Superfluids and the
Cosmological Constant
Problem

Adam R. Solomon
Carnegie Mellon University

With Justin Khoury & Jeremy Sakstein (UPenn)

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Outline

1. Intro to the cosmological constant problem(s)

2. Proposed solutions and obstructions

3. Our proposal: finite-temperature superfluid/Lorentz-violating massive gravity
“The cosmological constant problem is the unwanted child of two pillars of twentieth century physics: quantum field theory and general relativity.”

Tony Padilla
What is the cosmological constant?

- Einstein’s equations allow a cosmological constant:
  \[ G_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{1}{M_{\text{Pl}}^2} T_{\mu\nu} \]

- Most pronounced effect is on long distances/cosmology:
  - \( \Lambda > 0 \): late-time acceleration
  - \( \Lambda < 0 \): collapse

- Observations indicate the Universe is accelerating, consistent with a cosmological constant of size
  \[ \Lambda \sim \mathcal{O} \left( \frac{\text{meV}^4}{M_{\text{Pl}}^2} \right) \]
What could produce $\Lambda$?

- One known theoretical source of $\Lambda$: *vacuum energy*
  
- Equivalence principle: the vacuum gravitates
  
- Lorentz invariance: $T_{\mu\nu} \sim \frac{\rho_{\text{vac}}}{M_{\text{Pl}}^2} g_{\mu\nu}$

- Massive fields contribute $\rho_{\text{vac}} \sim m^4$

- Observed $\Lambda$ is about the vacuum energy due to neutrino masses, tiny compared to any other particles in Standard Model (and beyond)

  - Electron vacuum energy alone would lead to de Sitter horizon $\sim 10^6$ km

- Pauli: the radius of the world “nicht einmal bis zum Mond reichen würde” (would not even reach the Moon!)
Where is the vacuum energy?

- The problem is even worse with more particle species:

\[
\begin{align*}
\frac{\rho_{\text{vac,electron}}}{\rho_{\text{vac,obs}}} & \sim 10^{32} \\
\frac{\rho_{\text{vac,SM}}}{\rho_{\text{vac,obs}}} & \sim 10^{54} \\
\frac{\rho_{\text{vac,Planck}}}{\rho_{\text{vac,obs}}} & \sim 10^{121}
\end{align*}
\]

- “The worst theoretical prediction in the history of physics!”
Cosmological constant problems old and new

- Cosmic acceleration poses two (logically distinct?) problems:
  - “Old problem”: why does an enormous vacuum energy not gravitate?
  - “New problem”: why is there some residual acceleration anyway?
- Often treated separately! Solve one while ignoring the other
- This talk: focus on the old problem
Approaches to the old problem

- There are *many* proposed solutions. I will discuss a small sample. For comprehensive lists, see reviews by
  
  - **Weinberg, Rev. Mod. Phys. 61 (1989) 1-23**
  
  - Nobbenhuis, gr-qc/0411093
  
  - Martin, 1205.3365
  
  - **Burgess, 1309.4133**
  
  - Padilla, 1502.05296
Some approaches

- Anthropics: if $\Lambda$ were bigger, we wouldn’t be around to remark on it

- Could follow from string landscape + eternal inflation

- Dimopoulos: danger of “premature application”

- Modifications of gravity: leave $\Lambda$ alone, but change how it gravitates

- Degravitation: weaken gravitational response to long-wavelength sources

- Self-tuning: introduce new field(s) which dynamically counteract $\Lambda$
Self-tuning and our modest goal

- We will set a modest goal: field equations solved by Minkowski for arbitrary $\Lambda$

- Necessary but not sufficient condition for solving old CC problem

- Other criteria:
  - UV insensitivity, radiative stability, no pathologies, agreement with experiments, reproduce observed cosmological history, etc.!

- Self-tuning runs into a famous obstruction due to Weinberg
Weinberg’s famous no-go theorem

- Weinberg (1988); see also Padilla review 1502.05296

- Assume some fields $\phi^A$ “eat up” vacuum energy,

  $$T^{\phi}_{\mu\nu} = - T^\Lambda_{\mu\nu}$$

- Assume Poincaré-invariant vacua,

  $$\phi^A = \text{const}, \quad g_{\mu\nu} = \eta_{\mu\nu}$$
Weinberg’s famous no-go theorem: two paths

- Add a potential: \( \mathcal{L} = -\sqrt{-g}(V(\phi^A) + 2\Lambda) + \text{derivatives} \)
  - Fine-tuning: only cancels out one specific value of \( \Lambda \)

- Scaling symmetry: \( \mathcal{L} = -\sqrt{-g}V_0e^{4\tilde{\phi}} + \text{derivatives} \)
  - Particle masses also vanish; not physical
Evading Weinberg’s theorem

- Any no-go theorem has assumptions, pointing the way forward

- Our (and others’) approach: break Poincaré invariance

\[ \Phi^A = \Phi^A(x^\mu), \quad g_{\mu\nu} = \eta_{\mu\nu} \]

- e.g., in cosmology, we might have fields with time dependence

- This implies, at a minimum, that the fields must be accompanied by derivatives

- To leading order in EFT, one derivative per field
Warmup: one scalar

- Can we degravitate with a single scalar?
- No.
- Why?

- At leading order in derivatives, most general action is

\[ \mathcal{L} = \frac{M_{\text{Pl}}^2}{2} \left[ R - 2\Lambda + m^2 P(X) \right], \quad X \equiv (\partial \Phi)^2 \]

- NB: this is the EFT of a zero-temperature superfluid
The total stress tensor is
\[ \frac{1}{M_{\text{Pl}}^2} T_{\mu\nu} = \frac{1}{2} m^2 \left( P g_{\mu\nu} - 2 P_X \partial_\mu \Phi \partial_\nu \Phi \right) - \Lambda g_{\mu\nu}, \quad P_X \equiv \frac{\partial P}{\partial X} \]

In order to have flat solutions for arbitrary \( \Lambda \), this must vanish:

1. \( P_X = 0 \)

2. \( m^2 P = 2\Lambda \)

The latter requires fine-tuning.

E.g., ghost condensate, \( P(X) = X + \lambda X^2/2 \)

Solution: \( \Phi = \lambda^{-1/2} t \). But \( m^2 P = 2\Lambda \) only if \( \lambda \) is carefully tuned against \( \Lambda \! \! \! \! = \frac{-m^2}{4\Lambda} \)
Four scalars

- Fine-tuning can be alleviated with four scalars $\Phi^A$
- Can eliminate problem of inhomogeneous term in stress tensor
- Consider a simple (and trivially wrong) model:

$$\mathcal{L} = \frac{M_{\text{Pl}}^2}{2} \left[ R - 2\Lambda - m^2 \eta_{AB} \partial_{\mu} \Phi^A \partial^{\mu} \Phi^B \right]$$
The stress tensor no longer has an inhomogeneous term:

\[ \frac{1}{M_{\text{Pl}}^2} T_{\mu\nu} = \frac{1}{2} m^2 \left( -\eta_{AB} \partial_\alpha \Phi^A \partial^\alpha \Phi^B g_{\mu\nu} + 2 \eta_{AB} \partial_\mu \Phi^A \partial_\nu \Phi^B \right) - \Lambda g_{\mu\nu} \]

This admits flat solutions \( \Phi^A = \alpha x^A \), with

\[ \alpha = \frac{\sqrt{-\Lambda}}{m} \]

Degravitates any negative \( \Lambda \) without fine-tuning!

Key: instead of tuning theory parameters against \( \Lambda \),
tune integration constant \( \alpha \) to \( \Lambda \)

Tuning is achieved *dynamically*
- Fatal problem: $\Phi^0$ is a **ghost**

\[
\mathcal{L} = \frac{M_{Pl}^2}{2}\left[R - 2\Lambda - m^2\eta_{AB}\partial_\mu \Phi^A \partial^\mu \Phi^B\right]
\]

\[
= \frac{M_{Pl}^2}{2}\left[R - 2\Lambda + m^2(\partial \Phi^0)^2 - m^2 \sum_{i=1}^{3} (\partial \Phi^i)^2\right]
\]

- Direct result of internal Lorentz symmetry, which is what we used to remove inhomogeneous term!

- **Blessing and a curse**

- Can we self-tune without a ghost?
Ghosts and massive gravity

- The ghost is easy to understand if we recognize this is a theory of massive gravity

- Why? Consider adding to GR a mass term of the form

\[
\frac{\mathcal{L}}{\sqrt{-g}} = \frac{M^2_{\text{Pl}}}{2} \left[ R - 2\Lambda - m^2 g^{\mu\nu} \eta_{\mu\nu} \right]
\]

- This breaks diff invariance due to \( \eta_{\mu\nu} \). Can restore diffs by introducing Stückelberg fields \( \Phi^A \),

\[
\eta_{\mu\nu} \rightarrow \eta_{AB} \partial_\mu \Phi^A \partial_\nu \Phi^B
\]

- And we recover the action discussed in the previous slides
Massive gravity and degravitation

- This connects to an old and venerable story:
  - Massive graviton $\rightarrow$ finite range of gravity $\rightarrow$ gravity acts as "high-pass filter" screening out sources with wavelengths $>> m^{-1}$
  - $\Lambda$ is infinite-wavelength source!

- Massive gravity at linear level: Fierz-Pauli (1939) $h_{\mu\nu}h^{\mu\nu} - (h_{\mu}^{\mu})^2$

- Other linear mass terms have ghosts, just like the example we discussed
Massive gravity and degravitation

- Non-linear: another ghost! (Boulware-Deser, 1972)

- Unique non-linear, ghost-free, Lorentz-invariant massive gravity: de Rham-Gabadadze-Tolley (dRGT, 2010)

- dRGT cannot degravitate large $\Lambda$ without violating solar system tests of GR (1010.1780)

- This means we cannot use a theory of the form

$$\mathcal{L} = \frac{M_{\text{Pl}}^2}{2} \left[ R - 2\Lambda + m^2 U(\partial_{\mu} \Phi^A, \eta_{AB}, \epsilon_{ABCD}) \right]$$
Is Lorentz invariance too strong a requirement?

- For cosmology, we only need SO(3), not SO(3,1)

- Idea: break internal boosts

\[
\frac{\mathcal{L}}{\sqrt{-g}} = \frac{M_{\text{Pl}}^2}{2} \left[ R - 2\Lambda + m^2 U(\partial_\mu \Phi^0, \partial_\mu \Phi^i, \delta_{ij}, \varepsilon_{ijk}) \right]
\]

- Aim: use newfound freedom to avoid ghosts (and other pathologies) while retaining degravitation

- We will analyze this theory for physically sensible degravitating models with

\[
\Phi^0 = \alpha t, \quad \Phi^i = \beta x^i \quad \text{such that} \quad T_{\mu\nu}^{\Phi} \big|_{g=\eta} = M_{\text{Pl}}^2 \Lambda \eta_{\mu\nu}
\]
Interpreting our theory

- Two physical interpretations of this type of theory:
  1. Lorentz-violating massive gravity
  2. Low-energy EFT of self-gravitating fluid
- Difference hinges only on coordinate choice
Lorentz-violating massive gravity as a fluid EFT

- Consider the fields $\Phi^A = \Phi^A(t,x)$ to be **comoving** (Lagrangian) coordinates of a fluid

- Fluid rest frame is a coordinate system in which $\Phi^A = \alpha x^A$

- EFT describing excitations of fluid is a derivative expansion in $\Phi^A$ obeying any relevant symmetries
Building blocks and symmetry

- At leading order in derivatives, action is built out of
  \[ C^{AB} \equiv g^{\mu\nu} \partial_\mu \Phi^A \partial_\nu \Phi^B \implies \mathcal{L} = U(C^{00}, C^{0i}, C^{ij}) \]

- Choice of operators determines symmetry-breaking pattern and hence fluid, e.g.,
  - Solids: \( \mathcal{L} = U(C^{ij}) \)
  - Zero-temperature superfluids: \( \mathcal{L} = U(C^{00}) \)
  - Finite-temperature superfluids:
    \[ \mathcal{L} = U(C^{00}, \det C^{ij}, \det C^{AB}) \]
The importance of coordinates

- If we move to the fluid rest frame, $\Phi^A = \alpha x^A$, then we recover the Lorentz-violating massive gravity picture:

$$U(C^{00}, C^{0i}, C^{ij}) \xrightarrow{\Phi^A = \alpha x^A} U(h_{00}, h_{0i}, h_{ij})$$

- We refer to this as unitary gauge

- e.g., $X = C^{00} = g^{\mu\nu} \partial_\mu \Phi^0 \partial_\nu \Phi^0 \xrightarrow{\Phi^A = \alpha x^A} C^{00} = \alpha^2 g^{00}$

- $\rightarrow$ Potential for $g^{00}$
Criteria for degravitation

- **Existence of a Minkowski (degravitating) solution:**
  equations of motion must be solved by $g = \eta$ for arbitrary $\Lambda$

- **No fine-tuning:** Tune integration constants, not parameters, against $\Lambda$

- **Massless tensors:** Tensor mass generically is huge, $m \sim O(\Lambda^{1/2})$, unless they are exactly massless. (LIGO: $m < 10^{-22}$ eV)

- **No pathologies:** No ghosts, tachyons, gradient instabilities, infinite strong coupling, instantaneous modes

- **UV insensitivity:** Higher-derivative EFT corrections should not introduce new modes at low energy
Strategy

1. Identify parameter space of Lorentz-violating massive gravity which satisfies these criteria

2. Look for *symmetries* that protect our parameter choice

3. Determine fluid building blocks

4. Solve cosmological constant problem
Analysis

- Work in unitary gauge at linear level,

  \[ \Phi^A = (\alpha t, \beta x^i), \quad g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \]

- Most general SO(3)-invariant mass term
  Dubovsky hep-th/0409124

  \[ \mathcal{L}_{\text{mass}} = \frac{M_{\text{Pl}}^2}{2} \left( m_0^2 h_{00}^2 + 2m_1^2 h_{0i}^2 - m_2^2 h_{ij}^2 + m_3^2 h_{ii}^2 - 2m_4^2 h_{00} h_{ii} \right) \]

- Massless tensors: \( m_2 = 0 \)

- Stability: \( m_1 = 0 \) (see our paper for gory details! 1805.05937)
Searching for a symmetry

- With our parameter choice in hand, $m_1 = m_2 = 0$, we need to find a symmetry to protect it
  - Otherwise we’re just fine-tuning and not solving anything!

- Several candidate symmetries
  - Most either don’t degravitate or are UV-sensitive

- Only symmetry that works: **time-dependent, volume-preserving spatial diffeomorphism**
Time-dependent, volume-preserving spatial diffeomorphisms

- Fluid language:
  \[ \Phi^i \rightarrow \Psi^i(\Phi^0, \Phi^i) \quad \text{with} \quad \det \left( \frac{\partial \Psi^i}{\partial \Phi^j} \right) = 1 \]

- Massive gravity language: break diffs while leaving
  \[ x^i \rightarrow x^i + \xi^i(t, x^j) \quad \text{with} \quad \partial_i \xi^i = 0 \]

- Closely related to time-independent volume-preserving spatial diffs, which set \( m_2 = 0 \) and forbid massive tensors

- Adding time dependence further restricts \( m_1 = 0 \), as necessary for stability
Building blocks of degravitation

- The building blocks invariant under our symmetry,

$$X = (\partial \Phi^0)^2$$

$$Yb = \frac{\text{det}(\partial \Phi)}{\sqrt{-g}}$$

- Our degravitating theory is therefore

$$\frac{\mathcal{L}}{\sqrt{-g}} = \frac{M_{\text{Pl}}^2}{2} \left[ R - 2\Lambda + m^2 U(X, Yb) \right]$$

- **Unique** theory that satisfies our criteria

- This describes a finite-temperature superfluid!
Particle content

- Two massless tensors \((\omega^2 = p^2)\)
- No vectors
- One global scalar \((\omega^2 = 0)\)
  - Propagates due to higher-derivative EFT corrections
  - Sounds weird, but well-known from ghost condensation
Hints from the UV

- Including next-to-leading operators in derivative expansion, scalar dispersion relation becomes

\[
\left( m_0^2 - \frac{m_4^4}{m_3^2} \right) \omega^2 = ap^4
\]

- \( a \): dimensionless constant determined by UV physics

- Ghost-free condition: \( m_0^2 - \frac{m_4^4}{m_3^2} > 0 \)

- No gradient instability: \( a > 0 \)

- UV insensitive: \( \omega^2 \propto \frac{p^4}{\mathcal{M}^2} \), \( \mathcal{M} \sim \) cutoff
Aside: relation to unimodular gravity

- Well-known attempt to solve cosmological constant problem: unimodular gravity

\[ R_{\mu\nu} - \frac{1}{4} R g_{\mu\nu} = \frac{1}{M_{Pl}^2} \left( T_{\mu\nu} - \frac{1}{4} T g_{\mu\nu} \right) \]

- Traceless version of Einstein equations: \( \Lambda \) reintroduced as integration constant

- This is one of a class of theories invariant under transverse diffeomorphisms (TDiff), corresponding to U(Yb)
Aside: relation to unimodular gravity

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<th>GR:</th>
<th>Unimodular:</th>
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<td>[ \mathcal{L} \sqrt{-g} = \frac{M_{\text{Pl}}^2}{2} R ]</td>
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<td>[ \mathcal{L} \sqrt{-g} = \frac{M_{\text{Pl}}^2}{2} \left[ R + m^2 U(Yb) \right] ]</td>
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<th>Ghost condensate:</th>
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Our theory is to unimodular as ghost condensate is to GR!
Same symmetry-breaking pattern
UV sensitivity of unimodular/TDiff gravity

- Aside: we find that TDiff theories are **UV-sensitive**
- Not a good solution to the CC problem!
- TDiff: $m_0 = m_3 = m_4$
- Lowest order, no mode: \( \left( m_0^2 - \frac{m_4^4}{m_3^2} \right) \omega^2 = 0 = 0 \times \omega^2 \)
- Next order in EFT: Lorentz invariance implies a massless mode, \( \frac{ap^4 + b\omega^2 p^2 + c\omega^4}{\mathcal{M}^2} = 0 \propto \frac{(\omega^2 - p^2)^2}{\mathcal{M}^2} \)
Degravitating solutions in practice

- Finally, we can see how this all works! Example:

$$U(X, Yb) = \frac{K_1}{2} (X + 1)^2 + \frac{K_2}{2} (Yb)^2$$

Ghost condensate plus term quadratic in Yb

- Has degravitating solutions!

$$g_{\mu\nu} = \eta_{\mu\nu}, \quad \Phi^0 = t, \quad \Phi^i = \left(-\frac{4\Lambda}{K_2 m^2}\right)^{1/6} x^i$$

- Can degravitate any positive $\Lambda$ for $K_2 < 0$ and vice versa

- No ghost: $K_1 > 0$
Degravitating solutions in practice

- Another simple example:

\[ U(X, Yb) = -X + \gamma XYb - \frac{\lambda}{2}(Yb)^2 \]

- Degravitating solutions:

\[
\begin{align*}
  g_{\mu\nu} &= \eta_{\mu\nu}, \\
  \Phi^0 &= \sqrt{\frac{2\Lambda}{m^2} - \frac{\lambda}{2\gamma^2}} t, \\
  \Phi^i &= \left( \frac{2\gamma^2 \Lambda}{m^2} - \frac{\lambda}{2} \right)^{-1/6} x^i
\end{align*}
\]

- Can degravitate any \( \Lambda > \lambda m^2/4\gamma^2 \)

- Ghost-free: \( \lambda > 0 \)
Future directions

- Cosmology! Embed this model in a realistic scenario
- Is the Minkowski solution an attractor?
- Hints we may need to (softly) break shift symmetry
Summary

- New method for self-tuning $\Lambda$ by breaking Lorentz
  - Circumvent Weinberg
- Unique theory: finite-temperature superfluid
- Next step: see whether this cancellation can occur dynamically
Bonus slides
Weinberg’s famous no-go theorem

- Weinberg (1988); see also Padilla review 1502.05296

- Assume some field $\phi$ “eats up” vacuum energy density for arbitrary $\Lambda$, i.e.,

$$T^\phi_{\mu\nu} = - T^\Lambda_{\mu\nu}$$

- We’ll consider a single scalar field, but proof extends to arbitrary spins/number of fields

- Assume Poincaré-invariant vacua,

$$\phi = \text{const}, \quad g_{\mu\nu} = \eta_{\mu\nu}$$

- For constant $\phi$ and $g_{\mu\nu}$, the field equations are

$$\left. \frac{\partial \mathcal{L}}{\partial \phi} \right|_{g,\phi=\text{const.}} = 0, \quad \left. \frac{\partial \mathcal{L}}{\partial g_{\mu\nu}} \right|_{g,\phi=\text{const.}} = 0$$
- **Scenario 1:** equations of motion hold independently. Translation invariance:

\[
\delta_M x^\mu = M^\mu_\nu x^\nu \implies \delta_M g_{\mu\nu} = 2M_{(\mu\nu)}, \quad \delta_M \mathcal{L} = \mathcal{L} \text{ Tr } M
\]

- General variation of \( \mathcal{L} \), assuming \( \phi \) eom, is

\[
\delta \mathcal{L} = \frac{\partial \mathcal{L}}{\partial g_{\mu\nu}} \delta g_{\mu\nu} + \frac{\partial \mathcal{L}}{\partial \phi} \delta \phi \implies \mathcal{L} g^{\mu\nu} M_{\mu\nu} = 2 \frac{\partial \mathcal{L}}{\partial g_{\mu\nu}} M_{\mu\nu}
\]

- This implies (on-shell)

\[
\frac{\partial \mathcal{L}}{\partial g_{\mu\nu}} = \frac{1}{2} \mathcal{L} g^{\mu\nu} \implies \mathcal{L} = -\sqrt{-g} (V(\phi) + 2\Lambda) + \text{ derivatives}
\]

- The metric eom requires \( V(\phi) = -2\Lambda \) at min; *fine-tuning*
Scenario 2: equations of motion imply each other,

\[ 2g_{\mu\nu} \frac{\partial L}{\partial g_{\mu\nu}} = f(\phi) \frac{\partial L}{\partial \phi} \]

\[ \equiv \frac{\partial L}{\partial \tilde{\phi}}, \quad \tilde{\phi} \equiv \int \frac{d\phi}{f(\phi)} \]

For constant fields, this implies scaling symmetry,

\[ \delta_\epsilon g_{\mu\nu} = 2\epsilon g_{\mu\nu}, \quad \delta_\epsilon \tilde{\phi} = -\epsilon \quad \Longrightarrow \quad \delta_\epsilon L = 0 \]

This requires

\[ L = -V_0 \sqrt{-\hat{g}} + \text{derivatives}, \quad \hat{g}_{\mu\nu} \equiv e^{2\tilde{\phi}} g_{\mu\nu} \]

implying (by \( g_{\mu\nu} \) equation)

\[ V_0 e^{4\tilde{\phi}} = 0 \]
Our equation of motion (dropping tildes),

\[ V_0 e^{4\phi} = 0, \]

has two solutions:

- \( V_0 = 0 \): fine-tuning again
- \( e^{\phi} = 0 \): all particles massless
- QED