e^{-\lambda l} = \frac{\lambda l}{2}.$$

In contrast to $c(l|\infty)$, $c(l|1)$ is now linear in step length.

Rate & Difficulty A second choice of firms is the production rate at which performers operate. A higher production rate increases performer output per unit time but also raises the overall

[10] The right-hand side of (1) holds as an upper bound for complexity for small n. To see this for all n and for $\rho \geq 1$ we have:

$$n^{1/\rho}(E[X^\rho])^{1/\rho} = \left(E\left[\sum_{j=1}^{n}(X_j)^\rho\right]\right)^{\frac{1}{\rho}} \geq E\left[\left(\sum_{j=1}^{n}(X_j)^\rho\right)^{\frac{1}{\rho}}\right] = \mathbf{X}(n|\rho),$$

with equality holding when $\rho = 1$ or $n = 1$.

difficulty of a step. We denote with $r \geq \underline{r}$ the rate in terms of the number of repetitions of a step per unit time (the unit of time can, for example, be the length of a shift.)

We can now define the overall difficulty of a step of complexity c performer at rate r. Step difficulty is generated by an aggregator function $D : \mathbb{R}^2 \to \mathbb{R}$. A step with complexity $c(l|\rho)$ performed by a performer of type $o \in \{h, m\}$ with rate r is associated with a difficulty $D\left(c(l|\rho), r|o\right)$.[11] We assume the following for the difficulty function D.

Assumption 2. *The function D is differentiable, strictly increasing and convex in both arguments. Denote with D'_r the derivative of D with respect to r. We assume: $D'_r(c, r, |h) > D'_r(c, r, |m)$ and $D''_r(c, r, |h) > D''_r(c, r, |m)$ for all $c > 0$, and $r \geq \underline{r}$.*

The conditions in Assumption 2 formalize the differences between a human and machine performer with respect to sensitivity to rate: as the rate of the step grows, eventually the difficulty for a human performer overtakes that of a machine performer. The difficulty function D encodes two important properties. First is the substitutability between step length and rate for a given difficulty level. Second is the sensitivity of D with respect to rate r, defined as: $\sigma = 1 + r\frac{D''_r}{D'_r}$. To fix ideas, an example of a functional form that satisfies Assumption 2 is:

$$D\left(c(l|\rho), r|o\right) = c(l|\rho)(\underline{c} + r^{\varsigma_o}), \tag{3}$$

with $\underline{c} > 0$ and $\varsigma_o > 1$. The above specification features a constant $\sigma = \varsigma_o$ and a lower bound on the difficulty, $\underline{c} \cdot c(l|\rho)$, which is independent of r.

3.3 Performer Ability and Costs

So far, we have discussed two differences between performers: the ease with which a performer addresses problems of increased complexity (ρ), and the tolerance to an increase in rate (σ). In general, these characteristics vary between performer types (human vs. machine) and among performers of the same type. We next assume that performers are heterogeneous along a single-dimensional ability level (denoted with a). When assigning an operator to a step, the ability level of the performer needs to be commensurate with the difficulty of the step. Formally, a performer of type $o \in \{h, m\}$ with degree of generality ρ can execute a step of length l with rate r if $a \geq D\left(c(l|\rho), r|o\right)$.

Divisibility The final dimension characterizing performers is their divisibility. Performers vary in how easily they can divide their time across steps and reallocate their effort. Indeed, when asked to perform a different set of tasks, a human can more easily switch while a machine needs

[11] Job difficulty originates from the interaction between tasks and the type of performer. In general, when compared to machines, humans are better able to solve a variety of issues and experience a smaller increase in errors as complexity increases (Wickens, Hollands, Banbury, and Parasuraman 2015). Dividing complex work into smaller steps, reduces human advantage and allows competition with machines. Also, humans experience sharp increases in failure-rates as they are made to perform tasks faster; machines typically outperform humans in terms of the error-effect of repeating simple tasks faster.

to be reprogrammed and refitted (Korsah et al., 2013).[12] A highly divisible human performer is able to easily switch to a different set of tasks once the initial set of tasks are completed. For example, a human computer programmer can quickly switch to answering emails once their programming tasks are completed. As a consequence, this performer is then not idle even when they can finish their tasks quickly. In contrast, say a robotic welding machine, cannot switch to different tasks when the welding tasks are completed. The firm must pay for the performer (the rental price of capital in this case) even when they are idle.[13] Divisibility thus introduces an additional trade-off for firms. Requiring performers to work quickly reduces the amount of time a performer spends per unit produced; however, it also increases the possibility of having idle performers.

Denote with Y the number of products that must be produced. We encode divisibility using the function: $g(Y,r) : \mathbb{R}_+ \times \mathbb{R} \to \mathbb{R}_+$. This function is used to scale the cost of production for a given step. The function depends on the rate (r) at which a step is performed. A higher r implies a shorter amount of performer time that is devoted to the step, thus a lower performer cost for the step (a lower value for g). The function g also depends on output (Y), this is because the cost-saving done by raising r ultimately depends on the number of products to be processed and on the ability to reallocate the performer to a different task. To fix ideas, consider the following examples:

1. **Perfectly divisible performer:** In this case we set: $g(Y,r) = 1/r$. So that any increase in r translates into a proportional reduction in performer-costs. In this case, cost reductions from higher r are independent of Y.

2. **Indivisible performer:** This is the case of a performer that cannot be reallocated to a different task when idle. For this type of performer we set: $g(Y,r) = \frac{1}{Y}\left\lceil \frac{Y}{r} \right\rceil$. In this case, the gains from higher rate r are limited by the number of products produced, Y.

Function $g(Y,r)$ is assumed to be differentiable with respect to the second argument and assumed to have the following properties:

Assumption 3. *For human ($o = h$) and machine ($o = m$) performers, the function $g^o(Y,r)$ is such that:*

1. *For all Y, there exists an $\bar{r}(Y)$ such that $g^o(Y,r) = g^o(Y,\bar{r}(Y))$ for all $r \geq \bar{r}(Y)$;*

2. *Threshold $\bar{r}(\cdot)$ is strictly increasing in Y and $\lim_{Y\to\infty} g^o(Y,r) = 1/r$;*

3. *If $r > r'$ then $g^o(Y,r) \leq g^o(Y,r')$ for all Y (inequality is strict when $r < \bar{r}(Y)$);*

[12] The degree of divisibility of a performer can also be influenced by policy, for example minimum shift labor laws. It can also be impacted by institutional and organizational constraints. For example Schmitz and Teixeira (2008), in the Brazilian iron ore industry, document the productivity impact of organizational changes within the firm; specifically, they document how allowing repair staff to perform repairs outside their job classification (increasing the divisibility of the performer) increases labor productivity of these workers.

[13] The inability to fully use the capacity of a performer is a common concern in the systems engineering literature. Refer to Hopp and Spearman (2011) for an extensive analysis.

4. *For all* Y, $\bar{r}_h(Y) \geq \bar{r}_m(Y)$.

Condition 1 in the above Assumption formalizes the idea that above a certain rate there are no further cost savings that can occur.[14] We refer to $\bar{r}_i(Y)$ as a divisibility threshold for the performer. When $r > \bar{r}$, all output is produced with a single performer within the minimum time increment, and so increasing r further cannot reduce the costs associated with the performer. *Condition* 2 states that as the output quantity grows, the constraint on the minimum time a performer can be allocated to the task becomes progressively looser ($\bar{r}(\cdot)$ is increasing) and eventually is non-binding. *Condition* 3 highlights that increasing the rates weakly reduces the operator costs. Finally, *Condition* 4 encodes the idea that moving a human performer to a different task is easier than re-tasking a machine performer.

We can now determine the total cost of assigning a performer to a step. Total step-costs are determined by the ability-price of the performers, $w(a)$ for humans and $k(a)$ for machines, as well as the cost saving associated with increasing the rate in which a step is executed, $g^o(Y, r)$. We have that the price of a performer to complete a step with ability a, rate r, and total number of products produced Y is given by:

$$p(a, r, Y|o_i) = \begin{cases} w(a)g^h(Y, r), & \text{if } o_i = h \\ k(a)g^m(Y, r), & \text{if } o_i = m \end{cases} . \tag{4}$$

We assume the following conditions for functions $w(\cdot)$ and $k(\cdot)$.[15]

Assumption 4. *The functions* $w(\cdot)$ *and* $k(\cdot)$ *are positive, strictly increasing and weakly convex.*

We now have all the model ingredients needed to define the problem of the firm.

3.4 Firm Optimization

Firms take as given the number of units it needs to produce Y and the operator prices $w(\cdot)$ and $k(\cdot)$. . The firm chooses how to subdivide the production process by choosing the number and positions of steps and which performer to assign to each step. For each step, the firm also determines the required completion rate. We begin by taking output Y and the number of steps T as given and finding the cost minimizing step thresholds, s_i, operator, o_i, ability, a_i, and rate, r_i, for each step i. In this case the firm solves:

$$C(Y, T) = \min_{\{s_i\}_{i=1}^T, \{r_i, a_i, o_i\}_{i=1}^T} \sum_{i=1}^{T} p(a_i, r_i, Y|o_i) + \sum_{i=1}^{T} f(s_i, o_i), \tag{5}$$

[14] This insight is commonly represented in the engineering literature by assuming that performers are "dedicated" to a process or to a step, meaning that their unused capacity cannot be productively used elsewhere.

[15] Assumption 4 on operator costs is fairly general. It can also accommodate the common assumption in the task-assignment literature where it is unfeasible for a machine to perform certain tasks. This case can be modeled with a level of $k(\cdot)$ sufficiently high for a certain ability level.

subject to:

$$l_1 = s_1; \quad l_i = s_i - s_{i-1}, \quad \forall\, i = 2, \ldots, T; \tag{6}$$

$$a_i \geq D(c(l_i|\rho_{o_i}), r|o_i), \qquad \forall\, i = 1, \ldots, T; \tag{7}$$

$$s_i \in [0, \bar{v}]; \qquad s_T = \bar{v}; \qquad o_i \in \{h, m\}; \qquad r_i \geq \underline{r}, \qquad \forall\, i = 1, \ldots, T. \tag{8}$$

The two terms driving costs in (5) represent the performer and fragmentation costs associated with a given choice of T (and performer characteristics). The per-unit cost of producing Y units is then determined by choosing the cost minimizing number of steps T.

$$C(Y) = \min_{T \geq 1} C(Y, T). \tag{9}$$

3.5 Discussion

Table 1: Model interpretation of a variety of technology changes.

Technology Change	Period	Theory Interpretation	Labor Impact
Mechanization: Substitution of human performers by machines	1870s-1890s	Machine work faster than humans but less able to perform varied work: $\rho^m < \rho^h$ and $\sigma^m < \sigma^h$. Machines less divisible than humans, $\bar{r}_m < \bar{r}_h$.	Human ability demand polarized. Empirically: growth of higher skill professional jobs (Chandler, 1990), more demand for unskilled labor (Atack et al., 2019)
Interchangeable Parts and Assembly Line: Increased standardization of parts facilitate transfer of work and minimize refitting requirements	1870s-1910s	Increased process complexity, leading to $\lambda \uparrow$, but facilitation of transfer and reduced post-processing of parts driving $f \downarrow$	Upper bound of human ability demand increases, lower bound of demand decreases. In data: creation of new managerial jobs and of simple production jobs. (Hounshell, 1985; Womak et al., 1990)
Consolidation of Parts: Formerly discrete parts fabricated as one	1970s-2010s	Joint fabrication of parts makes some fabrication tasks indivisible, driving $f \uparrow$, allows simpler design and reduced assembly, driving $\lambda \downarrow$	Upper bound of human ability demand decreases, lower bound increases. In data: convergence of skill towards middle, reduced division of production (Combemale et al., 2020).
Automation and Computerization: Substitution of human labor by computer and machine performers	1960s-2010s	Machines able to repeat tasks faster than humans but unable to perform highly varied work: $\rho^m < \rho^h$ and $\sigma^m < \sigma^h$. Compared to mechanization, performers are more general ($\rho \uparrow$), intense ($\sigma \downarrow$) and divisible ($\bar{r} \uparrow$)	Polarization of worker ability demand at low volumes, shifting to high skill at high volumes. In data: up-skilling of skill demand (especially in manufacturing), aggregate polarization in conjunction with lower automation in services (Goos et al., 2019; Willcocks and Lacity, 2016).

The goal of the theory is to offer a unified explanation of the labor implications of technological change, capable of rationalizing the impacts of many historical and modern technological trends. Table 1 highlights how some of the different parameters of the theory can be used to develop a sort of taxonomy of technology change. First, technological change can be described as a change of the tasks that need to be solved (a product development).[16] In the cases when a change in the set of tasks is not a key change, a technology change may be described in terms of its effects on process complexity (λ) and task separability (f) and on performer characteristics such as divisibility (g), sensitivity of performers to rate (σ), and generality (ρ).

The implications of the model can be analyzed considering any combination of the parameters above that describe changes in technology. Given the data available in Section 5, we next focus on the choice of automation (the choice of performers) and changing fragmentation costs.

4 Analysis

We analyze the model in four steps. First, we provide conditions for the division of tasks to occur. Second, we establish a relationship between step length and ability demand, rate, and performer costs. Third we study the conditions for firms to automate steps. We conclude by analyzing the effect of changes in fragmentation costs on the division of production and on ability demand.

4.1 The Structure of Production: Division of Tasks

We begin analyzing when it is optimal to divide steps. Since the price of performers is strictly increasing in their ability, it follows that constraint (7) binds at the optimum. Given this, a necessary condition for tasks to be divided into more than one step is the existence of at least one feasible step-length l, performers o', o'' and rates r', r'' such that:

$$p(D\left(c(\overline{v}|\rho_o), r^*|o\right), r^*, Y|o) > p(D\left(c(l|\rho_{o'}), r'|o'\right), r', Y|o') + p(D\left(c(\overline{v} - l|\rho_{o''}), r''|o''\right), r'', Y|o'') \tag{10}$$

where r^* is the optimal rate without any division of tasks. The above inequality is strict since fragmentation costs are nonzero. Two forces lead firms to divide steps: convexity of operator ability-prices and high output. The intuition for why convexity of operator prices lead to fragmentation is straightforward. A sufficiently convex wage or capital cost makes it extremely expensive for a firm to hire a worker or a machine to execute a large non-fragmented step. Formally, this is described as follows.

Proposition 1. *Suppose that $f(\cdot,\cdot)$ is sufficiently low and that $w(\cdot)$ or $k(\cdot)$ is sufficiently convex. Then division of tasks is optimal.*

Proof. In Appendix A.2. □

The previous result considered division of tasks as a way to reduce the cost of production for sufficiently convex operator prices, trading off against fragmentation costs. The notion of connecting division of tasks to increases in production efficiency dates to Adam Smith in the *Wealth*

[16] This is the focus of papers such as Acemoglu and Restrepo (2018b) which is mostly interested in studying the role of automation on the aggregate equilibrium level of employment and wages.

of Nations in his discussion of the division of labor (Smith, 1776). Smith himself argues that the degree of specialization may also be limited by market size; we turn to this channel for division of labor next. The following proposition sharpens the trade-off present between fragmentation costs and the size of output (related to the size of the market).

Proposition 2. *Suppose that: $f(\cdot,\cdot)$ is sufficiently low and Y is sufficiently high. If $D'_r = \frac{\partial D}{\partial r}$ is sufficiently small (or $\overline{v}$ is sufficiently large), then division of tasks is optimal.*

Proof. In Appendix A.2. □

4.2 Step-Length, Costs and Ability

Next we explore how changing the length of a step impacts the overall costs and the ability required for a performer executing that step. The next proposition provides conditions in which the rate of execution and performer costs are inversely related to the length of a step. The result requires that the difficulty function D is sufficiently super-modular in rate and step length.

Proposition 3. *Suppose that Assumptions 2 and 4 hold. Suppose that $D''_{cr}/D'_c \geq -g'_r/g$. Given two steps i and j with the same performer: $o_i = o_j$. Denote with r_i, r_j (a_i, a_j) the optimal choice for rate (ability) in step i and j. Then if $l_i > l_j$, we have that (i) $r_i \leq r_j$, (ii) $p(a_i, r_i, Y|o_i) > p(a_j, r_j, Y|o_i)$.*

Proof. In Appendix A.3. □

To gain intuition on the Assumption on D and g in the previous proposition, consider the case in which D and g are given by the following Assumption:

Assumption 5. *For human ($o = h$) and machine ($o = m$) performers we have: $D(c(l|\rho), r|o) = c(l|\rho)(\underline{c} + r^{\varsigma})$ for $r > \underline{r}$ with $\varsigma > 1$ and $\underline{c} > 0$. In addition, $g^o(Y, r)$ is given by:*

$$g^o(Y,r) = \begin{cases} 1/r, & \text{if } r \leq \overline{r}_o(Y) \\ 1/\overline{r}_o(Y), & \text{if } r > \overline{r}_o(Y) \end{cases},$$

with $\overline{r}_o(Y) = \underline{r} + Y\overline{r}_o$ and $\overline{r}_h > \overline{r}_m$.

If this Assumption holds, we have that the condition in Proposition 3: $\frac{D''_{cr}}{D'_c} \geq -\frac{-g'_r}{g}$ implies $1 \geq \frac{\underline{c}+r^{\varsigma}}{\varsigma r^{\varsigma}}$. So that the condition in Proposition 3 holds whenever, for example, ς is high enough so that the sensitivity to increased effort is high enough so that a lower rate becomes optimal with longer steps. The previous result does not imply a specific ability demand for steps of different length. The relationship between step length and ability can be sharpened once we assume that costs relative to ability grow fast enough. In this case we determine a monotone relationship between step-length and ability.

Assumption 6. *The function $\mathbf{w}$, defined for $o_i = h$ (and similarly for $o_i = m$) as $\mathbf{w}(x, o_i) = xw'(x)/w(x)$, is increasing for all $x > 0$.*

Under the above Assumption (that holds for example with log-linear wages) we have that:

Proposition 4. *Suppose that Assumptions 5 and 6 hold. Given two steps i and j with the same performer: $o_i = o_j$. Then if $l_i > l_j$, we have that $a_i > a_j$.*

Proof. In Appendix A.3. □

The result of the proposition relies on showing that the increase in costs due to ability demand are not outweighed by the increase in cost due to operating with a lower rate. The result will then hold for any functional form for D and g for which the function $-\frac{g'}{g}\frac{D}{D_r}$ is decreasing in r.

4.3 Automation

We next describe the conditions under which a firm automates (choosing a machine performer rather than a human). We show how automation impacts labor demand by showing that step length (l) and production quantity (Y) are key determinants for understating patterns of automation. When combined, the results in this section generate a pattern of automation referred to as the *cone of automation* as displayed in Figure 2. The cone of automation implies that a minimum amount of output (Y_1 in the Figure) is necessary for automation to be an economically valuable option for firms. When automation occurs, it is likely to occur for steps performed by middle ability workers (thus automation is polarizing in ability demand). As output grows so does the range of steps that are automated. Ultimately, for high enough output (past Y_2 in the Figure) only the highest ability steps are not automated.

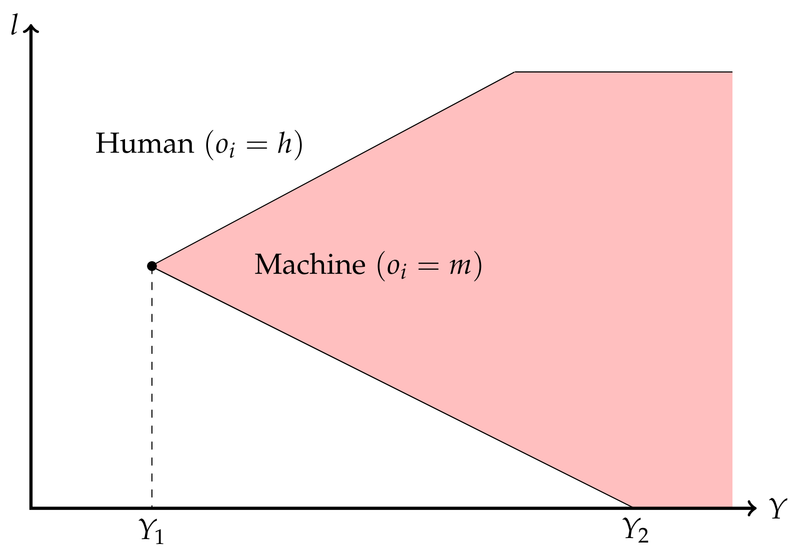


Figure 2: Automation patterns: production quantity (Y) and step length (l).

In this section, we assume that when a firm is indifferent between automating a step or not, it chooses not to automate.[17] The first result shows that if a step is long enough, then it will not

[17]This assumption can be made directly by assuming that fragmentation costs are arbitrarily close but lower for humans (as documented in Korsah et al. 2013).

be automated. The result provides upper bounds of a region of step-lengths that are automated. This result is driven by the relatively higher generality (higher ρ) of humans versus machines.

Proposition 5 (Upper Bound on Automation). *Suppose Assumptions 1, 3, and 4 hold. There exists $\bar{l}$ such that $o_i = h$ for all i with $l_i > \bar{l}$.*

Proof. In Appendix A.4. □

The previous result is stated in terms of l sufficiently high for a given step. A similar result holds for any l if λ (the intensity of the Poisson process for issues) is instead sufficiently high. In both cases the step will likely feature a high number of issues during execution. We now consider the case of automation of small steps. The result below provides a lower bound of automated step-lengths, it is driven by the lower ability of machines to be redeployed to different tasks. This feature is encoded in Assumption 3 Part 4. An additional natural assumption for the result to occur is that operator costs for humans are sufficiently low for lowest ability levels.

Proposition 6 (Lower Bound on Automation). *Suppose the g function satisfies Assumption 3. Suppose there exists a step i with l_i sufficiently small. Also suppose that $\lim_{c \to 0, r \to \underline{r}} w(D(c, r|h)) \leq k(D(c, r|m))$. Then if Y is sufficiently low, we have that $o_i = h$.*

Proof. In Appendix A.4. □

The proof is straightforward and relies on the idea that, for low Y, the advantage of a machine performer operating at a high rate is eliminated. This, of course, requires a minimum cost for human workers that is sufficiently low else automation at the bottom is always preferred. This floor for human wages is, in general, not something observed in practice as firms endogenously choose the rate of operations for humans and machines typically above their minimum level. Hence the demanded ability level for the operator is above the theoretical minimum ability level needed to execute the step.

The previous results focus on step length. A second dimension important in driving automation is the level of output (Y). A key advantage that machines have relative to humans is the lower sensitivity to rate (r). For sufficiently high levels of Y, we expect automation to be optimal since in this case the cost-minimizing machine rate is higher than the human rate and hence machine-operator costs are lower. On the other hand, for low levels of output, the advantage of a machine is lost as a machine with high operational rate remains idle for a significant amount of time. The following results describe optimality of automation for different step lengths and for different levels of output.

Proposition 7. *Suppose Assumption 5 holds. Denote with $o_i(Y)$ the optimal operator choice for l_i at output value Y. (i) If Y is sufficiently large then for all $Y' > Y$ we have that $o_i(Y) = o_i(Y')$; (ii) Let $o_i(Y) = m$. Suppose that $w' \geq k'$, $w'' \geq k''$ and that $w(D(c(l|\rho_h), \underline{r}|h)) < k(D(c(l|\rho_m), \underline{r}|m))$ for all $l > 0$, then for all Y if $Y' > Y$ we have that steps of length l_i also feature $o_i(Y') = m$; (iii) Assume wages and cost of capital are log-linear. Let $o_i(Y) = m$, under the assumption of (ii), if Y is sufficiently large, then for all steps with $l_j' < l_i$ we have that $o_j(Y) = m$.*

Proof. In Appendix A.4. □

The previous Proposition provides three results. First it shows how for sufficiently high levels of output only step length and not changes in output matter for the choice of operator. This is because, eventually, only the characteristics of the operator and not the amount of output matters for the optimal rate of the operator. This is true for both humans and machines. The second result specializes this intuition for machines also for smaller levels of output. Finally, under additional assumptions on costs, part *(iii)* highlights the optimality of automation of small steps for high levels of outputs.

Combining the three previous Propositions provides a characterization of where we might expect automation when considering steps of different length and different output requirements. The result can, diagrammatically, be represented with a *cone of automation* as highlighted in Figure 2. This pattern is a key theoretical aspect we will map to data in Section 5.4. The result on the cone of automation is significant as it implies that the phenomenon of skill polarization (automation impacting middle length steps as between output level Y_1 and Y_2 in Figure 2) is not only a relatively recent ICT-related phenomenon but is connected to any type of automation and is potentially observable already at firm level.

Remark 1. *The previous results are formulated in terms of output Y. Changes in output impact the choice of the firm by affecting the minimum divisibility threshold $\bar{r}$. Given Assumption 3 on the monotonicity of $\bar{r}$ with respect to Y, it is then possible to recast the preceding results directly in terms of $\bar{r}$. This interpretation of the preceding results will be helpful for the quantitative counterpart in Section 5.4.*

4.4 Fragmentation Costs and Division of Tasks

Over time, technological change has also impacted fragmentation costs. This section expands the analysis between changes in fragmentation costs (f) and the implied changes in the division of tasks and ability demand. We consider two benchmarks: a change in uniform fragmentation costs, and arbitrary fragmentation costs affected by a uniform shift. As a first step we show that variation in fragmentation costs is a necessary condition for (within plant) wage inequality: without variation in fragmentation costs, steps are uniform and hence so is the ability demand.

Lemma 3. *Suppose that the Assumptions of Proposition 1 and Proposition 4 hold. Consider the case in which $f(\cdot,\cdot) = \overline{f}$. Then $l_i = \bar{l}$ and $a_i = \bar{a}$ for all i.*

Proof. In Appendix A.5. □

While constant fragmentation costs do not create heterogeneity in skill demand, the level of skill is impacted by the level of fragmentation costs even when these costs are homogeneous. We now look at the impact of a reduction in fragmentation costs (this strengthens Proposition 1 where it was shown that sufficiently low fragmentation costs increase the number of steps). The reduction of fragmentation cost can be modeled as a proportional reduction in costs, when this occurs, the next Corollary shows a reduction in the lowest ability level demanded.

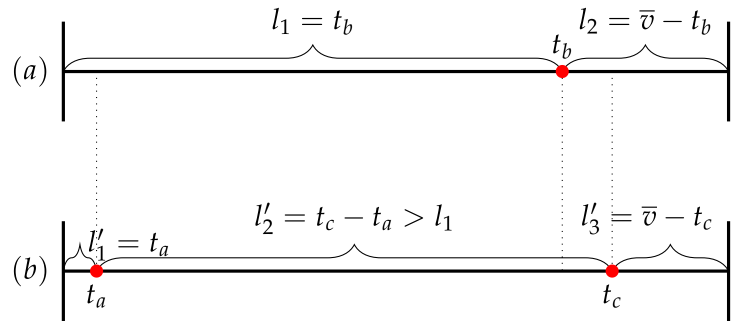


Figure 3: Maximum Step Length and T. (a) Case $T = 1$, (b) Case $T = 2$.

Corollary 1. *Suppose that the Assumptions of Proposition 4 hold. Let $w(\cdot)$ and $k(\cdot)$ be sufficiently convex. Consider arbitrary fragmentation costs f and let f' so that $f'(t,\cdot) = \overline{f} \cdot f(t,\cdot)$ for $\overline{f} > 0$. Let a_{min} and a'_{min} be the lowest ability demanded under f and f', respectively. If $\overline{f}$ is sufficiently low, then $a'_{min} \leq a_{min}$.*

Proof. In Appendix A.5. □

This Corollary shows how a sufficiently large reduction in fragmentation costs results in a decrease in the minimum step length. There is no equivalent property for the maximum step length. Indeed, it is possible for the maximum step length to increase due to an increase in T, even if the total cost of production decreases. The following example and Figure 3 make this point.

Example 2. *In this example we show how it is possible for the longest step length to increase as the number of steps increases. Let fragmentation costs be arbitrarily high for all t, except for three points, t_a, t_b, t_c, with $f(t_a,\cdot) = f(t_c,\cdot)$ and let $f(t_b,\cdot) = f(t_a,\cdot) + d$, with $d > 0$. Let $t_a < t_c - t_b$ and $t_b > \frac{\overline{v}}{2}$. For $T = 1$ we have $s_1 = t_b$, the more centrally located cut. This is the case whenever the convexity of costs with respect to length dominates the higher fragmentation costs at t_b. For $T = 2$ we have $s_1 = t_a$ and $s_2 = t_c$. This occurs if the reduction in performer costs from placing step thresholds at t_a, t_b or t_b, t_c relative to t_a, t_c are less than a. Figure 3 summarizes the example when $T = 1$ and $T = 2$.*[18]

The previous example featured regions where fragmentation costs are arbitrarily high. In many instances, it is natural to think of step lengths being defined by regions of tasks which are indivisible or have arbitrarily high fragmentation costs, such as in highly controlled processes (e.g., material deposition as described in Combemale et al. 2020), continuous processing (e.g., in steel production), or highly interconnected tasks (e.g., indivisible loads in computing as in Berenbrink et al. 2015). To formalize this phenomenon, we next consider sets of *lumpable tasks* defined as a set of tasks $V = [t_i, t_j]$ such that $f(t,\cdot)$ is arbitrarily high for $t \in V$. In the presence of a set of lumpable tasks such as V we have that the maximum step length will never be less than $t_j - t_i$. We next exploit the presence of lumpable tasks to think about technological changes that change the difference between the least and highest ability demand.

[18]The parametric scenario with $p(l) = l^{1.088}$, $t_a = .4, t_b = 7, t_c = 7.5, \bar{v} = 10, d = 0.05$ delivers the required properties for the example. Formally, we require t_a, t_b, t_c be such that $p(t_b) + p(\bar{v} - t_b) < \min\{p(t_c) + p(\bar{v} - t_c) - a, p(t_a) + p(\bar{v} - t_a) - a\}$. Let t_a, t_c be such that $p(t_c - t_a) + p(\bar{v} - t_c) < p(t_c - t_b) + p(\bar{v} - t_b) + a$ and $p(t_c - t_a) + p(t_a) < p(t_b - t_a) + p(t_b) + a$.

Changes in Issue Arrival Important historic technological changes have simultaneously affected the complexity and divisibility of processes.[19] These technological changes can be described by a simultaneous change in fragmentation costs f and in the parameter governing the average number of issues λ.[20] In what follows, we consider the ability demand implications of a change in technology which increases issue arrival and sufficiently decreases fragmentation costs. We show that this change generates an increase in the upper bound of ability demanded.

Corollary 2. *Suppose that the Assumptions of Proposition 4 hold. Suppose there exists a set of lumpable tasks $\widehat{V}$ of length $\widehat{l}$. Also suppose that under issue arrival λ, the maximum step length is $\widehat{l}$. Consider an issue arrival $\lambda' > \lambda$. Let a_{max} and a'_{max} be the lowest ability demanded under λ and λ', respectively. If the performer for the longest step remains the same, we then have $a'_{max} > a_{max}$.*

Proof. In Appendix A.5. □

The previous Corollary requires a constant performer type for the longest step. If the longest step is sufficiently long, then by Proposition 5 this Assumption is automatically satisfied, as human performers are assigned to this step before and after the change in issue arrival rate. Together Corollary 1 and 2 provide a theoretical basis to understand how within-firm inequality might increase or decrease given different types of technological change. For example, the previous Corollaries imply that in the presence of technological change that simultaneously lowers fragmentation costs and raises issue arrival, we will expect an increase of within-firm inequality.

5 Empirical Analysis

This Section provides empirical counterparts to the theoretical results described thus far. In order we show that: *(i)* increasing complexity of production requires performers with higher ability working at lower rates; *(ii)* a reduction in fragmentation costs leads to an increase in the number of steps; *(iii)* a reduction in fragmentation costs and an associated increase in issue arrival rates leads to an increase in the upper bound of ability demand, and a decline in the lower bound of ability demand; *(iv)* our main empirical finding, we show that a *cone of automation* similar to Figure 2 forms where automation substitutes for workers of middle ability at low volumes, and the range of ability substituted widens as production volumes increase.

[19]The development of the assembly line in manufacturing permitted a finer division of tasks but entailed a more complex overall process with greater logistical and managerial requirements (Hounshell, 1985; Chandler, 1990). The more recent phenomenon of design modularity in programming and design allows for easier separation of work but increases system complexity (Baldwin and Clark, 2003). The inverse is also possible. In modern manufacturing, parts consolidation, when formerly discrete parts are fabricated as one piece, makes dividing tasks more costly but reduces the number of issues that might arise in assembly (Selvaraj et al., 2009; Combemale et al., 2020).

[20]Technological changes affecting multiple dimensions are also intuitive from an adoption perspective. For example, a firm will not adopt a technology increasing fragmentation costs or issue arrival without an opposing effect reducing costs, such as reduced fragmentation cost or fewer issues.

5.1 Data Sources

We use three datasets that provide detailed information on production: the *Hand and Machine Labor Study* of 1898 (Wright, 1898); data on optoelectronic semiconductor manufacturing from Combemale, Whitefoot, Ales, and Fuchs (2020); and data on contemporary auto-body assembly from Fuchs, Field, Roth, and Kirchain (2008).

The Hand and Machine Labor Study (HML) The original data collection for this study was conducted by the Bureau of Labor Statistics between 1894 and 1898, with the goal of investigating the effect of the use of machines on labor.[21] The study covers 672 products across the agricultural, manufacturing, mining, and transportation sectors. Detailed descriptions of production steps (ranging from one to hundreds) are provided for all products. Every product recorded in the HML is described twice in two separate processes: a "hand" process (a relatively more manual process), and a "machine" process (a relatively more mechanized process). Taken together the two descriptions represent a change in process structure and performer type to produce the same good with identical characteristics.[22] The data characterizes each process step-by-step, analogously to the structure of steps in our model: for example, the hand process for producing hay consists of 1) mowing grass, 2) tending hay, 3) raking hay, 4) cocking hay, 5) hauling hay, 6) bailing hay and 7) weighing hay. The data includes the occupations employed in each step, the number of employees for each occupation for the step, the task content of the step and the motive power used in the step (e.g., hand, water, steam). Wages and operations data consist of the time worked per step cycle, the output per cycle of a process step, the number of workers required per step and the number of workers required per workstation. Each process step has a detailed task description, and coding to identify which step (or steps) in the hand process contains the same tasks as the machine process. For example, the machine process for making a sleigh (Product 183) includes steps coded 2 and 3 for sanding panels and setting up the sleigh body, while the hand process has a step for setting up and sanding the body, coded as (2,3) to indicate that it contains the same tasks as the other process but combined into one step. Refer to Appendix B for further details.

The remaining two datasets contain modern direct measurement of plant-level production processes. This data is collected to identify the technical parameters of a highly detailed production model. These models, called Process Based Cost Models (PBCMs) in the industrial engineering and operation management literature, are used across a variety of industries to inform engineering and production decisions. This modeling approach provides the benefit of isolating the effects of technology changes at the level of individual process inputs, for example the effect of using a human or a machine to perform a specific production step on output.

[21] The dataset is also described in Atack, Margo, and Rhode (2019).

[22] The original authors note rare exceptions, such as slabs of granite of different final weight or an 8-inch versus 9-inch pipe. These products are of the same composition, but different dimensions.

Optoelectronic Semiconductor Manufacturing This second dataset looks at the production of optoelectronic semiconductor transceivers for communications. We use data ranging from the fabrication of semiconductor components to their assembly into a final package. The optoelectronic semiconductor industry is a useful case study for the effects of technology change because optoelectronic transceivers have a common form factor and end-use, so that they are functionally homogeneous while varying significantly in their internal design and method of production (i.e., in terms of the technological parameters in our theory). This dataset (originally collected and described in Combemale et al. 2020) allows us to compare step-level demand for worker ability (captured using the same methodology as the O*NET database) under different technological scenarios. These scenarios vary in the level of automation and the level of consolidation of product designs (increasing in the number of internal components which are jointly fabricated).

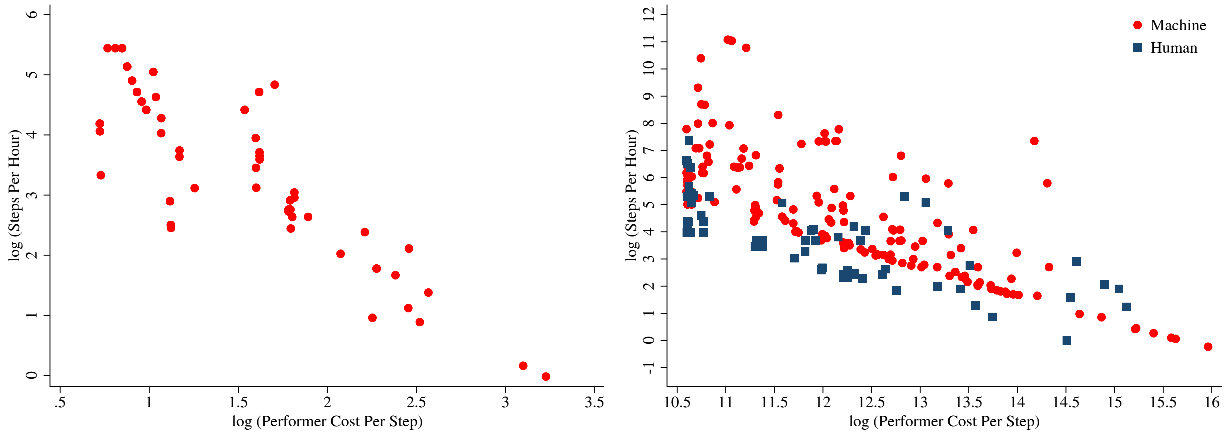
Automobile Body Fabrication and Assembly The final dataset is from automobile body fabrication and assembly. This dataset was originally collected and presented in Fuchs et al. (2008). The data which we use in this paper characterize process flow and step-level process inputs for automobile body assembly. For each assembly process step, the data includes capital and labor inputs (demand, price) for each process cycle as well as operations parameters, specifically batch size and cycle time. The dataset also includes data for each step on the number of welding joins required for each part of the automobile body.

5.2 The Relationship Between Ability, Rate and Step Length

We first use the optoelectronic semiconductor and automobile body production data to provide an empirical analogue to Proposition 3 and Proposition 4. These results relate rate (r) and ability (a) to step length (l). Using Proposition 3 we can use performer costs as a proxy for step length. In the data, human performer costs are given by the compensation of workers divided by the worker time needed per unit output. Machine performer costs are given by the cost of the machine used, scaled by the time of use per part and the length of service life of the machine. The empirical results from both contemporary contexts, presented in Figure 4, are consistent with Proposition 3 by showing that rate is decreasing in step length. In the optoelectronic semiconductor context (Figure 4, panel (b)), the same wire-bonding machine takes longer to complete more complex configurations while preserving the same proportion of successful versus failed outputs. In the automobile body assembly context (Figure 4, panel (a)), more complex welding operations require more expensive machines (see Figure C.1 in Appendix) or require the same machines to operate more slowly.

We next use the worker-level dexterity ability measures from the optoelectronic semiconductor data to explore the relationship between a and l. Recall that Proposition 4 provides conditions in which a is increasing in l. As before we proxy step length l with performer costs.[23] For finger

[23]In the optoelectronics contexts, we use a constant operator hourly wage across all steps based on the average hourly wage observed at each plant. To sharpen the focus on human ability we consider only steps in which human



(a) Automotive

(b) Optoelectronics

Figure 4: Rates of production and performer costs per step. Data for (a) is from Fuchs et al. (2008). Data for (b) is from Combemale et al. (2020).

dexterity ability information, our data provides information on ability levels as defined in the O*NET database. We compare across all steps in the dataset which have either a dexterity ability-level rating of 1 (the lowest value) or a level of 5 (the highest value recorded in the data).[24] For context, level 1 indicates that the task is easier than putting coins into a parking meter, and level 5 means that it is harder than assembling small knobs onto stereo equipment in an assembly line. The distribution of labor costs associated with steps of high and low ability is consistent with Proposition 4 that highlights a negative relationship between ability and length. We have that the average labor cost per unit for the low-ability steps is $0.19, while the average labor cost per unit for the high-ability steps is $0.52.

5.3 Fragmentation Costs and Division of Tasks

The time period covered by the HML dataset is characterized by a reduction of fragmentation costs and the onset of automation.[25] This dataset thus offers a useful empirical counterpart to the results of Sections 4.1 and 4.4. First, we look at how the reduction in fragmentation costs leads to changes in the number of production steps. In the HML dataset we look at mappings between hand and machine process steps to capture intervals which are affected by an increase in the division of tasks (for detailed information on the processing of data, refer to Appendix B.2). Since the HML dataset does not contain information on the fragmentation costs, we focus on the overall distribution of the number of steps across all processes. Results are in Figure 5. The figure displays a reduction in the number of processes that feature a small number of steps,

labor costs are at least 70 percent of total performer costs (human and machine combined).

[24]As a check for robustness to ability type, we also performed a comparison between all steps whose maximum ability was 1 across the abilities captured in the data (operations and control, dexterity, near vision) and steps whose maximum ability was 5. We found comparable results.

[25]See for example the analysis on interchangeable parts as documented by Hounshell (1985), Chandler (1990).

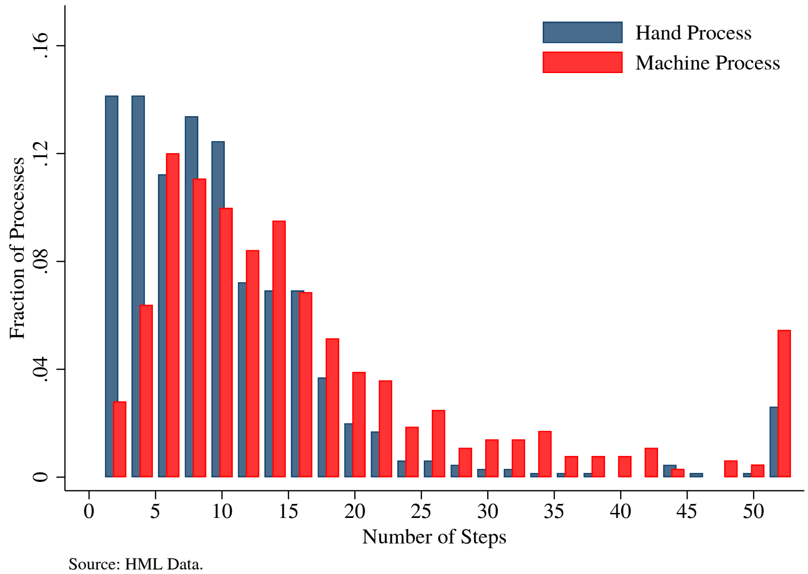


Figure 5: Fragmentation costs and step divisibility. HML data.

consistent with the increased division of tasks discussed in Section 4.1 following a reduction in fragmentation costs.

Within the HML dataset we next look at wages. We restrict the analysis to steps with constant performer type (i.e., manual motive power regardless of whether the process is characterized as a hand or machine process) this is to make sure we capture the impact of changing fragmentation costs as opposed to automation (analyzed below). Table 2 displays data from four distinct distributions. With the leftmost two columns, we compare the distribution of wages when changing from the hand to the machine process does not incur changes in the number of steps (constant T). To compare across plants, relative wages are calculated using the wage of a performer divided by the average wage in the plant. With the rightmost two columns, we look at the case in which changing from the hand to the machine process leads to an increased number of steps (T increasing). In either case we compare the distribution of relative wages for the hand and machine processes. Consistent with Corollary 1, for the case of increasing T, we observe a decrease of the lowest wages. As wages are monotone in ability this also suggests a decrease of ability demanded.[26] The Corollary emphasizes how in the presence of decreasing fragmentation costs we expect downward pressure in demand for the lowest skills. To confirm that the changes in wages are indeed driven by changes in the number of steps, consider the case of constant T in the first two columns of Table 2. In this case, the widening of the distribution of relative wages is much smaller than the case of increasing T.[27] The HML dataset, while covering a variety of different industries and products, lacks precise controls on fragmentation cost and lacks direct

[26] This pattern appears in direct industry observations see for example Womak et al. (1990).

[27] The initial distribution of wages is narrower for tasks that show increasing T than for those that have constant T between the Hand and Machine Process. This difference is expected because the processes which in practice see the greatest increase in T initially had less division of labor.

Table 2: Fragmentation costs and wages. HML data.

Relative Wage	Constant T Hand Process	Constant T Machine Process	Increasing T Hand Process	Increasing T Machine Process
10th Percentile	0.58	0.53	0.66	0.48
25th Percentile	0.85	0.76	0.88	0.70
50th Percentile	1.00	1.00	1.00	0.91
75th Percentile	1.23	1.22	1.10	1.15
90th Percentile	1.54	1.61	1.32	1.45
Number of Steps	1440	1440	770	2445

measurement of ability levels. To overcome these limitations, we next look at the optoelectronic semiconductor production data. This data provides information on different levels of automation and consolidation (different number of steps). For all levels of automation and consolidation, the final products are functionally homogeneous and perfect substitutes on the market. Changing the level of consolidation of the design drives step consolidation: the more consolidated the design, the fewer number of steps (lower T). Consolidation of parts leads to an increase in fragmentation costs (f) but also a reduction in assembly issues, captured in our theory by reduced issue arrival (λ). The case of consolidation allows us to look for an empirical analogue of Corollary 1 and Corollary 2 for constant performer type. Taking these two Corollaries together, we expect a convergence in ability demand (decline at the top and at the bottom), as fragmentation costs increase and issue arrival decrease. We use the skill-ratings collected for each step by Combemale et al. (2020) as a measure of a.[28] Holding performer type constant across levels of consolidation, Figure 6 shows the effects of two changes in consolidation (from low to medium consolidation and then from medium to high consolidation) on the distribution of skill demand.[29] We see that with consolidation skill demand converges toward middle skills. This is similar to the convergence in ability demand anticipated in Corollary 1 and 2.

5.4 Automation

We next move to our main empirical results looking at which steps in a production process are most likely to be automated. The results in Section 4.3 provide guidance on what steps are more likely to be automated when considering steps of different length or production processes with different levels of output. Together, the results of Section 4.3 describe what we refer to as a *cone of automation* where automation is more likely for higher level of output and for middle length steps (see Figure 2). In what follows we first look at the HML data and then at the optoelectronic

[28]Specifically we focus on the O*NET defined skill *Operation and Control*. A skill of 1 is rated low, a skill of 5 or greater (levels 6 and 7 were not observed) high, and 2-4 medium. As shown in Combemale et al. (2020), this result is robust across different types of skills and without aggregation of skill rankings.

[29]For the optoelectronic product studied, some components are fabricated independently and then assembled onto a common platform. The different levels of consolidation refer to the number of sub-components that are merged into one. Refer to Combemale et al. 2020 for additional details.

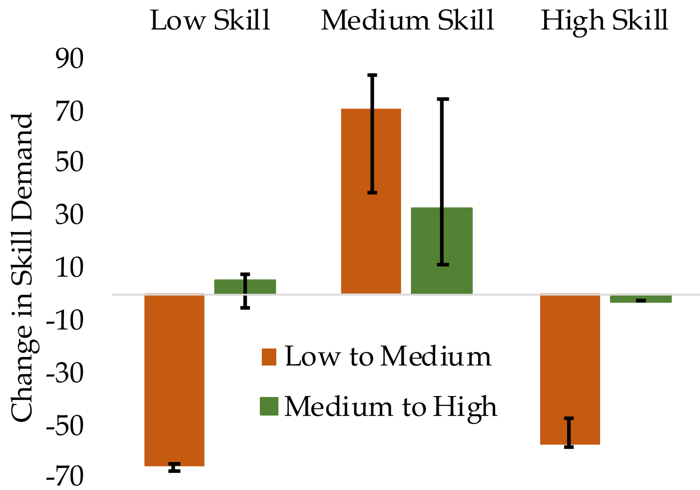


Figure 6: Impact of change in parts consolidation on skill demand. Different color bars denote different levels of parts consolidation. Data from Combemale et al. (2020).

semiconductor data to find evidence for this pattern of automation.

We begin with the HML dataset. An analogous of Figure 2 can either be constructed by observing a single plant at different output levels or by combining observations across different plants. The HML dataset provides a rich set of observations that allows us to implement the latter strategy. First it provides labor costs at step-level. This data, combined with Proposition 3, can be used to recover information on the length of a step. Then, to compare across production processes, cost for each step is normalized by the average cost observed in that process. The second dimension driving automation in Figure 2 is output level Y. For this variable, the connection with the data when dealing with multiple plants is more involved (as the absolute measure of output for different goods and services is not comparable across plants). The key insight is to use the capacity utilization of each step instead of the overall output level. As stated in Remark 1, the pattern in Figure 2 can be recast in terms of $\bar{r}$ as opposed to Y. The intuition on the role of $\bar{r}$ is that at a high enough rate, performers are underutilized. This is because the performer will run out of tasks that need to be performed. The HML data allows us to compute the degree of utilization (hence $\bar{r}$) across plants. We explain this procedure next (refer to Appendix B.2 for additional details.)

The HML dataset provides information on the number of workers involved in a step and the amount of time the step requires to be completed. For each process, following Hopp and Spearman (2011), we identify the bottleneck in production by looking at the step that requires the longest time to be completed. We determine the fractional utilization of a step by comparing its completion time to the completion time of the bottleneck. For example, if a bottleneck requires 10 hours to be completed and a preceding step requires 1 hour to be completed, the fractional utilization of the preceding step is 1/10. Finally, using the information on the number of performers on a given step we recover the fractional utilization of performers in a step. In

Wage Bin	1	2	3	4	5	6	7	8
8	30%	40%	48%	42%	46%	70%	58%	66%
7	28%	37%	46%	63%	62%	57%	71%	68%
6	32%	50%	42%	57%	44%	63%	67%	74%
5	30%	49%	70%	53%	76%	74%	69%	81%
4	55%	50%	46%	55%	67%	65%	81%	85%
3	26%	53%	46%	58%	57%	59%	63%	67%
2	25%	33%	35%	50%	76%	65%	75%	60%
1	34%	41%	49%	44%	58%	40%	56%	68%

Step Utilization Bin

Figure 7: Patterns of automation over wage and utilization bins. Numbers in each cell denote the percentage of steps automated in each step. HML data.

the previous example, if the step has two workers assigned to it, it implies that the fractional utilization of performers in the step is 1/5 of a worker. This fractional performer utilization rate can be compared across steps and across processes and is used as one of the two key drivers for automation in Figure 7.

When comparing plants an additional consideration is required. Figure 2 describes upper and lower bounds on automation for a given set of structural parameters. In our approach we compare different products in the HML dataset. Intuitively, the different products are heterogeneous in the production structural parameters (for example, they might differ in ρ or σ). Given this unobserved heterogeneity we expect to observe a probability of automation that varies as we vary wages and capacity utilization as opposed to a strict demarcation. In the HML dataset, for each product we consider pairs of steps with identical task content between the hand and machine processes. We select steps from hand processes that were performed manually. We then measure whether a step has been automated in the machine process using a binary indicator of a change in motive power.[30] Figure 7 displays the results.

In Figure 7, each cell is ordered in terms of percentile of performer utilization and wage of the performer. The number in each cell denotes the percentage of steps in each range of utilization and wage that is automated. As expected from Figure 2, the pattern that emerges displays characteristics of a *cone of automation*: automation occurs more often at middle wage steps, and the range of middle wage steps that are likely to be automated becomes progressively larger for higher utilization steps. Intuitively, the most automated steps in the HML data are the ones in which a large fraction of worker time is devoted to a step thus allowing a machine in that step to be less rate constrained. Additionally, automation is more likely when wages are not too high or too low.

We next turn to optoelectronic semiconductor data. In this dataset we observe different pro-

[30]We do not observe any instances of a hand step shifting to a less mechanized form of motive power in the equivalent step in the machine process. For additional details refer to Appendix B.2.

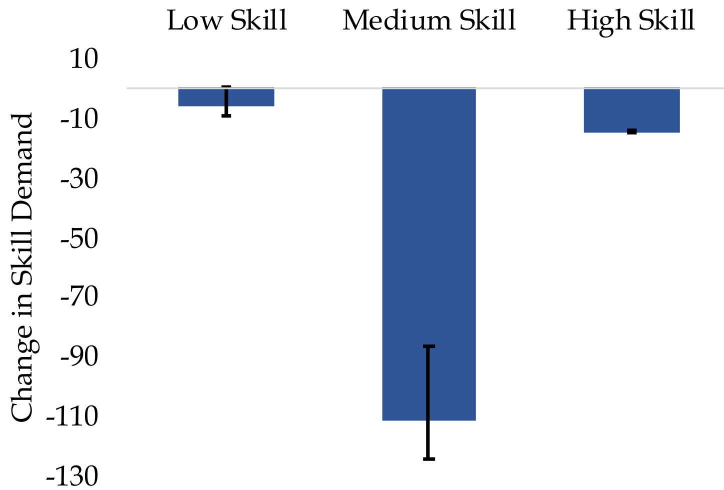


Figure 8: Impact of automation on skill demand. Data from Combemale et al. (2020).

duction scenarios with different levels of automation.[31] This level of detail allows us to precisely determine if a step has been automated or not. In addition, the available data allows us to determine the ability of each operator directly. As with the case with consolidation we focus on *Operation and Control* (as defined in the O*NET database) as a skill level. In Figure 8 we display the results as we move from a low to medium level of automation. The data displayed is for a single output level. This Figure can then be considered as a vertical slice of Figure 2 for middle output levels. The vertical axis denotes the number of displaced workers being automated at a given skill level. As anticipated by our theory, the impact of automation is more evident for middle-skill workers.

6 Conclusion

This paper provides a general theory relating technology change and labor demand. The theory provides a model of the firm that can be easily mapped to various types of technological change. We emphasize three dimensions of the problem of the firm affected by technology: the ease of fragmenting the production process into smaller steps; the costs of relocating the same performer (human or machine) across multiple steps; and the trade-off between step complexity and rate of completion. We show that automation has a polarizing effect on skill demand at low output and an upskilling effect at higher output. Technological changes that reduce fragmentation costs and increase process complexity can increase the dispersion of labor abilities demanded. We find that these implications of the theory are supported by empirical evidence across a wide range of technologies and industry contexts from the late 19^{th} century and contemporary manufacturing. The theory offers multiple broad insights about technology change and the division of labor. First, the division of production into different steps is the origin of heterogeneous ability demand.

[31] The change in level of automation is characterized using a taxonomy of automation (see Combemale et al. 2020).

Heterogeneous ability demand does not occur unless some steps are more costly to divide than others. Second, as it is easier to maintain machines at a high utilization rate (e.g., cloud computing), the effect of automation becomes less skill-polarizing and more upskilling. Third, the paper makes the point that the phenomenon of skill polarization is not only a relatively recent IT-related phenomenon that holds at the aggregate. Instead, we make the case that polarization naturally occurs with any form of automation and can occur within a firm. Several extensions of the model can be considered. A natural extension is to relax the assumption that firms set their ability demand to ensure that a step is completed in expectation. This extension allows firms to choose ability greater or less than step difficulty at the cost of higher or lower success (yield) rates. This extension can help explain empirical cases where high costs of failing to solve issues in a specific step would warrant higher demand for operator ability so that failure is less frequent.

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Appendix

A Proofs

A.1 Proofs of Section 3

Lemma 1. *Suppose Assumption 1 holds. Then: (i) for all $n > 1$, $\mathbf{X}(n|\rho_m) > \mathbf{X}(n|\rho_h)$. (ii) $\lim_{n\to\infty} \mathbf{X}(n|\rho_m) - \mathbf{X}(n|\rho_h) = \infty$.*

Proof. *(i)* For any realization of $\{X_i\}_{i=1}^n$, from Lemma 2.1 in Kirmaci et al. (2008) and Assumption 1 we have that:

$$\left(\sum_{j=1}^{n}(X_j)^{\rho_m}\right)^{\frac{1}{\rho_m}} > \left(\sum_{j=1}^{n}(X_j)^{\rho_h}\right)^{\frac{1}{\rho_h}},$$

the result then follows immediately. For *(ii)*, an application of Hölder inequality gives

$$\left(\sum_{j=1}^{n}(X_j)^{\rho_m}\right)^{\frac{1}{\rho_m}} \leq \left(\sum_{j=1}^{n}(X_j)^{\rho_h}\right)^{\frac{1}{\rho_h}} n^{\frac{1}{\rho_m}-\frac{1}{\rho_h}},$$

so that $\mathbf{X}(n|\rho_m) \leq \mathbf{X}(n|\rho_h)n^{\frac{1}{\rho_m}-\frac{1}{\rho_h}}$. We then have:

$$\mathbf{X}(n|\rho_m) - \mathbf{X}(n|\rho_h) \geq \mathbf{X}(n|\rho_m)\left[1 - \left(n^{\frac{1}{\rho_m}-\frac{1}{\rho_h}}\right)^{-1}\right].$$

Since $\mathbf{X}(n|\rho_m)$ diverges, the result follows. □

Lemma 2. *The function $c(l|\rho)$ is: (i) strictly increasing, and (ii) strictly concave in step length l; (iii) Suppose Assumption 1 holds, then $\lim_{l\to\infty}[c(l|\rho_m) - c(l|\rho_h)] = \infty$.*

Proof. We have that for all n, $\frac{dP_n(l)}{dl} = P_n(l)\left(\frac{n}{l} - \lambda\right)$. Since $\mathbf{X}(n|\rho)$ is strictly increasing in n, we have that:

$$\frac{dc(l)}{dl} = \sum_{n=1}^{\infty} P_n(l)\left(\frac{n}{l} - \lambda\right)\mathbf{X}(n|\rho) > \sum_{n=1}^{\infty} P_n(l)\left(\frac{n}{l} - \lambda\right)\mathbf{X}(1|\rho),$$

so that:

$$\frac{dc(l)}{dl} > \mathbf{X}(1|\rho)\left[\frac{1}{l}\sum_{n=0}^{\infty} nP_n(l) - \lambda(1 - P_0(l))\right] = \mathbf{X}(1|\rho)\lambda P_0(l) > 0.$$

Showing *(i)*. We next prove *(ii)*. Since $\mathbf{X}(0|\rho) = 0$ and since $\frac{n}{l}P_n(l) = \lambda P_{n-1}(l)$, we have that

$$\frac{dc(l)}{dl} = \lambda\sum_{n=0}^{\infty} P_n(l)(\mathbf{X}(n+1|\rho) - \mathbf{X}(n|\rho)) > 0,$$

so that:

$$\frac{d^2c(l)}{dl^2} = \lambda\sum_{n=0}^{\infty} P_n(l)\left(\frac{n}{l} - \lambda\right)(\mathbf{X}(n+1|\rho) - \mathbf{X}(n|\rho)),$$

so that:

$$\frac{d^2c(l)}{dl^2} = \lambda^2 \sum_{n=0}^{\infty} P_{n-1}(l)(\mathbf{X}(n+1|\rho) - \mathbf{X}(n|\rho)) - \lambda^2 \sum_{n=0}^{\infty} P_n(l)(\mathbf{X}(n+1|\rho) - \mathbf{X}(n|\rho)) =$$

$$< \lambda^2 \sum_{n=1}^{\infty} P_n(l) \left[(\mathbf{X}(n+2|\rho) - \mathbf{X}(n+1|\rho)) - (\mathbf{X}(n+1|\rho) - \mathbf{X}(n|\rho))\right] < 0.$$

Where the first inequality holds since $\lambda^2 P_0(l)(\mathbf{X}(1|\rho) - \mathbf{X}(0|\rho)) > 0$ and the second inequality holds since $\mathbf{X}(n+1|\rho) - \mathbf{X}(n|\rho)$ is decreasing in n.To show *(iii)* we have that:

$$\lim_{l\to\infty} [c(l|\rho_m) - c(l|\rho_h)] = \lim_{l\to\infty} \sum_{n=0}^{\infty} P_n(l) \left[\mathbf{X}(n|\rho_m) - \mathbf{X}(n|\rho_h)\right].$$

From the definition of $P_n(l)$ and from Lemma 1 *(iii)*, we have that the elements of the sum diverge, proving the result. □

A.2 Proofs of Section 4.1

Proposition 1. *Suppose that $f(\cdot,\cdot)$ is sufficiently low and that $w(\cdot)$ or $k(\cdot)$ is sufficiently convex. Then division of tasks is optimal.*

Proof. If fragmentation costs $f(\cdot,\cdot)$ are sufficiently low, then the condition in (10) is also sufficient. Suppose that, by contradiction, for all l, o', o'' and all r', r'' we have:

$$p\Big(D\left(c(\overline{v}|\rho_o), r^*|o\right), r^*, Y|o\Big) \leq p\Big(D\left(c(l|\rho_{o'}), r'|o'\right), r', Y|o'\Big) + p\Big(D\left(c(\overline{v}-l|\rho_{o''}), r''|o''\right), r'', Y|o''\Big).$$

To reach a contradiction, set $l = \overline{v}/2$ and $o'' = o' = o$, using (4) we have for $o = h$ (and proceeding similarly for $o = m$):

$$w(D\left(c(\overline{v}|\rho_h), r^*|h\right))g^h(Y, r^*) \leq w(D\left(c(\overline{v}/2|\rho_h), r'|h\right))g^h(Y, r') + w(D\left(c(\overline{v}/2|\rho_h), r''|h\right))g^h(Y, r''),$$

setting $r' = r'' = r^*$ the above implies:

$$w(D\left(c(\overline{v}|\rho_h), r^*|h\right)) \leq w(D\left(c(\overline{v}/2|\rho_h), r^*|h\right)) + w(D\left(c(\overline{v}/2|\rho_h), r^*|h\right)). \tag{A.1}$$

If w is sufficiently convex, so is the function $\tilde{w}(l) = w(D\left(c(l|\rho_h), r^*|h\right))$, reaching a contradiction. □

Proposition 2. *Suppose that: $f(\cdot,\cdot)$ is sufficiently low and Y is sufficiently high. If $D'_r = \frac{\partial D}{\partial r}$ is sufficiently small (or $\overline{v}$ is sufficiently large), then division of tasks is optimal.*

Proof. If fragmentation costs $f(\cdot,\cdot)$ are sufficiently low, then the condition in (10) is also sufficient. Suppose that, by contradiction, for all l, o', o'' and all r', r'' we have:

$$p\Big(D\left(c(\overline{v}|\rho_o), r^*|o\right), r^*, Y|o\Big) \leq p\Big(D\left(c(l|\rho_{o'}), r'|o'\right), r', Y|o'\Big) + p\Big(D\left(c(\overline{v}-l|\rho_{o''}), r''|o''\right), r'', Y|o''\Big).$$

Set $l = \overline{v}/2$ and $o'' = o' = o$, using (4) we have for $o = h$ (and proceeding similarly for $o = m$):

$$w(D\left(c(\overline{v}|\rho_h), r^*|h\right))g^h(Y, r^*) \leq w(D\left(c(\overline{v}/2|\rho_h), r'|h\right))g^h(Y, r') + w(D\left(c(\overline{v}/2|\rho_h), r''|h\right))g^h(Y, r''). \tag{A.2}$$

Set the values of $r' = r'' = \widehat{r}$ so that $D\left(c(\overline{v}|\rho_h), r^*|h\right) = D\left(c(\overline{v}/2|\rho_h), \widehat{r}|h\right)$. Totally differentiating $D\left(c(l|\rho), r|h\right)$ and keeping a constant difficulty level we get: $\frac{dr}{dl} = -c_l'(l|\rho)D_c'/D_r'$. So that $\widehat{r} = r^* + c_l'(l|\rho)\frac{\overline{v}}{2}(D_c'/D_r')$. From Assumption 3, for Y sufficiently high, we have $g(Y, r) \approx 1/r$ for all r. Equation (A.2) evaluated at $r' = r'' = \widehat{r}$ simplifies to $\widehat{r} \leq 2r^*$. From the definition of $\widehat{r}$ we see that this condition is violated when D_r' is sufficiently small or $\overline{v}$ is sufficiently large thus reaching a contradiction. □

A.3 Proofs of Section 4.2

Proposition 3. *Suppose that Assumptions 2 and 4 hold. Suppose that $D_{cr}''/D_c' \geq -g_r'/g$. Given two steps i and j with the same performer: $o_i = o_j$. Denote with r_i, r_j (a_i, a_j) the optimal choice for rate (ability) in step i and j. Then if $l_i > l_j$, we have that (i) $r_i \leq r_j$, (ii) $p(a_i, r_i, Y|o_i) > p(a_j, r_j, Y|o_i)$.*

Proof. Suppose that $o_i = o_j = h$ (similar arguments follow for a machine performer). Abusing notation let $p(l_i, r_i) = w(D(c(l_i|\rho_h)), r_i|h)g^h(Y, r_i)$. We first show *(i)*. From the statement of the proposition on the optimality of r_j, we have that $p(l_j, r_j) \leq p(l_j, r_i)$. Suppose statement *(i)* is not true so that $r_i > r_j$ we then have:

$$p(l_i, r_i) < p(l_i, r_j). \tag{A.3}$$

We next show how the above leads to a contradiction. If the function p satisfies the strict increasing difference property then, since $l_i > l_j$ and $r_i > r_j$, we have that:

$$0 > p(l_i, r_i) - p(l_i, r_j) > p(l_j, r_i) - p(l_j, r_j),$$

where the first inequality follows from (A.3). From the above we then have that $0 > p(l_j, r_i) - p(l_j, r_j)$ contradicting the optimality of r_j. It remains to be shown that the function p has the strict increasing difference property. From Topkis (1998) Chapter 2, it suffices to show that $\frac{\partial^2 p}{\partial r \partial l} > 0$. We have that:

$$\frac{\partial^2 p}{\partial r \partial l} = \frac{\partial^2 [w(D\left(c(l|\rho_h), r|h\right))g^h(r, Y)]}{\partial r \partial l} = \frac{\partial}{\partial r}\left[w' D_c' c_l' g\right] =$$
$$= w'' D_c' D_r' c_l' g + w' D_{cr}'' c_l' g + w' D_c' c_l' g_r' > 0.$$

where the inequality follows from properties of w, D, c and the assumption that $D_{cr}''/D_c' \geq -g_r'/g$. We next look at *(ii)*. From *(i)*, since $l_i > l_j$, we have that $r_i \leq r_j$. If $r_i = r_j$ result *(ii)* follows immediately. Suppose *(ii)* is not true then: $w(D(c(l_i), r_i))g(r_i) \leq w(D(c(l_j), r_j))g(r_j)$. We then have $w(D(c(l_j), r_j))g(r_j) \geq w(D(c(l_i), r_i))g(r_i) > w(D(c(l_j), r_i))g(r_i)$ which is a contradiction given the optimality of r_j for step-length l_j. □

Proposition 4. *Suppose that Assumptions 5 and 6 hold. Given two steps i and j with the same performer: $o_i = o_j$. Then if $l_i > l_j$, we have that $a_i > a_j$.*

Proof. If $r_i \geq r_j$ the result follows immediately. Let $r_i < r_j$. Suppose the statement is not true so that $a_i \leq a_j$. Suppose that $o_i = o_j = h$ (similar arguments follow for a machine performer). From (5), for step length l, the choice for r solves:

$$\min_{\underline{r} \leq r \leq Y\bar{r}_h} w\left(D\left(c(l|\rho_h), r|h\right)\right) g^h(Y, r). \tag{A.4}$$

If both r_i and r_j are interior, from the first order conditions follows that (suppressing notation):

$$\mathbf{w}_s = \frac{D(c(l_s), r_s) w'(D(c(l_s), r_s))}{w(D(c(l_s), r_s))} = -\frac{g_r'(Y, r_s)}{g(Y, r_s)} \frac{D(c(l_s), r_s)}{D_r(c(l_s), r_s)}, \quad s = i, j. \tag{A.5}$$

Where $\mathbf{w}_s = \mathbf{w}(D(c(l_s), r_s))$. From Assumption 6 and the contradicting assumption $a_i \leq a_j$ we have that $\mathbf{w}_i \leq \mathbf{w}_j$. From the functional forms specified in Assumption 5 we have that:

$$\mathbf{w}_i = -\frac{g_r'(Y, r_i)}{g(Y, r_i)} \frac{D(c(l_s), r_i)}{D_r(c(l_i), r_i)} = \frac{1}{\varsigma} + \frac{\underline{c}}{\varsigma r_i^{\varsigma}} > \frac{1}{\varsigma} + \frac{\underline{c}}{\varsigma r_j^{\varsigma}} = -\frac{g_r'(Y, r_j)}{g(Y, r_j)} \frac{D(c(l_j), r_j)}{D_r(c(l_j), r_j)} = \mathbf{w}_j.$$

Where the strict inequality follows from $r_i < r_j$. We thus reach a contradiction. We now consider the corner solutions. Since $r_i < r_j$ We have that $r_j > \underline{r}$ and $r_i < Y\bar{r}_h$. Remaining cases include $r_j = Y\bar{r}_h$ and $r_i = \underline{r}$. For these cases have that: $\mathbf{w}_i \geq \frac{1}{\varsigma} + \frac{\underline{c}}{\varsigma r_i^{\varsigma}} > \frac{1}{\varsigma} + \frac{\underline{c}}{\varsigma r_j^{\varsigma}} \geq \mathbf{w}_j$. Where the strict inequality as before is from $r_i < r_j$. We then reach a contradiction with the contradicting assumption that implies $\mathbf{w}_i < \mathbf{w}_j$. □

A.4 Proofs of Section 4.3

Proposition 5 (Upper Bound on Automation). *Suppose Assumptions 1, 3, and 4 hold. There exists $\bar{l}$ such that $o_i = h$ for all i with $l_i > \bar{l}$.*

Proof. Suppose not, then given our indifference assumption for all $\bar{l}$ there exists a j with $l_j > \bar{l}$ such that $o_j = m$. Let r_j^m be the optimal choice for rate for step j, this implies that for all r_j^h:

$$k(a_j^m) g^m(Y, r_j^m) < w(a_j^h) g^h(Y, r_j^h). \tag{A.6}$$

From Assumption 4, $k(\cdot)$ and $w(\cdot)$ are increasing. Given Assumption 3 Part 1, the optimal r for either performer is always $\underline{r} \leq r \leq \bar{r}$. We then have $D(c(l_j|\rho_h), r_j^h|h) = a_j^h \leq D(c(l_j|\rho_h), \bar{r}|h) \equiv \bar{a}(l_j)$, and $D(c(l_j|\rho_m), r_j^m|m) = a_j^m \geq D(c(l_j|\rho_m), \underline{r}|m) \equiv \underline{a}(l_j)$. Substituting the previous inequalities in (A.6) we have:

$$k(\underline{a}(l_j)) < w(\bar{a}(l_j)) \frac{g^h(Y, \bar{r})}{g^m(Y, \underline{r})}. \tag{A.7}$$

From Lemma 2 *(iii)* with D continuous, we have that: $\lim_{l \to \infty} D(c(l_j|\rho_m), \underline{r}|m) - D(c(l_j|\rho_h), \bar{r}|h) = \infty$. Reaching a contradiction with (A.7). □

Proposition 6 (Lower Bound on Automation). *Suppose the g function satisfies Assumption 3. Suppose there exists a step i with l_i sufficiently small. Also suppose that $\lim_{c\to 0, r\to \underline{r}} w(D(c,r|h)) \leq k(D(c,r|m))$. Then if Y is sufficiently low, we have that $o_i = h$.*

Proof. Suppose not, then for step i: $k(D(c(l_i|\rho_m), r_m|m))g^m(Y, r_m) < w(D(c(l_i|\rho_h), r_h|h))g^h(Y, r_h)$. If Y is sufficiently small we have that $r_m = \overline{r}_m(Y) < \overline{r}_h(Y) = r_h$. From Assumption 3 this implies that $g^h(Y, r_h) < g^m(Y, r_m)$. From the contradicting assumption we have $k(D(c(l_i|\rho_m), r_m|m)) < w(D(c(l_i|\rho_h), r_h|h))$. If l_i is sufficiently small, we then have that $c(l_i|\rho) \approx 0$; in addition, from Assumption 3 Part 2, we have that $\lim_{Y\to 0} r_j = \underline{r}$ for $o_j = h, m$. We then reach a contradiction with the assumption in the Proposition stating that $\lim_{c\to 0, r\to \underline{r}} w(D(c,r|h)) \leq k(D(c,r|m))$. □

Proposition 7. *Suppose that the g function satisfies Assumption 5. Denote with $o_i(Y)$ the optimal operator choice for l_i at output value Y. We then have: (i) If Y is sufficiently large then for all $Y' > Y$ we have that $o_i(Y) = o_i(Y')$; (ii) Let $o_i(Y) = m$. Suppose that $w' \geq k'$, $w'' \geq k''$ and that $w(D(c(l|\rho_h), \underline{r}|h)) < k(D(c(l|\rho_m), \underline{r}|m))$ for all $l > 0$, then for all Y if $Y' > Y$ we have that steps of length l_i also feature $o_i(Y') = m$; (iii) Assume wages and cost of capital are log-linear. Let $o_i(Y) = m$, under the assumption of (ii), if Y is sufficiently large, then for all steps with $l'_j < l_i$ we have that $o_j(Y) = m$.*

Proof. We begin with *(i)*. Denote with $r^*_o(l_i)$ the optimal choice for rate for operator o assuming no concern for the amount of output produced (assuming $\overline{r}_o(Y) = \infty$). If Y is sufficiently large, for operator o we have that the optimal choice for $r_i = r^*_o(l_i)$. Since $\overline{r}_o(Y)$ is increasing in Y, the optimal choice of rate for both types of operators o performing step l_i is unchanged. It immediately follows that $o_i(Y') = o_i(Y)$ for all $Y' > Y$.

We next look at *(ii)*. For this case we cannot rely on the rate being set at the optimum (output unconstrained) rate. Since step i is automated it implies:

$$\min_{r \leq \overline{r}_m(Y)} \left\{ \frac{k\left(D\left(c(l_i|\rho_m), r|m\right)\right)}{r} \right\} < \min_{r \leq \overline{r}_h(Y)} \left\{ \frac{w\left(D\left(c(l_i|\rho_h), r|h\right)\right)}{r} \right\}, \tag{A.8}$$

The proof proceeds by contradiction. Suppose that with $Y' > Y$ the step of length l_i is not automated. The contradicting assumption implies that:

$$\min_{r \leq \overline{r}_m(Y')} \left\{ \frac{k\left(D\left(c(l_i|\rho_m), r|m\right)\right)}{r} \right\} \geq \min_{r \leq \overline{r}_h(Y')} \left\{ \frac{w\left(D\left(c(l_i|\rho_h), r|h\right)\right)}{r} \right\}, \tag{A.9}$$

Let $\widetilde{r}$ be defined as the rate such that $k(D(c(l_i|\rho_m), \widetilde{r}|m)) = w(D(c(l_i|\rho_h), \widetilde{r}|h))$. This $\widetilde{r}$ exists and is unique given Assumption 2 and our stated assumptions on w and k including the assumption that $w(D(c(l_i|\rho_h), \underline{r}|h)) < k(D(c(l_i|\rho_m), \underline{r}|m))$. For any $r < \widetilde{r}$ we have that:

$$\frac{k\left(D\left(c(l_i|\rho_m), r|m\right)\right)}{r} > \frac{w\left(D\left(c(l_i|\rho_h), r|h\right)\right)}{r},$$

hence we have that for (A.8) to hold it must be the case that $\widetilde{r} \leq \overline{r}_m(Y)$. Given Assumption 2 and $w(D(c(l_i|\rho_h), \underline{r}|h)) < k(D(c(l_i|\rho_m), \underline{r}|m))$, it also follows that $D'_r(c(l_i|\rho_m), r|m) < D'_r(c(l_i|\rho_h), r|h)$

and $D''_r(c(l_i|\rho_m), r|m) < D''_r(c(l_i|\rho_h), r|h)$ for all $r \geq \tilde{r}$. Since difficulty increases with respect to r at a faster rate for humans relative to machine performers, given the assumption on first and second derivatives of k and w, we reach a contradiction with (A.9).

We finally look at *(iii)*. As notation let the cost of capital be given by $k(D) = k_0 e^{\gamma_k D}$ (and for labor $w(D) = w_0 e^{\gamma_l D}$). As for case *(i)*, in this case we can set the operator rate optimally. From first order conditions for capital (similarly for labor) we get that the optimal machine rate r_m when step-length is l_i satisfies:

$$\gamma_k c(l_i|\rho_m)\sigma_m r_m^{\sigma_m} = 1 \rightarrow r_m = \left[\frac{1}{\gamma_k c(l_i|\rho_m)\sigma_m}\right]^{\frac{1}{\sigma_m}}. \tag{A.10}$$

Denote with r'_m the optimal choice with step length l'_j. We have that:

$$\log r'_m - \log r_m = \frac{1}{\sigma_m}\left[\log(c(l_i|\rho_m)) - \log(c(l'_j|\rho_m))\right].$$

Since $\sigma_m < \sigma_h$ and $\left[\log(c(l_i|\rho_m)) - \log(c(l'_j|\rho_m))\right] > \left[\log(c(l_i|\rho_h)) - \log(c(l'_j|\rho_h))\right]$, it follows that

$$(\log r'_m - \log r_m) > (\log r'_h - \log r_h). \tag{A.11}$$

Denote with k and k' the capital cost for step l_i and l'_j (similarly for labor). The result holds if:

$$\log k - \log k' + (\log r'_m - \log r_m) > \log w - \log w' + (\log r'_h - \log r_h).$$

Given (A.11), we need to show next that the relationship $\log k - \log k' > \log w - \log w'$ holds. Given the assumption on wages and the functional form for D, the above relationship implies:

$$\gamma_k \left(c(l_i|\rho_m)(\underline{c} + r_m^{\sigma_m}) - c(l'_j|\rho_m)(\underline{c} + r_m'^{\sigma_m})\right) > \gamma_w \left(c(l_i|\rho_h)(\underline{c} + r_h^{\sigma_h}) - c(l'_j|\rho_h)(\underline{c} + r_h'^{\sigma_h})\right).$$

Substituting the optimal r from (A.10) we get

$$\gamma_k \underline{c}\left(c(l_i|\rho_m) - c(l'_j|\rho_m)\right) > \gamma_w \underline{c}\left(c(l_i|\rho_h) - c(l'_j|\rho_j)\right)$$

which holds as long as $\gamma_k > \gamma_w$. □

A.5 Proofs of Section 4.4

Lemma 3. *Suppose that the Assumptions of Proposition 1 and Proposition 4 hold. Consider the case in which $f(\cdot,\cdot) = \overline{f}$. Then $l_i = \bar{l}$ and $a_i = \bar{a}$ for all i.*

Proof. Suppose not, then there exist two consecutive steps i, j such that without loss of generality $l_i > l_j$. Consider the alternative allocation with $\bar{l} = (l_i + l_j)/2$. For this allocation not to be optimal it must be the case that: $p(D\left(c(l_i|\rho_h), r_i|h\right), r_i, YR|h) + p(D\left(c(l_j|\rho_h), r_j|h\right), r_j, Y|h) \leq$

$2p(D(c(\bar{l}|\rho_h),\bar{r}|h),\bar{r},Y|h)$. The contradiction is then reached as in the proof of Proposition 1 exploiting sufficiently convex wages. The result for ability follows from Proposition 4. □

Corollary 1. *Suppose that the Assumptions of Proposition 4 hold. Let $w(\cdot)$ and $k(\cdot)$ be sufficiently convex. Consider arbitrary fragmentation costs f and let f' so that $f'(t,\cdot) = \overline{f} \cdot f(t,\cdot)$ for $\overline{f} \geq 0$. Let a_{min} and a'_{min} be the lowest ability demanded under f and f', respectively. If $\overline{f}$ is sufficiently low, then $a'_{min} \leq a_{min}$.*

Proof. Let T, T' be the optimal number of thresholds under f, f' respectively. For sufficiently small $\overline{f}$ it follows from Proposition 1 that $T' > T$. Let l_{min} (l'_{min}) be the length of the shortest step given T (T'). We have that $l_{min} \leq \frac{\bar{v}}{T}$. If not $\sum_{i=1}^{T} l_i > \bar{v}$, reaching a contradiction. It then follows that $l'_{min} < l_{min}$. The result then follows from Proposition 4. □

Corollary 2. *Suppose that the Assumptions of Proposition 4 hold. Suppose there exist a set of lumpable tasks $\widehat{V}$ of length $\widehat{l}$. Also suppose that under issue arrival λ, the maximum step length is $\widehat{l}$. Consider an issue arrival $\lambda' > \lambda$. Let a_{max} and a'_{max} be the lowest ability demanded under λ and λ', respectively. If the performer for the longest step remains the same, we then have $a'_{max} > a_{max}$.*

Proof. Since $\widehat{V}$ is lumpable, the maximum step length cannot be smaller than $\widehat{l}$. From the definition of complexity in (2) we observe that step length and issue arrival are perfect substitutes in their effect on complexity. The result then follows the proof of Proposition 4 substituting changes in l with changes in λ. □

B Hand and Machine Labor Data

B.1 Overview

For each product, the Hand and Machine Labor (HML) dataset includes general information on the process as well as detailed information on the steps required to create the product using different methods: either by hand (hand process) or using a machine (machine process). Products vary greatly in the complexity of production: the observed number of steps range from one to over two hundred and fifty. The data used in this paper comprise 612,625 step-level entries and 21,120 process-level entries. The HML data is publicly available in a non-digitized form. The entire dataset was digitized from scanned and physical copies of the data by undergraduate students at Carnegie Mellon University between 2019 and 2023. The type and definition of variables present in the HML dataset are described below.

Table B.1 describes process-level variables, which apply across all steps and methods. For ease of comparison between processes of each method, the dataset reports observed production volumes for each process. The dataset also reports the input requirements to meet a conformed volume which is consistent across the hand and machine methods. Table B.2 describes variables which are reported for each step.

Table B.1: Process variables in HML data.

Variable Name	Definition	Example
Unit	Product name	Potatoes
Unit Volume	Volume of product for each full cycle	880 bushels
Conformed Volume	Volume of product per cycle used in presentation of step-level data	220 bushels
Method	Level of process mechanization	Hand/Machine
Total Employment	Number of people employed in process	4 people
Total Animals	Number and type of animals used	2 horses
Time Worked	The number of hours worked per day	10 hours
Year	Date of production process	1893
Unit Characteristics	Additional product details	From grafts

Table B.2: Step-level variables in HML data.

Variable Name	Definition	Example
Operation Number	Code for the set of tasks in a process step	{2, 3}
Work Done	Type of the activities performed in a step	Planting seed
Machine, Implement or Tool Used	Description of primary equipment used to complete step	Steam shovel
Motive Power	Source of power for operations described	Steam; Horse
Persons Necessary on One Machine	Number of workers required per machine or station	2 workers
Animals Necessary on One Machine	Number of animals required per machine (type recorded in motive power)	2 horses
Number of Workers	Number of workers required in a process step across all stations	4 workers
Sex	Sex of workers	M, F
Occupation	Occupational title of workers	Laborer
Age	Age (or age range) of workers	21-30
Time Worked	Total hours and minutes to complete step	1hr 15m
Animal Time Worked	Total hours and minutes to complete step	2hr 30m
Worker Pay Rate	Per-period pay (nominal dollars)	$1.00
Animal Pay Rate	Per-period cost (nominal dollars)	$0.375
Worker Pay Period	Payment cycle for workers	1 Day
Animal Pay Period	Cost cycle for animals	1 Day
Labor Cost	Total labor cost	$.125
Animal Cost	Total animal cost	$.0938

B.2 Mapping Hand and Machine Processes

We next describe additional steps taken to map the data to the model. In the original data, entries concerning animal labor in production are given a distinct line with otherwise identical step information (tools, task content). Since there are never animals used in production without workers, we condense animal information into the same step as the human workers that manage them. Some steps also include workers with multiple occupational titles. When this occurs, the dataset provides separate entries in the data. When mapping the task content between hand and machine methods, distinct occupations are kept as separate steps with the same task content. Any step containing multiple occupations (7.4 percent of steps observed) is excluded from our analysis of step automation or changes in the division of tasks among steps, because the division of tasks within occupations within a single step is not specified (and to avoid double-counting steps). For all products, we build a mapping between hand and machine processes. We index the tasks in hand and machine processes as $\mathcal{V}^H$ and $\mathcal{V}^M$ respectively. In terms of notation, superscripts H, M indicate either hand or machine process-types. Every step i contains a set $\mathcal{S}_i$ of tasks.[1] Any given step belongs to exactly one of the following six possible cases:

1. **1 to 1:** Steps i^H and j^M belong to this case if they have the same task content and do not share task content with any other steps.A 1 to 1 mapping is useful when analyzing a change in performer type or performer characteristics, independently of changes in the division of production.

2. **1 to 0:** a step i^H belongs to this case if $S_i^H \cap \mathcal{V}^M = \varnothing$. These steps capture activities that are no longer performed in the machine case (e.g., post-processing work made unnecessary by process improvement).

3. **0 to 1:** a step i^H belongs to this case if $S_i^M \cap \mathcal{V}^H = \varnothing$. These steps represent activities which are new to a process (e.g., firing a boiler, which would be unnecessary in a hand process without a steam engine).

4. **1 to N:** step i^H belongs to this case if: *(a)* all of its tasks are contained in the machine process, *(b)* tasks in the hand step are contained in more than one machine step, and *(c)* no machine step with a task set intersecting the hand step contains tasks that are contained in any other hand step. ($\forall j$ such that $\mathcal{S}_j^M \cap \mathcal{S}_i^H \neq \varnothing$ we have $\mathcal{S}_j^M \cap (\mathcal{V}^M \setminus \mathcal{S}_i^H) = \varnothing$) Step j^M belongs to this case if $\mathcal{S}_j^M \subset \mathcal{S}_i^H$ such that i^H satisfies the above conditions. This case allows us to capture an increase in the division of tasks.

5. **M to 1:** a step j^M belongs to this case if: *(a)* all of its tasks are contained in the hand process, *(b)* tasks in the machine step are contained in more than one hand step, and *(c)* no hand step with a task set intersecting the machine step contains tasks that are contained in any

[1] Note that it is possible for two steps $i \neq j$ to exist such that $\mathcal{S}_i^M \cap \mathcal{S}_j^M \neq \varnothing$: for example, steps with content 1a and 1b in the hand process are identical in task content to step 1 in the machine process, and to each other.

other machine step. Step i^H belongs to this case if $\mathcal{S}_i^H \subset \mathcal{S}_j^M$ such that j^M satisfies the above conditions. This case allows us to capture a decrease in the division of tasks.

6. **M to N:** any remaining step not included above belongs to this case.

Table B.3 reports the number and share of process steps for each method which belong to each of the six cases described above. We have that 77% of Hand steps and 56% of Machine steps belong to mappings which can be interpreted as changes in T or o for fixed $\mathcal{V}$, allowing them to be used to explore technological cases which vary or hold constant the division of tasks.

Table B.3: Mapping between steps of different methods recovered from HML data.

Process Mapping	**Hand Steps**	**Share of Hand**	**Machine Steps**	**Machine Share**
0 to 1	0	0	4,890	.37
1 to 0	832	.11	0	0
1 to 1	3,849	.49	3,849	.29
1 to N	948	.12	3,086	.23
M to 1	1,327	.17	477	.04
M to N	915	.12	881	.07
Missing Alternate	9	.001	0	0
Total	7,880		13,183	

The "*Missing Alternate*"row indicates steps from processes which do not have a corresponding process of the opposite method: in our data, one hand process had a counterpart machine process for which the authors of the Hand and Machine study could not compare task content and thus could not encode operation numbers.

Automation and Utilization When analyzing the impact of automation, we focus on the rate of automation of steps belonging to the 1 to 1 case described in the previous section (so to keep the task content of each step constant across the human and machine processes). The HML dataset features no observations of motive power in hand processes such as steam or water shifting to "less mechanized" motive powers such as hand or animal power in the respective machine processes. Given this we treat all changes in motive power as a shift toward automation.

To look at the implications of $\bar{r}$ on the rate of automation, we construct a measure of the utilization of performers in each process step, $u = Y/\bar{r}$. The lower the utilization of performers, the lower the returns to increasing rate and the closer the performer is to $\bar{r}$. To compare between process steps which were or not automated, we use the parameters of a step's performer in the hand process to determine utilization given the volume in the machine process, as a proxy for $\bar{r}$. For each product, we can recover an upper bound on the possible output of each process step: $Y_j^H = rn$, where r is the rate of output per performer shift and n is the number of performers demanded per shift. The maximum effective output of any step in a production process cannot be greater than the maximum output of every other step (bottlenecks), giving us $\overline{Y}_i^H = \min_{i=1}^N(Y_i^H)$.

When analyzing the division of tasks, to remove the effect of other changes beyond the division of tasks, we control for task content and for the level of automation. For the former we only consider the case of steps mapping from 1 to N (the decrease in the division of tasks is characterized by a M to 1 mapping. As only 55 machine processes have any steps that exhibit this property with a constant motive power, we do not consider this scenario). To control for the level of automation we further restrict the sample selection to steps in which the motive power is unchanged.

C Additional Figures

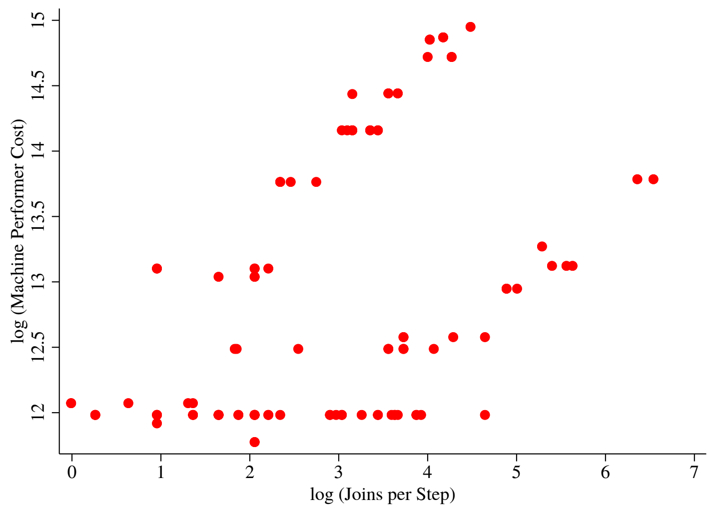

Figure C.1: Machine costs and step complexity. Prices in 2006 Dollars. For information on the data refer to Fuchs et al. (2008).