Computational investigation of generalized intersection cuts

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INTRODUCTION

Objective used with the cut LP and compare strength of cuts obtained from different test effect of cutting rays by additional various options for choosing

Computational

\[ \alpha \]

PHA: Intersect each ray of \( C(\xi) \) with a hyperplane, activating it (partially) on that ray alone

Valid cuts: Consider the system, for \( \beta \in [-1,1] \):

\[ a^T \rho \geq \beta, \quad \rho \in P \]

\[ a^T \rho < \beta, \quad \rho \in R. \]

Here, \( P \) and \( R \) are points and rays generated by PHA. Any feasible solution \( \tilde{\alpha} \) with \( \beta = \tilde{\beta} \), such that \( \tilde{a}^T \tilde{\alpha} < \tilde{\beta} \) yields a valid cut \( \tilde{a}^T \tilde{\alpha} \geq \beta \) for \( \tilde{\beta} \).

Computational experiment: Investigate with various options for choosing hyperplanes in PHA, test effect of cutting rays by additional hyperplanes, and compare strength of cuts obtained from different objectives used with the cut LP

RESULTS

Experimental setup: Instances selected from MIPLIB 3 based on time taken to test one set of parameters. Compared generalized intersection cuts (GICs) to standard intersection cuts (SICs), which are known to be strong.

Hyperplane selection. Choose hyperplane that:

Cut selection.

1. \((\xi_1)\) Intersects ray first

2. \((\xi_2)\) Gives intersection points with best average depth

3. \((\xi_3)\) Creates largest number of final intersection points (final means the point in \( P \))

Number of hyperplanes cutting a ray: Tested effect of activating up to three hyperplanes per ray (+1H, +2H, +3H). First hyperplane selected by one of rules above; additional ones activated to maximize number of final intersection points.

In the (separable) bilinear program, \( \bar{P} \) refers to \( P \) intersected with all the standard intersection cuts. It is solved iteratively over each of the variable sets, which only appear together in the objective.

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Table 1: Percentage gap closed by hyperplane activation procedure

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Average    18.70 23.27 4.57 4.27 4.21 3.83 4.33 2.51

Table 2: Percentage gap closed by objectives used for cut LP

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CONCLUSION

Activating an additional hyperplane per split increases the strength of cuts. However, activating a third hyperplane sometimes leads to worse cuts.

Investigation showed that strategy C performs nearly as well as more sophisticated methods (A, B).

Future research will aim to address the questions:

1. Why are there so few GICs generated?

2. Why does the third hyperplane, while adding more deep and final points, lead to worse cuts?

REFERENCES


