Harmful Signaling in Matching Markets

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Abstract

Several labor markets, including the job market for new Ph.D. economists, have recently developed formal signaling mechanisms. We show that such mechanisms are harmful for some environments. While signals transmit previously unavailable information, they also facilitate information asymmetry that leads to coordination failures. In particular, we consider a two-sided matching game of incomplete information between firms and workers. Each worker has either the same "typical" known preferences with probability close to one or "atypical" idiosyncratic preferences with the complementary probability close to zero. Firms have known preferences over workers. We show that under some technical condition if at least three firms are responsive to some worker's signal, the introduction of signaling strictly decreases the expected number of matches.

JEL classification: C72, C78, D80, J44.

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1 Introduction

In December 2006, the Ad Hoc Committee of American Economic Association implemented signaling as an actual instrument to facilitate match formation in the job market for new Ph.D. economists. Using this service, Ph.D. candidates have an opportunity to send signals to up to two departments to indicate their interest. The introduction of signals is supported by Roth (2008) and Coles et al. (2013) who suggest that a limited number of signals can credibly transmit information about candidates' preference, which could help to reduce the coordination failures faced by the market participants and facilitate better match formation.[1]

In addition to the job market for new Ph.D. economists, some versions of signaling mechanisms have emerged in other markets. For example, Skydeck360, a student-operated company at Harvard, offers a signaling service for MBA students in their search for internships and full-time jobs. Each registered student can send up to ten signals to employers via their secure website.[2] Early college admission in the U.S. can also be viewed as a form of signaling. Many schools require that applicants send early applications to one school and view an early application as a signal of a student's enthusiasm for a particular school.[3]

To study signaling in matching markets, we consider one-to-one decentralized matching model between firms and workers without transfers similar to the one of Coles et al. (2013).[4] Each agent (firm or worker) knows its own preferences over agents on the other side of the market, but is uncertain about the preferences of other agents. Each worker has either the same "typical" commonly known preferences with a probability close to one or "atypical" preferences taken from some distribution with the complementary probability close to zero.[5] The preferences of workers are ex-ante independently distributed. Firms have some fixed and commonly known preferences over workers that need not to be the same.

We consider a decentralized matching game with three stages. First, agents' preferences are realized and each worker chooses a firm, to which she sends *one private costless signal*. Each signal is a fixed message; that is, the only decision of workers is whether and to whom to send a signal. No decision can be made about the content of the signal. Signals are sent simultaneously, and are observed only by firms who have received them. Second, firms make decisions about job offers by taking into account signals received. Each firm can make only one offer. Finally, each worker chooses one offer among the available ones.

We show that signals induce information asymmetry among firms. This leads to coordina-

[1]For more discussion see "Signaling for Interviews in the Economics Job Market" AEA (2005), a document created by the Ad Hoc Committee to provide advice to participants in the job market for new Ph.D. economists.

[2]See http://skydeck360.posterous.com for details.

[3]See Avery and Levin (2010) for the analysis of match formation in the U.S. college admission market. For additional examples of markets that employ signaling mechanisms see Coles et al. (2013).

[4]Our model differs from Coles et al. (2013) only in the assumption on the distribution of agent preferences.

[5]We assume that typical workers rank firms according to some public ranking. For example, typical candidates in the job market for new Ph.D. economists rank departments of economics in their field according to the *U.S. News and World Report* ranking.

tion failure at the offer stage and decrease in the expected number of matches. In particular, under some technical assumption if at *least three firms* respond to some worker's signal in our environment, i.e. treat signals informatively, the introduction of signaling *strictly decreases* the expected number of matches.

The overall effect on firm and worker welfare due to the introduction of a signaling mechanism is *ambiguous*. On the one hand, signals help to secure "better" matches between some workers and firms, which positively affects the welfare of agents on both sides of the market. On the other hand, the introduction of signals leaves some workers and firms unmatched, which negatively affects the welfare of agents on both sides of the market.

The main contribution of this paper is twofold. First, this paper shows that the signaling mechanism that has been implemented in some real-life markets impedes match formation in some environments. Second, this paper contributes to the cheap talk literature (see Crawford and Sobel, 1982) by exemplifying an environment in which new cheap talk equilibria may be welfare inferior compared to babbling equilibria.[6] To the best of our knowledge, only Farrell and Gibbons (1989) have results that new cheap talk equilibria can be welfare inferior to babbling equilibria. They consider costless communication with limited type space in two-agent bargaining model. Since there is no coordination problem in their two-agent model, their intuition differs from ours.[7]

We want to note here that we do not aim to capture the most realistic environment with the above model. We rather want to show our main point that the introduction signals can be harmful for some matching markets. We discuss the robustness of our results to the introduction of additional periods of interaction between firms and workers, additional signals, as well as public signals in the conclusion.

Our results are complementary to Coles et al. (2013) who show that signaling is beneficial for matching markets in which agents have disperse distribution of preferences. They show that the introduction of signaling increases both the expected number of matches and the expected welfare of workers in their environment. Parendo (2010) also analyzes a complete information version of this model with workers being indifferent among firms and firms have perfectly aligned preferences. On empirical side, Coles et al. (2010) provide suggestive evidence regarding the job market for new Ph.D. economists. They show that, on average, sending a signal of interest increases the chances of receiving an interview from an economics department.[8]

The current paper is also related to the study of preference signaling in centralized matching markets. Abdulkadiroglu et al. (2012) show that the introduction of a signaling mechanism can improve the ex-ante efficiency of the deferred acceptance algorithm (see Gale

[6]Note that signaling that we analyze in this paper is a form of costless communication, or cheap talk. There is no penalty attached for lying, and claims do not directly affect payoffs.

[7]We are thankful for Lones Smith who drew our attention to this comparison.

[8]The economic departments conduct most of their interviews during the annual Allied Social Science Associations meeting in January.

and Shapley, 1962) in presence of indifferences. Lee and Schwarz (2007) analyze preferences signaling in centralized matching markets in a three step matching formation process: preference signaling, investments in information acquisition, and the formation of matches based on available information. They show that agents reveal their preference truthfully in equilibrium under some assumptions on agent utilities.

The paper proceeds as follows. Section 2 outlines our general model and introduces some notation. Equilibrium analysis is presented in Section 3. Sections 4 and 5 provide the comparison of the expected number of matches and agent welfare between the models with and without signals. Section 6 compares these implications with the results in the previous literature and examines two controversial roles of signals in matching markets. Finally, Section 7 discusses some extensions and concludes. All proofs are postponed to the Appendix.

2 Model

We consider a two-sided matching market with workers and firms. The set of workers and the set of firms are denoted as $\mathcal{W}$ and $\mathcal{F}$ respectively with $|\mathcal{W}|=W$ and $|\mathcal{F}|=F$. We assume that $W \geq F$.

Each worker can fill at most one position, and each firm has the capacity to hire at most one worker. Worker w ranks firms according to some strict preference list θ_w, and Θ_w denotes the set of possible worker's preference lists. We use the convention that the firm of rank one is the most preferred firm, while the firm of rank F is the least preferred firm. The set of all workers' preference profiles is denoted as $\Theta_{\mathcal{W}} = (\Theta_w)^W$. Similarly, we define θ_f, Θ_f and $\Theta_{\mathcal{F}}$ for firms.

Each agent a has cardinal utility compatible with preference list θ_a. For simplicity of the exposition, we assume that firms and workers have the same utility function. In addition, utility depends only on the rank of the matching partner. Specifically, the utility of an firm (worker) from being matched with a worker (firm) on the kth position in her/its preference list equals $u(k)$. An agent's cardinal utility from being unmatched is normalized to zero. We also assume that there is no worker whom firms do not want to hire, and there is no worker who prefers being unemployed to being matched with some firm, i.e. for any k, $u(k) > 0$.

Each firm f has some *fixed publicly known* preference list θ_f. Firms' preferences need not to be the same. Each worker can be one of two types: "typical" or "atypical". All workers of typical type have *the same commonly known* preference list θ_0. The preferences of atypical workers are identically and independently distributed according to some distribution $A(\Theta_w)$ with full support, i.e. each possible worker preference profile has a positive probability of realization. Each worker is ex-ante typical with probability $1 - \varepsilon$ and atypical with complementary probability ε, for some $\varepsilon \in (0, 1)$. Our main analysis considers the case of small ε.

For convenience, we name firms according to the typical preference list of workers $\theta_0 = (f_1, ..., f_F)$; i.e. f_1 is the best firm, f_2 is the second best, etc. We also impose a convenient notation for workers: worker w_1 is the best worker among all workers $\mathcal{W}$ according to firm f_1's preferences, and for each $i = 2, ..., F$ worker w_i is the best worker among $\mathcal{W}\setminus\{w_1, ..., w_{i-1}\}$ according to firm f_i's preferences. The rest of the workers, $\mathcal{W}\setminus\{w_1, ..., w_F\}$, are named according to some prespecified order.[9]

We analyze two settings. The first one, *the game without signals*, consists of two stages. First, agents' preferences are realized, and each firm can make up to one offer to some worker. Second, workers choose one offer among available ones.

In the second setting, *the game with signals*, before offers are made, each worker has the opportunity to send *one private costless signal* to a firm, which may use this signal to partially infer worker preferences. Each signal is a fixed message; that is, the only decision of workers is whether and to whom to send a signal. No decision can be made about the content of the signal. Signals are sent simultaneously, and are observed only by firms who have received them.

Note that sequential rationality ensures that workers will always select the best available offer at the last stage of the both games. Hence, we take this workers' behavior at the last stage as given and focus on the reduced games with one and two stages respectively.

We now describe agents' strategies and the equilibrium concept for the game with signals. The corresponding notions for the game without signals can be adapted accordingly. In the game with signals, a pure strategy of worker w (at the first stage) is a map from the set of all possible preference lists to the union of the set of firms and no-firm option, denoted by $\mathcal{N}$, $\sigma_w : \Theta_w \to \mathcal{F} \cup \mathcal{N}$. A pure strategy of firm f (at the second stage) is a map from the set of all possible combinations of received signals, 2^W, which is the set of all subsets of workers, to the union of workers and the no-worker option. That is, $\sigma_f : 2^W \to \mathcal{W} \cup \mathcal{N}$, where with some abuse of notation we denote $\mathcal{N}$ of being no-worker option. The dependence of firm strategy on preferences is omitted, because we assume that each firm has some fixed publicly known preferences. Mixed strategies of workers and firms are defined accordingly. We denote a profile of all workers' strategies as $\sigma_W = (\sigma_{w_1}, ...\sigma_{w_W})$ and the set of worker's strategies as Σ_w. Similarly we define σ_F and Σ_f.

We denote the utility of agent a given strategy profile $\sigma = (\sigma_{\mathcal{W}}, \sigma_{\mathcal{F}})$ and profile of types θ as $\pi_a(\sigma, \theta)$. The interim expected payoff of worker w with preferences θ_w from strategy σ_w when the other agents follow a strategy profile σ_{-w} equals $u_w(\sigma_w|\sigma_{-w}, \theta_w) = \sum_{\theta_{-w}} t(\theta_{-w})\pi_w((\sigma_w, \sigma_{-w}), (\theta_w, \theta_{-w}))$, where $t(\theta_{-w})$ denotes the joint distribution of all agents except worker w preferences.

Note that signals are private and each firm does not observe a worker's action unless it receives a signal from the worker. Hence, the game with signals is the game of in-

[9] This notation implies that if all firms have the same preferences θ^*, workers are named according to this preference list $\theta^* = \{w_1, ..., w_W\}$.

complete information and unobserved actions (see Fudenberg and Tirole, 1991). For each possible set of received signals, $\mathcal{W}_f^S$, firm f forms beliefs about the distribution of both workers' types and actions. Namely, $\mu_f(\theta, \mathcal{W}_{-f}^S|\mathcal{W}_f^S)$ specifies the probability firm f assigns to outcome $(\theta, \{\mathcal{W}_f^S\}_{f\in\mathcal{F}})$ conditional on receiving signals from set $\mathcal{W}_f^S$ of workers. The interim expected payoff of firm f given a subset of received signals $\mathcal{W}_f^S \subset \mathcal{W}$, beliefs $\mu_f(\cdot|\mathcal{W}_f^S)$, and other agents' strategy profile σ_{-f} is given by $u_f(\sigma_f|\sigma_{-f}, \mathcal{W}_f^S, \mu_f) = \sum_\theta \sum_{\mathcal{W}_{-f}^S \in (2^W)^{F-1}} \mu_f(\theta, \mathcal{W}_{-f}^S|\mathcal{W}_f^S)\pi_f(\sigma_f, \sigma_{-f}, \theta)$. To conduct our formal analysis we use the notion of sequential equilibrium.

Definition 1 *A strategy profile $(\hat{\sigma}_\mathcal{W}, \hat{\sigma}_\mathcal{F})$ and firm beliefs $\{\hat{\mu}_f\}_{f\in\mathcal{F}}$ form a sequential equilibrium if*

- *for any $w \in \mathcal{W}$, $\theta_w \in \Theta_\mathcal{W} : \hat{\sigma}_w(\theta_w) \in \arg\max_{\sigma_w \in \Sigma_w} u_w(\sigma_w|\hat{\sigma}_{-w}, \theta_w)$ and*
- *for any $f \in \mathcal{F}$, $\mathcal{W}_f^S \subset \mathcal{W} : \hat{\sigma}_f(\mathcal{W}_f^S) \in \arg\max_{\sigma_f \in \Sigma_f} u_f(\sigma_f|\hat{\sigma}_{-f}, \mathcal{W}_f^S, \hat{\mu}_f)$,*

  where beliefs are defined using Bayes' rule whenever possible.[10]

3 Equilibrium analysis

As a benchmark, we first consider the setting without signals. For sufficiently small ε there is a unique equilibrium in this game. The top firm, firm f_1, makes an offer to its best worker, i.e. worker w_1. The second top firm, firm f_2, anticipates that worker w_1 is likely to accept firm f_1's offer and, hence, optimally makes an offer to worker w_2, its favorite worker among $\mathcal{W}\setminus\{w_1\}$. The same logic extends to the other firms.

Theorem 1 *For every $\varepsilon \in (0, \bar{\varepsilon})$ there exists a unique equilibrium in the game without signals.*[11] *In this equilibrium firm f_j, $j = 1, ..., F$, makes an offer to worker w_j.*

Therefore, the number of matches in the equilibrium of the game without signals is maximal and equals to F (since $W \geq F$). We further refer to this match as "no signaling" match.

Now, we proceed to the analysis of the game with signals. We say that firm f responds to worker w's signal if there exists a subset of workers $\mathcal{W}_f^S \subset \mathcal{W}$, $w \notin \mathcal{W}_f^S$, such that f receives signals from *exactly* sets $\mathcal{W}_f^S$ and $\mathcal{W}_f^S \cup w$ with *positive probability* and $\sigma_f(\mathcal{W}_f^S) \neq \sigma_f(\mathcal{W}_f^S \cup w)$. Intuitively, f responding to w's signal means that w's signal changes the firm's strategy with positive probability. This happens only if the signal transmits useful information about w's preferences, and f is ready to take this information into account and act upon it. We will also use phrase "a firm responds to a worker's signal in equilibrium" meaning that the firm responds to the worker's signal when the worker follows the equilibrium strategies.

[10]As usual in a sequential equilibrium, permissible off-equilibrium beliefs are defined by considering the limits of completely mixed strategies.

[11]$\bar{\varepsilon} = \min(\min_j(\frac{u(j)-u(j+1)}{u(j)}), \frac{u(F)}{u(F)+u(1)})$.

Note that there always exists a babbling equilibrium in which firms do not respond to signals. Since firms do not take into account signals the only possible outcome of any babbling equilibrium is no signaling match. If firms respond to signals in equilibrium they make their offers based on the set of signals they receive. This might change the matching outcome. The following theorem establishes the existence of such an equilibrium.[12] Moreover, it provides a condition when there exists an equilibrium when at least 3 firms respond to some worker signals. As we will see later in section 4, this property of the equilibrium will prove to be very useful.

To state the theorem we denote the set of firms that weakly prefer worker w_i to their no signaling match as $\Delta(w_i) = \{f_j \in \mathcal{F} : w_i \succeq_{f_j} w_j\}$ and the set of workers that are weakly preferred by firm f_j to its no signaling match as $\Delta(f_j) = \{w \in \mathcal{W} : w \succeq_{f_j} w_j\}$.

Theorem 2 *For every $\varepsilon \in (0, \varepsilon^*)$ the strategies*

- $\sigma_{w_i}(\theta_{w_i}) = \max_{\theta_{w_i}}(f \in \Delta(w_i))$ *for* $i = 1, ..., W$,
- $\sigma_{f_j}(\mathcal{W}_f^S) = \begin{cases} \max_{\theta_{f_j}}(w \in \mathcal{W}_f^S) & \text{if } \mathcal{W}_f^S \cap \Delta(f_j) \neq \varnothing \\ \max_{\theta_{f_j}}(w \in \mathcal{W} \backslash \{w_1, ..., w_j\}) & \text{if } \mathcal{W}_f^S \cap \Delta(f_j) = \varnothing \end{cases}$, *for* $j = 1, ..., F-1$,
- $\sigma_{f_j}(\mathcal{W}_f^S) = \begin{cases} \max_{\theta_{f_j}}(w \in \mathcal{W}_f^S) & \text{if } \mathcal{W}_f^S \cap \Delta(f_j) \neq \varnothing \\ w_j & \text{if } \mathcal{W}_f^S \cap \Delta(f_j) = \varnothing \end{cases}$, *for* $j = F$,

and firms' beliefs consistent with the agents' strategies constitute an equilibrium of the game with signals.[13] *In addition, if there are at least 3 firms that weakly prefer some worker w' to their no signaling match, $|\Delta(w')| \geq 3$, then there are at least 3 firms that respond to worker w' signal in this equilibrium.*

The equilibrium strategies outlined in the theorem prescribe each worker to send a signal to the worker's best firm among those that weakly prefer her to its no signaling match (note that in case $W > F$ some workers never send signals). Each firm also makes an offer to its best signaled worker that is weakly better than its no signaling match. For each firm except the worst one, if the firm does not receive any signal from workers that is weakly better than its no signaling match, the firm makes an offer to its best worker among the other workers. The worst firm always makes an offer to its no signaling match if it does not receive signals from better workers. Note that since w' sends her signal to firms in $\Delta(w')$ at least 3 firms respond to w' signal.

We illustrate the above result by way of an example with 3 firms and 3 workers. For simplicity, we assume that all firms rank the workers in the same way (w_1, w_2, w_3), i.e. each

[12] Note that standard refinements (see Cho and Kreps, 1987; Banks and Sobel, 1987) cannot guarantee the uniqueness of equilibrium in our model.

[13] $\varepsilon^* = \min(\min_j \frac{u(j)-u(j+1)}{(F-1)u(j)}, \frac{u(F)}{(F-1)(u(F)+u(1))})$. For a specific construction for off-equilibrium beliefs see the proof of the theorem in Appendix.

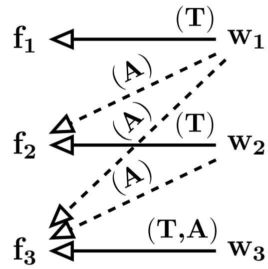

Figure 1: Workers' signaling behavior prescribed by Theorem 2

firm strictly prefers w_1 to w_2 to w_3. Hence, all three firms weakly prefer worker w_1 to their no signaling matches. The equilibrium strategies of Theorem 2 reduce to the following ones. Worker w_1 sends her signal to her best firm. Worker w_2 sends her signal to her best firm among $\{f_2, f_3\}$. Worker w_3 always sends her signal to her no signaling match, i.e. firm f_3. Each firm makes its offer to w_1 only if the firm receives a signal from her. If f_1 does not receive a signal from w_1, it makes its offer to w_2. If f_2 does not receive a signal from w_1 and receives a signal from w_2, it makes an offer to w_2. If f_2 receives a signal neither from w_1 nor from w_2, it makes an offer to w_3. Finally f_3 makes an offer to w_3 unless it receives a signal from a better worker; in that case f_3 makes an offer to the best such worker.[14]

Intuitively, each firm believes that it is the best firm in worker w_1's preference list only if it receives a signal from her. Hence, each firm optimally makes an offer to w_1 only when it receives a signal from her. If firm f_1 does not receive w_1's signal it believes that w_1 has sent a signal to her best firm that will ultimately make her an offer. Hence, firm f_1 can secure a match with w_2 with probability at least equal $1 - \varepsilon$. If firm f_2 does not receive w_1's signal and receives w_2's signal, it can secure the match with w_2 with probability at least equal $1-\varepsilon$. If firm f_2 receives neither w_1's nor w_2's signals, it is unlikely to secure the match with either of them. Hence, firm f_2 makes an offer to w_3 in this case. Similar logic applies to firm f_3. Note that all firms ignore worker w_3's signal. Worker w_3 is the worst worker for the firms and her signal transmit no useful information.

4 Number of matches

In this section we present a comparison between the game with and without signals in terms of the expected number of matches.

As we observed above, there is a maximum match in the offer game without signals for sufficiently small probability of atypical preferences. Hence, the introduction of a signaling cannot increase the expected number of matches. The next theorem provides a condition

[14] Beliefs can be obtained using Bayes' rule whenever possible. See the proof of Theorem 2 for the explicit construction off-equilibrium beliefs.

when the expected number of matches in an equilibrium of the game with signals is strictly smaller than in no signaling match.

Theorem 3 *Consider an equilibrium of the game with signals. If there exists a subset of firms $\mathcal{F}'$ and a subset of workers $\mathcal{W}'$, $|\mathcal{F}'| \geq |\mathcal{W}'|$, such that*

- *all firms in $\mathcal{F}'$ make offers only to workers in $\mathcal{W}'$,*
- *at least 3 firms (one of which is in $\mathcal{F}'$) respond to signals of some worker in $\mathcal{W}'$*

then the expected number of matches in this equilibrium is strictly smaller than in the unique equilibrium of the game without signals.

If we assume that $\max_{\theta_{f_j}}(w \in \mathcal{W}\backslash\{w_1, ..., w_j\}) \in \{w_{j+1}, ..., w_F\}$ for each $f_j \in \mathcal{F}$ the strategies of Theorem 2 prescribe firms making offers only to workers in $\{w_1, ..., w_F\}$. Hence, we can take $\mathcal{F}' = \mathcal{F}$ and $\mathcal{W}'=\{w_1, ..., w_F\}$, and Theorem 2 provides a sufficient condition for the existence of equilibrium where the expected number of matches is strictly smaller than in the unique equilibrium of the game without signals. However, this assumption on firm preferences is not necessary and Theorem 3 applies to more general environments.

To illustrate the theorem, let us consider an example discussed after Theorem 2. Consider $\mathcal{F}'$ be the set of *all* firms and $\mathcal{W}'$ be the set of *all* workers. Consider a realization of workers' preferences when w_1 is atypical and w_2 and w_3 are typical. Since $A(\Theta_w)$ has the full support f_3 is the most favorite firm of w_1 with positive probability. Hence, w_1 sends her signal to f_3, worker w_2 sends her signal to f_2, and w_3 sends her signal to f_3. Firm f_3 responds to w_1's signal and makes an offer to w_1. Firm f_1 anticipates that w_1 is atypical and makes an offer to w_2. Firm f_2 also makes its offer to w_2. Hence, f_2 ends up unmatched because the typical type of w_2 prefers f_1 to f_2. Figure 2 illustrates the above argument. Note that the coordination failure arises because f_2 has no information about w_1's type and cannot anticipate f_1's behavior. Thus, the number of matches for some realization of preferences is

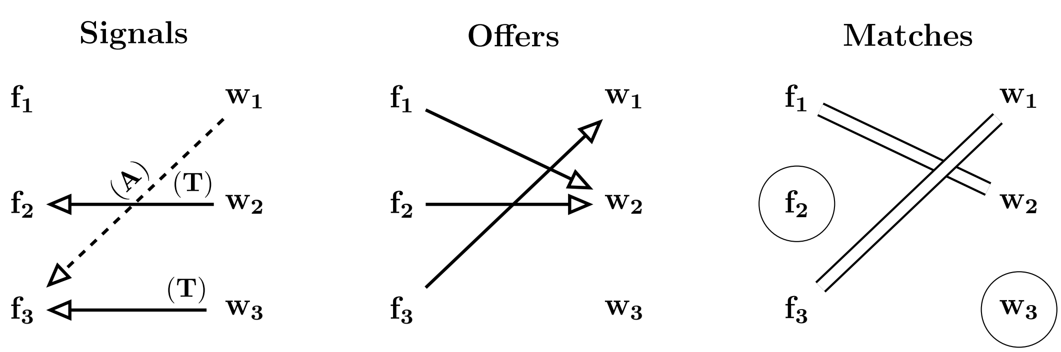

Figure 2: Mismatches

strictly smaller than the maximum match. Therefore, the expected number of matches in this equilibrium is strictly smaller than in no signaling match.

Note that if only two firms respond to some worker signals, the expected number of matches in the equilibrium of the game with signals might be equal to maximum match. In such an equilibrium if some firm f_j secures a better match with some atypical worker w_i, $i < j$, firm f_i always makes its offer to w_j. Therefore, firms exchange their partners that does not decrease the number of matches.

We need the condition that there exists a subset of firms $\mathcal{F}'$ and a subset of workers $\mathcal{W}'$, $|\mathcal{F}'| \geq |\mathcal{W}'|$, such that all firms in $\mathcal{F}'$ make offers only to workers in $\mathcal{W}'$, only if $W > F$ and firms have a very special preference structure. For instance, consider an environment when firms are matched to their first best workers in no signaling match, and their second best workers are unmatched in this match. Then there is an equilibrium of the game with signals when firms make offers to their second best workers conditional on not receiving signals from their first best workers. Responding to signals does not create mismatches because the second best workers remain unmatched unless firms do not receive signals from their first best workers. Workers receive offers with probability less than one, but more than F workers are approached in equilibrium. Hence, the expected number of matches could still equal the maximum one.

5 Welfare

In this section we show that the effect from the introduction of signaling on welfare depends on the relative magnitudes of firms' and workers' cardinal utilities. The intuition is that signals play two roles in equilibria when firms respond to signals. On the one hand, signals help to secure "better" matches between some atypical workers and firms, which positively affects the welfare of agents on both sides of the market. On the other hand, the introduction of signals leaves some workers and firms unmatched, which negatively affects the welfare of agents on both sides of the market.

Example 1 illustrates that the introduction of signals is beneficial for a matching market according to egalitarian welfare criterion if and only if the decrease in the number of matches is offset by better matches of atypical workers. A similar calculation shows that the total welfare of firms changes ambiguously.

Example 1 *Let us consider an example similar to the one discussed after Theorem 2. Firm f_1 ranks workers as (w_1, w_3, w_2) and firms f_2 and f_3 rank workers according to (w_1, w_2, w_3). We assume that preferences of atypical workers are independently uniformly distributed among all possible preference order lists. Workers' cardinal utilities from being matched to first, second, and third choice are $\delta + \lambda, \delta$, and $\delta - \lambda$ $(\delta > \lambda)$ respectively. The expected* *total welfare of workers in no signaling match equals*

$$E[W_{worker}^{nosignals}] = \sum_{i=1}^{3} \left[(1-\varepsilon)\, u(i) + \varepsilon \tfrac{1}{3}\sum_{l=1}^{3} u(l)\right] = 3\delta.$$

The expected total welfare of workers in the equilibrium of the game with signals stated in Theorem 2 equals (terms of the order of ε^2 and ε^3 are omitted)

$$E[W_{worker}^{signals}] = 3\delta + \left(-\tfrac{1}{3}\delta + \tfrac{7}{2}\lambda\right)\varepsilon$$

Hence, the expected total welfare of workers increases only if the difference in utilities between adjacent firms is large enough, $\lambda > \frac{2}{21}\delta$.

6 Role of signals in matching markets

Coles et al. (2013) show that the introduction of signals increases the expected number of matches and the welfare of workers in the model similar to the one of this paper, where, however, agents' preferences are block-correlated. Specifically, they assume that there exists a partition $\mathcal{F}_1, \ldots, \mathcal{F}_B$ of the firms into *blocks* and

1. For any $b < b'$, where $b, b' \in \{1, \ldots, B\}$, each worker prefers every firm in block $\mathcal{F}_b$ to any firm in block $\mathcal{F}_{b'}$;
2. Each worker's preferences within block $\mathcal{F}_b$ are uniform and uncorrelated, for all b;
3. Firm preferences over workers are uniform and uncorrelated.

We have shown above that the results of Coles et al. (2013) rely on the assumption that preferences are block-correlated. If the preferences of workers are almost aligned and the preferences of firms are known, there is a perfect match in the market without signals. Trying to help atypical workers through the introduction of a signaling mechanism we decrease the expected number of matches and ambiguously affect the welfare of agents. Overall, Table 1 summarizes the effect from the introduction of the signals for the two different environments: almost complete (this paper) and block-correlated distribution of preferences.

Preferences	No signals	*Matches*	$E[W_{\text{worker}}]$	$E[W_{\text{firm}}]$
Almost complete	0	$-$	$\pm$	$\pm$
Block-correlated	0	$+$	$+$	$\pm$

Table 1. Almost complete and block-correlated distribution preferences.

A natural question is why signals influence matching markets in different ways. We argue that the signals play two different roles: transmit information and introduce information asymmetry. On the one hand, the introduction of signals helps atypical workers to transmit information about their preferences and locate a better match. On the other hand, signals transmit information only to some firms, thus introducing information asymmetry. This information asymmetry leads to coordination failures that decrease the number of matches.

When there is ex-ante small amount of information about agents' preferences, information transmission plays a more important role in match formation. This happens when agents' preferences are block-correlated, as in Coles et al. (2013). However, when there is almost complete information about agents' preferences—as in the model of this paper—the introduction of signals leads to coordination failures. Table 2 presents this comparison.

Preferences	Transmit information	Introduce information asymmetry
Almost complete	Small	**Large**
Block-correlated	**Large**	Small

Table 2. The roles of signals

Overall, the signals play opposing roles in the match formation process. This could make signals a less powerful tool than it was previously anticipated.

7 Conclusion

There is a general belief that the introduction of a signaling mechanism should facilitate match formation in matching markets (see Roth, 2008; AEA, 2005). This belief is also supported by Coles et al. (2013) who show that the introduction of signaling increases the expected number of matches and welfare of workers in the environment in which agents' preferences distribution is quite disperse. We show in this paper that this belief can be erroneous for some matching markets.

The main contribution of the current paper is twofold. First, we identify an environment in which the signaling mechanism that has been implemented in some real-life markets is actually harmful for matching outcomes. We also identify the exact way how private signals harm matching outcomes - *introduce information asymmetry*. Second, we contribute to the cheap talk literature by providing an environment in which new cheap talk equilibria may be welfare inferior compared to babbling equilibria. To the best of our knowledge, only Farrell and Gibbons (1989) have similar results, though their focus differs from ours.

We finally want to note that the negative signaling effects are robust to several extensions of our model. Several identical signals will allow transmitting information to more firms;

however make this information less precise. As long as the number of signals is smaller than the total number of firms, signaling still introduce information asymmetry. If agents have several periods of interaction they will be able to secure better matches. However, if the number of agents in the market is large compared to the number of interaction periods, as in real-life markets, the introduction of information asymmetry still leads to mismatches. One might also think about making signal public, i.e. observable to all firms. However, Kushnir (2010) shows that the introduction of public signals still lead to inefficiencies in the environment of our paper.

A Appendix

Proof of Theorem 1. Let us consider $\bar{\varepsilon} = \min(\min_j(\frac{u(j)-u(j+1)}{u(j)}), \frac{u(F)}{u(F)+u(1)})$. We show that firm strategies are optimal sequentially. If firm f_1 makes an offer to its best worker, w_1, worker w_1 accepts the offer with probability equal at least $1-\varepsilon$. Hence, firm f_1's expected payoff from making an offer to worker w_1 equals at least $(1-\varepsilon)u(1)$, which is greater than $u(k)$, $k \geq 2$, for any $0 < \varepsilon < \bar{\varepsilon}$. Hence, firm f_1 has a dominant strategy to make an offer to w_1.

Let us assume that each firm f_k, $k < j$, makes its offer to worker w_k. Now we consider the incentives of firm f_j. Given the strategies of firm f_k, $k < j$, the expected payoff from making an offer to some worker among $\{w_1, ...w_{j-1}\}$ equals at most $\varepsilon u(1)$. In addition, the expected payoff from making an offer to some worker $\mathcal{W}\backslash\{w_1, ...w_j\}$ equals at most $u(j+1)$. At the same time, the expected payoff from making an offer to worker w_j equals at least $(1-\varepsilon)u(j)$. Since for any $0 < \varepsilon < \bar{\varepsilon}$ we have $(1-\varepsilon)u(j) > \varepsilon u(1)$ and $(1-\varepsilon)u(j) > u(j+1)$ the optimal strategy of firm f_j is to make an offer to worker w_j. □

Proof of Theorem 2. We first provide a specific construction for off-equilibrium beliefs of firms. There are two possible off-equilibrium events: firm f does not expect to receive a signal from worker w, but receives the signal, and firm f expects to receive a signal from worker w with probability one, but does not receive the signal.[15] For both cases we define f's off-equilibrium beliefs regarding w's preferences to coincide with the firm's prior, i.e. $A(\Theta_w)$. In the latter case we also define f's beliefs regarding w's actions as w sends her signal to some fixed firm in $\mathcal{F}\backslash\{f\}$ with probability one. This construction of off-equilibrium beliefs corresponds to the limit of beliefs obtained through Bayes' rule in perturbed game when w sends her signal to f in the former case and to the fixed firm in $\mathcal{F}\backslash\{f\}$ in the latter case with infinitesimal small probability for each realization of w's preferences. We

[15]To illustrate consider the example discussed in the text immediately after Theorem 2. The former event happens when the best firm f_1 does not expect to receive a signal from the worst worker w_3, but receives the signal. The latter event happens when the worst firm f_3 expects to receive a signal from the worst worker w_3 independently on w_3 preferences, but does not receive the signal.

now show that the strategies, stated in the theorem, constitutes an equilibrium given the constructed off-equilibrium beliefs and on-equilibrium beliefs defined by Bayes' rule for any $\varepsilon < \varepsilon^* = \min(\min_j \frac{u(j)-u(j+1)}{(F-1)u(j)}, \frac{u(F)}{(F-1)(u(F)+u(1))})$.

Let us consider $w \in \{w_1, ..., w_{F-1}\}$. Note that firm f responds only to a signal from workers in $\Delta(f)$. Since $w \in \Delta(f)$ if and only if $f \in \Delta(w)$ sending a signal to some firm in $\mathcal{F}/\Delta(w)$ *does not* attract an offer, and, in addition, it might cause the loss of the offer from a firm in $\Delta(w)$. Hence, w cannot benefit from sending a signal to a firm outside $\Delta(w)$. Let us consider $f' = \max_{\theta_w}(f \in \Delta(w))$ that has rank r in w's preference list. Firm f' does not make an offer to w conditional on receiving her signal only if f' receives a signal from a better worker. This happens with probability less than $\varepsilon(F-1)$. Hence, w's expected payoff from sending a signal to f', conditional on not receiving an offer from firms in $\mathcal{F}/\Delta(w)$, equals at least $(1-\varepsilon(F-1))u(r)$, which is greater than $u(r+1)$ for $\varepsilon < \varepsilon^*$. Since f' is the best firm among $\Delta(w)$, and w will not receive an offer from f' if w does not send a signal to it the optimality of w's strategy follows. We finally note that firms do not respond to worker w_F's signal, and, hence, w_F is indifferent to which firm to send her signal. Hence, w_F's strategy is a best response.

Let us now consider firm $f_j \in \{f_1, ..., f_{F-1}\}$ and denote the set of workers who sent a signal to f_j as $\mathcal{W}_f^S$. In case $\mathcal{W}_f^S \cap \Delta(f_j) \neq \varnothing$ let us denote w_t be f_j's best worker among $\mathcal{W}_f^S$. Note that w_t necessarily belongs to $\mathcal{W}_f^S \cap \Delta(f_j)$. Using Bayes' rule f_j concludes that it is the best firm among $\Delta(w_t)$ according to w_t's preferences. Since w_t sends her signal to f_j only firms in $\{f_1, ..., f_{t-1}\}$ could make a better offer to w_t according to firm equilibrium strategies. Firm $f_k \in \{f_1, ..., f_{t-1}\}$ could make an offer to w_t only if neither of workers in $\Delta(f_k)$ (including w_k) sent a signal to f_k. Therefore, the probability that f_k makes an offer to w_t does not exceed the probability that f_k's no signaling match w_k is atypical, i.e. ε. Since there are $t-1$ firms in $\{f_1, ..., f_{t-1}\}$ the probability that w_t rejects f_j's offer equals at most $(t-1)\varepsilon$. If we denote w_t's rank in f_j's preferences as r firm f_j's expected payoff from the offer to w_t equals at least $(1-(t-1)\varepsilon)u(r)$.

The utility from making an offer to a worker in $\mathcal{W}/\{w_1, ..., w_t\}$ equals at most $u(r+1)$, which is smaller than $(1-(t-1)\varepsilon)u(r)$ for $\varepsilon < \varepsilon^*$. We finally calculate the utility of f_j from making an offer to w_k, $k < t$, who did not send a signal to f_j, and could have a smaller rank. The atypical type of w_k accepts f_j's offer with at most unit probability. The typical type of w_k accepts f_j's offer only if w_k does not receive a better offer. The probability of this event is smaller than the probability w_k does not receive an offer from her no signaling match f_k. Firm f_k does not make an offer to the typical type of w_k only if f_k receives a signal from some worker in $\Delta(f_k)/\{w_k\}$, which happens with probability less than $(k-1)\varepsilon$. Taking into account the probabilities of w_k's types, the probability that w_k accepts f_j's offer is less than $k\varepsilon$. Hence, f_j's expected payoff from making an offer to w_k, $k < t$, is less than $\varepsilon(t-1)u(1)$, which is less than $(1-(t-1)\varepsilon)u(r)$ for $\varepsilon < \varepsilon^*$. Therefore, firm f_j's optimal strategy is to make an offer to worker $\max_{\theta_{f_j}}(w \in \mathcal{W}_f^S)$ if $\mathcal{W}_f^S \cap \Delta(f_j) \neq \varnothing$.

Let us now consider the case when $\mathcal{W}_f^S \cap \Delta(f_j) = \varnothing$ and denote w_t be f_j's best worker among $\mathcal{W}\backslash\{w_1, ..., w_j\}$. We again denote w_t's rank in f_j's preferences as r. The atypical type of w_t rejects f_j's offer with at most unit probability. The typical type of w_t rejects f_j's offer only if she receives an offer from some better firm $f_k \in \{f_1, ..., f_{j-1}\}$. Firm f_k could make an offer to w_t only if neither of workers in $\Delta(f_k)$ sent a signal to f_k. Similarly to the argument above some firm in $\{f_1, ..., f_{j-1}\}$ makes an offer to w_t with probability at most $(j-1)\varepsilon$. Taking into account the probabilities of w_t's types, the utility of f_j from making an offer to w_t equals at least $(1-j\varepsilon)u(r)$. Since $(1-j\varepsilon)u(r) > u(r+1)$ for $\varepsilon < \varepsilon^*$ firm f_j prefers making an offer to w_t rather than to any other worker in $\mathcal{W}\backslash\{w_1, ..., w_j\}$. In addition, firm f_j believes that each worker $w \in \{w_1, ..., w_j\}$ sends her signal to a firm that w weakly prefers to her no signaling match. This firm does not make an offer to worker w only if this firm receives a signal from some better worker, which happens with probability less than $\varepsilon(F-1)$. Hence, the utility of f_j from making an offer to w equals at most $\varepsilon(F-1)u(1)$, which is less than $(1-j\varepsilon)u(r)$ for $\varepsilon < \varepsilon^*$.

We now consider firm f_F. Note that $\Delta(w_F) = \{f_F\}$, and w_F always sends her signal to f_F in equilibrium. Therefore, we have $\mathcal{W}_f^S \cap \Delta(f_j) \neq \varnothing$ on equilibrium path and the above analysis applies. The event $\mathcal{W}_f^S \cap \Delta(f_j) = \varnothing$ happens only on off-equilibrium path, for which the beliefs are constructed above. Since any firm $f_k \in \{f_1, ..., f_{F-1}\}$ does not respond to w_F's signal, and make an offer to her only if neither of workers in $\Delta(f_k)$ sent a signal to f_k the payoff to f_F from making an offer to worker w_F equals at least $(1-(F-1)\varepsilon)u(r)$, where r is rank of worker w_F in f_F's preferences. For $\varepsilon < \varepsilon^*$ this payoff is again greater than the payoff from any other offer.

Finally, we note that the condition that there are at least 3 firms that weakly prefer some worker w' to their no signaing match guarantees that worker w' sends signals to at least 3 firms in the stated equilibrium. Given the strategies of firms this guarantees that at least 3 firms respond to w' signal in this equilibrium. □

Proof of Theorem 3. On the contrary, let us assume that all firms are matched for all outcomes in an equilibrium of the game with signals. Then each worker always receives at most one offer at the offer stage. We show that all firms use pure strategies in such equilibrium. If firm f uses a mixed strategy it makes an offer to w' and w'' with positive probability for a given set of received signals $\mathcal{W}_f^S$. Since all firms are matched for all outcomes no other firm makes an offer to either w' or w'' given that f receives signals from $\mathcal{W}_f^S$. Therefore, w' and w'' accept f's offer with probability one. Since f is not indifferent between workers this contradicts the optimality of f's strategy. Firm f would benefit from making an offer to the best worker between w' and w''. Contradiction.

Let us now consider a subset of firms $\mathcal{F}'$ and a subset of worker $\mathcal{W}'$ such that $|\mathcal{F}'| \geq |\mathcal{W}'|$ and all firms in $\mathcal{F}'$ make offers only to workers in $\mathcal{W}'$. Firms in $\mathcal{F}'$ being matched for all outcomes is possible only if $|\mathcal{F}'| = |\mathcal{W}'|$. Hence, all workers in $\mathcal{W}'$ are also matched for all

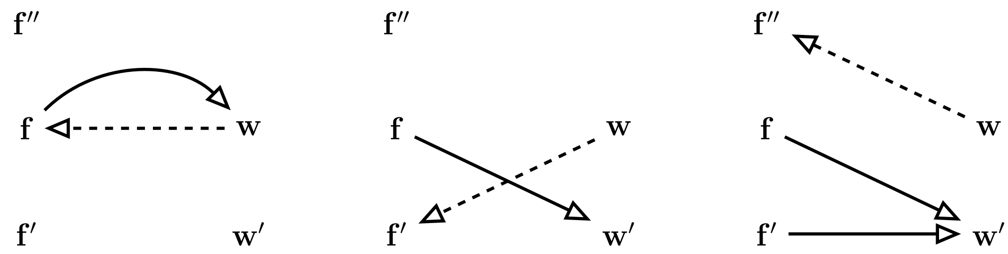

Figure 3: Firm f makes an offer to w upon receiving her signal. Signals and offers for profiles $\{\tilde{\mathcal{W}}_f^S\}_{f\in\mathcal{F}}$ (left), $\{\dot{\mathcal{W}}_f^S\}_{f\in\mathcal{F}}$ (middle), and $\{\ddot{\mathcal{W}}_f^S\}_{f\in\mathcal{F}}$ (right).

outcomes. Consider worker w in $\mathcal{W}'$ such that at least 3 firms (one of which in $\mathcal{F}'$) respond to her signals. Let firm f be in $\mathcal{F}'$ and f *responds* to w's signals. If f responds to w's signals there exists $\mathcal{W}_f^S$, which does not include w, such that f receives signals from exactly $\mathcal{W}_f^S$ and $\mathcal{W}_f^S \cup w$ with positive probability and $\sigma_f(\mathcal{W}_f^S) \neq \sigma_f(\mathcal{W}_f^S \cup w)$. We now consider some profile of received signals $\{\tilde{\mathcal{W}}_f^S\}_{f\in\mathcal{F}}$ such that f receives signals from $\mathcal{W}_f^S \cup w$, i.e $\tilde{\mathcal{W}}_f^S = \mathcal{W}_f^S \cup w$. There are two possibilities either $\sigma_f(\mathcal{W}_f^S \cup w) = w$ or $\sigma_f(\mathcal{W}_f^S \cup w) \neq w$.

For the former case f makes an offer to w for profile $\{\tilde{\mathcal{W}}_f^S\}_{f\in\mathcal{F}}$ (see Figure 3 for an illustration of the argument). Note that workers' preferences are independently distributed, and workers cannot condition their signaling behavior on the signaling behavior of the other workers. Hence, we can fix the signaling behavior of all workers except w, and consider the change of only w's signaling behavior. Since firm f receives signals from exactly $\mathcal{W}_f^S$ with positive probability there exists a profile of received signals $\{\dot{\mathcal{W}}_f^S\}_{f\in\mathcal{F}}$ that differs from the previous one only in w's signaling outcome and w does not send her signal to f, $\dot{\mathcal{W}}_f^S = \mathcal{W}_f^S$. Since f changes its strategy $\sigma_f(\mathcal{W}_f^S) \neq \sigma_f(\mathcal{W}_f^S \cup w)$ firm f makes its offer to some other worker w' in $\mathcal{W}'$ for profile $\{\dot{\mathcal{W}}_f^S\}_{f\in\mathcal{F}}$. Since workers in $\mathcal{W}'$ are matched for all outcomes worker w' received an offer from some other firm f' for profile $\{\tilde{\mathcal{W}}_f^S\}_{f\in\mathcal{F}}$.

If w does not send a signal at all for profile $\{\dot{\mathcal{W}}_f^S\}_{f\in\mathcal{F}}$ both f and f' (since f' receives the same set of signals and, hence, makes the same offer) make their offers to w'. This contradicts to the assumption that all firms are matched for all outcomes. Therefore, w sends her signal to f', which changes its behavior (f' responds to w's signal). Both firms f and f' are matched for profile $\{\dot{\mathcal{W}}_f^S\}_{f\in\mathcal{F}}$.

Since there are at least 3 firms that respond to w's signals there exists the third firm $f'' \notin \{f, f'\}$ that receives w's signal with positive probability. Therefore, there exists the third profile of received signals for all firms $\{\ddot{\mathcal{W}}_f^S\}_{f\in\mathcal{F}}$ that differs from the previous two only in w's signaling outcome such that w sends her signal to f''. For profile $\{\ddot{\mathcal{W}}_f^S\}_{f\in\mathcal{F}}$, firm f receives signals from $\mathcal{W}_f^S$ and firm f' receives signals from $\tilde{\mathcal{W}}_{f'}^S$, and, hence, both firms make an offer to w'. This contradicts to the assumption that all firms are matched for all outcomes.

The case when f does not make an offer to w can be treated similarly (see Figure 4).

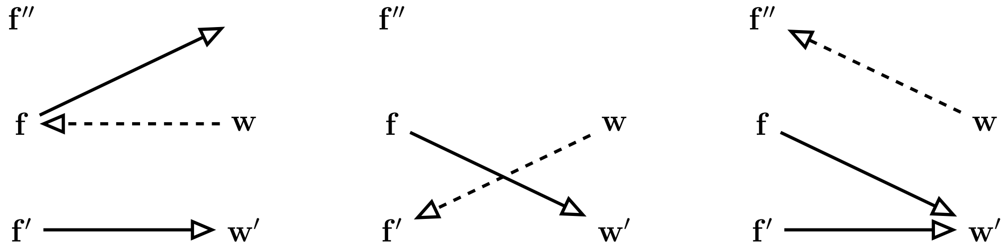

Figure 4: Firm f does not make an offer to w upon receiving her signal. Signals and offers for profiles $\{\tilde{\mathcal{W}}_f^S\}_{f\in\mathcal{F}}$ (left), $\{\dot{\mathcal{W}}_f^S\}_{f\in\mathcal{F}}$ (middle), and $\{\ddot{\mathcal{W}}_f^S\}_{f\in\mathcal{F}}$ (right).

Since all firms are matched for all outcomes f makes an offer to some worker for profile $\{\tilde{\mathcal{W}}_f^S\}_{f\in\mathcal{F}}$. We again consider a profile $\{\dot{\mathcal{W}}_f^S\}_{f\in\mathcal{F}}$ that differs from $\{\tilde{\mathcal{W}}_f^S\}_{f\in\mathcal{F}}$ only in w's signaling outcome. Such profile exists since f receives signals from exactly $\mathcal{W}_f^S$ with positive probability. Since f changes its strategy $\sigma_f(\mathcal{W}_f^S) \neq \sigma_f(\mathcal{W}_f^S \cup w)$ and it belongs to $\mathcal{F}'$ firm f makes an offer to some other worker w' in $\mathcal{W}'$ (that can be w) for profile $\{\dot{\mathcal{W}}_f^S\}_{f\in\mathcal{F}}$. Since all workers in $\mathcal{W}'$ are matched for all outcomes w' received an offer from some other firm f' for profile $\{\tilde{\mathcal{W}}_f^S\}_{f\in\mathcal{F}}$. If w does not send a signal at all for profile $\{\dot{\mathcal{W}}_f^S\}_{f\in\mathcal{F}}$ both f and f' (since f' receives the same set of signals and, hence, makes the same offer) make their offers to w'. This contradicts to the assumption that all firms are matched for all outcomes. Therefore, w sends her signal to f' that changes its behavior (f' responds to w's signal). Both firms f and f' are matched for profile $\dot{\mathcal{W}}$.

Since there are at least 3 firms that respond to w's signal there exists the third firm $f'' \notin \{f, f'\}$ that receives w's signal with positive probability. Therefore, there exists a profile of received signals for all firms $\{\ddot{\mathcal{W}}_f^S\}_{f\in\mathcal{F}}$ that differs from previous two only in w's signaling outcome such that w sends her signal to f''. For profile $\{\ddot{\mathcal{W}}_f^S\}_{f\in\mathcal{F}}$, firm f receives signals from $\mathcal{W}_f^S$ and firm f' receives signals from $\tilde{\mathcal{W}}_{f'}^S$, and, hence, both firms make an offer to w'. This contradicts the assumption that all firms are matched for all outcomes. □

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References

Abdulkadiroglu, A., Che, Y.-K., and Yasuda, Y. (2012). Expanding 'choice' in school choice. Working paper.

AEA (2005). Signaling for interviews in the economics job market. Technical report, American Economic Association. http://www.aeaweb.org/joe/signal/signaling.pdf.

Avery, C. and Levin, J. (2010). Early admissions at selective colleges. *American Economic Review*, 100(5):2125–2156.

Banks, J. and Sobel, J. (1987). Equilibrium selection in signaling games. *Econometrica*, 55(3):647–661.

Cho, I. and Kreps, D. (1987). Signaling games and stable equilibria. *The Quarterly Journal of Economics*, 102(2):179–221.

Coles, P., Cawley, J., Levine, P. B., Niederle, M., Roth, A. E., and Siegfried, J. J. (2010). The job market for new economists: A market design perspective. *Journal of Economic Perspectives*, 24(4):187–206.

Coles, P., Kushnir, A., and Muriel, N. (2013). Preference signaling in matching markets. *American Economic Journal: Microeconomics*, forthcoming.

Crawford, V. and Sobel, J. (1982). Strategic information transmission. *Econometrica*, 50(6):1431–1451.

Farrell, J. and Gibbons, R. (1989). Cheap talk can matter in bargaining. *Journal of Economic Theory*, 48(1):221–237.

Fudenberg, D. and Tirole, J. (1991). *Game Theory*. MIT Press.

Gale, D. and Shapley, L. (1962). College admissions and the stability of marriage. *American Mathematical Monthly*, 69(1):9–15.

Kushnir, A. (2010). *Preference Signaling in Matching Markets*. PhD thesis, The Pennsylvania State University.

Lee, R. and Schwarz, M. (2007). Signaling preferences in interviewing markets. In Cramton, P., Muller, P., Tardos, E., and Bisin, M., editors, *Computational Social Systems and the Internet*.

Parendo, S. (2010). Costless signaling in matching markets. Dissertation paper, University of California, Santa Barbara.

Roth, A. (2008). What have we learned from market design? *The Economic Journal*, 118(527):285–310.