1. Enzyme Kinetics

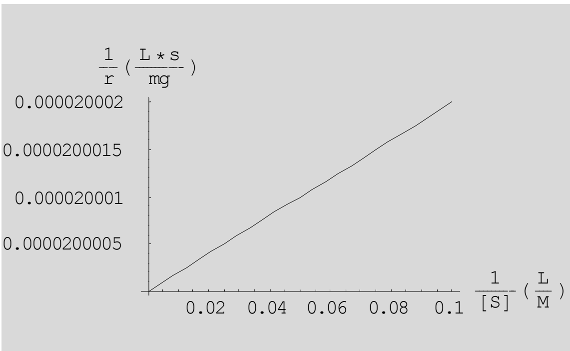
a) $V_m = k * [E_o] = (10^4\ s^{-1}) * (5\ mg/L) = 50000\ mg / (s*L)$

b) In solving this problem we assume that $K_d \sim K_m$:

$$r = \frac{V_{\max}[S]}{K_m + [S]}$$

We rearrange this to get:

$$\frac{1}{r} = \frac{K_m}{V_{\max}}\frac{1}{[S]} + \frac{1}{V_{\max}}$$



The y-intercept is $1/V_m$, and the slope is K/V_m.

c) One can use the plot to find V_m and then K. Using V_m, K, and the amount of enzyme, one can then find k. This process is the reverse of what we have done in parts a) and b).

2. Metabolic Pathway Control

a) Four enzymes

b) Two enzymes

c) Yes, this drug may help to reduce symptoms. More S3 will be unbound and push the reaction forward. It will trigger S5 production.

3. Enzyme Inhibition

a) $K_m = k_{-1} / k_1$ and $K_I = k_{I,-1} / k_{I,1}$

Thus, an increasing K_x indicates an increasingly weak association. In this problem, $K_m > K_I$, therefore the inhibitor binds more strongly than the substrate.

b)

$$v = \frac{v_{\max}[S]}{K_m\left(1+\frac{[I]}{K_I}\right)+[S]} = \frac{0.67\cdot 3.7}{4.9\cdot\left(1+\frac{3.7}{1.2}\right)+3.7} = 0.104562\ \left(\frac{\text{mM}}{\text{min}}\right)$$

So the answer is **0.10 mM/min** (two significant figures).

c) In this problem we are given V_m, K_m, I, and K_i. Additionally, we are given the initial condition S = 0.22 mM at t=0. Thus, we solve the differential equation:

$$\frac{dS}{dt} = -\frac{aS}{b+S}$$, note that we are consuming S, so the rate is negative. $a = V_m$ and $b = K_m(1+I/K_i)$

$$\left(\frac{b+S}{S}\right)dS = -adt$$

$$b\ln(S) + S = -at + c1$$

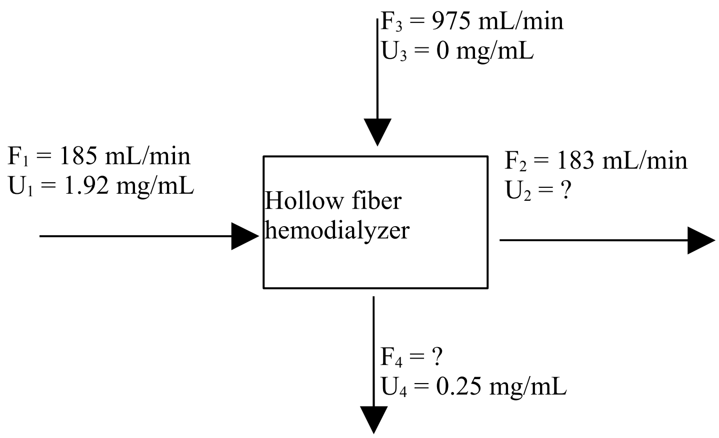
Using the initial condition we solve for c1, and now we can also plug in the values for a and b. We find that:

$$8.49333\ln(S) + S = -0.67t + 12.64$$

Now we can solve the problem. If 90% of substrate is consumed then 0.022 mM remains. Thus, we substitute this value for S and calculate t. We find that t = 29.4845 min, or **29.5 min** (3 significant figures).

4. Mass Balance on a Hollow Fiber Hemodialyzer

a)



There are no chemical reactions occurring in this system, therefore $mass_{in} = mass_{out}$ for all species. Additionally, all flow into and out of the system must be conserved. Using a flow rate balance we find that:

$0 = F_1 + F_3 - F_2 - F_4$

$F_4 = F_3 + F_1 - F_2 = 977$ mL/min

Note that the volume difference of blood at arterial and venous side is due to the fact that water and urea are displaced from blood to the exiting dialysate stream.

Finding rate of urea removal:

The urea mass rate coming into the system from arterial blood is:

$$F_1 * U_1 = 355.2\frac{mg}{\min}$$

The urea mass rate exiting the system from the dialysate is:

$$F_4 * U_4 = 244.25\frac{mg}{\min}$$

This result is the rate of urea removal, so this rate is **240 mg/min** to two significant figures.

Finding concentration of urea in venous blood:

The urea mass rate exiting the system in venous blood is:

$$355.2 - 244.25 = 110.9\frac{mg}{\min}$$

We calculate U_2 as:

$U_2 = \dfrac{110.95}{183}$ = **0.61 mg/ml** (to two significant figures)

As mentioned above, there is a 2 mL/min removal of fluid from the blood. To find the rate of removal of water, assume that the density of the removed fluid is 1 g/mL. Then,

$$\frac{2mL}{\min}\cdot\frac{1g}{mL}\cdot\frac{1000mg}{g} = \frac{2000mg}{\min}$$

$$\frac{2000mg}{\min} - \frac{240mg}{\min} = \frac{1760mg}{\min}$$

Thus, the rate of removal of water is **2 mg/min** (one significant figure).

b) Initial concentration of urea in blood: U_1 = 2.95 mg/mL
Final concentration of urea in blood: U_2 = 0.61 mg/mL

Initial amount of urea in blood: M_1 = $\dfrac{2.95mg}{mL}\cdot 4900mL = 14455mg$, at t_1 = 0 min

Final amount of urea in blood: M_2 = $\dfrac{0.60mg}{mL}\cdot 4900mL = 2940mg$, at t_2 = ? min

The rate of removal of urea, as found in part a), is 240 mg/min. Thus,

$t_1 - t_2 = \dfrac{M_1 - M_2}{-240\,mg/\min} = -47.9792$, so t_2 = 47.9792 min

So the patient should be put on dialysis for **48 minutes** (two significant figures).

5. Biomechanics – Stride Analysis

We set up the problem as $P_{t,body} = P_{t,body+pack}$. Note that $M_1 = M_{body}$, while $M_2 = M_{body+pack}$. $M_2 = 1.2\ M_1$.

Assuming optimal stride length, and using equation 10.6 from MMD, we find that

$$P_{t,body} = 2(\alpha\beta)^2(g/L)^{1/2}M_1v_1^2 = 2(\alpha\beta)^2(g/L)^{1/2}1.2M_1v_2^2 = P_{t,body+pack}$$

This reduces to $v_1^2 = 1.2v_2^2$

Thus, the factor by which our walking speed compares to our speed without the pack is $\dfrac{v_2}{v_1} = \sqrt{\dfrac{1}{1.2}}$ = 0.912871, or **0.91** to two significant figures.

6. Biofluid Mechanics – Inviscid Flow

Given:

$$d_1 = 1.0cm$$

$$v_1 = 55\, {}^{cm}\!/_{s}$$

$$P_1 = 100mmHg$$

$$d_2 = 1.5cm$$

Pressure drop in aneurysm:

First determine velocity in aneurysm:

$$Q = v_1 \cdot A_1 = v_1 \cdot \pi \cdot \left(\frac{d_1}{2}\right)^2 = 50\, {}^{cm}\!/_{s} \cdot \pi \cdot \left(\frac{1.0cm}{2}\right)^2 = 43.20\, {}^{cm^3}\!/_{s}$$

$$v_2 = \frac{Q}{\pi \cdot \left(\frac{d_2}{2}\right)^2} = \frac{43.20\, {}^{cm^3}\!/_{s}}{\pi \cdot \left(\frac{2.1cm}{2}\right)^2} = 12.47\, {}^{cm}\!/_{s}$$

Use Bernoulli's Equation to relate pressures:

$$\mathrm{P}_1 + \rho \frac{\mathrm{v}_1^2}{2} + \rho \mathrm{g}\mathrm{z}_1 = \mathrm{P}_2 + \rho \frac{\mathrm{v}_2^2}{2} + \rho \mathrm{g}\mathrm{z}_2$$

Assume that the change in height between 1 and 2 is negligible (ρgz term cancel out)

Assume density:

$$\rho = 1.056\, {}^{\mathrm{g}}\!/_{\mathrm{cm}^3}$$

The equation is now:

$$\mathrm{P}_1 + \rho \frac{\mathrm{v}_1^2}{2} = \mathrm{P}_2 + \rho \frac{\mathrm{v}_2^2}{2}$$

$$P_2 = P_1 + \frac{\rho}{2}\left(\mathrm{v}_1^2 - \mathrm{v}_2^2\right)$$

$$P_2 = 100mmHg + \frac{1.056\, {}^{g}\!/_{cm^3}}{2}\left(\left(55\, {}^{cm}\!/_{s}\right)^2 - \left(12.47\, {}^{cm}\!/_{s}\right)^2\right) \cdot \left(\frac{1kg}{1000g} \cdot \frac{100cm}{m} \cdot \frac{1Pa}{kg\!/_{m \cdot s^2}} \cdot \frac{760mmHg}{1.01 \times 10^5\, Pa}\right)$$

P_2 = 101 mmHg

Now for vigorous exercise:

$$v_1 = 200\, {}^{cm}\!/_{s}$$

Pressure drop in aneurysm during vigorous exercise

First determine velocity in aneurysm:

$$Q = v_1 \cdot A_1 = v_1 \cdot \pi \cdot \left(\frac{d_1}{2}\right)^2 = 200\, {}^{cm}\!/_{s} \cdot \pi \cdot \left(\frac{1.0cm}{2}\right)^2 = 157.1\, {}^{cm^3}\!/_{s}$$

$$v_2 = \frac{Q}{\pi \cdot \left(\frac{d_2}{2}\right)^2} = \frac{157.1 \, {}^{cm^3}\!/\!_s}{\pi \cdot \left(\frac{1.5cm}{2}\right)^2} = 45.35 \, {}^{cm}\!/\!_s$$

Use Bernoulli's Equation to relate pressures, making the same assumptions as above:

$$P_2 = P_1 + \frac{\rho}{2}\left(\mathrm{v}_1^2 - \mathrm{v}_2^2\right)$$

$$P_2 = 100mmHg + \frac{1.056 \, {}^{g}\!/\!_{cm^3}}{2}\left(\left(200 \, {}^{cm}\!/\!_s\right)^2 - \left(45.35 \, {}^{cm}\!/\!_s\right)^2\right) \cdot \left(\frac{1kg}{1000g} \cdot \frac{100cm}{m} \cdot \frac{1Pa}{{}^{kg}\!/\!_{m \cdot s^2}} \cdot \frac{760mmHg}{1.01 \times 10^5 \, Pa}\right)$$

P_2 = 115 mmHg

If the aneurysm increases in size then v_2 would decrease and P_2 would increase.