Cosmology and a Massive Graviton

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The cosmological constant problem is hard

- **Old CC problem**: why isn’t the CC enormous?
  - Vacuum energy induces large CC
  - Universe accelerates long before structure can form

- **New CC problem**: why isn’t it zero?
  - The Universe is accelerating! Implies tiny but non-zero CC
  - Require *technical naturalness*, otherwise reintroduce old CC problem
The CC problem is a problem of gravity

- Particle physics tells us the vacuum energy, and gravity translates that into cosmology.
- The CC affects gravity at ultra-large distances, where we have few complementary probes of GR.
- Does the CC problem point to IR modifications of GR?
- Can address the old problem, new problem, or both.

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Modifying gravity is also hard
Modifying gravity is also hard

- It is remarkably tricky to move away from GR in theory space
- Ghosts/instabilities
- Long-range fifth forces
Massive gravity is a promising way forward

- Conceptual simplicity
  - GR: unique theory of a massless spin-2
  - Mathematical simplicity? Depends on your aesthetics

- Has potential to address both CC problems
  - Old CC problem: *degravitation*
    - Yukawa suppression lessens sensitivity of gravity to CC
    - Degravitation in LV massive gravity: work in progress (with Justin Khoury, Jeremy Sakstein)

- Technically natural small parameter
  - Graviton mass protected by broken diffs
How to build a massive graviton

Step 1: Go linear

Consider a linearized metric

\[ g_{\mu\nu} = \eta_{\mu\nu} + \frac{1}{M_{\text{Pl}}} h_{\mu\nu} \]

The Einstein-Hilbert action at quadratic order is

\[ \mathcal{L} = -\frac{1}{4} h^{\mu\nu} (\mathcal{E} h)_{\mu\nu} + \frac{1}{2M_{\text{Pl}}} h_{\mu\nu} T^{\mu\nu} \]

with the Lichnerowicz operator defined by

\[ (\mathcal{E} h)^{\mu\nu} = -\frac{1}{2} \varepsilon^{\mu\alpha\rho} \varepsilon_{\nu\beta\sigma} \partial_{\alpha} \partial_{\beta} h_{\rho\sigma} \]

This action is invariant under linearized diffeomorphisms

\[ h_{\mu\nu} \rightarrow h_{\mu\nu} + 2 \partial_{(\mu} \xi_{\nu)} \]
How to build a massive graviton

Step 1: Go linear

- Unique healthy (ghost-free) mass term (Fierz and Pauli, 1939):

\[
\mathcal{L} = -\frac{1}{4} h^{\mu\nu} (\mathcal{E} h)_{\mu\nu} - \frac{1}{8} m^2 \left( h_{\mu\nu} h^{\mu\nu} - h^2 \right) + \frac{1}{2M_{Pl}} h_{\mu\nu} T^{\mu\nu}
\]

- This massive graviton contains five polarizations:
  - 2 x tensor
  - 2 x vector
  - 1 x scalar

- Fierz-Pauli tuning: Any other coefficient between \( h_{\mu\nu} h^{\mu\nu} \) and \( h^2 \) leads to a ghostly sixth degree of freedom
The unique non-linear action for a single massive spin-2 graviton is

\[ S = -\frac{M_g^2}{2} \int d^4x \sqrt{-\det g} R \]

\[ + m^2 M_g^2 \int d^4x \sqrt{-\det g} \sum_{n=0}^{4} \beta_n e_n \left( \sqrt{g^{-1} f} \right) \]

where \( f_{\mu\nu} \) is a reference metric which must be chosen at the start

\( \beta_n \) are interaction parameters; the graviton mass is \( \sim m^2 \beta_n \)

\( e_n \) are elementary symmetric polynomials given by…

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\[ S = -\frac{M_g^2}{2} \int d^4x \sqrt{-\det g} R + m^2M_g^2 \int d^4x \sqrt{-\det g} \sum_{n=0}^{4} \beta_n e_n \left( \sqrt{g^{-1}} f \right) \]

For a matrix \( X \), the elementary symmetric polynomials are \( [\cdot] = \text{trace} \)

\[
\begin{align*}
    e_0 (X) & \equiv 1, \\
    e_1 (X) & \equiv [X], \\
    e_2 (X) & \equiv \frac{1}{2} \left( [X]^2 - [X^2] \right), \\
    e_3 (X) & \equiv \frac{1}{6} \left( [X]^3 - 3 [X] [X^2] + 2 [X^3] \right), \\
    e_4 (X) & \equiv \det (X)
\end{align*}
\]
An aesthetic aside

The potentials in the last slide are **ugly**

Lovely structure in terms of **vielbeins and differential forms**:

\[ g_{\mu\nu} = \eta_{ab} e^a_{\mu} e^b_{\nu}, \quad f_{\mu\nu} = \eta_{ab} f^a_{\mu} f^b_{\nu} \]

\[ e_1(X) \sim \varepsilon_{abcd} e^a \wedge e^b \wedge e^c \wedge f^d \]

\[ e_2(X) \sim \varepsilon_{abcd} e^a \wedge e^b \wedge f^c \wedge f^d \]

\[ e_3(X) \sim \varepsilon_{abcd} e^a \wedge f^b \wedge f^c \wedge f^d \]
There is a simple (heuristic) reason that massive gravity needs a second metric: you can’t construct a non-trivial interaction term from one metric alone:

\[ g^{\mu\alpha} g_{\nu\alpha} = \delta^{\mu}_{\nu}, \quad (g_{\mu\nu})^2 = 4, \ldots \]

We need to introduce a second metric to construct interaction terms.

Can be Minkowski, (A)dS, FRW, etc., or even dynamical

This points the way to a large family of theories with a massive graviton

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Theories of a massive graviton

Examples of this family of massive gravity theories:

- $g_{\mu\nu}, f_{\mu\nu}$ dynamical: **bigravity** (Hassan, Rosen: 1109.3515)
  - One massive graviton, one massless

- $g_{\mu\nu}, f_{1,\mu\nu}, f_{2,\mu\nu}, \ldots, f_{n,\mu\nu}$, with various pairs coupling à la dRGT: **multigravity** (Hinterbichler, Rosen: 1203.5783)
  - n-1 massive gravitons, one massless

- Massive graviton coupled to a scalar (e.g., **quasidilaton, mass-varying**)
The search for viable massive cosmologies

- No stable FLRW solutions in dRGT massive gravity
- Way out #1: large-scale inhomogeneites
- Way out #2: generalize dRGT
  - Break translation invariance (de Rham+: 1410.0960)
  - Generalize matter coupling (de Rham+: 1408.1678)
- Way out #3: new degrees of freedom
  - Scalar (mass-varying, f(R), quasidilaton, etc.)
  - Tensor (bi/multigravity) (Hassan/Rosen: 1109.3515)
Massive bigravity has self-accelerating cosmologies

Consider FRW solutions

\[ ds_g^2 = a^2 \left( -d\tau^2 + d\bar{x}^2 \right), \]
\[ ds_f^2 = -X^2 d\tau^2 + Y^2 d\bar{x}^2 \]

NB: g = physical metric (matter couples to it)

Bianchi identity fixes X

New dynamics are entirely controlled by \( y = Y/a \)
Massive bigravity has self-accelerating cosmologies

The Friedmann equation for $g$ is

$$3H^2 = \frac{a^2 \rho}{M_g^2} + m^2 a^2 \left( \beta_0 + 3\beta_1 y + 3\beta_2 y^2 + \beta_3 y^3 \right)$$

The Friedmann equation for $f$ becomes algebraic after applying the Bianchi constraint:

$$\beta_3 y^4 + (3\beta_2 - \beta_4) y^3 + 3(\beta_1 - \beta_3) y^2$$

$$+ \left( \frac{\rho}{M_{Pl}^2 m^2} + \beta_0 - 3\beta_2 \right) y - \beta_1 = 0$$

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Massive bigravity has self-accelerating cosmologies

\[ 3 \mathcal{H}^2 = \frac{a^2 \rho}{M_g^2} + m^2 a^2 \left( \beta_0 + 3 \beta_1 y + 3 \beta_2 y^2 + \beta_3 y^3 \right) \]

\[ \beta_3 y^4 + (3 \beta_2 - \beta_4) y^3 + 3(\beta_1 - \beta_3) y^2 \]

\[ + \left( \frac{\rho}{M_{Pl}^2 m^2} + \beta_0 - 3 \beta_2 \right) y - \beta_1 = 0 \]

At late times, \( \rho \rightarrow 0 \) and so \( y \rightarrow \text{const.} \).

The mass term in the Friedmann equation approaches a constant – dynamical dark energy.

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Massive bigravity vs. $\Lambda$CDM

Y. Akrami, T. Koivisto, and M. Sandstad [arXiv:1209.0457]
See also F. Könnig, A. Patil, and L. Amendola [arXiv:1312.3208];
ARS, Y. Akrami, and T. Koivisto [arXiv:1404.4061]

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Beyond the background

Cosmological perturbation theory in massive bigravity is a huge cottage industry and the source of many PhD degrees. See:

Cristosomi, Comelli, and Pilo, 1202.1986
ARS, Akrami, and Koivisto, 1404.4061
Könnig, Akrami, Amendola, Motta, and ARS, 1407.4331
Könnig and Amendola, 1402.1988
Lagos and Ferreira, 1410.0207
Cusin, Durrer, Guarato, and Motta, 1412.5979

and many more for more general matter couplings!
Scalar perturbations in massive bigravity

Our approach (1407.4331 and 1404.4061):

- Linearize metrics around FRW backgrounds, restrict to scalar perturbations \( \{E_{g,f}, A_{g,f}, F_{g,f}, \text{and } B_{g,f}\} \):

\[
\begin{align*}
\frac{ds^2}{g} &= a^2 \left\{ -(1 + E_g) d\tau^2 + 2 \partial_i F_g d\tau dx^i + [(1 + A_g) \delta_{ij} + \partial_i \partial_j B_g] dx^i dx^j \right\} \\
\frac{ds^2}{f} &= -X^2 (1 + E_f) d\tau^2 + 2XY \partial_i F_f d\tau dx^i + Y^2 [(1 + A_f) \delta_{ij} + \partial_i \partial_j B_f] dx^i dx^j
\end{align*}
\]

- Full linearized Einstein equations (in cosmic or conformal time) can be found in ARS, Akrami, and Koivisto, arXiv:1404.4061
Scalar fluctuations can suffer from instabilities

- Usual story: solve perturbed Einstein equations in subhorizon, quasistatic limit: $k^2 \Phi \gg H^2 \Phi \sim H \dot{\Phi} \sim \ddot{\Phi}$

- This is valid only if perturbations vary on Hubble timescales

- Cannot trust quasistatic limit if perturbations are unstable

- Check for instability by solving full system of perturbation equations
Scalar fluctuations can suffer from instabilities

- **Degree of freedom count:** ten total variables
  - Four $g_{\mu\nu}$ perturbations: $E_g$, $A_g$, $B_g$, $F_g$
  - Four $f_{\mu\nu}$ perturbations: $E_f$, $A_f$, $B_f$, $F_f$
  - One perfect fluid perturbation: $\chi$

- **Eight** are redundant:
  - Four of these are *nondynamical/auxiliary* ($E_g$, $F_g$, $E_f$, $F_f$)
  - Two can be gauged away
  - After integrating out auxiliary variables, one of the dynamical variables **becomes** auxiliary – related to absence of ghost!

- **End result:** only **two** independent degrees of freedom

**NB:** This story is deeply indebted to *Lagos and Ferreira*

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Scalar fluctuations can suffer from instabilities

Choose g-metric Bardeen variables:

\[ \Phi \equiv A_g - H (F_g + B'_g) \]

\[ \Psi \equiv E_g - H (F_g + B'_g) - F'_g - B''_g \]

Then entire system of 10 perturbed Einstein/fluid equations can be reduced to two coupled equations:

\[ X''_i + F_{ij} X'_j + S_{ij} X_j = 0 \]

where

\[ X_i = \{ \Phi, \Psi \} \]
Scalar fluctuations can suffer from instabilities

Ten perturbed Einstein/fluid equations can be reduced to two coupled equations:

\[ X''_i + F_{ij} X'_j + S_{ij} X_j = 0 \]

where

\[ X_i = \{ \Phi, \Psi \} \]

Under assumption (WKB) that \( F_{ij}, S_{ij} \) vary slowly, this is solved by

\[ X_i = X_i^0 e^{i \omega N} \]

with \( N = \ln a \)
Scalar fluctuations can suffer from instabilities

- $B_1$-only model – simplest allowed by background

$$\omega_{B_1} = \pm \frac{k}{H} \frac{\sqrt{-1 + 12y^2 + 9y^4}}{1 + 3y^2}$$

- **Unstable** for small $y$ (early times)

- NB: Gradient instability
Scalar fluctuations can suffer from instabilities

- B₁-only model – simplest allowed by background

\[ \omega_{B_1} = \pm \frac{k}{H} \frac{\sqrt{-1 + 12y^2 + 9y^4}}{1 + 3y^2} \]

- Unstable for small y (early times)

- For realistic parameters, model is only (linearly) stable for \( z < \sim 0.5 \)
Instability does not rule models out

- Instability $\rightarrow$ breakdown of linear perturbation theory
  - Nothing more
  - Nothing less

- Cannot take quasistatic limit

- Need nonlinear techniques to make structure formation predictions
Do nonlinearities save us?

- Helicity-0 mode of the massive graviton has nonlinear derivative terms (cf. galileons)

- Linear perturbation theory misses these

- Some evidence that the gradient instability is cured by these nonlinearities! (Aoki, Maeda, Namba 1506.04543)
  - Morally related to the Vainshtein mechanism

- How can we study nonlinear perturbations in detail?
Strategy for “quasilinear” perturbations

- Bigravity’s degrees of freedom:
  - 1 massive graviton (two helicity-2, two helicity-1, one helicity-0)
  - 1 massless graviton (two helicity-2)
- Goal: keep nonlinearities in spin-0 mode while leaving other fluctuations linear
Cosmology as a perturbation of flat space

- Helicity decomposition only well-defined around flat background

Q: How can we isolate helicity-0 mode in cosmological perturbations?

A: Use Fermi normal coordinates, for which

\[ g_{\mu\nu}^{\text{FRW}}(x) = \eta_{\mu\nu} + \mathcal{O}(H^2x^2) \]
Cosmology as a perturbation of flat space

Starting from the FRW metric in comoving coordinates,

\[ ds^2 = -dt_c^2 + a^2(t_c) d\bar{x}_c^2 \]

the coordinate change

\[ t_c = t - \frac{1}{2} H \bar{x}^2 \]

\[ \bar{x}_c = \frac{\bar{x}}{a(t)} \left( 1 + \frac{1}{4} H^2 \bar{x}^2 \right) \]

yields

\[ ds^2 = - \left[ 1 - (\dot{H} + H^2) \bar{x}^2 \right] dt^2 + \left( 1 - \frac{1}{2} H^2 \bar{x}^2 \right) d\bar{x}^2 + \mathcal{O}(H^4 \bar{x}^4) \]
Cosmology as a perturbation of flat space

So within the horizon we can write FRW as Minkowski space plus a small perturbation,

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu},$$

$$h_{\mu\nu} = \text{diag} \left( \dot{H} + H^2, -\frac{1}{2} H^2 \delta_{ij} \right) \vec{x}^2$$
Bimetric Fermi normal coordinates

In bigravity with two FRW metrics,

\[ ds_g^2 = -dt_c^2 + a^2 d\vec{x}_c^2 \]
\[ ds_f^2 = -X^2 dt_c^2 + Y^2 d\vec{x}_c^2 \]

we clearly need two different Fermi coordinates.

Fermi coordinates \( \Phi^a \) for f metric. **Method**: start with f-metric comoving coordinates \( \Phi_c^a \) such that

\[ ds_f^2 = -d(\Phi_c^0)^2 + Y^2 d\vec{x}_c^2 \]

and then build FNC \( \Phi^a \) from \( \Phi_c^a \) as in previous slide.

We identify \( \Phi^a \) as the **Stückelberg fields** associated to broken diff invariance. 

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Bimetric Fermi normal coordinates

The FNC for the bimetric FRW backgrounds are related by the helicity-0 Stückelberg,

\[ \Phi^a = x^a + \partial^a \pi \]

\[ \pi = \int \int (1 - X) \, dt^2 + \frac{1}{2} (y - 1) \bar{x}^2 \]
To the decoupling limit

This is all suggestive of the **decoupling limit** of bigravity:

\[
g_{\mu\nu}(x) = \eta_{\mu\nu} + \frac{1}{M_{Pl}} h_{\mu\nu}
\]

\[
f_{ab}(\Phi) = \eta_{ab} + \frac{1}{M_f} v_{ab}
\]

\[
\Phi^a = x^a + \Lambda_3^{-3} \partial^a \pi
\]

with \( \Lambda_3^3 = m^2 M_{Pl} \), and take the scaling limit

\[
M_{Pl}, M_f \to \infty, \quad m \to 0
\]

while keeping \( \Lambda_3, M_{Pl}/M_f \), and \( \beta_n \) fixed

**DL leaves leading interactions** between helicity modes
The decoupling limit

The action in the DL is

\[ \mathcal{L} = -\frac{1}{4} h^{\mu \nu} \varepsilon_{\mu \nu}^{\alpha \beta} h_{\alpha \beta} - \frac{1}{4} v^{\mu \nu} \varepsilon_{\mu \nu}^{\alpha \beta} v_{\alpha \beta} + \frac{\Lambda^3}{2} \left( h_{\mu \nu} X^{\mu \nu} + \frac{M_{Pl}}{M_f} v_{\mu \nu} \tilde{Y}^{\mu \nu} \right) \]

The first two terms are linearized Einstein-Hilbert, and

\[ X^{\mu \nu} = -\frac{1}{2} \sum_{n=1}^{3} \frac{\beta_n}{(3-n)!n!} \varepsilon^{\mu \cdots \nu} (\eta + \Pi)^n \eta^{3-n} \]

\[ \tilde{Y}^{\mu \nu} = -\frac{1}{2} \sum_{n=1}^{3} \frac{\beta_n}{(4-n)!(n-1)!} \varepsilon^{\mu \cdots \nu} (\eta + \Sigma)^{4-n} \eta^{n-1} \]

where

\[ \Pi_{\mu \nu} \equiv \Lambda^{-3} \partial_\mu \partial_\nu \pi, \quad \Sigma_{\mu \nu} \equiv \Lambda^{-3} \partial_\mu \partial_\nu \rho \]

\[ \frac{\partial \pi(x)}{\partial x^\mu} = -\frac{\partial \rho(\Phi)}{\partial \Phi^\mu} \]
Cosmological perturbations in the decoupling limit

- The DL equations of motion yield correct background cosmological equations ✓

- **Nonlinear subhorizon structure** formation by perturbing in DL

\[
\begin{align*}
    h_{\mu\nu} &= \bar{h}_{\mu\nu} + \chi_{\mu\nu} \\
    v_{ab} &= \bar{v}_{ab} + w_{ab} \\
    \pi &= \bar{\pi} + \phi \\
    \rho &= \bar{\rho} + \psi
\end{align*}
\]

- Can consistently keep $\chi_{\mu\nu}$ and $w_{ab}$ linear while retaining nonlinearities in $\phi$ and $\psi$!
Example: $\beta_2$-only model

- The action in the DL is

$$\mathcal{L} = -\frac{1}{4} h\mathcal{E}h - \frac{1}{4} v\mathcal{E}v - \frac{\Lambda_3^3}{8} \beta_2\mathcal{E}\mathcal{E} \left( h(\eta + \Pi)^2\eta + v(\eta + \Sigma)^2\eta \right)$$

- Perturb to second order:

$$\mathcal{L}_2 = -\frac{1}{4} \chi\mathcal{E}\chi - \frac{1}{4} w\mathcal{E}w -$$

$$-\frac{\Lambda_3^3}{8} \beta_2\mathcal{E}\mathcal{E} \left( \bar{h}\Phi^2\eta + 2\chi\Phi(\eta + \bar{\Pi})\eta + \bar{v}\Psi^2\eta + 2w\Psi(\eta + \bar{\Sigma})\eta \right)$$

- Can be fully diagonalized:

$$\chi_{\mu\nu} = \hat{\chi}_{\mu\nu} + \beta_2\phi(\eta + \bar{\Pi})_{\mu\nu}$$

$$w_{\mu\nu} = \hat{w}_{\mu\nu} + \beta_2\psi(\eta + \bar{\Sigma})_{\mu\nu}$$
Example: $\beta_2$-only model

This leaves us with (after int by parts, removing dual galileon)

$$\mathcal{L}_2 = -\frac{1}{4} \dot{\chi} \mathcal{E} \dot{\chi} - \frac{1}{4} \dot{\mathcal{E}} \dot{\mathcal{E}} + c_k \phi^2 - c_g \nabla^2 \phi$$

where the coefficients of the kinetic and gradient terms are

$$c_k = \frac{6}{X} \left[ \beta_2 y (1 + X y) - H^2 (X + y) \right]$$

$$c_g = \frac{2}{y} \left[ \beta_2 (X + 2y + 2X y^2 + y^3) - (X + 5y) H^2 - 4 \dot{H} y \right]$$

$c_k > 0$ is equivalent to the Higuchi bound: useful consistency check! NB $\beta_2$ is *not* a linearly unstable model
Summary

・ Massive gravity is a promising approach to the cosmological constant problems

・ Bimetric massive gravity has cosmological backgrounds competitive with $\Lambda$CDM

・ These models have linear instabilities

・ A deeper look into quasilinear behavior of perturbations can shed light on endpoint of instability

・ Resuscitate massive (bi)gravity as a target for observations?

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