Looking for TransPlanckia in the CMB

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The exponential stretching of length scales during inflation leads to the possibility that effects occurring at the smallest distance scales might be imprinted on large scale observables such as the CMB power spectrum. To be able to make credible predictions, we need to be able to construct an effective theory that absorbs the effects of new physics that could modify the initial state of inflaton quantum fluctuations. We provide such a formalism in terms of an *effective* initial state. We describe the formalism, the divergence structure of the effective theory and describe how it will be applied to computing TransPlanckian effects in inflation

1 Introduction

Observational data lend credence to the idea that the primordial metric perturbations observed in the CMB[1] were produced during an inflationary epoch in the early universe. In fact, the data are strong enough to be able to constrain various models of inflation; as an example, WMAP3 data[1] together with large scale structure information places severe pressure on inflationary models based on certain power law potentials for the inflaton[2].

While only 60-70 e-folds of inflation are necessary to solve the various fine tuning problems inflation was devised to deal with, most models tend to inflate for many more e-folds than this. As a consequence, length scales corresponding to the largest cosmological structures today had their origin in quantum fluctuations whose length scales corresponded to sub-Planckian sizes during inflation. A natural question to ask is the following: if the length scales can be stretched from sub-Planckian sizes to cosmological, can *physical* effects from scales smaller than the Planck length be imprinted on cosmological observables such as the CMB? This is the "trans-Planckian" problem[?] of inflation; we will take "trans-Planckian" here to mean energy scales larger than the Hubble scale during inflation.

While it has been dubbed a problem, this peculiar effect of inflation could just as easily be called an opportunity. The downside of this effect is that if it did happen, it would be an egregious violation of the notion of decoupling of scales which underlies most of physics. If such an infiltration of short-distance physics could propagate to the longest distance measured, we would have a difficult time in understanding why we have been able to neglect this effect in high energy collisions, for example. On the other hand, there is an incredible upside if the physics scales up. In this case, measurements of the CMB would be able to access physics well beyond any currently envisage accelerator.

It is this possibility that motivates us to find ways to accurately estimate the size of *generic* trans-Planckian signals. Our approach is to develop an effective theory description of this effect.

This is based on a perturbative expansion that uses the smallness of the ratio of the two natural scales–H, the Hubble scale during inflation and M, the scale associated with new physics. If the signal were only suppressed by H/M, it could conceivably be observed in the not too distant future.

From the perspective of the effective theory principle, new physics can appear in either the time evolution of the inflaton and its fluctuations or in their "initial" states. The first of these—how the system evolves—is more familiar since the evolution of the quantum fluctuations of the inflaton is determined by its interaction Hamiltonian. The form of the general set of possible corrections that we can add to this Hamiltonian, encoding the effects of the unknown physics, is rather constrained by the space-time symmetries. Given these constraints, the size of the corrections from the unknown physics relative to the leading prediction for an inflationary model is usually suppressed by a factor of $(H/M)^2$ [4]. The other ingredient—the state of the inflaton—is more directly related to the trans-Planckian problem because it is the details of the initial state we have chosen which are being stretched to vast scales. The leading correction from these effects is typically much less suppressed, scaling instead as H/M [5].

In this article, we will concentrate on the effect of trans-Planckian physics on the initial state[6]. We give a general parametrization of this initial state that contains corrections to the standard initial state in both the IR as well as the UV. We then discuss how the renormalization program is modified in the presence of this state and discuss the effects of this state on the problem of back-reaction onto the stress energy tensor.

2 An Effective Theory of Initial Conditions

Let's recall how the power spectrum is usually computed in inflation. We consider the gauge invariant variable Φ which is a linear combination of inflaton and metric perturbations; it behaves as a free field during inflation. We quantize it in the usual way: first decompose Φ in terms of the appropriate modes (solutions of the massless, minimally coupled, free scalar field equations of motion) and then convert the coefficients to operators acting on the relevant Fock space. If we write

$$\Phi(\eta,\vec{x}) = \int \frac{d^3k}{(2\pi)^3}\left[\mathcal{U}_k(\eta)\ e^{i\vec{k}\cdot\vec{x}}a_{\vec{k}} + \mathcal{U}_k^*(\eta)e^{-i\vec{k}\cdot\vec{x}}a_{\vec{k}}^\dagger\right], \tag{1}$$

where η is conformal time, then

$$\left[\frac{d^2}{d\eta^2} + 2H\frac{d}{d\eta} + k^2 - \frac{1}{6}a^2(\eta)R\right]\mathcal{U}_k = 0. \tag{2}$$

How do we pick the appropriate solution of Eq. 2? The standard answer is to pick the linear combination that matches to the flat space vacuum as $\eta \to -\infty$. This is the so-called Bunch-Davies (BD) vacuum $|BD\rangle$. The reasoning here is that at short-distances, the field should not be able to tell that the spacetime is curved so that the vacuum should just be the Minkowski one.

We have to take into account the possibility that as we try to match to the flat space ground state at short distance, it may be that the dynamics itself changes. For example, suppose the inflaton is a fermionic composite particle, with a scale of compositeness M. Then at distances shorter than M^{-1}, the scalar field description is completely inappropriate in terms of looking for the vacuum state. If $M \sim M_{\rm Pl}$, then we expect that at distances shorter than M^{-1}, we would expect both the field dynamics and gravity to be described by more complicated operators. What we can glean from this discussion is that assuming that we can make use of the scalar mode equations to arbitrarily short distances, and hence to argue that the BD vacuum is the "natural" choice of ground state, is a *radical* statement. A *much* more conservative statement is that we should choose a ground state that is general enough to encode possible corrections to the Klein-Gordon equation of motion coming from physics at energy scales larger than M.

We implement this by the imposition of boundary conditions at the initial time hypersurface $\eta = \eta_0$. If we start with the BD modes, $\mathcal{U}_k^{\rm BD}$, we write our modes as

$$\begin{aligned} \mathcal{U}_k(\eta) &= N_k \left[\mathcal{U}_k^{\rm BD}(\eta) + e^{\alpha_k}\mathcal{U}_k^{\rm BD*}\right], \\ e^{\alpha_k} &= \frac{\omega_k - \varpi_k}{\omega_k + \varpi_k} \\ N_k &= \frac{1}{\sqrt{1 - e^{\alpha_k + \alpha_k^*}}}. \end{aligned} \tag{3}$$

Here ω_k is the frequency appearing in the scalar field equation, while

$$\partial_\eta \mathcal{U}_k(\eta_0) = -i\varpi_k \mathcal{U}_k(\eta_0). \tag{4}$$

The structure function e^{α_k} describes how the state differs from the assumed vacuum at different scales. At very large distances, if we are considering an excited state the structure function need not vanish; but the signals of new physics should not be very apparent since the approximation that the theory is that of a nearly free scalar field is good far below M. In this regime, it is natural for the effects of new physics to be suppressed by powers of k/M.

Our goal here is to implement an effective theory description of the initial state. From this perspective, the state in Eq. 3 is only meant to be appropriate for observables measured at scales well below M, and not that for a complete theory which is applicable to measurements made at any scale. As a consequence, the effective states can contain structures which are the analogues of the nonrenormalizable operators used in an effective field theory Lagrangian. In both cases, the theory remains predictive at long distances since there is a natural small parameter given by the ratio of the energy or momentum of the process being studied to the scale of new physics M. In both cases too, renormalization of the theory can introduce further higher order corrections so that an infinite number of constants is often needed to make a prediction to arbitrary accuracy; but to any finite accuracy, only a small number are needed since the rest are suppressed by high powers of the small ratio of scales. At scales near M, the effective Lagrangian description breaks down but at these energies we should be able to observe the dynamics which produced the nonrenormalizable operators in the low energy effective theory. Similarly, once we probe short distances directly, we should see corrections to the Klein-Gordon equation and the modes $U_k(\eta)$ given in Eq. 3 should be replaced with the correct short-distance eigenmodes.

The effective theory description of the initial state relies upon the smallness of the measured scale, $k_{\rm exp}$, compared with the scale of new physics, M, but this ratio is also influenced by the expansion,

$$\frac{a(\eta_{\rm now})}{a(\eta_0)} \frac{k_{\rm exp}}{M} \ll 1, \tag{5}$$

and the earliest time for which perturbative calculation works is one which does quite saturate this bound,

$$\frac{a(\eta_0^{\rm earliest})}{a(\eta_{\rm now})} \sim \frac{k_{\rm exp}}{M}. \tag{6}$$

Although this time dependence of scales limits the applicability of the effective theory, it should not be seen as anything mysterious or that η_0 must be chosen either at this bound or at a time when some nontrivial dynamics is occurring. To study the inflationary prediction for the cosmic microwave background power spectrum, for example, it is sufficient to choose an "initial time" when all of the features of the currently observed power spectrum are just within the horizon during inflation and which still satisfies the condition in Eq. 5 for a well behaved perturbation theory. What the effective theory approach accomplishes is not a complete description of the theory to an arbitrarily early time, but rather it provides a completely generic parameterization of the effects of these earlier epochs or of higher scale physics once the state has entered a regime where they can be treated perturbatively.

3 Renormalization

If the initial state deviates from the standard vacuum state the renormalizability of the theory has to be checked. A major difference from the flat space case is that the information of interest concerns time evolution of the field modes as opposed to what happens in the S-matrix approach. The correct approach to this problem involves the Schwinger-Keldysh or closed-time path formalism[7].

The divergences that arise when using the general initial state described above are of two types. The first, which we call bulk divergences, are the usual ones that would be encountered even with the standard vacuum. They can be absorbed by the standard local counterterms and the coupling constants in the theory acquire their usual scale dependence.

The more interesting divergences are those arising from the change of initial state. What we have shown is that they only appear on the initial time hypersurface and can be absorbed by counterterms associated with this hypersurface. The fields inherit their mass dimension from the full $3+1$ theory but the operators contained in the boundary counterterm Lagrangian are classified according to whether their mass dimension is greater than, less than or equal to the dimension of the boundary surface. The structure function can be expanded as

$$e^{\alpha_k^*} = \sum_{n=0}^{\infty} d_n \frac{H^n(\eta_0)}{\Omega_k^n(\eta_0)} + \sum_{n=1}^{\infty} c_n \frac{\Omega_k^n(\eta_0)}{a^n(\eta_0) M^n}. \tag{7}$$

where $\Omega_k(\eta_0)$ is a generalized frequency. The two series are associated with IR and UV aspects of the initial state; the IR part can be viewed as a non-vacuum excitation of the BD state since at large distances we expect the Klein-Gordon description to be a good one. The divergences associated with this part of the initial state can be absorbed by renormalizable (on the initial time hypersurface) counterterms. Trans-Planckian effects are encoded in the second series and these divergences require irrelevant operators on the boundary to absorb them.

This formalism can also be used to renormalize the expectation value of the stress energy tensor and thus to understand the issue of the backreaction of the initial state on inflationary dynamics. If the expansion rate during inflation is denoted by H, and $M_{\rm Pl}$ and M correspond respectively to the Planck mass and to the scale of new physics responsible for that structure, we find that the size of this back-reaction, relative to the vacuum energy sustaining the inflation, is suppressed at least by

$$\frac{H^2}{M_{\rm pl}} \frac{H}{M}. \tag{8}$$

4 Conclusion

The basic idea of the effective theory of an initial state is that a discrepancy can exist between what is the true state of the system and the state we have chosen to use in a quantum field theory, which thereby defines the propagator and the matrix elements of operators. Over distances where have a good empirical understanding of nature and a reasonable knowledge about the relevant dynamics, we can usually make an appropriate choice for this state. Yet there always exist shorter distances where the behavior of nature is unknown and the correct state might not match with that we obtained by extrapolating our understanding at long distances down to these much shorter scales. This discrepancy is particularly important for inflation, where the relevant fields and their dynamics have not been observed directly and where the natural energy scale, the Hubble scale H, can be an appreciable fraction of the Planck scale, $M_{\rm pl}$. At this scale, gravity becomes strongly interacting and so we do not even have a predictive understanding of the behavior of space-time. Since inflation naturally produces a set of primordial perturbations through the inherent quantum fluctuations of a field, it is important to determine, from a very general perspective, the observability of the features at short distances compared with $1/H$ through their imprint on this primordial spectrum. While this imprint is not expected to be observed in the most recent experiments[1], future observations of the microwave background and of the large scale structure over large volumes of the observed universe should be able to extract the spectrum of primordial perturbations to a far better precision.

The ultimate goal of this program is to compute the power spectrum of inflationary perturbations as a function of the structure function of the initial state. Once this is done, we will be able to fit the coefficients $c_n,\ d_n$ in Eq. 7 and determine, in a controlled fashion, whether or not trans-Planckian physics changes in the initial state can affect the CMB.

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