

OPTIMAL INCOME TAXATION WITH ENDOGENOUS PRICES*

Alexey Kushnir

Robertas Zubrickas

May 2020

Abstract

We study an optimal income taxation problem in a Mirrleesian setting with endogenous product prices and positive firm profits. When firm profits are progressively distributed among agents, we show that the public authority uses the price level as a redistributive tool favoring lower prices in competitive markets. This leads to higher marginal taxes in the optimum. Using simulations, we show that this price effect adds around 4 percentage points to marginal income tax rates. In oligopolistic markets, market power adds an additional non-competitive component to the price effect that exerts downward pressure on optimal tax rates. Using simulations, we show that, as the market structure varies, changes in non-competitive and redistributive components almost cancel each other out for medium and high degrees of equity concerns. Thus, optimal income tax policy remains stable across various market structures. We also analyze how commodity and profit taxation influence the price and non-competitive effects.

Keywords: Optimal income taxation, endogenous price, competitive market, conjectural variation, oligopolistic market.

JEL Classification: H21, H23, D43.

**Kushnir:* Carnegie Mellon University, Tepper School of Business, Pittsburgh, USA; National Research University, Higher School of Economics, Moscow, Russia; and New Economic School, Moscow, Russia, akushnir@andrew.cmu.edu; *Zubrickas:* University of Bath, Department of Economics, Bath, UK, r.zubrickas@bath.ac.uk. This paper supersedes an earlier version entitled “Market Power and Optimal Income Taxation.” We are grateful to Marcus Berliant, Simon Board, Craig Brett, Christian Bredemeier, David Childers, Dennis Epple, Ed Green, Emanuel Hansen, David Jinkins, Louis Kaplow, Dirk Niepelt, Peter Norman, Nicola Pavoni, Markus Reisinger, Luca Rigotti, Florian Scheuer, Ali Schourideh, Chris Sleet, Ron Siegel, Stefanie Stantcheva, and John Weymark; seminar participants at Carnegie Mellon University, Copenhagen Business School, Frankfurt School of Finance and Management, Higher School of Economics, New Economic School, Nottingham University, UNC-Duke, University of Bern, University of Cologne, University of Nottingham, University of Pittsburgh, and University of Zurich; participants at the Association of Public Economic Theory Conference, Econometric Society European Meeting, International Conference on Game Theory in Stony Brook; and all others who generously provided feedback and offered insights at various stages of this project. Thanks to their precious help, we were able to significantly improve the paper.

1 Introduction

The design of income taxes is among the most important economic issues. At the same time, the income taxation literature commonly assumes that product markets are perfectly competitive. While this assumption is convenient for analysis, it is far from an accurate description of most markets. According to recent studies (Azar et al., 2017; De Loecker et al., 2020), one third of U.S. industries are highly concentrated and over 75% of them are operating in markets that are more concentrated than 20 years ago.¹ Similar market structures are observed in Europe, where the top five food retailers typically have a 43% to 69% share of the market (European Commission, 2015).

What are the implications for optimal income taxation if markets are not perfectly competitive? What is the size of optimal income tax change due to the presence of endogenous prices? Our goal in this paper is to answer these questions and, thus, to bridge a gap in the public economics literature by incorporating endogenous prices into the analysis of income taxation.²

We consider the standard Mirrlees (1971) framework with a continuum of agents who differ in their productivity types. Agents earn labor income and care about the consumption of two goods: the numeraire good and the “main” good. The numeraire good is produced with a constant returns to scale technology and has a perfectly elastic supply function, whereas the main good is produced with a decreasing returns to scale technology and has a strictly increasing supply function. As a consequence, the price of the numeraire good is constant and the price of the main good is endogenously determined by the market equilibrium condition. Also, firms producing the numeraire good realize zero profits and firms producing the main good realize positive profits. Motivated by empirical evidence, we assume that firm profits are progressively distributed among agents in the economy with more productive agents receiving larger profit shares (see Alvaredo et al., 2013 and Saez and Zucman, 2016).

The public authority problem is to design the optimal income tax schedule that maximizes a weighted sum of agent utilities subject to three constraints: *the resource constraint*, which demands that the public authority raises a fixed level of public funds; *the incentive compatibility constraint*, which requires agents to reveal their productivity types; and *the market equilibrium condition*, which determines the price level. We assume that the public authority has equity concerns with low productivity agents receiving larger social weights.

When agent productivity is imperfectly observable and the public authority has a limited set of tax instruments, not every Pareto optimum allocation can be supported with competitive

¹See also Grullon et al. (2019) and articles in the popular press by Theo Francis and Ryan Knutson, “Wave of Megadeals Tests Antitrust Limits in U.S.” *The Wall Street Journal*, October 18, 2015, and the Editorial Board, “How Mergers Damage the Economy” *The New York Times*, October 31, 2015.

²Atkinson (2012, p. 775) offers an extensive discussion of how the existing taxation literature mostly fails to take the underlying market structure and endogenous prices into account. Two recent exceptions are Costa and Maestri (2018) and Kaplow (2019). We provide an extensive discussion of these and other relevant papers in the literature review section.

prices. To illustrate, let us consider an example in which all agents have the same productivity type but different profit shares. If an agent’s utility is linear in income, the difference in profit shares does not create any difference in optimal labor supply. Then, all agents have the same labor income and are subject to the same income tax. Hence, their income cannot be freely redistributed. Overall, a tax policy based on labor income alone does not allow for full discretion in redistribution, which leads to the failure of the second welfare theorem.³

The failure of the second welfare theorem leads to a binding market equilibrium condition in the optimum. The public authority, motivated by equity concerns, uses then the price level as an additional redistributing tool: a decrease in price level benefits low-productivity agents as they can afford to consume more products, but hurts high-productivity agents as their utility is mainly influenced by the decrease in firm profits.⁴ The public authority, however, cannot control the price level directly. Instead, it imposes a higher marginal income tax schedule that decreases agent labor supply and lowers agent income. The lower agent income decreases agent demand for products leading to a smaller product prices in equilibrium. In addition, if we assume that a share of firm profits belongs to agents outside the economy, efficiency concerns influence the choice of income tax policy.⁵ In this case, motivated by both efficiency and equity concerns, the public authority favors even lower price levels and an even higher marginal income taxation in the optimum.

To understand whether the price effect can significantly alter the optimal income tax policy, we estimate it using numerical simulations based on the U.S. housing market. This market is particularly suitable for illustrating our theoretical results because it is not perfectly competitive (Cosman and Quintero, 2018) and housing costs comprise the largest share of overall household expenditures.⁶ To estimate the size of the price effect, we compare the optimal income tax schedule with the Mirrleesian tax schedule that arises in *a self-confirming policy equilibrium* (see Rothschild and Scheuer, 2013, 2016). Using a calibrated model of the housing market (based on Miles and Sefton, 2018; Saiz, 2010), we found that the price effect increases optimal marginal income taxes by approximately *4 percentage points* on average in competitive markets.

³The second welfare theorem also fails if tax policy is based on total income and if firm profits can be separately taxed (see p. 11).

⁴This argument is in line with recent empirical evidence that connects individual earnings growth with real stock returns. Guvenen et al. (2017) show that individual earnings growth is especially sensitive to a change in real stock returns for high- and low-income levels. This relationship for high-income levels supports our reasoning that a decrease in firm profits hurts more high-income agents than moderate- and low-income agents. For low-income levels, the sensitivity can be explained by a high correlation between real stock returns and GDP growth. That is, labor earnings are higher in booms and lower in recessions.

⁵A substantial share of equities is held by foreigners: the share of foreign equity holdings amounts to 13.6% in the U.S. (U.S. Treasury, 2017) and, on average, 38% in European countries (Davydoff et al., 2013).

⁶See Consumer Expenditure Survey, 2017, Table 1203. *Income Before Taxes: Annual Expenditure Means, Shares, Standard Errors, and Coefficients of Variation* and Bureau of Economic Analysis, 2016, Table 2.3.5U. *Personal Consumption Expenditures by Major Type of Product and by Major Function* that report approximately 25% as the share of housing costs (including utilities) in U.S. household expenditure. Housing costs account for a similar average share of household expenditure in Europe (Eurostat, 2016).

We also consider markets with various forms of oligopolistic competition. In oligopolistic markets, the presence of market power leads to under-production in equilibrium. As a countermeasure, the public authority seeks to stimulate labor income and, thus, aggregate demand by decreasing marginal income taxes. This non-competitive force works in the direction opposite to that of the redistribution force, explained above, leading to the second non-competitive component in the price effect. To understand the interplay between these two components, we use simulations. For medium and high degrees of equity concerns, we show that, as the market structure varies, changes in non-competitive and redistributive components almost cancel each other out. Thus, the optimal income taxation schedule remains stable across market specifications, the result that was not previously pointed out in the literature.⁷

Further, we investigate how *commodity* and *profit taxation* influence the price effect on income taxation. For both competitive and oligopolistic markets, we show that the price effect is present if the optimal commodity taxation is also at the discretion of the public authority. This result is similar in spirit to Naito (1999) who showed that the seminal result of “uniform commodity tax under nonlinear income taxation” by Atkinson and Stiglitz (1976) no longer holds if marginal costs of production are non-constant. At the same time, if firm profits are equally distributed among agents (no redistribution component), the combination of optimal commodity and income taxes brings production to the efficient level (see also Myles, 1996). We also show that the price effect on optimal income taxation persists in the presence of profit taxation in an extension of our model with firm entry and exit.

We want to highlight here that one should not regard our analysis as meaning that income tax design should correct for market inefficiencies. For this purpose the government might have better instruments such as commodity and profit taxation. Instead, we want to emphasize that the optimal income tax design should take into account market inefficiencies and market structure. This claim pertains even to settings where other tax instruments such as commodity and profit taxation are at the discretion of the public authority.

Next, we relate our findings to the three tests on policy relevance proposed by Diamond and Saez (2011). First, our results are based on an economic mechanism whereby the second welfare theorem does not hold in incomplete-information markets when only a limited set of tax instruments is available. Second, using numerical simulations, we show that the price effect is empirically relevant and of the first order by estimating it as equal to around 4 percentage points. As we show in the appendix, our results are also robust to various market structures, model specifications, and simulation assumptions. Third, our tax policy recommendation favors higher marginal income tax rates, which is socially acceptable on equity grounds. Lastly, we note that the price effect is relevant to any policy with implications for income redistribution

⁷To avoid any confusion, we want to highlight that for a large number of firms the oligopolistic model does not converge to the standard Mirrleesian setting. Instead, the oligopolistic model converges to our competitive model with upward sloping supply curve.

such as the design of the minimum wage, basic income, welfare benefits, and pensions.

We postpone a detailed literature review until Section 7 and, here, only briefly outline our main contributions in relation to previous research. Compared to the growing literature on optimal income taxation with endogenous wages in labor markets (e.g., Rothschild and Scheuer, 2013; Sachs et al., 2020), we consider endogenous prices in product markets. Though our model can be equivalently rewritten in terms of *endogenous wages* (see Appendix A.7), the price effect in our model is driven by the distribution of firm profits among agents inside and outside the economy and by the presence of market power. These effects are in contrast to general equilibrium effects studied in endogenous wages literature.

In relation to studies focused on optimal income taxation with externalities (e.g., Rothschild and Scheuer, 2016; Lockwood et al., 2017; Rothschild and Scheuer, 2014), agents in our paper do not impose any direct externality. Instead, in our model, agents impose only pecuniary externalities. The welfare theorems imply that pecuniary externalities do not limit the set of feasible outcomes in complete information markets. If agent productivity is not perfectly observable and taxes are based on labor income, we show that a Pareto optimum allocation is not generally supported by a competitive equilibrium. This leads to a binding market equilibrium constraint and the price effect on optimal income taxation.

Finally, the literature analyzing taxation in the presence of imperfect competition is rather thin. Kaplow (2019) considers income taxation for various market structures. He studies income taxation policies in economies with *exogenously given firm markups*. Hence, he does not consider the influence of income taxation policy on equilibrium product prices, which is the main subject of our analysis. Costa and Maestri (2018) study optimal income taxation in non-competitive labor markets. They assume that firm profits are spread uniformly across agents thus abstracting from firm profit distribution effect, the main subject of our analysis. There are also a few important papers on *commodity taxation* in oligopolistic markets (Auerbach and Hines, 2001; Myles 1987; Reinhorn, 2005). In contrast to these papers, we consider an incomplete information setting and highlight the influence of profit distribution among agents on the optimal tax schedule.

The remainder of the paper is organized as follows. In Section 2, we introduce the model. In Section 3, we consider competitive markets and analyze the properties of the optimal income tax schedule. In Section 4, we calibrate our model and estimate the size of the price effect. In Section 5, we present our analysis for oligopolistic markets. We investigate the price effect in the presence of commodity taxation in Section 6. In Section 7, we provide a detailed literature review, and in Section 8, we present our conclusion. The omitted proofs are postponed to Appendix A.1. Appendices A.2- A.7 comprise extensions of our main model, provide additional simulation results, and relate our model to the previous literature.

2 Model

There is a continuum of agents indexed by productivity type n and distributed according to the probability density function $f(n) > 0$ with support $[\underline{n}, \bar{n}]$. Agent n 's labor income is given by $z = n\ell$, where ℓ is the number of hours worked. The labor cost is represented by an increasing and convex function $c(\ell)$. The labor income is taxed according to schedule $T(z)$. After tax, the agent's disposable labor income is equal to $y = z - T(z)$.

In the economy, there are two goods: a numeraire good and good X (referred to as the main good in the introduction). The numeraire good is produced with a constant returns to scale technology that results in a fixed price normalized to 1 and zero firm profits. Good X is produced with a decreasing returns to scale technology that yields positive profits. We denote p and $\Pi(p)$ as the price and the profit, respectively, of firms producing good X.⁸

We assume that firm profits are distributed in the form of dividends with agent n receiving share $\xi(n) \geq 0$.⁹ Motivated by empirical evidence, we consider a progressive distribution of firm profits when agents with higher labor income possess a larger share of firm profits $\xi'(n) \geq 0$ (see Alvaredo et al., 2013 and Saez and Zucman, 2016). Although we mainly analyze the case when all profits are distributed inside the economy $\int \xi(n)f(n)dn = 1$, we also consider an extension when some firm profits are held by foreigners, as observed in both U.S. and European data (see U.S. Treasury, 2017; Davydoff et al., 2013).

For clarity, we do not consider explicitly profit taxation in the main theoretical analysis. To make our calibration exercise plausible, we assume that *firm profits are taxed* at a fixed rate of 15% in our simulations. This corresponds to the U.S. tax rate on qualified dividends at most income levels. In Appendix A.5, we also investigate the interaction between optimal profit and labor income taxation in a model with endogenous firm entry and exit.¹⁰

Overall, an agent's income consists of labor income and dividends (profit shares) $\tilde{y}(n) = y(n) + \xi(n)\Pi(p)$. Agents' preferences for consumption are represented by an indirect utility function $v(p, \tilde{y})$. When firm profits are unequally distributed, agent indirect utility $v(p, \tilde{y})$ can violate the single-crossing property (Mirrlees, 1976). To avoid this problem, we consider *homothetic* preferences in the main text. This case also corresponds to our numerical simulations in Section 4, where we consider the constant elasticity of substitution (CES) preferences.¹¹ Homothetic preferences imply linear indirect utility $v(p, \tilde{y}) = a(p)\tilde{y}$, where $a(p)$ is some strictly

⁸We do not model explicitly why firms that produce the numeraire good do not switch to producing the more profitable good X. A lack of technology and/or patents may function as a barrier to entry to an industry. The high degree of profit and performance differences even among similar firms is a well-documented phenomenon (see, for example, Syverson (2011) for a recent survey).

⁹Alternatively, one could assume that the distribution of dividends is a function of disposable labor income (see, e.g., Kaplow, 2019). In this case, the endogenous choice of stock holding should be modeled using a dynamic model. We avoid this route to keep the model tractable.

¹⁰See also Scheuer (2014) for a related analysis of optimal profit taxation under endogenous firm formation.

¹¹Recent empirical studies found small income effects on individual labor supply (see a discussion in Sorensen, 2009), which also justifies the use of homothetic preferences in our analysis.

decreasing function. Note that $a(p) \equiv v_y(p)$ and we use both expressions interchangeably. Appendix A.3 also provides a detailed analysis of general *non-homothetic* preferences.

Overall, the agent's net utility is defined as the difference between indirect utility from consumption and labor cost

$$U(p, \tilde{y}, \ell) = v(p, \tilde{y}) - c(\ell). \quad (1)$$

The social welfare function is given by

$$W = \int U(p, \tilde{y}(n), \ell(n)) \psi(n) f(n) dn, \quad (2)$$

where $\psi(n)$ is the welfare weight of an agent with productivity n and satisfies $\int \psi(n) f(n) dn = 1$. Assuming that the public authority has equity concerns, we take ψ to be a decreasing function.

The public authority wants to maximize social welfare W subject to three constraints. The first one is the resource constraint:

$$\int T(z(n)) f(n) dn = \int (n\ell(n) - \tilde{y}(n) + \xi(n)\Pi(p)) f(n) dn \geq R \quad (3)$$

that ensures the public authority covers its own expenses $R \geq 0$, which are spent solely on the numeraire good. The second one is the incentive compatibility constraint:

$$U(p, z(n) - T(z(n)) + \xi(n)\Pi(p), \ell(n)) \geq U(p, z(m) - T(z(m)) + \xi(n)\Pi(p), z(m)/n), \quad (4)$$

for all $n, m \in [\underline{n}, \bar{n}]$, which ensures that an agent with productivity n does not want to seek the labor income of an agent with productivity m .

The third one is a *market equilibrium condition* that determines price p . This condition varies across market structures. In *competitive markets* (Section 3), where we assume that firms are price-takers, the market equilibrium condition requires the market supply to be equal to the market demand for good X . In *oligopolistic markets* (Section 5), where each firm takes into account its influence on the level of product price, the market equilibrium is determined by the firm profit maximization condition.

Overall, the main difference between our framework and the one of Mirrlees (1971) is that one of the product prices is endogenously determined. In addition, in our model firms obtain positive profits that are progressively distributed among agents. The effect of endogenous prices and profit distribution on optimal income taxation is the main subject of our analysis.

Finally, we want to comment on our modeling choice. We utilize the Mirrleesian framework rather than a more recent framework by Saez (2001) who uses an empirically relevant income distribution to analyze the optimal income taxation. We follow this route because agent income distribution is endogenous to prices, and we are interested in the comparative statics of different price regimes. In Section 3, we relate our results to the ones of Saez (2001) in more detail.

3 Competitive Market

In this section, we analyze optimal income taxation in competitive markets, in which the price of good X is determined by the market equilibrium condition

$$S(p) = \int x(p, \tilde{y}(n))f(n)dn. \quad (5)$$

On the left-hand side we have market supply $S(p)$, and on the right-hand side the market demand for good X, where $x(p, \tilde{y})$ is the Walrasian demand of agent with disposable income \tilde{y} . We consider an increasing supply function $S'(p) > 0$ and zero fixed costs so that the total firm profits coincide with producer surplus: $\Pi(p) = \int_0^p S(\tilde{p})d\tilde{p}$. The Walrasian demand function can be determined using Roy's identity as $x(p, \tilde{y}) = -v_p(p, \tilde{y})/v_y(p, \tilde{y})$. We assume that good X is normal $x_y > 0$, and that the demand for good X satisfies the law of demand $x_p < 0$. In Appendix A.2, we show how our model can be supported with a labor market. We also explain why condition (5) clears both product and labor markets if the government expenditures are spent solely on the numeraire good.

Before we study the public authority's maximization problem, let us simplify incentive compatibility constraints (4). Denote an agent's utility from revealing his/her productivity type truthfully as

$$u(n) \equiv U(p, y(n) + \xi(n)\Pi(p), z(n), n) = v(p, y(n) + \xi(n)\Pi(p)) - c(z(n)/n).$$

If the truthful revelation is optimal, then

$$u(n) = \max_m (v(p, y(m) + \xi(n)\Pi(p)) - c(z(m)/n)). \quad (6)$$

We obtain from the envelope theorem the following first-order condition

$$u'(n) = c_\ell z(n)/n^2 + v_y(p)\xi'(n)\Pi(p). \quad (7)$$

Note that the presence of firm profits does not change the incentive compatibility condition when agent indirect utility is linear because firm profit shares can be taken out of the maximization problem (6). Hence, the second-order condition that ensures truth-telling to be optimal is that income schedule $y(n)$ is non-decreasing as in the standard case (see Mirrlees, 1976). At the same time, profit distribution $\xi(n)$ does influence the level of agent utility $u(n)$. This will be an important factor that influences optimal income taxation when the public authority has equity concerns.

The public authority problem is then to find a combination of price p , income schedule

$\tilde{y}(n)$, and labor supply schedule $\ell(n)$ that maximizes¹²

$$\max_{p, \tilde{y}(n), \ell(n)} \int (v(p, \tilde{y}(n)) - c(\ell(n)))\psi(n)f(n)dn \quad \text{subject to (3), (5), and (7).}$$

It is convenient to change variables $\{p, \tilde{y}(n), \ell(n)\}$ to $\{p, u(n), \ell(n)\}$ where utility level is defined by $u(n) = v_y(p)\tilde{y}(n) - c(\ell(n))$. From the latter expression, we can invert disposable income \tilde{y} and express it as $\tilde{y} = r(p, u, \ell) \equiv \frac{u+c(\ell)}{v_y(p)}$. The maximization problem can then be written as

$$\max_{p, u(n), \ell(n)} \int u(n)\psi(n)f(n)dn \quad (8)$$

s.t.

$$\int [n\ell(n) - r(p, u(n), \ell(n)) + \xi(n)\Pi(p)]f(n)dn \geq R \quad (9)$$

$$S(p) - \int x(p, r(p, u(n), \ell(n)))f(n)dn = 0 \quad (10)$$

$$u'(n) - v_y(p)\xi'(n)\Pi(p) - c_\ell\ell(n)/n = 0. \quad (11)$$

Let λ be the multiplier corresponding to budget constraint (9) and ζ be the elasticity of (un-)compensated labor supply.¹³ We also denote cumulative weights as $\Psi(n) = \int_{\underline{n}}^n \psi(m)f(m)dm$, $n \in [\underline{n}, \bar{n}]$, and covariance between welfare weights and profit shares as $\text{Cov}(\xi, \psi) = \int (\xi(n) - 1)(\psi(n) - 1)f(n)dn$. The optimal marginal income tax can then be found using the standard techniques by solving the optimization problem stated above (see Appendix A.1 for details).

Theorem 1. *In competitive markets, the optimal marginal income tax is determined by*

$$\frac{t}{1-t} = A(n)B(n) - \text{Cov}(\xi, \psi)(v_y(p)\Pi(p))' \frac{x_y}{\lambda(S'(p) - H'(p))}, \quad (12)$$

where

$$A(n) = \frac{1 + \zeta}{\zeta} \frac{1 - F(n)}{nf(n)}, \quad B(n) = \frac{v_y}{\lambda} \frac{\Psi(n) - F(n)}{1 - F(n)}.$$

The optimal marginal income tax formula (12) contains, in addition to the standard Mirrleesian term $A(n)B(n)$, a new term, which we refer to as the “price effect” term. Prior to giving an interpretation of this term, note that it does not depend on ability n (since $x_y = \text{const}$ for homothetic preferences) and changes ratio $\frac{t}{1-t}$ for all incomes in the same way. To establish its sign, note that $x_y > 0$, $S'(p) > 0$, and $H'(p) \leq 0$. We have the multiplier $\lambda > 0$ and the

¹²We maximize over p because the price is implicitly determined by equation (5). If we had an explicit price function, we could introduce it into indirect utility and maximize over $(\tilde{y}(n), \ell(n))$ alone.

¹³Note that the compensated elasticity of labor supply coincides with the uncompensated one in the absence of income effects (linear indirect utility).

derivative $(v_y(p)\Pi(p))' = v'_y(p)\Pi(p) + v_y(p)\Pi'(p) \geq 0$ because of

$$\Pi'(p) = S(p) = \int x(p, \tilde{y})f(n)dn = -\frac{v'_y(p)}{v_y(p)} \int \tilde{y}(n)f(n)dn \geq -\frac{v'_y(p)}{v_y(p)}pS(p) \geq -\frac{v'_y(p)}{v_y(p)}\Pi(p).$$

Above, the first equation follows from $\Pi(p) = \int_0^p S(\tilde{p})d\tilde{p}$, the second from the market equilibrium condition, and the third from the Roy's identity; the first inequality follows from an agent's individual budget constraint, and the second one from $pS(p) \geq \int_0^p S(\tilde{p})d\tilde{p}$. Overall, the sign of the price effect depends on the sign of the covariance between welfare weights $\psi(n)$ and profit shares $\xi(n)$. If the planner has equity concerns ($\psi(n)$ is decreasing) and if agents with higher abilities possess a larger share of firm profits ($\xi(n)$ is increasing) we obtain covariance $\text{Cov}(\xi, \psi)$ to be negative. In this case, the new price effect term is positive.

The price level p and the marginal value of public funds λ are still endogenous in the optimum. However, they enter $A(n)B(n)$ only through ratio v_y/λ . This ratio equals one in the Mirrleesian case and greater than one in the case of endogenous prices. Overall, we obtain higher optimal marginal income taxes in the economy with endogenous prices compared to the standard Mirrleesian case.

For intuition, let us consider a change in the tax policy where a small interval of labor incomes $[z(n), z(n) + dz]$ are subject to a small marginal tax increase. Using the terminology of Saez (2001), this policy change results in mechanical and substitution effects, captured by standard term $A(n)B(n)$.¹⁴ In our model, an additional effect emerges. The substitution effect changes total income in the economy, which also leads to a change in price proportional to $x_y/(S'(p) - H'(p))$. The change in price influences total welfare through the change of firm profits as $-(v_y(p)\Pi(p))'\text{Cov}(\xi, \psi)$ (see Appendix A.1). Combining these effects, we obtain the "price effect" term that is expressed in terms of the value of public funds (divided by λ).

The price effect can also be explained from a different perspective. Let us consider how a small decrease in product price influences agents' welfare. This price change benefits low-productivity agents because they can afford more consumption. At the same time, it hurts high-productivity agents whose utility is mostly influenced by the resultant decrease in dividend income. Therefore, the price level can serve as an additional instrument to redistribute welfare from high-productivity to low-productivity agents. The public authority, however, cannot enforce the price level directly. Instead, it uses an additional marginal income tax to decrease agent labor supply, which leads to lower labor income; lower labor income decreases the market demand for good X leading to a decrease in equilibrium price p .¹⁵ Overall, the public authority uses the equilibrium price level on par with income taxation to achieve its

¹⁴For details, see Saez (2001). He also distinguishes the income effect, which is absent in our setting due to homothetic preference assumption.

¹⁵A decrease in labor supply can also potentially influence the supply curve for good X. This channel is absent in our setting because any change in labor supply is fully absorbed by the sector producing the numeraire good (see Appendix A.2 for details).

redistributive objectives.

In general, the emergence of the price effect term is a consequence of the failure of the second welfare theorem in markets with incomplete information and a limited set of tax instruments. Taxation that is solely based on labor income restricts the set of feasible outcomes that the public authority can achieve. However, the market equilibrium condition is not binding ($\gamma = 0$) if firm profits are zero $\Pi(p) = 0$. In this case, for any constrained Pareto optimal profile of agent utilities and labor input $\{u(n), \ell(n)\}$, the public authority can find an income tax schedule that supports this profile with equilibrium price p . The same conclusion holds if firm profits are uniformly distributed $\xi'(n) = 0$, in which case the presence of firm profits can simply be eliminated with a uniform lump sum tax. In these two special cases, the second welfare theorem holds and the equilibrium price level has no influence on the optimal marginal income tax schedule.

As the price effect of optimal income taxation can be viewed as a consequence of the failure of the second welfare theorem, we explore the limits of this result in more detail. In Appendix A.4, we show that if we assume that income tax is based on *total agent income* (labor income plus dividends) instead of labor income alone, the market equilibrium condition remains binding. In this case, it becomes more difficult for high-income agents to deviate because they need to mimic not only smaller labor income of low-productivity agents but also smaller dividend income. This alleviates the incentive compatibility problem and leads to more progressive taxation in the optimum. In Appendix A.5, we also consider *optimal profit taxation* in an extension of our model where firms have idiosyncratic fixed costs and can freely enter and exit the market. We show that the optimal profit tax is below 100% and, as a consequence, firms earn positive profits in the optimum. The presence of firm profits leads again to the failure of the second welfare theorem and to the price effect on optimal income taxation.¹⁶ Overall, Theorem 1 and its extensions (Theorems A1, A2 and A3) establish that the analysis of optimal income taxation should take into account the market structure in economies with incomplete information and progressive distribution of firm profits.

We now consider an extension of our model to allow for foreign ownership of firm profits.¹⁷ This extension is motivated by empirical evidence that a substantial share of equities is held by foreigners in many countries: the share of foreign equity holdings amounts to 13.6% in the U.S. (U.S. Treasury, 2017) and, on average, 38% in European countries (Davydoff et al., 2013). Assuming that $\int \xi(n)f(n)dn = \Xi < 1$, we obtain the following extension of our main result.

¹⁶The second welfare theorem would be restored if the public authority could separately tax away all dividends in our model. This could hardly be empirically relevant. A dynamic variation of our model, where firms require agents' investments at the first stage and agents receive returns from their investments in the form of dividends at the second stage, could establish that 100% dividend taxation cannot be optimal. We leave the analysis of optimal dividend taxation outside the scope of this paper. See Chetty and Saez (2005, 2010) for recent papers on dividend taxation.

¹⁷We assume that foreigners spend their profit income on the numeraire good to keep the same market equilibrium condition. The foreigners also have a zero weight in welfare function.

Theorem 2. *In competitive markets with foreign ownership $1 - \Xi > 0$, the optimal marginal income tax is determined by*

$$\frac{t}{1-t} = A(n)B(n) + \left((1 - \Xi)\Pi'(p) - \text{Cov}(\xi, \psi)(v_y(p)\Pi(p))' \right) \frac{x_y}{\lambda(S'(p) - H'(p))} \quad (13)$$

with $A(n)$ and $B(n)$ as in Theorem 1, and $\text{Cov}(\xi, \psi) = \int (\psi(n) - 1)(\xi(n) - \Xi)f(n)dn$.

With foreign ownership, we observe an effect of pecuniary externalities on welfare that goes beyond equity concerns. In the optimal tax formula in (13), the price effect term contains an additional component, $(1 - \Xi)\Pi'(p) > 0$, that takes into account the production inefficiency related to foreign ownership. Specifically, the change in labor supply stemming from a tax policy change now contributes to firm profits that belong in part to foreigners; hence, competitive equilibrium results in production inefficiency (over-production) and welfare losses. To compensate for these losses, the public authority imposes an additional income tax or, from a different perspective, suppresses labor supply and thereby brings the competitive equilibrium outcome closer to the Pareto frontier.

Lastly, we analyze the optimal tax rate for top income earners. Saez (2001) derives an asymptotic tax rate formula for top income earners. He argues that the upper tail of U.S. income distribution is well described by a Pareto distribution, which also corresponds to an upper Pareto tail of agent productivity. Moreover, Pareto parameter \bar{a} of income distribution corresponds to Pareto parameter $\bar{a}(1 + \zeta)$ of agent productivity distribution for high productivity agents (see p. 218 in Saez (2001)). Following Saez (2001), we denote the asymptotic ratio of social marginal utility to the marginal value of public funds for the government for the top income earners as $\bar{g} = \frac{v_y}{\lambda} \lim_{n \rightarrow \bar{n}} \frac{1 - \Psi(n)}{1 - F(n)}$. Then, term $A(n)$ reduces to $1/(\zeta\bar{a})$ and $B(n)$ approaches $1 - \bar{g}$. Overall, the top tax rate can be written as

$$t^{top} = \frac{1 - \bar{g} + \text{PE} \zeta \bar{a}}{\zeta \bar{a} + 1 - \bar{g} + \text{PE} \zeta \bar{a}}, \quad (14)$$

where $\text{PE} = -\text{Cov}(\xi, \psi)(v_y(p)\Pi(p))' \frac{x_y}{\lambda(S'(p) - H'(p))}$ is the price effect term of optimal income taxation from Theorem 1.

We observe that the optimal tax rate for top earners in a competitive economy with endogenous prices coincides with the standard one when the public authority puts a smaller social weight on top earners, i.e., $\bar{g} \rightarrow \bar{g} - \text{PE} \zeta \bar{a}$. This correction, however, is not solely determined by the characteristics of the upper tail but rather depends on the parameters of the whole population. Furthermore, we also note that unlike in the standard case, the tax rate for top earners remains strictly positive even if the upper tail of productivity distribution becomes thinner, i.e., $\bar{a} \rightarrow \infty$.

4 Numerical Simulations

In this subsection, we provide numerical simulations to estimate the size of the price effect on optimal income taxes. To accomplish this goal, we consider the U.S. housing market that suits well to quantify our results as housing is the largest consumption item that accounts for about 25% of total household expenditure.¹⁸

To incorporate housing demand, we consider a model estimated by Miles and Sefton (2018) with a constant elasticity of substitution (CES) utility function:

$$u(x_o, x) = (ex_o^{1-1/\rho} + (1-e)x^{1-1/\rho})^{\frac{\rho}{\rho-1}}, \quad (15)$$

where x denotes the consumption of housing, x_o denotes the consumption of the other goods, e is the weight on the other goods (relative to housing consumption) in utility, and ρ is the elasticity of substitution between housing and the other goods. Following Miles and Sefton (2018), we take $e = 0.85$ and $\rho = 0.6$.¹⁹ This utility specification results in the absolute value of the price elasticity of demand of 0.7 and the unit income elasticity of demand in our simulations, which are consistent with elasticity values estimated in the literature.²⁰

We model the supply of housing using the standard constant price elasticity function $S(p) = sp^\varepsilon$, where s is a scale parameter and ε is the price elasticity of supply.²¹ The estimates of the price elasticity of supply ε vary significantly across countries and even across cities. In particular, Saiz (2010) shows that ε highly depends on geographical and regulatory constraints within U.S. metropolitan areas. Drawing on his estimates for the average U.S. metropolitan area, we take $\varepsilon = 1.75$.²² We calibrate scale parameter s to match the average share of housing expenditure of 25%, which renders $s = 18.5$. In Appendix A.6, we present the results for inelastic supply $\varepsilon = 0.01$ that better describe housing supply in large U.S. coastal cities (e.g., Boston and San-Francisco) and in countries with a rigid housing planning system (e.g., the UK).²³ We also present results for the price elasticity of supply $\varepsilon = 3$ that are closer to the estimates obtained by Epple and Romer (1991).

Following Saez et al. (2012), we consider cost function $c(\ell) = \ell^4/4$ with a constant labor

¹⁸See Consumer Expenditure Survey, 2017, Table 1203. *Income Before Taxes: Annual Expenditure Means, Shares, Standard Errors, and Coefficients of Variation* and Bureau of Economic Analysis, 2016, Table 2.3.5U. *Personal Consumption Expenditures by Major Type of Product and by Major Function*. Similar numbers are also observed in the European Union, for which Eurostat (2016) reports that on average housing accounts for 24.4% of household expenditure.

¹⁹For other estimates of the CES model for housing, see Määttä and Terviö (2014) and Li et al. (2016).

²⁰For the U.S. housing market, the estimates of price elasticity of demand are in the range 0.3 to 1 in absolute numbers and the estimates of income elasticity of demand are in the range of 0.4 to 1 (see Albouy et al., 2016). For the European housing market, the corresponding estimates are in the range of 0 to 1.43 and 0.74 to 1.23, respectively (see Arrazola et al., 2015).

²¹This functional form corresponds to the Cobb-Douglas production function (see, e.g., Epple et al. (2010)).

²²For similar estimates, see also DiPasquale and Wheaton (1994) and Harter-Dreiman (2004).

²³See Hilbert and Schöni (2016). For a discussion of the U.K. housing crisis in the popular press, see “How to solve Britain’s Housing Crisis,” *The Economist*, 2017, August 5.

Distribution of dividends (in %) held by income percentiles:						
Bottom 90%	Top 10%	Top 5%	Top 1%	Top 0.5%	Top 0.1%	Top 0.01%
0.4%	99.6%	95.9%	76.0%	67.1%	50.0%	31.2%

Table 1: The Distribution of Dividends in the U.S., 2012

Note: The table shows the percentage shares of dividends held by the U.S. population. Source: Saez and Zucman (2016, Table B23).

supply elasticity of $\zeta = 1/3$. We take the distribution of agent productivities from Mankiw et al. (2009), who proxy it with the distribution of hourly wages in the U.S. Specifically, this distribution consists of several parts. There is a 5% mass of agents with $n = 0$ which matches the percentage of total employees on public disability insurance in the U.S. The productivities of agents between the 5th and 95th percentiles follow the lognormal distribution with mean 2.757 and standard deviation 0.5611. The remaining top 5% levels of productivity follow the Pareto distribution with the shape parameter of 2.²⁴ We used a standard kernel smoother to merge the log-normal and Pareto parts of the distribution together. We set public expenditures at $R = 0$ and use welfare weights $\Psi(n) = 1 - (1 - F(n))^\tau$, where τ is a parameter for redistribution. If $\tau = 1$ the public authority has no equity concerns and if $\tau > 1$ public authority has equity concerns. Following Rothschild and Scheuer (2013), we consider a moderate level of equity concerns and set $\tau = 1.3$.

Next, we set the share of profits held by agents in the economy at $\Xi = 0.85$, which approximately equals the 86.4% domestic share of equity holdings in the U.S. (see U.S. Treasury, 2017). This estimate should be considered an upper bound for developed countries. In Europe, on average, only 62% of equity shares are held by domestic investors, whereas the corresponding figure is 50% in the UK, 60% in France, and 70% in Germany (Davydoff et al., 2013).

Lastly, we approximate the distribution of firm profits among agents by the empirical distribution of dividends across U.S. households in 2012, as presented in Table 1.²⁵ To make our estimates more realistic, we slightly depart from our theoretical analysis and assume a flat profit tax at 15% (as in U.S. for most income levels). The proceeds from the profit tax go to financing government expenditures and they are included in the resource constraint.²⁶

²⁴This shape parameter corresponds to $\bar{a}(1 + \zeta)$ derived by Saez (2001) from income distribution, where $\bar{a} = 1.5$ is the shape parameter of the Pareto tail of the U.S. income distribution (see Diamond and Saez, 2011).

²⁵The cumulative distribution of profits is approximated well by functional form $\Xi(n) = \exp(\hat{b}(-\log F(n))^{\frac{1}{2}})$, where $F(n)$ is the cumulative distribution of agent productivities and $\hat{b} = -18.1$; the coefficient of determination equals $R^2 = 0.95$. We obtain function $\xi(n)$ as the derivative of $\Xi(n)$ and by normalizing $\int \xi(n)f(n)dn = 0.85$. Our estimation results of the price effect do not significantly change if we consider instead the distribution of capital income (Saez and Zucman, 2016, Table B21) or the distribution of wealth and financial resources (Wolff, 2017) to approximate the distribution of profit shares.

²⁶Supplementary materials "OIT_simulation_documentation.pdf" present how the first order condition change in the presence of the profit tax.

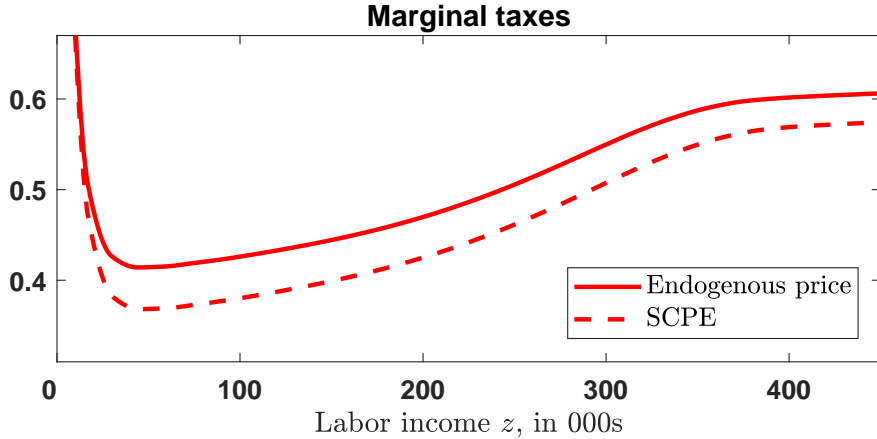


Figure 1. Optimal income taxation with endogenous prices.

Note: The solid line presents the optimal marginal income tax rates for an economy with a competitive market and endogenous prices (see Theorem 2). The dashed line presents the Mirrleesian tax rates in self-confirming policy equilibrium for the same economy.

Figure 1 presents our main simulation results. The solid line shows the optimal marginal income tax schedule as determined by Theorem 2. In this case, the public authority fully internalizes how income taxes influence the equilibrium price level and sets the optimal tax schedule. The dashed line corresponds to the marginal tax schedule in *self-confirming policy equilibrium* (SCPE) when public authority takes the price level in the economy as given. In particular, the public authority assumes that the price level is fixed at some level \hat{p} and imposes the standard Mirrleesian tax schedule, i.e., the marginal taxes are determined by term $A(n)B(n)$ in (13). Given the imposed income taxes, the market reaches an equilibrium with the price level \hat{p} , thus confirming the initial guess of the public authority. SCPE was originally developed and applied by Rothschild and Scheuer (2013, 2016).

We note that the estimated optimal income tax schedules are of a shape similar to those obtained by previous studies. The high tax rates at the bottom correspond to the phasing-out of the guaranteed income level (Mankiw et al. 2009; Saez, 2001).²⁷ Due to the Pareto tail, at top income levels the optimal marginal income taxes flatten out and are equal to approximately 62% for the case with endogenous prices and 59% for the SCPE case with fixed prices. Similarly to Mankiw et al. (2009), the minimum of the optimal marginal income tax schedule is achieved around labor income of \$50,000.

Our numerical simulations show that the endogenous price has an upward effect on optimal marginal income tax rates, adding *4.2 percentage points* on average. The price effect for top earners is around 3%. In Appendix A.6, we also show that the size of the price effect is robust to other assumptions about the distribution of productivity types. We show that the price effect becomes slightly larger 4.4% if we remove the Pareto tail and consider only the lognormal

²⁷The high optimal tax rate at the bottom disappears if one considers a labor participation (see Saez, 2002).

distribution of productivities as in Mankiw et al. (2009). We also present the estimates for two other values of the price elasticity of supply $\varepsilon = 0.01$ and $\varepsilon = 3$.

The overall increase in marginal income tax can be attributed to two factors: the presence of progressive distribution of firm profits $\xi'(n) > 0$, which leads to a tax correction due to equity concerns, and the presence of foreign ownership $\Xi < 1$, which leads to a tax correction based on efficiency concerns. However, if we reestimate our model without foreign ownership $\Xi = 1$, we obtain that the optimal marginal income tax change equals 4 percentage points on average. Hence, the foreign ownership is responsible only for 0.2 percentage points. This is a rather small effect. This can be explained by the fact that the share of foreign ownership in U.S. is less than 15%. With a larger share of foreign ownership, like 50% in the case of the UK, the effect of foreign ownership on income tax rates would become larger. On contrary, the degree of inequality of dividends in U.S. is rather large, and we should expect the price effect due to progressive profit distribution to be smaller for UK and other European countries.

5 Oligopolistic Competition

In this section, we consider markets with various degrees of oligopolistic competition. As the effect of foreign ownership on optimal income taxation is relatively small, we assume that all firm profits remain inside the economy, $\int \xi(n)f(n)dn = 1$. We consider $M \geq 1$ identical firms with each firm i having a convex cost function $K(X_i)$ of producing X_i units of good X. We also denote the inverse aggregate demand function by $p(X)$, where $X = \int x(p, \tilde{y}(n))f(n)dn$.²⁸

We can write firm i 's profit as $X_i p(X) - K(X_i)$, where the market clearing condition ensures $\sum_{i=1}^M X_i = X$. To model various forms of oligopolistic competition, we assume that when firm i maximizes profits it forms a belief, or a *conjectural variation*, about the other firms' responses to a unit change in its output level,

$$\frac{d(\sum_{j \neq i} X_j)}{dX_i} = \theta, \text{ where } -1 \leq \theta \leq M - 1. \quad (16)$$

The conjectural variation was introduced by Bowley (1924) to capture a wide variety of oligopolistic competition models. For instance, the competitive equilibrium corresponds to $\theta = -1$ when firms expect the rest of the industry to absorb exactly its output expansion, the conjectural variation $\theta = 0$ represents the Cournot-Nash model where each firm expects the output of the other firms in its industry to remain unchanged, and the collusive behavior of firms maximizing their joint profits leads to $\theta = M - 1$.²⁹

²⁸The assumption $x_p < 0$ ensures that the inverse aggregate demand function $p(X)$ is well defined.

²⁹For an excellent overview of various conjectural variational parameters, see Perry (1982).

The first-order condition for the profit maximization problem can then be expressed by

$$p(X) - K'(X_i) + (1 + \theta)X_i p'(X) = 0. \quad (17)$$

We assume that the firm maximization problem is well-behaved and has a unique maximum.³⁰ For the rest of our analysis, it is convenient to consider product price being an independent variable. We also limit our attention to symmetric equilibria $X_i = X/M$. After the change of variable, the market equilibrium condition then reads as

$$p - K'(X/M) + \frac{(1 + \theta)}{M} \frac{X}{X_p} = 0, \quad (18)$$

where $X_p = \int x_p(p, \tilde{y}(n))f(n)dn$. We note that the assumption of homothetic preferences implies that market demand X is a linear function of aggregate income. Therefore, the price elasticity of market demand is a function of price only, $\varepsilon_d(p) = X_p p/X$. Then the equilibrium condition can be transformed into a condition for the equilibrium markup $m(p)$

$$p - K'(X/M) = -\frac{(1 + \theta)}{M} \frac{p}{\varepsilon_d(p)} \equiv m(p). \quad (19)$$

To relate the oligopolistic case to our previous analysis, we rewrite the last equation in terms of the quantity produced by all firms in equilibrium

$$\tilde{S}(p) \equiv M(K')^{-1}(p - m(p)). \quad (20)$$

To be able to write formula (20), we assume $p - m(p)$ is an increasing function, which also implies $\tilde{S}'(p) > 0$. This property can be guaranteed in equilibrium, for example, if $\varepsilon'_d(p) \leq 0$.³¹ This assumption is trivially satisfied for CES preferences as $\varepsilon'_d(p) = 0$. Note that the market clearing condition has to ensure that quantity supply by firms equals the market demand $\tilde{S}(p) = X$. Lastly, we note that the total firm profit function equals $\Pi(p) = p\tilde{S}(p) - MK(\tilde{S}(p)/M)$ with its derivative $\Pi'(p) = \tilde{S}(p) + m(p)\tilde{S}'(p)$. With this transformation, the public authority's problem becomes very similar to the one studied for competitive markets.

Theorem 3. *In oligopolistic markets, the optimal marginal income tax is determined by*

$$\frac{t}{1-t} = A(n)B(n) + \left(-\lambda m(p)\tilde{S}'(p) - \text{Cov}(\xi, \psi)(v_y(p)\Pi(p))' \right) \frac{x_y}{\lambda(\tilde{S}'(p) - H'(p))} \quad (21)$$

with $A(n)$ and $B(n)$ as in Theorem 1.

With oligopolistic markets, we observe that the price effect term contains an additional *non-*

³⁰For an insightful discussion of problems associated with the firm objective function being non-concave, see Guesnerie and Laffont (1978).

³¹If equilibrium price satisfying (19) exists, we have $1 - m'(p) = 1 - \frac{p-K'}{p} - \frac{1+\theta}{M} \frac{p\varepsilon'_d(p)}{\varepsilon_d^2} \geq 0$ if $\varepsilon'_d(p) \leq 0$.

competitive component $-\lambda m(p)\tilde{S}'(p) < 0$ that corrects for production inefficiency. But unlike in Theorem 2, where the additional component is positive and corrects for over-production, here the non-competitive component is negative and corrects for the under-production of imperfectly competitive outcome relative to the Pareto optimum. To obtain an outcome closer to the Pareto optimum, the public authority stimulates market demand by lowering income tax rates. This result is closely related to the previous studies in the commodity taxation literature where subsidy is used in order to remedy production inefficiency (see Myles, 1989).

In equation (21), the two components of the price effect term work in opposite directions. The non-competitive component $-\lambda m(p)\tilde{S}'(p)$ advocates for lower marginal income taxes while the redistributive component $-\text{Cov}(\xi, \psi)(v_y(p)\Pi(p))'$ advocates for higher marginal income taxes. We illustrate the interplay of these two forces in the following subsection.

Numerical Simulations: Oligopolistic Competition

We consider the numerical simulations to analyze optimal income tax rates in oligopolistic markets for various market structures. For this purpose, we use the framework introduced in Section 3. To have an analysis comparable with the previous section, we infer production cost function $K(\cdot)$ from the specification of the competitive market. The profit maximization problem in competitive market yields $p - K'(\frac{X}{M_0}) = 0$, where X is equal to the market supply $S(p) = sp^\varepsilon$ in market equilibrium and M_0 is some fixed number of firms. Thus, for the marginal cost function, we obtain $K'(X_i) = (\frac{X_i M_0}{s})^{\frac{1}{\varepsilon}}$ and, hence,

$$\tilde{S}(p) = s(p - m(p))^\varepsilon$$

where $s = 18.5$ and $\varepsilon = 1.75$ as used in our analysis of the competitive market. As the degree of competition is captured by the conjectural variation parameter θ , we set $M_0 = 2$. All other parameters of the model remain the same.

To understand how optimal income tax policy depends on the market structure and to illustrate both the non-competitive and redistributive components, we consider optimal income tax schedules for various degrees of equity concerns: $\tau = 1.05$ (small), $\tau = 1.3$ (medium), $\tau = 2$ (large), as shown in Figure 2. In all three cases, we also vary the conjectural variation parameter $\theta = -1$ (perfect competition), $\theta = -0.5$, and $\theta = 0$ (Cournot-Nash).

In the case of small equity concerns (the left diagram of Figure 2), the redistributive component is barely present, leading to smaller optimal marginal tax rates in less competitive markets (larger θ). In this case, the public authority lowers income tax rates in order to offset the increase in production inefficiency.

For medium and large levels of equity concerns (the medium and right diagrams of Figure 2), the redistributive component plays an important role. Both the non-competitive component

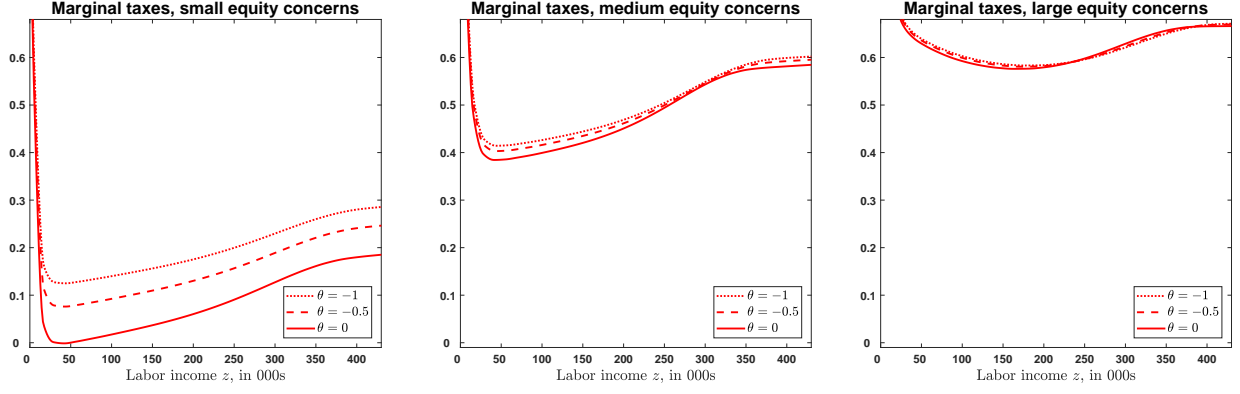


Figure 2. Optimal income taxation for various market structures and several degrees for equity concerns of public authority.

Note: The figure illustrates the optimal marginal income tax schedules for various market structures by $\theta \in \{0, -0.5, -1\}$, where $\theta = 0$ stands for the Cournot-Nash competition model and $\theta = -1$ for perfect competition. The three diagrams depicts optimal marginal income tax schedules for three degrees of equity concerns: $\tau = 1.05$ (small), $\tau = 1.3$ (medium), and $\tau = 2$ (large).

and the redistributive component increase in the size of market power due to larger production inefficiency and larger firm profits. However, these two effects work in opposite direction and almost cancel each other. As a result, when equity concerns are medium or large, the optimal income taxation policy remains stable for various market structures—the result that has not be highlighted in the previous literature.

6 Commodity Taxation

In the previous sections, we demonstrate that the optimal income tax policy has to take into account market structure and the distribution of firm profits. The question then arises as to whether the price effect survives in the presence of other types of taxation. We study the question of optimal commodity taxation in this section. The question of optimal profit taxation is postponed until Appendix A.5 where we consider an extension of our main model allowing for endogenous firm exit and entry.

We use the framework from Section 5 which allows us to consider simultaneously the cases of competitive markets ($m(p) = 0$) and oligopolistic markets ($m(p) = -\frac{1+\theta}{M} \frac{p}{\varepsilon_d(p)} > 0$). Assume the public authority can impose a commodity tax b paid by producers for every sold unit of good X. Negative commodity tax $b < 0$ is interpreted as a subsidy. An individual firm's profits read as $(p-b)X_i - K(X_i)$ and the analogue of the market equilibrium condition in (19) becomes

$$p - b - K'(X/M) = m(p).$$

The equilibrium quantity supplied by all firms can then be defined as

$$\tilde{S}(p, b) = M(K')^{-1}(p - b - m(p)).$$

Aggregate firm profits are given by $\Pi(p, b) = (p - b)\tilde{S}(p, b) - MK(\tilde{S}(p, b)/M)$ and their partial derivatives by $\Pi_p = \tilde{S} + m(p)\tilde{S}_p$ and $\Pi_b = -\tilde{S} + m(p)\tilde{S}_b$. The public authority maximization problem can then be written as

$$\begin{aligned} & \max_{p, b, u(n), \ell(n)} \int \psi(n)u(n)f(n)dn & (22) \\ \text{s.t.} & \begin{cases} \int (n\ell(n) - r(p, u(n), \ell(n))) f(n)dn + \Pi(p, b) + b\tilde{S}(p, b) \geq R, \\ \tilde{S}(p, b) - \int x(p, r(p, u(n), \ell(n)))f(n)dn = 0, \\ u'(n) - v_y(p)\xi'(n)\Pi(p, b) - c_\ell\ell(n)/n = 0. \end{cases} \end{aligned}$$

The next theorem presents our findings about optimal income and commodity taxation.

Theorem 4. (Commodity taxation) *With income and commodity taxation, the optimal marginal income tax is determined by*

$$\frac{t}{1-t} = A(n)B(n) - \frac{\text{Cov}(\psi, \xi) v_{yp}(p)\Pi(p, b) + v_y(p)m'(p)\tilde{S}(p, b)}{\lambda - H'(p)} x_y, \quad (23)$$

where $A(n)$ and $B(n)$ as in Theorem 1. The optimal commodity tax equals

$$b = -m(p) + \frac{\text{Cov}(\xi, \psi)}{\lambda} \left(\frac{v_{yp}(p)\Pi(p, b) + v_y(p)m'(p)\tilde{S}(p, b)}{-H'(p)} - \frac{v_y(p)(-\tilde{S}(p, b) + m(p)\tilde{S}_b(p, b))}{\tilde{S}_b(p, b)} \right). \quad (24)$$

When profits are positive $\Pi(p) > 0$ and unequally distributed $\xi'(n) \neq 0$, commodity taxation and the price effect of income taxation coexist in the optimum. This result is similar in spirit to Naito (1999), who showed that the seminal result of “uniform commodity tax under nonlinear income taxation” by Atkinson and Stiglitz (1976) no longer holds if marginal costs of production are non-constant. We extend the findings of Naito (1999) by showing that in addition to demand and supply properties, the distribution of firm profits plays an important role in determining optimal income and commodity taxes.

For competitive markets ($m(p) = 0, m'(p) = 0$), commodity tax b is positive in the optimum as $\xi'(n) > 0$ and consists of two terms that correspond to consumer and producer tax burden

$$b = -\frac{\text{Cov}(\xi, \psi)}{\lambda} \left(\frac{v_{yp}(p)\Pi(p, b)}{H'(p)} + \frac{v_y(p)(-\tilde{S}(p, b))}{\tilde{S}_b(p, b)} \right) > 0.$$

At the same time, the price effect term of optimal income taxation becomes negative and is the

opposite of the consumer tax burden of the commodity tax. Thus, the role of the price effect changes to compensating agents for the consumer tax burden when the optimal commodity tax is imposed. Consequently, only the producer burden of the commodity tax effectively remains and its role is to tax firm profits and redistribute the proceeds more evenly in the economy. This intuition becomes more apparent in the special case of perfectly elastic aggregate demand. With $H'(p) \rightarrow \infty$, the consumer tax burden of the commodity tax disappears and so does the price effect of optimal income taxation, but the producer tax burden term remains.

For oligopolistic markets ($m(p) > 0$, $m'(p) \neq 0$), the commodity tax also aims to correct for market inefficiency along the same lines as discussed in Section 5. Specifically, the public authority reduces the optimal commodity tax by the amount of firm markup $m(p)$. The presence of market power and non-zero mark-ups also has an effect on consumer and producer tax burden that may affect the size and even sign of the commodity tax and the price effect of optimal income taxation. Finally, the commodity tax does not disappear in the optimum even when firm profits are equally distributed, $\xi'(n) = 0$. In this case, the optimal tax is the subsidy $b = -m(p)$ that exactly brings the economy to the efficient level of production. This result is similar to the one by Myles (1996), who showed that a combination of ad valorem tax and commodity tax can eliminate the welfare loss that arises from oligopolistic competition.

7 Literature Review

The results of the present paper are connected to several strands in the literature. First, our analysis is closely related to the study of optimal income taxation in the presence of endogenous wages in labor markets. Stiglitz (1982) was one of the first to consider a setting in which workers are not perfect substitutes in production. In this case, the general equilibrium effects imply that the optimal tax policy should subsidize high-talent workers and tax low-talent workers. Extending this analysis to a setting where workers have two-dimensional skill characteristics and an occupational choice, Rothschild and Scheuer (2013) found that the ability of workers to select their occupation involves a more progressive tax schedule than a model without occupational choice. Ales et al. (2015) use a related multi-task assignment model with finite one-dimensional agent types to study a change in optimal income policy in response to a technical change. In a study of the continuum of one-dimensional agent types and general constant returns to scale production functions, Sachs et al. (2020) show that the optimal labor supply in an equilibrium is determined by a complicated integral equation. Using this equation, they analyze the incidence of a tax reform with the actual U.S. tax code as a starting point. We show that our model can be equivalently parameterized through endogenous wages (see Appendix A.7). In contrast to endogenous wages literature, however, the price effect in our model arises because of the distribution of firm profits among agents and the presence of

market power, rather than general equilibrium effects studied in this literature.

The literature with endogenous wages in labor markets also studies the impact of externalities on the optimal income tax schedule.³² Rothschild and Scheuer (2016) study the corrective role of income taxation in a setting where agents can engage in rent-seeking activity whereby private returns are different from social returns. Lockwood et al. (2017) integrate tax considerations into an assignment model of agents to professions in which high-paying professions have negative externalities and low-paying professions have positive externalities. They estimate that the welfare gains from an optimal income taxation policy targeted to compensate for these externalities are small. See also Rothschild and Scheuer (2014) for a unifying framework to analyze the effects of externalities on optimal income taxation. Compared to this literature, agents do not impose direct externalities in our model. Instead, agents impose pecuniary externalities. Pecuniary externalities do not influence the set of feasible outcomes when agent types are perfectly observable - the consequence of the second welfare theorem. When agent types are not perfectly observable and tax policy can be based only on labor income (or total income), the second welfare theorem fails. In this case, a Pareto optimum allocation cannot be generally supported with competitive product prices. A similar effect of pecuniary externalities arises in the analysis of the optimal taxation of profits and labor income by Scheuer (2014).

Our analysis is also closely related to the seminal production efficiency theorem of Diamond and Mirrlees (1971), who assert that optimal income taxation entails efficient production in equilibrium.³³ This seminal result holds, however, only in the absence of firm profits or when firm profits are fully taxed.³⁴ The contribution of the present paper is to analyze optimal income taxation in the *Mirrleesian setting in the presence of progressively distributed firm profits*.

Only a few papers consider models in which firm profits are not fully taxed. Stiglitz and Dasgupta (1971) were among the first to show that production efficiency may not be desirable when the maximum profit tax rate is limited.³⁵ Dasgupta and Stiglitz (1972) show that the government might not wish to tax all profits away in the economy with non-identical consumers when lump-sum taxes are not allowed. Guesnerie and Laffont (1978) and Iwamoto and Konishi (1991) analyze the optimal commodity tax rule in a setting with firm profits and many consumers. In contrast to these papers, we consider an *incomplete information environment* of optimal income taxation. We both derive analytically and estimate numerically the influence of firm profit distribution on the optimal income tax schedule. We also establish that the government does not want to tax all profits when firms have idiosyncratic costs and can freely

³²Stantcheva (2014) also studies how adverse selection in labor markets influences optimal income taxation. Stantcheva (2017) provides a joint analysis of optimal income tax and optimal human capital policies.

³³See also Scheuer and Werning (2016) for an insightful connection between the seminal model of Diamond and Mirrlees (1971) on commodity taxation and Mirrlees (1971) on income taxation.

³⁴Dasgupta and Stiglitz (1972) also establish that production efficiency is desirable if the government can set different percentage profit taxes for different producers; see also Mirrlees (1972).

³⁵See also Atkinson and Stiglitz (1976, 2015), Diamond and Mirrlees (1971), Munk (1978, 1980), and Kopczuk and Slemrod (2006).

enter and exit the market.

Further, the absence of firm profits is an important assumption in the commodity taxation literature. Under this assumption, Atkinson and Stiglitz (1976) establish that commodity taxation is unnecessary in the presence of optimal income tax given weak separability of utility between labor and all consumption goods (see also Mirrlees, 1976). Naito (1999) shows that this result depends on the assumption of marginal cost of production being constant. Similarly, we show that once this assumption is abandoned and firm profits are progressively distributed, the commodity taxation plays an important role on par with income taxation for both competitive and oligopolistic markets.

We also want to mention an important paper by Scheuer and Werning (2017) who consider an assignment model to study the taxation of top income earners.³⁶ They establish that the conditions for the efficiency of a marginal tax rate schedule do not depend on the production technology even when firm profits are not fully taxed. However, for a given social objective function, the optimal marginal tax policy does depend on the production technology and the distribution of firm profits. Scheuer and Werning (2017) also do not consider endogenous product prices, which is the main subject of our analysis.

As mentioned in the introduction, there is almost no literature relating optimal income taxation to various market structures. One exception is a recent work by Kaplow (2019) who studies the influence of market power on income taxation. One major difference between his approach and ours is that the former analyzes income taxation in an economy with *exogenously given firm markups*. Hence, Kaplow (2019) does not consider how income taxation policy influences equilibrium prices and firm markups, which is the main subject of our analysis. The other exception is Costa and Maestri (2018) who study optimal income taxation in non-competitive labor markets. They assume that firms' ownership is spread uniformly across agents, thus abstracting from firm profit distribution effect on optimal income taxation, the main subject of our analysis.

Our paper is also related to a small body of literature focused on analyzing commodity taxation in oligopolistic markets. Auerbach and Hines (2001) and Myles (1987, 1989) show the benefit of a corrective subsidy to offset producer markups in the presence of market power (see also Reinhorn, 2005, 2012). Similar to a corrective subsidy, a decrease in marginal income tax stimulates market demand that brings the equilibrium consumption closer to a Pareto-efficiency frontier. Though our results can be seen as an extension of the previous analysis to optimal income taxation settings, we also study the distributional effect of firm profits, a consideration that is absent from previous papers. We also establish that the price effect influences optimal income tax schedule even in the presence of optimal commodity taxation.

³⁶See also Ales and Sleet (2016), Kleven et al. (2013), Piketty et al. (2014), and Shourideh (2014) for analyses of optimal taxation of top labor income and capital income.

Our paper is also closely related to an important strand of literature focused on the effects of relativity concerns on optimal income taxation; see Boskin and Sheshinski (1978), Ireland (2001), Jinkins (2016), Kanbur and Tuomala (2013), Oswald (1983).³⁷ These papers are typically motivated by empirical observations showing that people care not only about their absolute level of consumption but also how it compares with that of others. In our paper, relativity concerns arise endogenously through equilibrium product prices.

8 Conclusion

In this paper, we considered how endogenous prices affect optimal income taxation policy in both competitive and oligopolistic markets. We focused on a Mirrleesian framework with imperfectly observable agent productivities and firm profits distributed among agents inside and outside the economy. We established that progressive distribution of firm profits is associated with the price effect on optimal income taxation. Using numerical simulations, we estimated that the price effect increases optimal marginal income tax by 4 percentage points on average.

The presence of market power in oligopolistic markets is associated with an additional non-competitive component in the price effect term. Using numerical simulations, we showed that, as the market structure varies, changes in the redistributive and non-competitive components almost cancel each other out for medium and large degrees of equity concerns. Thus, the optimal income taxation policy remains stable across various market structures. We also studied the price and non-competitive effects in the presence of commodity and profit taxation.

Our study of the price effect is only the first step in incorporating an underlying market structure into the analysis of optimal income taxation. There is only one income tax schedule, but many goods. Hence, to estimate the overall price effect on optimal income taxation, one would need to develop and carefully calibrate a model with several goods and industries. The consumption of these goods could also vary among income levels. As the next step, one could consider a market with two markets and numeraire, where poor agents consume mostly the first good and rich agents consume the second good. It could be illuminating to understand how the labor income tax schedule should be modified in this economy.

Many sectors, including healthcare, can be better described with dynamic models (Grossman, 1972). Hence, combining the literature on dynamic optimal income taxation (e.g., Albanesi and Sleet, 2006) and the insights of the present paper to analyze the interaction between market structure and income taxation in a dynamic model will be an important exercise.

Finally, we want to highlight that endogenous prices in product markets should be an important consideration beyond income taxation policies. The welfare assessment of subsidies, welfare benefits, pensions, and the minimum wage, etc., would be biased unless the public

³⁷A similar idea according to which individuals may seek a more equal income distribution in order to improve the terms of trade is explored by Zubrickas (2012).

authority takes into account the price effect analyzed in this paper.

A Appendix

A.1 Proofs

Proof of Theorem 1. Let λ, γ , and $\mu(n)$ be multipliers corresponding to constraints (9), (10), and (11), respectively. After integration by parts and taking the transversality condition $\mu(\underline{n}) = \mu(\bar{n}) = 0$ into account, we express the Lagrangian of the maximization problem as

$$\int \left\{ [u(n)\psi(n) + \lambda(n\ell(n) - r(p, u(n), \ell(n)) + \xi(n)\Pi(p) - R) + \gamma(S(p) - x(p, r(p, u(n), \ell(n))))] f(n) - \mu'(n)u(n) - \mu(n)(v_y(p)\xi'(n)\Pi(p) + c_\ell\ell(n)/n) \right\} dn.$$

Note that $r_u = 1/v_y$, $r_\ell = c_\ell/v_y$, and $r_p = -v_p/v_y = x$. Let $H(p)$ denote the aggregate Hicksian demand function and its slope $H'(p) = \int (x_p + x_y x) f(n) dn$. Hence, the first-order conditions can then be written as

$$u(n) : \left[\psi(n) - \frac{\lambda + \gamma x_y}{v_y} \right] f(n) - \mu'(n) = 0, \quad (\text{A.1})$$

$$\ell(n) : \left[\lambda n - \frac{(\lambda + \gamma x_y)c_\ell}{v_y} \right] f(n) - \mu(n)(c_\ell + c_{\ell\ell}\ell(n))/n = 0, \quad (\text{A.2})$$

$$p : \gamma(S'(p) - H'(p)) - (v_y(p)\Pi(p))' \int \mu(n)\xi'(n)dn = 0. \quad (\text{A.3})$$

The assumption of homothetic preferences implies that expression $(\lambda + \gamma x_y)/v_y$ is constant and it is equal to 1, which is obtained by integrating first-order condition (A.1) and using the transversality conditions $\mu(\underline{n}) = \mu(\bar{n}) = 0$. The same first-order condition yields the formula $\mu(n) = \Psi(n) - F(n)$, where $\Psi(n) = \int_{\underline{n}}^n \psi(m)f(m)dm$, $n \in [\underline{n}, \bar{n}]$ are cumulative weights. Taking this expression into account, the integration by parts yields

$$\int \mu(n)\xi'(n)dn = -\text{Cov}(\xi, \psi), \quad (\text{A.4})$$

where covariance $\text{Cov}(\xi, \psi) = \int (\xi(n) - 1)(\psi(n) - 1)f(n)dn$. Taking into account that optimal marginal income tax equals $t(z) = 1 - c_\ell/(nv_y)$ and first-order condition (A.2), we find

$$\frac{t}{1-t} = \left(1 + \frac{\ell c_{\ell\ell}}{c_\ell} \right) \frac{v_y \mu(n)}{\lambda n f} + \frac{\gamma}{\lambda} x_y. \quad (\text{A.5})$$

Let us show that $1 + \ell c_{\ell\ell}/c_\ell = (1 + \zeta)/\zeta$, where ζ is the elasticity of uncompensated labor supply. The individual utility maximization condition implies $v_y(p, \tilde{y}) n(1-t) - c_\ell(\ell) = 0$. Implicitly differentiating the latter expression with respect to net wage, denoted by $w = (1-t)n$, and taking into account that indirect utility v is linear in income, we obtain $\partial\ell/\partial w = v_y/c_{\ell\ell}$. Thus, we find $\zeta = w/\ell (\partial\ell/\partial w) = c_\ell/(\ell c_{\ell\ell})$, where we also use that $v_y w - c_\ell = 0$. The expression for

$1 + \ell c_{\ell\ell}/c_\ell$ then follows.

From first-order condition (A.3) and equation (A.4) we find multiplier

$$\gamma = -\frac{\text{Cov}(\xi, \psi)(v_y(p)\Pi(p))'}{S'(p) - H'(p)}.$$

Substituting the derived expressions together with $\mu(n) = \Psi(n) - F(n)$, $n \in [\underline{n}, \bar{n}]$, into (A.5), we obtain the tax formula in Theorem 1. \square

Proof of Theorem 2. The public authority's optimization problem for competitive markets with foreign ownership is the same as without foreign ownership (8)–(11). The first-order conditions with respect $u(n)$ and $\ell(n)$ remain unchanged. The first order condition with respect to price p is slightly different due to $\int \xi(n)f(n)dn = \Xi < 1$

$$p : \lambda(-S(p) + \Xi\Pi'(p)) + \gamma(S'(p) - H'(p)) - (v_y(p)\Pi(p))' \int \mu(n)\xi'(n)dn = 0.$$

Given that $\Pi'(p) = S(p)$, the statement of the theorem follows from the same steps as in the proof of Theorem 1. \square

Proof of Theorem 3. With oligopolistic markets, the first-order conditions with respect to $u(n)$ and $\ell(n)$ remain the same as with competitive markets. The first-order condition with respect to price is

$$p : \lambda(-\tilde{S}(p) + \Pi'(p)) + \gamma(\tilde{S}'(p) - H'(p)) - (v_y(p)\Pi(p))' \int \mu(n)\xi'(n)dn = 0.$$

Given that $\Pi'(p) = \tilde{S}(p) + m(p)\tilde{S}'(p)$, the statement of the theorem follows from the same steps as in the proof of Theorem 1. \square

Proof of Theorem 4. The first-order conditions with respect to $u(n)$ and $\ell(n)$ remain the same as before. The conditions with respect to price p and commodity tax b are

$$p : \lambda(m(p) + b)\tilde{S}_p + \gamma(\tilde{S}_p - H'(p)) - (v_y(p)\Pi(p, b))'_p \int \mu(n)\xi'(n)dn = 0 \quad (\text{A.6})$$

$$b : \lambda(m(p) + b)\tilde{S}_b + \gamma\tilde{S}_b - v_y(p)\Pi_b(p, b) \int \mu(n)\xi'(n)dn = 0. \quad (\text{A.7})$$

Recall that $\int \mu(n)\xi'(n)dn = -\text{Cov}(\xi, \psi)$ (see (A.4)). Multiplying the first equation by \tilde{S}_b and the second one by \tilde{S}_p and subtracting one from the other, we obtain

$$\gamma(-H'(p))\tilde{S}_b + \text{Cov}(\xi, \psi)\left(v_{yp}\Pi\tilde{S}_b + v_y\Pi_p\tilde{S}_b - v_y\Pi_b\tilde{S}_p\right) = 0$$

Given expressions for $\Pi_p = \tilde{S} + m(p)\tilde{S}_p$, $\Pi_b = -\tilde{S} + m(p)\tilde{S}_b$, and $\tilde{S}_p = -(1 - m'(p))\tilde{S}_b$ we obtain

$$\gamma = -\text{Cov}(\xi, \psi)v_{yp}(p)\Pi(p, b) + v_y(p)m'(p)\tilde{S}(p, b)/(-H'(p))$$

To derive the formula for the optimal income tax, we follow the same steps as in Theorem 1. To obtain the expression for b , we substitute the above expression into (A.7). \square

A.2 Supporting Equilibrium Model

In this section, we show that our model of competitive markets can be supported with a labor market and a consumer's utility maximization problem. In particular, we consider two competitive industries: one producing the numeraire good G and the other producing good X. We label these industries as G and X respectively. We assume that agents can earn wage w for effective labor hours supplied (i.e., $n\ell(n)$) in both industries, the price of the numeraire good is normalized to $p_g = 1$, and the price of good X is equal to p .

Industry G has N_g firms with each having a homogeneous of degree one production technology $F_g(L) \equiv L$. Hence, the total output equals the total amount of labor involved in industry G. As the price of the numeraire good is normalized to 1, the profit maximization condition implies that $w = p_g = 1$, *zero profits*, and *any level of the equilibrium labor demand* L_g^d .

Industry X has N_x firms with each having a production technology with decreasing returns to scale $F_x(L)$. Hence, if L_x is the total amount of labor involved in industry X, the total output of product X equals $N_x F_x(L_x/N_x)$. To ensure that the firm profit maximization problem has a well-defined interior solution, we assume that F_x is differentiable and strictly concave, and that it satisfies the Inada conditions, e.g., $F_x(L_x) = AL_x^a$, where $0 < A$, $0 < a < 1$ are constants. Hence, the firm profit maximization problem

$$\max_{L_x} pF_x(L_x) - wL_x.$$

The solution to this maximization problem leads to the equilibrium labor demand $L_x^d(p)$ in industry X. If the production function is $F_x(L_x) = AL_x^a$, we have $L_x^d(p) = N_x(aAp)^{\frac{1}{1-a}}$ (taking into account $w = 1$). The equilibrium market supply of good X then equals $S_x(p) = N_x F_x(L_x^d(p)/N_x)$ and total firm profits equal $\Pi(p) = \int_0^p S_x(\tilde{p})d\tilde{p}$. Firm profits are distributed among agents according to some distribution function $\xi(n)$ with $\int \xi(n)f(n)dn = 1$.³⁸

On the demand side of the economy, we assume that an agent's preferences can be summarized by utility function $u(c_x, c_g) - c(\ell)$, where (c_x, c_g) is the amount of good X and the numeraire good G consumed by the agent. Utility u is a continuous function representing locally non-satiated preferences. We also assume that the government spends all its resources

³⁸For the economy with foreign ownership, we assume that only share Ξ of profits is distributed among the agents in the economy. The remaining share $1 - \Xi$ belongs to "foreigners" who spend it on the consumption of numeraire good G.

R on numeraire good G.

Consider an agent with productivity n who works ℓ hours. With equilibrium wage $w = p_g = 1$ and tax schedule $T(n\ell)$ taken into account, her income equals $n\ell - T(n\ell)$. Hence, the agent's maximization problem is

$$\begin{aligned} \max_{c_x, c_g, \ell} \quad & u(c_x, c_g) - c(\ell) \\ \text{s.t.} \quad & pc_x + c_g \leq n\ell - T(n\ell) + \xi(n)\Pi(p) \end{aligned} \tag{A.8}$$

The solution to the above problem is labor supply $\ell^*(n, p)$ and consumption bundle $(c_x^*(n, p), c_g^*(n, p))$. Overall, aggregate labor supply and consumer demand equal

$$L^s(p) = \int n\ell^*(n, p)f(n)dn, \quad C_x(p) = \int c_x^*(n, p)f(n)dn, \quad C_g(p) = \int c_g^*(n, p)f(n)dn.$$

The economy must satisfy three market clearing conditions:

$$S_x(p) = C_x(p) \tag{A.9}$$

$$S_g(p) = C_g(p) + R \tag{A.10}$$

$$L^s(p) = L_x^d(p) + L_g^d(p), \tag{A.11}$$

where market clearing condition (A.10) requires market supply for good G to equal the sum of the market demand for G and the extent of government spending.³⁹

Let us show that condition (A.9) is the only one that we should consider in the optimal income taxation problem. As the constant return to scale technology for the numeraire good ensures that any level $L_g^d(p)$ satisfies the firm maximization problem (when $w = p_g = 1$), we are free to choose $L_x^d(p) = L^s(p) - L_g^d(p)$ to clear the labor market. Taking into account that $S_g(p) = L_g^d(p)$ and $\Pi(p) = pS_x(p) - L_x^d(p)$, we can rewrite condition (A.10) equivalently as

$$L_g^d(p) + L_x^d(p) = C_g(p) + pS_x(p) + R - \Pi(p)$$

Given conditions (A.9) and (A.11) this is equivalent to

$$\int n\ell^*(n, p)f(n)dn = C_g(p) + pC_x(p) + R - \Pi(p).$$

This condition follows from the budget constraint of agent's maximization problem (A.8) when the government spending constraint is binding $\int T(n\ell^*(n, p))f(n)dn = R$ (as we assume). Overall, the only independent market clearing condition that we should take into account in the optimization problem is (A.9) – the market clearing condition for good X.

³⁹For the economy with foreign ownership, the market clearing condition for good G is $S_g(p) = C_g(p) + (1 - \Xi)\Pi + R$. In this case, the other derivations should be modified accordingly.

A.3 Competitive Markets: Non-Linear Indirect Utility

In this section, we consider the general case of *non-homothetic preferences*. Let us again denote an agent's utility from revealing his productivity type truthfully as

$$u(n) \equiv U(p, y(n) + \xi(n)\Pi(p), z(n), n) = v(p, y(n) + \xi(n)\Pi(p)) - c(z(n)/n).$$

If revealing the agent's type truthfully is optimal then

$$u(n) = \max_m U(p, y(m) + \xi(n)\Pi(p), z(m), n). \quad (\text{A.12})$$

The envelope theorem implies the following first-order condition

$$u'(n) = U_n + U_y \xi'(n)\Pi(p). \quad (\text{A.13})$$

We note that the single-crossing condition does not generally hold when agent indirect utility is non-linear. Hence, we need to derive the second-order condition when truth-telling is optimal.

Proposition A1. *Condition (A.13) ensures that truth-telling is an optimal solution of (A.12) if and only if schedule $\{y(n), z(n)\}$ satisfies for each n*

$$\left[\frac{c_{\ell\ell}z(n)/n + c_\ell v_y}{n} \frac{v_y}{c_\ell} + v_{yy}\xi'(n)\Pi(p) \right] y'(n) \geq 0. \quad (\text{A.14})$$

Proof. Let us assume that $y(n)$ and $z(n)$ are differentiable. The second-order condition for maximization (A.12) is then

$$u''(n) - U_{nn} - (U_{yy}\xi'(n)\Pi(p) + 2U_{yn})\xi'(n)\Pi(p) - U_y\xi''(n)\Pi(p) \geq 0. \quad (\text{A.15})$$

Taking the derivative of (A.13) with respect to n we obtain

$$\begin{aligned} u''(n) &= U_{nn} + U_{ny}(y'(n) + \xi'(n)\Pi(p)) + U_{nz}z'(n) + \\ &(U_{yn} + U_{yy}(y'(n) + \xi'(n)\Pi(p)) + U_{yz}z'(n))\xi'(n)\Pi(p) + U_y\xi''(n)\Pi(p). \end{aligned}$$

Hence, condition (A.15) is equivalent to

$$U_{ny}y'(n) + U_{nz}z'(n) + (U_{yy}y'(n) + U_{yz}z'(n))\xi'(n)\Pi(p) \geq 0.$$

Given our separable utility specification, this reduces to

$$\frac{c_{\ell\ell}z(n)/n + c_\ell}{n^2} z'(n) + v_{yy}y'(n)\xi'(n)\Pi(p) \geq 0.$$

Maximization condition (A.12) implies $v_y y'(n) - c_\ell z'(n)/n = 0$, which allows rewriting the previous inequality in the form of (A.14). \square

Note that the first term in (A.14) is always positive because the cost function $c(\ell)$ is increasing and convex. Hence, the second-order condition reduces to income schedule $y(n)$ being non-decreasing if either profits are zero $\Pi(p) = 0$ or agent profits are equally distributed $\xi'(n) = 0$. When both $\Pi(p) > 0$ and $\xi'(n) > 0$ the second term in (A.14) is negative because the indirect utility function is concave.

Assuming the second-order condition (A.14) is satisfied, the public authority maximization problem can be written as

$$\begin{aligned} & \max_{p, u(n), \ell(n)} \int u(n) \psi(n) f(n) dn \\ & s.t. \\ & \int [n\ell(n) - r(p, u(n), \ell(n)) + \xi(n)\Pi(p)] f(n) dn \geq R \\ & S(p) - \int x(p, r(p, u(n), \ell(n))) f(n) dn = 0 \\ & u'(n) - v_y(p, r(p, u(n), \ell(n))) \xi'(n) \Pi(p) - c_\ell \ell(n)/n = 0 \end{aligned}$$

Having the same notation for Lagrange multipliers as before, the first-order conditions equal

$$u(n) : \left[\psi(n) - \frac{\lambda + \gamma x_y}{v_y} \right] f(n) - \mu'(n) - \mu(n) \frac{v_{yy}}{v_y} \xi'(n) \Pi(p) = 0 \quad (\text{A.16})$$

$$\ell(n) : \left[\lambda n - \frac{(\lambda + \gamma x_y) c_\ell}{v_y} \right] f(n) - \mu(n) \frac{v_{yy} c_\ell}{v_y} \xi'(n) \Pi(p) - \mu(n) (c_\ell + c_{\ell\ell} \ell(n))/n = 0 \quad (\text{A.17})$$

$$p : \gamma(S'(p) - H'(p)) - \int \mu(n) \xi'(n) ((v_{yp} + v_{yy} x) \Pi(p) + v_y \Pi'(p)) dn = 0. \quad (\text{A.18})$$

To calculate the optimal marginal income tax, we consider the individual utility maximization problem $\max_\ell (v(p, n\ell) - T(n\ell) + \xi(n)\Pi(p)) - c(\ell)$ as before. The first-order condition with respect to ℓ yields $t(z) = 1 - c_\ell/(v_y n)$. Using equation (A.17) we can then write

$$\frac{t}{1-t} = \frac{\mu(n) v_y}{\lambda n f} \frac{1 + \zeta^u}{\zeta^c} + \frac{\gamma x_y}{\lambda} + \frac{\mu(n) v_{yy} \xi'(n) \Pi(p)}{\lambda f}, \quad (\text{A.19})$$

where ζ^u is the elasticity of the uncompensated labor supply, ζ^c is the elasticity of the compensated labor supply, and we exploited that $1 + \ell c_{\ell\ell}/c_\ell = (1 + \zeta^u)/\zeta^c$.

Lagrange multiplier $\mu(n)$ is determined by first-order linear differentiation equation (A.16)

$$\mu'(n) + C(n)\mu(n) = D(n).$$

where $C(n) = \frac{v_y^{(n)}}{v_y} \xi'(n) \Pi(p)$ and $D(n) = \left(\psi(n) - \frac{\lambda + \gamma x_y^{(n)}}{v_y^{(n)}} \right) f(n)$ and superscript (n) means that functions are evaluated at productivity n . Taking into account the transversality condition $\mu(\underline{n}) = \mu(\bar{n}) = 0$ its solution equals

$$\mu(n) = - \int_n^{\bar{n}} E_{mn} D(m) dm = \int_n^{\bar{n}} E_{mn} \left(\frac{\lambda + \gamma x_y^{(m)}}{v_y^{(m)}} - \psi(m) \right) f(m) dm,$$

where $E_{mn} = \exp(\int_n^m C(m') dm') = \exp(\Pi(p) \int_n^m \frac{v_{yy}^{(m')}}{v_y^{(m')}} \xi'(m') dm')$.

Lagrange multiplier γ is then determined by equation (A.18). Denoting $G(n, p) = (v_{yp}^{(n)} + v_{yy}^{(n)} x^{(n)}) \Pi(p) + v_y^{(n)} \Pi'(p)$, we have

$$\gamma(S'(p) - H'(p)) = \int_{\underline{n}}^{\bar{n}} G(n, p) \xi'(n) \int_n^{\bar{n}} E_{mn} \left(\frac{\lambda + \gamma x_y^{(m)}}{v_y^{(m)}} - \psi(m) \right) f(m) dm dn.$$

Therefore,

$$\gamma = \frac{\int_{\underline{n}}^{\bar{n}} G(n, p) \xi'(n) \int_n^{\bar{n}} \lambda E_{mn} / v_y^{(m)} \left(1 - \frac{\psi(m) v_y^{(m)}}{\lambda} \right) f(m) dm dn}{S'(p) - H'(p) - \int_{\underline{n}}^{\bar{n}} G(n, p) \xi'(n) \int_n^{\bar{n}} E_{mn} x_y^{(m)} / v_y^{(m)} f(m) dm dn}.$$

Substituting the above expressions in (A.19), we obtain the following result.

Theorem A1. *In competitive markets with endogenous prices, the optimal marginal income tax is determined by*

$$\frac{t}{1-t} = A(n)B(n) + \frac{\gamma}{\lambda} \left(x_y^{(n)} + A(n)v_y^{(n)} \frac{\int_n^{\bar{n}} x_y^{(m)} E_{mn} / v_y^{(m)} f(m) dm}{1-F(n)} \right),$$

where

$$A(n) = \left(\frac{1 + \zeta_{(n)}^u}{\zeta_{(n)}^c} + n \frac{v_{yy}^{(n)}}{v_y^{(n)}} \xi'(n) \Pi(p) \right) \frac{1 - F(n)}{n f(n)},$$

$$B(n) = \frac{v_y^{(n)}}{1 - F(n)} \int_n^{\bar{n}} \frac{E_{mn}}{v_y^{(m)}} \left(1 - \frac{\psi(m) v_y^{(m)}}{\lambda} \right) f(m) dm,$$

$$\gamma = \frac{\int_{\underline{n}}^{\bar{n}} \xi'(n) G(n, p) \int_n^{\bar{n}} \lambda E_{mn} / v_y^{(m)} \left(1 - \frac{\psi(m) v_y^{(m)}}{\lambda} \right) f(m) dm dn}{S'(p) - H'(p) - \int_{\underline{n}}^{\bar{n}} \xi'(n) G(n, p) \int_n^{\bar{n}} E_{mn} x_y^{(m)} / v_y^{(m)} f(m) dm dn}.$$

Note that the optimal marginal tax formula is more complicated and depends on many endogenous variables in contrast to the case of homothetic preferences. The price effect generally depends on income level when agent indirect utility is non-linear. Still, the formula reduces to the one of Theorem 1 when $v(p, y) = a(p)y$. In this case, x_y and v_y does not depend on productivity, $(\lambda + \gamma x_y) / v_y = 1$, $E_{mn} = 1$, and $G(n, p) = (v_y(p) \Pi(p))'_p$.

A.4 Competitive Markets: Taxing Total Income

In this subsection, we consider the case when public authority can tax total agent income. We denote total income as

$$z(n) = n\ell(n) + \xi(n)\Pi(p).$$

The agent's disposable income then equals $y(n) = z(n) - T(z(n))$. Given that agents have different profit shares, it is now harder for agents to pretend to have different productivity levels compared to the case when taxes are based on labor income only. Now, if an agent of type n wants to pretend to be of type m , he must work more (or less) hours $\ell = (m\ell(m) + (\xi(m) - \xi(n))\Pi(p))/n$. Hence, the agent's utility from reporting type m is equal to

$$U(p, y(m), z(m), n) = v(p, y(m)) - c((z(m) - \xi(n)\Pi(p))/n).$$

We denote the agent's utility from revealing his productivity type truthfully as

$$u(n) \equiv U(p, y(n), z(n), n) = v(p, y(n)) - c((z(n) - \xi(n)\Pi(p))/n).$$

If revealing the agent's type truthfully is optimal, then

$$u(n) - U(p, y(n), z(n), n) = 0 \leq u(m) - U(p, y(n), z(n), m), \quad (\text{A.20})$$

which leads to the following first-order condition

$$u'(n) = U_n = c_\ell \frac{\ell(n) + \xi'(n)\Pi(p)}{n}. \quad (\text{A.21})$$

The latter condition coincides with the standard one when profits are zero $\Pi(p) = 0$ or $\xi'(n) = 0$. We also notice that the standard single-crossing condition is satisfied when agent total income is taxed. Hence, the second-order condition for truth-telling coincides with the one in the standard case, i.e., $z(n)$ must be increasing. In this case,

$$\frac{\partial}{\partial n \partial m} c((z(m) - \xi(n)\Pi(p))/n) < 0.$$

Finally, the first-order condition of maximizing (A.20) implies $v_y y'(n) - c_\ell z'(n)/n = 0$. Hence, agent income $z(n)$ is increasing if and only if disposable agent income $y(n)$ is increasing. Given the assumption that agent disposable income is increasing, the maximization problem of the public authority can be written as follows

$$\begin{aligned} & \max_{p, u(n), \ell(n)} \int u(n) \psi(n) f(n) dn \\ \text{s.t.} & \begin{cases} \int [n\ell(n) - r(p, u(n), \ell(n)) + \xi(n)\Pi(p)] f(n) dn \geq R \\ S(p) - \int x(p, r(p, u(n), \ell(n))) f(n) dn = 0 \\ u'(n) - c_\ell \frac{\ell(n) + \xi'(n)\Pi(p)}{n} = 0, \end{cases} \end{aligned}$$

where disposable income is determined by $y(n) = r(p, u, \ell)$ from $u(n) = U(p, y(n), \ell(n))$. Having the same notation for Lagrange multipliers as before, the first-order conditions equal

$$\begin{aligned} u(n) &: \left[\psi(n) - \frac{\lambda + \gamma x_y}{v_y} \right] f(n) - \mu'(n) = 0 \\ \ell(n) &: \left[\lambda n - \frac{(\lambda + \gamma x_y) c_\ell}{v_y} \right] f(n) - \mu(n) \left(c_\ell + c_{\ell\ell}(\ell(n) + \xi'(n)\Pi(p)) \right) / n = 0 \\ p &: \gamma(S'(p) - H'(p)) - \Pi'(p) \int \mu(n) \frac{\xi'(n) c_\ell}{n} dn = 0. \end{aligned}$$

The first-order condition with respect to $u(n)$ implies $\frac{\lambda + \gamma x_y}{v_y} = 1$ and $\mu(n) = \Psi(n) - F(n)$, $n \in [\underline{n}, \bar{n}]$. From the individual utility maximization problem, optimal marginal income is determined by $t(z) = 1 - c_\ell / (v_y n)$. Since $1 + \ell c_{\ell\ell} / c_\ell = \frac{1 + \zeta}{\zeta}$, we obtain

$$\frac{t}{1-t} = \left(\frac{1 + \zeta + \xi'(n)\Pi(p) / \ell(n)}{\zeta} \right) \frac{\mu(n) v_y}{\lambda n f(n)} + \frac{\gamma}{\lambda} x_y$$

Using the first-order condition with respect to p we obtain the following result, where and superscript (m) means that functions are evaluated at productivity m .

Theorem A2. *In competitive markets with endogenous prices when agent total income is taxed, the optimal marginal income tax is determined by*

$$\frac{t}{1-t} = A(n)B(n) + \Pi'(p) \int_{\underline{n}}^{\bar{n}} (\Psi(m) - F(m)) \frac{\xi'(m) c_\ell(\ell(m))}{m} dm \frac{x_y}{\lambda(S'(p) - H'(p))},$$

where

$$A(n) = \left(\frac{1 + \zeta + \xi'(n)\Pi(p) / \ell(n)}{\zeta} \right) \frac{1 - F(n)}{n f(n)}, \quad B(n) = \frac{v_y}{\lambda} \frac{\Psi(n) - F(n)}{1 - F(n)}.$$

Compared to Theorem 1, term $A(n)$ is now influenced by the distribution of firm profits and the equilibrium labor supply. For $\xi'(n) > 0$ and $\Pi(p) > 0$, the price effect remains positive. Intuitively, when tax is based on total income, it becomes more difficult for high income earners to deviate as they have higher profits. Hence, when total income is taxed, incentive compatibility is a lesser problem, which leads to higher optimum marginal income taxation.

A.5 Firm Profit Taxation

To analyze firm profit taxation, we consider a competitive model with firms having a possibility to enter and exit the market. There is a unit continuum of firms $[0, 1]$ indexed by i . Each firm has the same marginal costs that result in the same individual firm supply curve $S(p)$.⁴⁰ Therefore, if firm i enters the market, it supplies $S(p)$ amount of good X receiving accounting profit equal to $\pi(p) = \int_0^p S(\tilde{p})d\tilde{p}$.

In addition, each firm has firm-specific fixed costs k_i that are distributed according to differentiable distribution function G over $[\underline{k}, \bar{k}]$ with positive probability density function. We think about fixed costs as firm opportunity costs. These costs can be interpreted as earning possibilities in another country or the manager's earning potential in the labor market. These costs can also be frictions and risks associated with starting a new firm that cannot be captured by accounting costs. Overall, fixed costs enter firm economic profit and influence their entry and exit decisions, but they can not be accounted for in profit taxation.⁴¹

If the public authority sets profit tax rate $\tau \in [0, 1]$, firm i enters the market if and only if

$$\pi(p) - \tau\pi(p) - k_i \geq 0.$$

If the public authority imposes 100%-profit tax, no firm enters the market. This profit tax scheme cannot be optimal. Let us denote the fixed costs of the marginal firm that enters the market as $k^*(p, \tau) = (1 - \tau)\pi(p)$. The market supply is then given by $\tilde{S}(p, \tau) = S(p)G(k^*(p, \tau))$ and the total firm profits after taxes by $\Pi(p, \tau) = (1 - \tau)\pi(p)G(k^*(p, \tau)) = k^*(p, \tau)G(k^*(p, \tau))$. Given that the total firm shares satisfy $\int \xi(n)f(n)dn = 1$, the sum of labor income taxes and profit taxes equals

$$\begin{aligned} & \int [n\ell(n) - \tilde{y}(\ell(n)) + \xi(n)\Pi(p, \tau) + \tau\pi(p)G(k^*(p, \tau))]f(n)dn = \\ & = \int [n\ell(n) - \tilde{y}(\ell(n)) + \pi(p)G(k^*(p, \tau))]f(n)dn. \end{aligned}$$

We then express the public authority's maximization problem as follows.

$$\begin{aligned} & \max_{p, \tau, u(n), \ell(n)} \int u(n)\psi(n)f(n)dn \\ & s.t. \begin{cases} \int [n\ell(n) - r(p, u(n), \ell(n)) + \pi(p)G(k^*(p, \tau))]f(n)dn \geq R, \\ \tilde{S}(p, \tau) - \int x(p, r(p, u(n), \ell(n)))f(n)dn = 0, \\ u'(n) - v_y(p)\xi'(n)\Pi(p, \tau) - c_\ell\ell(n)/n = 0. \end{cases} \end{aligned}$$

⁴⁰The supply curve is determined by the short-run firm-profit maximization condition with price equal marginal costs in the optimum.

⁴¹For convenience, we normalize the accounting fixed costs to zero.

This maximization problem reminds the problem of commodity and income taxation studied in Theorem 4 (see equation (22)) as the public authority can control market supply through profit tax (similar to commodity tax) and market demand through labor income tax. Having the same notation for Lagrange multipliers, we obtain the following first-order conditions

$$\begin{aligned}
u(n) &: \left[\psi(n) - \frac{\lambda + \gamma x_y}{v_y} \right] f(n) - \mu'(n) = 0, \\
\ell(n) &: \left[\lambda n - \frac{(\lambda + \gamma x_y)c_\ell}{v_y} \right] f(n) - \mu(n)(c_\ell + c_{\ell\ell}\ell(n))/n = 0, \\
p &: \lambda G'(k^*)k_p^*\pi(p) + \gamma(\tilde{S}_p(p, \tau) - H'(p)) - (v_y(p)\Pi(p, \tau))'_p \int \mu(n)\xi'(n)dn = 0, \\
\tau &: \lambda G'(k^*)k_\tau^*\pi(p) + \gamma\tilde{S}_\tau(p, \tau) - (v_y(p)\Pi(p, \tau))'_\tau \int \mu(n)\xi'(n)dn = 0.
\end{aligned}$$

Using the first-order conditions and the same steps as in the proof of Theorem 1, we obtain

Theorem A3. (Profit taxation in competitive markets) *In competitive markets with the optimal profit taxation, the optimal marginal income tax is determined by*

$$\frac{t}{1-t} = A(n)B(n) - \text{Cov}(\xi, \psi) \frac{v_{yp}(p)\Pi(p, \tau)}{\lambda} \frac{x_y}{G(k^*)S'(p) - H'(p)}$$

with $A(n)$ and $B(n)$ as in Theorem 1.

Similarly to the case with commodity taxation, we note that with optimal profit taxation the price effect of optimal income taxation is negative when $\text{Cov}(\xi, \psi) < 0$ (since $v_{yp} < 0$). The public authority can now control the level of firm profits directly through profit tax. The profit tax revenue then relaxes the budget constraint. Hence, the public authority can now decrease optimal marginal income tax to remedy inefficiencies connected with low entry of firms in equilibrium.

A.6 Numerical Simulations: Further Results

In this appendix, we provide additional numerical simulations of the size of the price effect on optimal income tax rates. Within the same framework of U.S. housing market, we study the robustness of the price effect to different forms of housing supply and to income distribution.

In Section 4 we consider the competitive market with supply function $S = sp^\varepsilon$ and price elasticity $\varepsilon = 1.75$ which corresponds to the price elasticity of the average U.S. metropolitan area (Saiz, 2010). However, as also noted earlier, the price elasticity of housing supply widely differs across various countries and regions and, therefore, we reestimate the size of the price effect for the cases of (i) inelastic supply $\varepsilon = 0.01$ and (ii) elastic supply with $\varepsilon = 3$. The first case better describes housing supply in large U.S. coastal cities (e.g., Boston, San-Francisco) and in countries with a rigid housing planning system, e.g., the UK (see Hilbert and Schoni (2016) and Saiz and Salazar (2017)). In the second case, we draw on the estimates of the price elasticity of U.S. housing supply obtained by Epple and Romer (1991).

	$\varepsilon = 0.01$	$\varepsilon = 1.75$	$\varepsilon = 3$
Δt	16.2%	4.2%	2.7%

Table 2: The average change between the optimal marginal income tax rates and the Mirrleesian tax rates that arises in self-confirming policy equilibrium for various elasticities of housing supply.

Table 2 reports the average change between the optimal marginal income tax rate (as in Theorem 1) and Mirrleesian tax rate that arises in self-confirming policy equilibrium for various elasticities of housing supply.⁴² For the case of inelastic supply $\varepsilon = 0.01$, we immediately note a massive increase in the size of the price effect compared to $\varepsilon = 1.75$, considered in Section 4. Intuitively, with fixed supply any change in aggregate demand is solely translated into price change, which calls for stronger price corrective measures on the part of the public authority. In contrast, in the case of an elastic supply of housing, $\varepsilon = 3$, we see a reduction in the size of the price effect compared to $\varepsilon = 1.75$ as changes in demand lead to smaller changes in price.

Lastly, we also reestimate the price effect for the lognormal distribution of agent productivities with the parameter values of mean $m = 2.757$ and standard deviation $\sigma = 0.5611$. We adjust parameter $s = 16.2$ of supply function $S(p) = sp^\varepsilon$, $\varepsilon = 1.75$, in order to match the average housing expenditure share of 25%. Figure 3 presents our findings. Compared to the lognormal-Pareto case, the removal of Pareto tail leads to a decline of marginal tax rates at high income levels, which is in line with the previous literature (Saez, 2001). At the same time, the average change between the optimal marginal income tax rates and Mirrleesian tax rates in SCPE becomes equal to 4.4%, which is slightly larger than what we obtain in Section 4.

⁴²In our simulations, we calibrate parameter s of supply function $S(p) = sp^\varepsilon$ in order to match the average expenditure share of housing of 25%. In particular, we have $s = 15.5$ for $\varepsilon = 0.01$, $s = 18.5$ for $\varepsilon = 1.75$, and $s = 21.7$ for $\varepsilon = 3$.

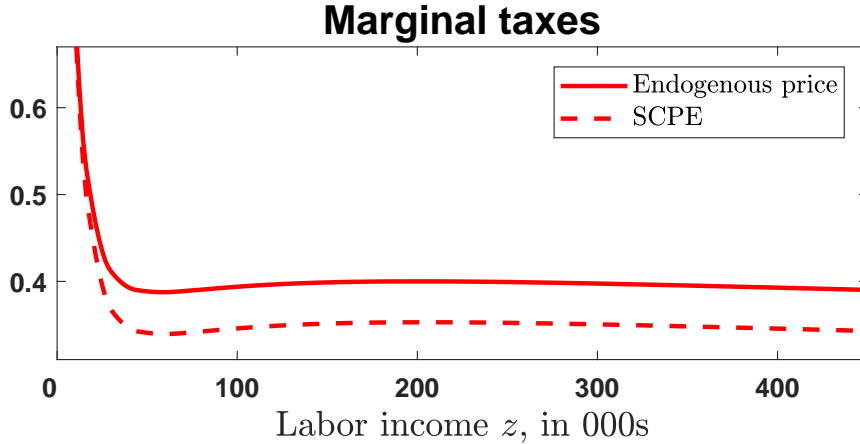


Figure 3. Optimal income taxation with lognormal distribution of productivities.

Note: The solid line presents the optimal marginal income tax rates for an economy with a competitive market and endogenous prices (see Theorem 2). The dashed line presents the Mirrleesian tax rates in self-confirming policy equilibrium for the same economy.

A.7 Relation to Literature on Optimal Income Taxation with Endogenous Wages

In this section, we consider how the framework of our paper can be reformulated using a model with endogenous wages. For this purpose, we assume that agents have homogeneous of degree one utility function. Except for this assumption, we mainly follow our supporting equilibrium model of Appendix A.2.

We consider two competitive industries: one producing the numeraire good G and the other producing good X . We label these industries as G and X respectively. Industry G has N_g firms that have a homogeneous of degree one production technology $F_g(L) \equiv L$. Hence, the total output of product G equals the total amount of labor supplied to industry G , which we denote as L_g . Industry X has N_x firms with each having a production technology with decreasing returns to scale $F_x(L)$. Hence, if L_x is the total amount of labor supplied to industry X , the total output of product X equals $N_x F_x(L_x/N_x)$ that we denote as $M(L_x)$. To ensure that the firm profit maximization problem has a well-defined interior solution, we assume that M is differentiable and strictly concave, and that it satisfies the Inada conditions.

We assume that an agent's preferences can be summarized by utility function $\tilde{u}(c_x, c_g) - c(m(\ell_x, \ell_g))$, where (c_x, c_g) are the amounts of goods consumed and (ℓ_x, ℓ_g) are agent labor supply to both industries. Utility \tilde{u} is a continuous function representing locally non-satiated preferences. We assume that \tilde{u} is *homogeneous of degree one*. We also assume that labor inputs are perfect substitutes with $\ell \equiv m(\ell_x, \ell_g) = \ell_x + \ell_g$. This implies that labor wages in both industries have to be the same in equilibrium. We denote labor wage as w .

Individual maximization problem pins down the ratio of product prices $\frac{\tilde{u}_x(c_x, c_g)}{\tilde{u}_g(c_x, c_g)} = \frac{p_x}{p_g}$. Using the assumption that agent's utility \tilde{u} is homogeneous of degree one, this expression can

be rewritten in terms of aggregate parameters

$$\frac{\tilde{u}_x(C_x/C_g, 1)}{\tilde{u}_g(C_x/C_g, 1)} = \frac{p_x}{p_g}, \quad (\text{A.22})$$

where $C_i = \int c_i(n)f(n)dn$ for $i = x, g$. Firm profit maximization problem also imply

$$w = p_g \quad w = p_x M'(L_x). \quad (\text{A.23})$$

Note that equilibrium conditions (A.22) and (A.23) allow for several equivalent parameterizations. We can normalize $w = p_g = 1$ and treat $p \equiv p_x$ as endogenous variable, as we did in the main text. In this case, since $M'(L_x)$ is strictly decreasing, price p uniquely determines L_x . Hence, price p pins down production $M(L_x)$ and, hence, aggregate consumption C_x . Consumption C_g is then determined by equation (A.22). Labor supply in each industry L_x and L_g and total labor supply $L = L_x + L_b = \int n\ell(n)f(n)dn$ can also be uniquely recovered.

Alternatively, we can normalize $p_x = 1$ and treat w as endogenous variable. In this case, wage w uniquely determines L_x . Hence, production of good X , which is $M(L_x)$, and its consumption C_x are uniquely determined. Consumption level C_g can then be uniquely recovered from (A.22) taking into account that $p_g = w$. Labor supply in each industry L_x and L_g and total labor supply $L = L_x + L_b = \int n\ell(n)f(n)dn$ can also be similarly recovered.

Overall, our main model with endogenous prices can be equivalently reformulated using a model with endogenous wages as in some previous papers in the literature (Rothschild and Scheuer 2013, 2016). Note that one could consider alternative parameterizations in terms of C_x, C_g, L_x, L_g , or L . In our model with symmetric firms, these parameterizations are equivalent. Note that to establish the equivalence we assumed that agent utility is homogeneous of degree one. The equivalence might no longer hold if more than one parameter determines the equilibrium outcome (as in the case of heterogeneous firms).

References

- Ábrahám, Á., Koehne, S., and Pavoni, N. (2016). Optimal income taxation when asset taxation is limited. *Journal of Public Economics*, 136, 14–29.
- Albouy, D., Ehrlich, G. and Liu, Y., 2016. *Housing demand, cost-of-living inequality, and the affordability crisis*. NBER Working Paper No. 22816.
- Albanesi, S., Sleet, C., 2006. Dynamic optimal taxation with private information. *Review of Economics Studies* 73, 1–30.
- Ales, L., Kurnaz, M., Sleet, C., 2015. Technical change, wage inequality, and taxes. *American Economic Review* 105, 3061–3101.
- Ales, L., Sleet, C., 2016. Taxing top CEO incomes. *American Economic Review* 106, 3331–66.
- Alvaredo, F., Atkinson, A.B., Piketty, T., Saez, S., 2013. The Top 1 Percent in International and Historical Perspective. *Journal of Economic Perspectives* 27, 3–20.
- Atkinson, A.B., 2012. The Mirrlees review and the state of public economics. *Journal of Economic Literature* 50, 770–780.
- Atkinson, A.B., Stiglitz, J.E., 1976. The design of tax structure: direct versus indirect taxation. *Journal of Public Economics* 6, 55–75.
- Atkinson, A.B., Stiglitz, J.E., 2015. *Lectures on Public Economics*. Princeton University Press.
- Arrazola, M., de Hevia, J., Romero, D., Sanz-Sanz, J. F., 2015. Long-run Supply and Demand Elasticities in the Spanish Housing Market. *Journal of Real Estate Research* 37, 371–404.
- Auerbach, A.J., Hines Jr., J. R., 2001. *Perfect taxation with imperfect competition*. NBER Working Paper No. 8138.
- Azar, J., Schmalz, M.C., Tecu, I., 2018. Anti-competitive effects of common ownership. *Journal of Finance* 73, 1513–1565.
- Boskin, M.J., Sheshinski, E., 1978. Optimal redistributive taxation when individual welfare depends upon relative income. *Quarterly Journal of Economics* 92, 589–601.
- Bowley, A.L., 1924. *Mathematical groundwork of economics*. California Press, Oxford.
- da Costa, C.E., Maestri, L.J., 2018. Optimal Mirrleesian taxation in non-competitive labor markets. *Economic Theory* 1–42.
- Chetty, R., Saez, E., 2005. Dividend taxes and corporate behavior: Evidence from the 2003 dividend tax cut. *Quarterly Journal of Economics* 120, 791–833.
- Chetty, R., Saez, E., 2010. Dividend and corporate taxation in an agency model of the firm. *American Economic Journal: Economic Policy* 2, 1–31.
- Cosman, J., Quintero, L., 2018. *Fewer players, fewer homes: concentration and the new dynamics of housing supply*. Carey Business School, Johns Hopkins University, Working Paper.
- Davydoff, D., Fano, D., Qin, L., 2013. *Who owns the European Economy? Evolution of the Ownership of EU-listed Companies Between 1970 and 2012*. European Savings Institute and INSEAD, Report for the European Commission and the Financial Services User Group.
- Dasgupta, P., Stiglitz, J., 1972. On optimal taxation and public production. *Review of Economic Studies* 39, 87–103.

- De Loecker, J., Eeckhout, J., Unger, G., 2020. The rise of market power and the macroeconomic implications. *Quarterly Journal of Economics* 135, 561–644.
- Diamond, P.A., Mirrlees, J.A., 1971. Optimal taxation and public production I: production efficiency. *American Economic Review* 61, 8–27.
- Diamond, P., Saez, E., 2011. The Case for a Progressive Tax: From Basic Research to Policy Recommendations. *Journal of Economic Perspectives* 25, 165–190.
- DiPasquale, D., Wheaton, W. C., 1994. Housing market dynamics and the future of housing prices. *Journal of Urban Economics* 35, 1–27.
- Epple, D., Gordon, B., Sieg, H., 2010. A new approach to estimating the production function for housing. *American Economic Review* 100, 905–924.
- Epple, D., Romer, T., 1991. Mobility and redistribution. *Journal of Political Economy* 99, 828–858.
- European Commission, 2015. *You are part of the food chain: Key facts and figures on the food supply chain in the European Union*. Agricultural Market Brief No 4.
- Eurostat, 2016. *Expenditure of households by consumption purpose: A quarter of household expenditure allocated to housing*. News release 236/2016, 29 November 2016.
- Grossman, M., 1972. On the concept of health capital and the demand for health. *Journal of Political Economy* 80, 223–255.
- Grullon, G., Larkin, Y., Michaely, R., 2019. Are U.S. industries becoming more concentrated? *Review of Finance* 23, 697–743.
- Guesnerie, R., Laffont, J.J., 1978. Taxing price makers. *Journal of Economic Theory* 19, 423–455.
- Guvenen, F., Schulhofer-Wohl, S., Song, J., Yogo, M., 2017. Worker betas: Five facts about systematic earnings risk. *American Economic Review* 107, 398–403.
- Harter-Dreiman, M., 2004. Drawing inferences about housing supply elasticity from house price responses to income shocks. *Journal of Urban Economics* 55, 316–337.
- Hilbert, C., Schöni, O., 2016. Housing Policies in the United Kingdom, Switzerland, and the United States: Lessons Learned. *Cityscape* 18, 291–332.
- Ireland, N.J., 2001. Optimal income tax in the presence of status effects. *Journal of Public Economics* 81, 193–212.
- Iwamoto, Y., Konishi, H., 1991. Distributional considerations of producers’ profit in a commodity tax design problem. *Economics Letters* 35, 423–428.
- Jinkins, D., 2016. Conspicuous consumption in the United States and China. *Journal of Economic Behavior and Organization* 127, 115–132.
- Kanbur, R., Tuomala, M., 2013. Relativity, inequality, and optimal nonlinear income taxation. *International Economic Review* 54, 1199–1217.
- Kaplow, L., 2019. *Market power and income taxation*. Harvard University, Working Paper.
- Kleven, H. , Landais, C., Saez, E., 2013. Taxation and international migration of superstars: Evidence from the European football market. *American Economic Review* 103, 1892–1924.
- Kopczuk, W., Slemrod, J., 2006. Putting Firms into Optimal Tax Theory, *American Economic*

- Review Papers and Proceedings* 96, 130–134.
- Li, W., Liu, H., Yang, F., Yao, R., 2016. Housing over time and over the life cycle: a structural estimation. *International Economic Review* 57, 1237–1260.
- Lockwood, B.B., Nathanson, C.G., Weyl, E. G., 2017. Taxation and the allocation of talent. *Journal of Political Economy* 125, 1635–1682.
- Määttänen, N., Terviö, M., 2014. Income distribution and housing prices: An assignment model approach. *Journal of Economic Theory* 151, 381–410.
- Mankiw, N.G., Weinzierl, M.C., Yagan, D., 2009. Optimal Taxation in Theory and Practice. *Journal of Economic Perspectives* 23, 147–174.
- Miles, D., Sefton, J.A., 2018. *Houses across time and across place*. CEPR Working Paper.
- Mirrlees, J.A., 1971. An exploration in the theory of optimum income taxation. *Review of Economic Studies* 38, 175–208.
- Mirrlees, J.A., 1972. On producer taxation. *Review of Economic Studies* 39, 105–111.
- Mirrlees, J.A., 1976. Optimal tax theory: A synthesis. *Journal of Public Economics* 6, 327–358.
- Munk, K.J., 1980. Optimal taxation with some non-taxable commodities. *Review of Economic Studies* 47, 755–765.
- Myles, G.D., 1987. Tax design in the presence of imperfect competition: an example. *Journal of Public Economics* 34, 367–378.
- Myles, G.D., 1989. Ramsey tax rules for economies with imperfect competition. *Journal of Public Economics* 38, 95–115.
- Myles, G.D., 1996. Imperfect competition and the optimal combination of ad valorem and specific taxation. *International Tax and Public Finance* 3, 29–44.
- Naito, H., 1999. Re-examination of uniform commodity taxes under a non-linear income tax system and its implication for production efficiency. *Journal of Public Economics* 71, 165–188.
- Oswald, A.J., 1983. Altruism, jealousy and the theory of optimal non-linear taxation. *Journal of Public Economics* 20, 77–87.
- Perry, M.K., 1982. Oligopoly and consistent conjectural variations. *Bell Journal of Economics* 13, 197–205.
- Piketty, T., Saez, E., Stantcheva, S., 2014. Optimal taxation of top labor incomes: A tale of three elasticities. *American Economic Journal: Economic Policy* 6, 230–271.
- Reinhorn, L.J., 2005. Optimal taxation with Cournot oligopoly. *The BE Journal of Economic Analysis and Policy*, 5(1).
- Reinhorn, L.J., 2012. Optimal taxation with monopolistic competition. *International Tax and Public Finance* 19, 216–236.
- Rothschild, C., Scheuer, F., 2013. Redistributive taxation in the Roy model. *Quarterly Journal of Economics* 128, 623–668.
- Rothschild, C., Scheuer, F., 2014. *A theory of income taxation under multidimensional skill heterogeneity*. NBER Working Paper No 19822.

- Rothschild, C., Scheuer, F., 2016. Optimal taxation with rent-seeking. *Review of Economic Studies* 83, 1225–1262.
- Sachs, D., Tsyvinski, A., Werquin, N., 2020. Nonlinear tax incidence and optimal taxation in general equilibrium. *Econometrica* 88, 469–493.
- Saez, E., 2001. Using elasticities to derive optimal income tax rates. *Review of Economic Studies* 68, 205–229.
- Saez, E., 2002. Optimal income transfer programs: intensive versus extensive labor supply responses. *Quarterly Journal of Economics* 117, 1039–1073.
- Saez, E., Slemrod, J., Giertz, S., 2012. The elasticity of taxable income with respect to marginal tax rates: a critical review. *Journal of Economic Literature* 50, 3–50.
- Saez, E., Zucman, G., 2016. Wealth inequality in the United States since 1913: Evidence from capitalized income tax data. *Quarterly Journal of Economics* 131, 519–578.
- Saiz, A., 2010. The geographic determinants of housing supply. *Quarterly Journal of Economics* 125, 1253–1296.
- Saiz, A., Salazar, A., 2017. *Real Trends: The Future of Real Estate in the United States*. Cambridge, MA: MIT.
- Scheuer, F., 2014. Entrepreneurial taxation with endogenous entry. *American Economic Journal: Economic Policy* 6, 126–63.
- Scheuer, F. and Werning, I., 2016. *Mirrlees meets Diamond-Mirrlees: Simplifying Nonlinear Income Taxation*. NBER Working Paper No 22076.
- Scheuer, F., Werning, I., 2017. The taxation of superstars. *Quarterly Journal of Economics* 132, 211–270.
- Shourideh, A., 2014. *Optimal taxation of wealthy individuals*. University of Pennsylvania, Working Paper.
- Sorensen, P. B., 2009. The theory of optimal taxation: New developments and policy relevance. Longobardi, E. and Petretto, A., *Saggi di economia della tassazione, Milano, FrancoAngeli per la Società italiana di economia pubblica*, pp. 39-74.
- Stantcheva, S., 2014. Optimal income taxation with adverse selection in the labour market. *Review of Economic Studies* 81, 1296–1329.
- Stantcheva, S., 2017. Optimal taxation and human capital policies over the life cycle. *Journal of Political Economy* 125, 1931–1990.
- Stiglitz, J.E., 1982. Self-selection and Pareto efficient taxation. *Journal of Public Economics* 17, 213–240.
- Stiglitz, J.E., Dasgupta, P., 1971. Differential taxation, public goods, and economic efficiency. *Review of Economic Studies* 38, 151–174.
- Syverson, C., 2011. What determines productivity? *Journal of Economic Literature* 49, 326–365.
- U.S. Treasury, 2017. *United States Holdings of Foreign Securities as of June 30, 2016*. Retrieved from <http://ticdata.treasury.gov/Publish/shla2016r.pdf>.
- Wolff, E.N., 2017. *Household Wealth Trends in the United States, 1962 to 2016: Has Middle*

Class Wealth Recovered? NBER Working Paper No. 24085.

Zubrickas, R., 2012. How exposure to markets can favor inequity-averse preferences. *Journal of Economic Behavior and Organization* 84, 174–181.